Breakdown of quantum relational dynamics in a nonintegrable cosmological model

Philipp Höhn

Perimeter Institute

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mainly based on PH, Kubalová, Tsobanjan PRD **86** 065014 (2012)
framework from Bojowald, PH, Tsobanjan PRD **83** 125023 (2011); Bojowald, PH, Tsobanjan CQG **28** 035006 (2011) (summary PH 1110.5631)

The $k = 1$ FRW model with massive scalar field

- canonically subject to Hamiltonian constraint ($\alpha = \ln a$)

$$C_H = p_\phi^2 - p_\alpha^2 - e^{4\alpha} + m^2\phi^2 e^{6\alpha} \approx 0$$

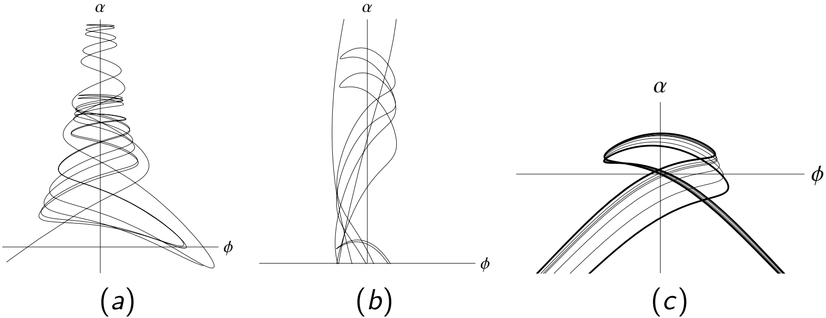
- generates EOMs

$$\begin{aligned}
\dot{\alpha} &= \{\alpha, C_H\} = -2p_\alpha \\
\dot{p}_\alpha &= \{p_\alpha, C_H\} = 4e^{4\alpha} - 6m^2\phi^2 e^{6\alpha} \\
\dot{\phi} &= \{\phi, C_H\} = 2p_\phi \\
\dot{p}_\phi &= \{p_\phi, C_H\} = -2m^2\phi e^{6\alpha}
\end{aligned}$$

- conceptual "problem of time" in QT: "frozen dynamics" $\hat{C}_H|\psi\rangle = i\hbar\frac{\partial}{\partial s}|\psi\rangle = 0.$

- conceptual "solution": relational dynamics $\Rightarrow$ dynamical DoFs as internal clocks, e.g. correlations $\alpha(\phi)$ gauge invariant observables $\Rightarrow$ translate into QT

Classical dynamics of k=1 FRW + ϕ



(a) typical solution, (b) close-up on (a), (c) defocussing of nearby trajectories in turning region

- model chaotic and non-integrable [Page '84, Cornish, Shellard '98]
- strong defocussing of classical solutions near α_{max}
- devoid of global clocks $\Rightarrow$ problem for QT

Non–integrability and relational dynamics

- only global Dirac observable Hamiltonian (constraint)
  $\Rightarrow$ ergodic orbits
- relational observables still (temporally) locally meaningful
- non-integrability generic in dynamical systems (and GR?)
- how to deal with this in QT?
  1. Hilbert space?,
  2. clock choice?, deal with generic imperfect clocks
  3. non-unitarity?,
  4. observables?...

$\Rightarrow$ so far only WKB approximations in Wheeler-DeWitt cosmology available [Hawking, Page, Kiefer,...'80s], but no study of relational dynamics

Effective description of constrained systems

underlying idea: avoid Hilbert space representation altogether (sidestep Hilbert space problem)

- instead: for canonial pairs $(\hat{q}_i, \hat{p}_i)$ use expectation values $\langle\hat{q}_i\rangle$ and $\langle\hat{p}_i\rangle$, and moments

$$\Delta(q_1^{a_1} p_1^{b_1} q_2^{a_2} p_2^{b_2}) := \langle(\hat{q}_1 - \langle\hat{q}_1\rangle)^{a_1}(\hat{p}_1 - \langle\hat{p}_1\rangle)^{b_1}(\hat{q}_2 - \langle\hat{q}_2\rangle)^{a_2}(\hat{p}_2 - \langle\hat{p}_2\rangle)^{b_2}\rangle_{\mathrm{Weyl}}$$

$$a_1 + b_1 + a_2 + b_2 \geq 2$$

  to describe states instead of wave functions or density matrices [Bojowald, Skirzewski '06]

- (quantum) phase space structure via Poisson bracket

$$\{\langle\hat{A}\rangle, \langle\hat{B}\rangle\} = \frac{\langle[\hat{A}, \hat{B}]\rangle}{i\hbar}, \quad \text{e.g.} \quad \{\langle\hat{q}_i\rangle, \langle\hat{p}_j\rangle\} = \delta_{ij}, \quad \{\langle\hat{q}_j\rangle, \Delta(\ldots)\} = 0$$

  extended also to moments

- Constraint $\langle\hat{C}\rangle = 0$, but also [Bojowald, Sandhöfer, Skirzewski, Tsobanjan '09; Bojowald, Tsobanjan '09]

$$C_{\mathrm{pol}} := \langle\widehat{\mathrm{pol}}\,\hat{C}\rangle = 0$$

- infinitely many constraints for infinitely many variables

- semiclassical order: assume $\Delta(q^a p^b) = O(\hbar^{(a+b)/2})$ and truncate at $\hbar$ (more general than Gaussians) $\Rightarrow$ finite system

$\Rightarrow$ flows/dynamics via Poisson structure

$$\dot{f}(q,p) = \{f, C_{\mathrm{pol}}\}$$

- effective and Hilbert space results coincide where latter available

Effective constraints for k=1 FRW + ϕ

- at order $\hbar$ retain 14 kinematical dofs
  - 4 expectation values $a = \langle\hat{a}\rangle$, $\quad a, b = \alpha, \phi, p_\alpha, p_\phi$
  - 4 spreads $\langle(\hat{a} - \langle\hat{a}\rangle)^2\rangle_{\mathrm{Weyl}}$, and
  - 6 covariances $\langle(\hat{a} - \langle\hat{a}\rangle)(\hat{b} - \langle\hat{b}\rangle)\rangle_{\mathrm{Weyl}}$

- $\hat{C} = \hat{p}_\phi^2 - \hat{p}_\alpha^2 - e^{4\hat{\alpha}} + m^2\hat{\phi}^2 e^{6\hat{\alpha}}$ translates into 5 constraint functions

$$\begin{aligned}
C &= p_\phi^2 + (\Delta p_\phi)^2 - p_\alpha^2 - (\Delta p_\alpha)^2 - e^{4\alpha} - 8e^{4\alpha}(\Delta\alpha)^2 + m^2\phi^2 e^{6\alpha} + m^2 e^{6\alpha}(\Delta\phi)^2 \\
&\quad + 12m^2\phi e^{6\alpha}\Delta(\alpha\phi) + 18m^2\phi^2 e^{6\alpha}(\Delta\alpha)^2, \\
C_\alpha &= 2p_\phi\Delta(\alpha p_\phi) - 2p_\alpha\Delta(\alpha p_\alpha) - i\hbar\, p_\alpha + 2m^2\phi e^{6\alpha}\Delta(\alpha\phi) + (6m^2\phi^2 e^{6\alpha} - 4e^{4\alpha})(\Delta\alpha)^2, \\
C_\phi &= 2p_\phi\Delta(\phi p_\phi) + i\hbar\, p_\phi - 2p_\alpha\Delta(\phi p_\alpha) + (6m^2\phi^2 e^{6\alpha} - 4e^{4\alpha})\Delta(\alpha\phi) + 2m^2\phi e^{6\alpha}(\Delta\phi)^2, \\
C_{p_\alpha} &= 2p_\phi\Delta(p_\alpha p_\phi) - 2p_\alpha(\Delta p_\alpha)^2 + (6m^2\phi^2 e^{6\alpha} - 4e^{4\alpha})\Delta(\alpha p_\alpha) + 2m^2\phi e^{6\alpha}\Delta(\phi p_\alpha) \\
&\quad - i\hbar(3m^2\phi^2 e^{6\alpha} - 2\, e^{4\alpha}), \\
C_{p_\phi} &= 2p_\phi(\Delta p_\phi)^2 - 2p_\alpha\Delta(p_\alpha p_\phi) + (6m^2\phi^2 e^{6\alpha} - 4e^{4\alpha})\Delta(\alpha p_\phi) + 2m^2\phi e^{6\alpha}\Delta(\phi p_\phi) - i\hbar\, m^2\phi e^{6\alpha}
\end{aligned}$$

- 5 (1st class) constraints generate 4 independent flows (degenerate Poisson structure)

Choice of local time — *Zeitgeist* [Bojowald, PH, Tsobanjan '11]

- e.g. α relational clock $\Rightarrow$ not represented as operator $\Rightarrow$ choose gauge/project clock to parameter

$$(\Delta\alpha)^2 = \Delta(\alpha\phi) = \Delta(\alpha p_\phi) = 0 \quad \Rightarrow \quad \text{1 Hamilt. flow left}$$

- choice of internal time is best described/interpreted in corresponding gauge: *Zeitgeist* $\Rightarrow$ corresponds to local deparametrization

- remaining DoFs $\phi, p_\phi, (\Delta\phi)^2, \Delta(\phi p_\phi), (\Delta p_\phi)^2$ and α:
EoMs via Poisson structure $\dot{\phi} = \{\phi, C_H\}, \ldots$ etc.

- local relational observables at effective level:
correlations of expectation values and moments with expectation value of clock

$$\phi(\alpha), \Delta(\phi p_\phi)(\alpha), \ldots$$

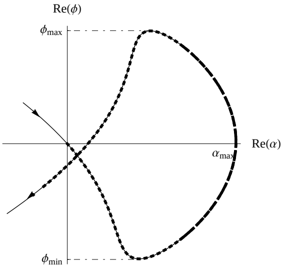
evaluated in corresponding Zeitgeist

Non-unitarity at effective level and switching clocks

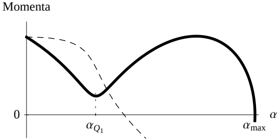
- classically, in turning region $\{\alpha, C\} = 2p_\alpha \to 0$
- EoMs in α–Zeitgeist feature factors p_α^{-n} $n \in \mathbb{N}_+ \Rightarrow$ effective equations diverge
  - e.g. evolving moments $(\Delta\phi)^2, \Delta(\phi p_\phi), (\Delta p_\phi)^2$ grow unboundedly
- clock too slow to appropriately resolve evolution of 'fast' DoFs
- *Zeitgeist* $(\Delta\alpha)^2 = \Delta(\alpha\phi) = \Delta(\alpha p_\phi) = 0$ incompatible with semiclassical expansion in turning region
- systematic formalism for switching internal clocks available: essentially gauge transformations [Bojowald, PH, Tsobanjan, '11]

$\Rightarrow$ translate between clock frameworks and patch up semiclassical trajectory ("physical coordinate transformation")

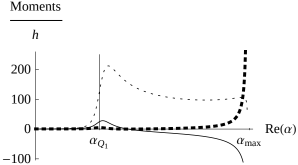
Numerical results: benign trajectories of k=1 FRW + ϕ



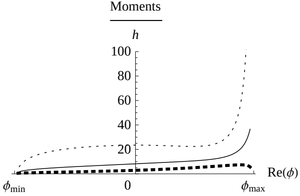
patched up semiclassical trajectory



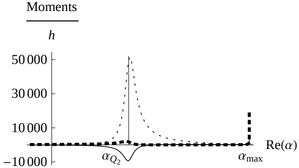
classical p_α , p_ϕ on incoming branch



α–Zeitgeist

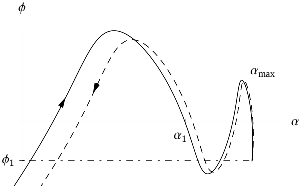


ϕ–Zeitgeist

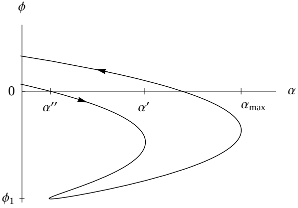


α–Zeitgeist

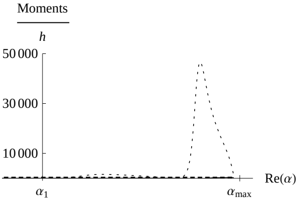
Generic breakdown of relational dynamics [PH, Kubalová, Tsobanjan '12]



classical solution

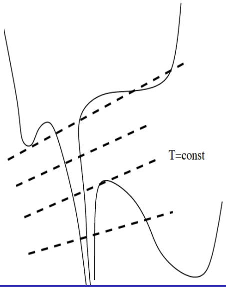


close-up on α_{max}



moments in initial α–Zeitgeist

- generic classical trajectory has structure below chosen quantum scale
- semiclassicality generically breaks down in region of maximal expansion ('too much structure' + defocussing)
- any clock 'bad' in this region, no clock change possible $\Rightarrow$ relational evolution breaks down



Conclusions

- effective approach to evaluate semiclassical relational dynamics
  - handles generic clocks
  - transient relational observables
  - can switch clocks via gauge transformation
- non-integrable k=1 FRW model with massive scalar field: generic breakdown of semiclassicality and relational evolution due to chaotic behaviour in region of maximal expansion
- a) 'good relational evolution' seems to be transient and semiclassical phenomenon
  b) non-integrability potential killer of relational paradigm

further reading: PH, Kubalová, Tsobanjan PRD **86** 065014 (2012); Bojowald, PH, Tsobanjan PRD **83** 125023 (2011); Bojowald, PH, Tsobanjan CQG **28** 035006 (2011)