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Antiscalar Dark Energy

Eduard G. Mychelkin

Fesenkov Astrophysical Institute of National Center for Space Research and Technology, Almaty 050020, Kazakhstan

Geometrical foundations of antiscalar approach.

We consider dark energy (DE) and dark matter (DM) as two-fold identifiable scalar fields $\varphi = \varphi(\varphi^{+}, \varphi^{-})$ and $\Phi(\nu, \bar{\nu})$. The geometrical justification of those follows from consideration of the space-time deformation tensor.

Deformation tensor $D_{\mu\nu}$ **defines the symmetries of spacetime and fundamental scalar field** $\xi \equiv (\xi_{\alpha}\xi^{\alpha})^{1/2}$**:**

$$\boxed{L_{\xi}(g_{\mu\nu}) = \xi_{\mu;\nu} + \xi_{\nu;\mu} = D_{\mu\nu}}, \quad \boxed{\xi^{\mu} = \xi u^{\mu}}, \quad \boxed{u_{\alpha}u^{\alpha} = 1},$$

Simplest examples: 1. $D_{\mu\nu} = 0 \rightarrow$ **the Killing equations,**

2. $D_{\mu\nu} = \Phi g_{\mu\nu} \rightarrow$ **conformal symmetries,**

3. $D_{\mu\nu} = \Psi h_{\mu\nu}, \quad h_{\mu\nu} = g_{\mu\nu} - u_{\mu}u_{\nu} \rightarrow$ **space-conformal symmetries.**

Using the definition of curvature tensor $\xi_{\mu;\nu\alpha} - \xi_{\mu;\alpha\nu} = \frac{1}{2} R^{\beta}{}_{\mu\nu\alpha}\xi_{\beta}$, **we get the Ehler's-type identity with fundamental geometrical scalar** ξ **(related to redshift, inverse temperature, etc.):**

$$\boxed{R_{\alpha\beta}u^{\alpha}u^{\beta} = -\xi^{-1}\Box\xi + u_{\alpha;\beta}u^{\alpha;\beta} + \xi^{-1} f_{\alpha}u^{\alpha}},$$

$$\boxed{f^{\mu} \equiv -\frac{1}{2}\left(D^{\alpha}{}_{\alpha}\right)^{;\mu} + D^{\mu\alpha}{}_{;\alpha}}, \quad \Box\xi \equiv \xi_{;\alpha}{}^{;\alpha}.$$

There must exist a correspondence:

$$\textbf{(Geometry)}\ \ \xi \equiv (\xi_\alpha \xi^\alpha)^{1/2} \Leftrightarrow \phi\ \ \textbf{(Physics).}$$

Papapetrou's ideology: $g_{\mu\nu} = g_{\mu\nu}(\phi(x^\alpha))$ **and** $\xi = \xi(\phi(x^\mu))$.

<u>Universal free gravitating scalar field</u> generating an effective Riemannian space is just what people recognize as <u>gravity.</u>

The Newtonian limit as a direct property of Ehler's-type identity shown above:

$$\boxed{R_{\alpha\beta}u^\alpha u^\beta = -\xi^{-1}\Box\xi + u_{\alpha;\beta}u^{\alpha;\beta} + \xi^{-1} f_\alpha u^\alpha},$$

$$\boxed{f^\mu \equiv -\frac{1}{2}\left(D^\alpha{}_\alpha\right)^{;\mu} + D^{\mu\alpha}{}_{;\alpha}},\ \Box\xi \equiv \xi_{;\alpha}{}^{;\alpha}.$$

By using the known static relation: $\xi = e^{-\phi} \Rightarrow\ -\xi^{-1}\Box\xi\ \rightarrow\ \xi^{-1}\Delta\xi \cong \Delta\phi$ **for <u>any</u> combination of EMT-components replacing Ricci-projector so that**

$\boxed{R_{\mu\nu}u^\mu u^\nu \rightarrow 4\pi G\rho}$ **(** ρ **- density of matter) master-identity (2) gives rise in non-relativistic case exactly to <u>the Poisson equation of Newtonian gravity</u>**

$$\boxed{R_{\alpha\beta}u^\alpha u^\beta = -\xi^{-1}\Box\xi + u_{\alpha;\beta}u^{\alpha;\beta} + \xi^{-1} f_\alpha u^\alpha} \Rightarrow \Delta\phi = -4\pi\rho. \qquad \textbf{(*)}$$

Thus we get the Poisson equation before establishing the definite Einstein-type equations (and so without taking the corresponding limit in those).

$$\xi = e^{-\phi} \Rightarrow\ -\xi^{-1}\Box\xi\ \ \xi = e^{-\phi} \Rightarrow (-\xi^{-1}\Box\xi \rightarrow$$

$$\xi = e^{-\phi} \Rightarrow (-\xi^{-1}\Box\xi \rightarrow \xi^{-1}\Delta\xi \cong \Delta\phi)$$

1. Antiscalar principle [1]

- ***"Antiscalar"*** **means *negative sign* (with respect to the usual matter) of EMT (energy-momentum tensor) for the universal scalar field (*USF*).**
- **This follows from requirement of *thermodynamic stability* of systems described by the Einstein equations and from *conformity to experiments.***
- **We reinterpret GR theory as the one in space-time with unremovable USF background which is identifiable with DE (dark energy) or, in general, with DE plus DM (dark matter).**

GENERAL RELATIVITY (GR)

TRADITIONAL APPROACH	ANTISCALAR APPROACH
$G_{\mu\nu} = 8\pi G\{T_{\mu\nu}{}^{matter} + T_{\mu\nu}{}^{scalar}(\phi)\}$	$G_{\mu\nu} = 8\pi G\{-T_{\mu\nu}{}^{scalar}(\phi) + T_{\mu\nu}{}^{matter}\}$
THERE ARE VACUUM EQUATIONS $G_{\mu\nu} = 0$	**NO VACUUM EQUATIONS** $G_{\mu\nu} = -8\pi G T_{\mu\nu}{}^{scalar}(\phi)$
VACUUM SOLUTIONS: **(SCHWARZSCHILD, KERR)** $ds^2 =$ $= (1-2GM/r)dt^2 - (1+2GM/r)dr^2 + r^2 d\Omega^2$ **YES BLACK HOLES**	**ONLY SOLUTIONS OF TYPE:** $g_{\mu\nu} = g_{\mu\nu}(\phi(x^\alpha))$ **(PAPAPETROU, SZEKERES)** $ds^2 = e^{-2\phi}dt^2 - e^{2\phi}(dr^2 + r^2 d\Omega^2)$ $= e^{-2GM/r}dt^2 - e^{2GM/r}(dr^2 + r^2 d\Omega^2)$ **NO BLACK HOLES**

GRAVITATIONAL WAVES, etc.

TRADITIONAL APPROACH	ANTISCALAR APPROACH
YES $\Box g_{\mu\nu} = 0$	NO $\Box g_{\mu\nu} \neq 0$
ANY SELF-INTERACTIONS ARE ADMITTED, INCLUDING PHANTOM FIELDS	NO SELF-INTERACTION EXCEPT MASS-TERM, NO PHANTOM FIELDS
DIFFERENT TYPES OF NON-MINIMAL INTERACTIONS ARE PERMITTED	EXCEPT CONFORMAL, ONLY MINIMAL INTERACTIONS ARE PERMITTED

Antiscalar Papapetrou's solution is more realistic than the Schwarzschild one:

- **It contains all 'crucial effects'.**
- **It leads to correct formulae of lensing [1].**
- **There are no black holes (BH) but for the compact objects with a scale of order *$2GM/c^2$* we get usual results of BH-thermodynamics [2].**
- **Antiscalarity satisfies the thermodynamic stability conditions and is justified by electrostatic origin of background scalar field [3].**

Universal scalar field ϕ (*USF*), with $\phi_\mu \equiv \partial\phi / \partial x^\mu$:

$$T_{\mu\nu}^{scalar} = \phi_\mu \phi_\nu - \tfrac{1}{2} g_{\mu\nu} (\phi_\alpha \phi^\alpha - \mu^2 \phi^2) + (\Lambda / 8\pi G) g_{\mu\nu}$$

USF is a superposition of quasi-static electric fields $\phi \sim \phi_+ + \phi_-$ emulating the instant action expressible by 'transcendent tachyon' condition

$$\mu^2 = -m^2 < 0.$$

ELECTROVACUUM SPACES

TRADITIONAL APPROACH	ANTISCALAR APPROACH
THE EINSTEIN-MAXWELL EQUATIONS $G_{\mu\nu} = 8\pi G T^{EM}_{\mu\nu}$ $T^{EM}_{\mu\nu} = \frac{1}{4\pi}(-F_{\mu\alpha}F^{\alpha}{}_{\nu} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta})$	**MINIMAL (ANTI)SCALAR FIELD** $G_{\mu\nu} = -8\pi G T^{\min}_{\mu\nu}(\phi)$ $T^{\min}_{\mu\nu} = \phi_\mu \phi_\nu - \frac{1}{2} g_{\mu\nu}\phi_\alpha \phi^\alpha$
ANTISCALARITY IN STATIC LIMIT $T^{EM}_{ij} = -(\phi^{\pm}_i \phi^{\pm}_j - \frac{1}{2} g_{ij}\phi^{\pm}_k \phi^{k\pm})$	**ANTISCALARITY SUPPLEMENTED WITH** $\phi \overset{def}{=} (\phi^- + \phi^+)/\sqrt{2}$

So, we get equivalent equations in both cases if ϕ-field is identified as a neutral superposition of quasi-static electric (electro-vacuum) fields [2]: $\phi = (\phi^- + \phi^+)/\sqrt{2}$.

2. Transition from scalar to antiscalar case, masses as scalar charges:

From *Fisher's* [*JETP,1948*] or *JNW* [*Janis A., Newman E., Winicour J., PRL 20 (1968) 878*] asymptotically flat, spherically-symmetric solution of Einstein equations for scalar field ϕ with scalar charge q:

$$ds^2 = \left(1 - \frac{b}{r}\right)^{\gamma} dt^2 - \left(1 - \frac{b}{r}\right)^{-\gamma} dr^2 - \left(1 - \frac{b}{r}\right)^{1-\gamma} r^2 (d\theta^2 + \sin^2\theta d\varphi^2),$$

$$\phi = \frac{q}{b}\ln\left(1 - \frac{b}{r}\right), \quad \textbf{where} \quad \gamma = 2m/b,\ b = 2\sqrt{m^2 + q^2},$$

we get for $b \to 0$ the fundamental antiscalar Papapetrou solution ($G = 1$):

$$ds^2 = e^{-2\phi}dt^2 - e^{2\phi}(dr^2 + r^2 d\Omega^2)$$

$$= e^{-2m/r}dt^2 - e^{2m/r}(dr^2 + r^2 d\Omega^2),$$

and simultaneously it follows that $\phi^2 \to -\phi^2$ because the scalar charge q transforms to source mass,

$$q^2 \to -m^2 \;\leftrightarrow\; |q| \to |m|,$$

and effectively

$$\phi \to i\phi = im/r\,.$$

The appearance of imaginary unit indicates that the scalar EMT as quadratic function of field changes its sign (becomes automatically *antiscalar*).

3. Dark energy as background for the compact objects

Integrability condition (which is the manifestation of inseparability of Λ and mass–terms):

$$|\Lambda| = -(2/3)\mu^2 \;\to\; |\mu| = m \approx 10^{-33} eV \approx 10^{-65} g\;,$$

and the corresponding (tachyon-type) Klein-Gordon equation,

$$\partial_\mu(\sqrt{-g}\,g^{\mu\nu})\partial_\nu\phi - m^2\sqrt{-g}\,\phi = 0$$

lead to the cosmological solution appropriate for the description of DE [3]:

$$\begin{aligned} ds^2 &= dt^2 - a^2(\phi(t))(dr^2 + r^2 d\Omega^2) \\ &= dt^2 - \exp\{-|\Lambda|(t-t_0)^2\}(dr^2 + r^2 d\Omega^2)\;, \end{aligned}$$

From that follows the effective equation of state for DE [3]:

$$p = w\varepsilon\,,\;\; \varepsilon = T_0^0\,,\;\; p = -\tfrac{1}{3}T_i^i\,,\quad w = \frac{-3\Lambda(t-t_0)^2/2+1}{3\Lambda(t-t_0)^2/2}\,.$$

Asimptotically at $t \to \infty$ it goes to the de Sitter state $w = -1$.

The case when $w < -1/3$ leads to the accelerated phase for the Universe. After t_0, the expansion will always change to contraction. There are no phantom fields with $w < -1$ in the given equation of state.

The local and cosmological effects might be joined in metric:

$$ds^2 = e^{-2\phi} dt^2 - e^{2\phi} a^2(\phi(t))(dr^2 + r^2 d\Omega^2), \quad e^{\pm 2\phi} \approx 1 \pm 2\phi .$$

In particular for both the crucial effects and DE we get the solution [5]:

$$ds^2 = e^{-2GM/r} dt^2 - e^{2GM/r} \exp\{-|\Lambda|(t - t_0)^2\}(dr^2 + r^2 d\Omega^2).$$

This corresponds to compact objects on the background of dark energy.

We suppose that DM effects may in general be included into ϕ:

$$\phi = \phi_{Newton}(r) + \phi_{dm}(r).$$

Appendix I. Quasi-Newtonian equations:

Field equations in the Einstein-Grossmann form always include the four-velocity u^{μ} (observer's frame):

$$R_{\alpha\beta}u^{\alpha}u^{\beta} = \kappa\{(T_{\mu\nu}^{matter}\, u^{\mu}u^{\nu} - \tfrac{1}{2}\, T^{matter}) - \frac{1}{4\pi}[(\phi_{\alpha}u^{\alpha})^{2} - \frac{1}{2}(\mu^{2}\phi^{2} + \frac{\Lambda}{G})]\}$$

where $\kappa = 8\pi G/c^{4}, c = 1$**, and** $R_{\alpha\beta}$ **– the Ricci tensor.**

Quasi-Newtonian equations reduce to the following non-linear form:

$$\Delta\phi = -4\pi G\{\rho - \frac{1}{2\pi}[(\dot{\phi})^{2} - \frac{1}{2}(\mu^{2}\phi^{2} + \frac{\Lambda}{G})]\}.$$

Or, if the scalar field choose to be geometrized,

$$\Delta\varphi - [2(\dot{\varphi})^{2} - (\mu^{2}\varphi^{2} + \Lambda)] = -4\pi G\rho,$$

where potential ϕ **got to be positive, and** $\dot{\varphi} = \partial/\partial(ct)$**.**

It is essential the tachyon mass-factor μ **and** Λ**-term,** $\Lambda = |\Lambda|$**, obey the relation** $\Lambda = -(2/3)\mu^{2}$ **as before.**

Appendix II. Scalar thermodynamics and gravitation [5–13]

<u>Foundations of «ω-law»-thermodynamics.</u> Thermodynamic quantities – functions of ξ **(modulus of the t-like Killing), and effective number density** n **(here the Boltzmann constant** $k=1$**):**

Temperature T**,** $T = 1/\xi$**; Energy density** ε**,** $\varepsilon = -\partial n/\partial\xi$**;**

Pressure p**,** $p = n/\xi$**; Number density,** $n = n(\xi)$**.**

<u>Gibbs relation</u> **(**s **– entropy density):**

$$\boxed{dq = Td(s/n) = d(\varepsilon/n) - p(1/n)},$$

in absence of heat sources $dq = 0 \Leftrightarrow ds/s = dn/n$**, transforms into:**

$$\boxed{nn'' + nn'/\xi - (n')^2 = 0},\ n = n(\xi)\ .$$

The first integral of this equation will be just '*ω-law*':

$$\boxed{-\omega\partial n/\partial\xi = n/\xi}\ \Leftrightarrow\ \boxed{p = \omega\varepsilon},$$

with ω **to be a constant of integration. Integrating yet once we get:**

$$\boxed{n = C\xi^{-1/\omega}},\ \text{where}\ C = n_0\xi_0^{\ 1/\omega}.$$

In terms of reduced temperature:

$T/T_0 \to T \ \Leftrightarrow\ \xi/\xi_0 \to \xi,\qquad \xi_0 = 1/T_0,\ C = n_0,$ **setting**

$\boxed{z = 1 + 1/\omega}$ **(entropy per particle,** $z = s/n$**,** $\xi = 1/T$**), if chemical potential** $\mu = 0$**:**

$$\varepsilon = -\partial n/\partial\xi = (C/\omega)\xi^{-z},$$

$$p = n/\xi = C\xi^{-z},$$

$$s = dp/dT = \xi(\varepsilon + p) = Cz\xi^{-1/\omega};$$

[in opposite case $\mu \neq 0$**:** $s = \xi(\varepsilon + p - \mu n) = C(z-\mu)\xi^{-1/\omega}$ **].**

<u>Stability conditions for string gas and scalar field.</u>

- **For if** $\omega = -1/3$ **and** $C = -B$ **(string gas) we get:**

$$\varepsilon = 3B\,\xi^2,\qquad p = -B\,\xi^2,\qquad s = 2B\,\xi^3.$$

- **For** $\omega = 1$ **(stiff state of scalar field) we have:**

$$\varepsilon = -\partial n / \partial \xi = C\xi^{-2}, p = n / \xi = C\xi^{-2},$$

$$s(\mu = 0) = dp / dT = \xi(\varepsilon + p) = 2C\xi^{-1}.$$

- **To get the explicit <u>thermodynamic stability</u> <u>condition</u> it is necessary the relation**

$$p = \omega\varepsilon - B$$

to substitute into the identity

$$p = n\partial\varepsilon / \partial n - \varepsilon .$$

Then the <u>thermodynamic</u> stability condition

$$\partial^2 \varepsilon / \partial n^2 > 0$$

proves to be

$$\omega(\omega + 1) n^{\omega - 1} > 0. \qquad \textbf{(*)}$$

- **For if** $\omega = -1/3$,

(*) is broken, so a string gas cannot be stable.

- **For** $\omega = 1$

(*) is fulfilled, thus scalar field is thermodynamically stable.

<u>Scalar thermodynamics, black-holes and cosmology:</u>

At the *big scales* for the temperature and the entropy density:

$$T \propto const\sqrt{\Lambda}, \quad s \propto const\sqrt{\Lambda}$$

just as in Hawking's case for the de Sitter horizon.

But now that is a general asymptotic relation for any metrics.

Nearby the compact objects (black-holes analogs) there is no sense take into account the negligible values of Λ **(and mass) terms:**

$$T = \frac{1}{4\sqrt{\pi GC}}\sqrt{-R+4\Lambda} \cong \frac{1}{4\sqrt{\pi GC}}\sqrt{-R},$$

$$s = 2CT = \left(\frac{C}{4\pi G}\right)^{1/2}\sqrt{-R+4\Lambda} \cong \left(\frac{C}{4\pi G}\right)^{1/2}\sqrt{-R}.$$

In the Papapetrou-Yilmaz approximation:

$$ds^2 = e^{-2\phi}dt^2 - e^{2\phi}(dr^2 + r^2 d\Omega^2) =$$

$$e^{-2GM/r}\,dt^2 - e^{2GM/r}\,(dr^2 + r^2 d\Omega^2)$$

for scales of order 'Schwarzschild's gravitational radius', we get:

$$R = -\frac{2}{r^2}\phi^2 e^{-2\phi} = -2\frac{G^2M^2}{r^4}e^{-2GM/r} = -2\frac{G^2M^2}{r^4}g_{00}(r)$$

$$\xrightarrow{r=2GM} R = -\frac{1}{8G^2M^2}g_{00}(r_g).$$

<u>Invariant temperature</u> (with a red shift to be included)

of scalar field *on the arbitrary equipotential surface* will be the same as the Hawking black-hole temperature:

$$T = \frac{\sqrt{g_{00}}}{\sqrt{8\pi}}\left(\frac{G}{C}\right)^{1/2}\frac{M}{r^2} \xrightarrow{r=r_g} T_g = \frac{\sqrt{g_{00}(r_g)}}{8\sqrt{2\pi}}(CG^3)^{-1/2}\frac{1}{M} = \frac{const}{M},$$

where C – an arbitrary constant defined by initial conditions. Analogously, for entropy $S = sV$ of scalar field in the volume restricted by the given surface, we get in accord with black-hole thermodynamics the well known formulae:

$$s \propto T \propto 1/M \Rightarrow S = sV \approx sr^3 \propto M^2.$$

This is similar to Stephan-Boltzmann's law $W = \sigma T^4$ which was found at first by classic theory, and then factor σ was calculated by quantum methods.

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