Domains in zero resistance states

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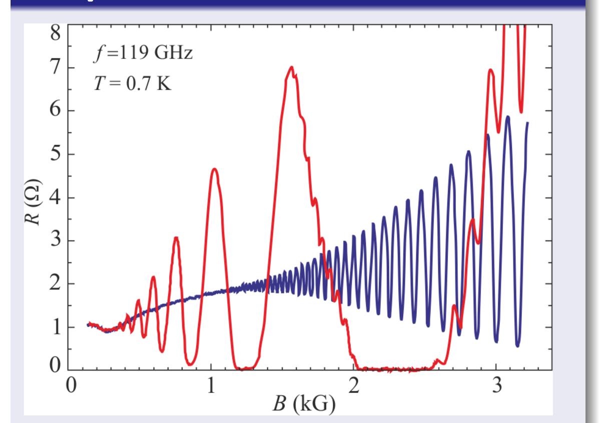
Microwave Induced Resistance Oscillations (MIRO) and associated Zero Resistance States (ZRS)

Nonequilibrium phenomena in high Landau levels

2001-present: Discovery of integer and fractional microwave-induced resistance oscillations, zero-resistance states in semiconductor quantum Hall systems and on electrons on surface of liquid He, magnetooscillations induced by strong dc current and resonant interaction with acoustic phonons, photovoltaic effects…

Review: Dmitriev, Mirlin, Polyakov, Zudov, *Rev. Mod. Phys. 84, 1709 (2012)*

Experiments: MIRO & ZRS



ZRS: Mani,Smet,von Klitzing, Narayanamurti,Johnson,Umansky'02, Zudov,Du,Pfeiffer,West'03, Dorozhkin'03, Willett,Pfeiffer,West'04 .. MIRO: Zudov,Du,Simmons,Reno'01, Ye,Tsui,Simmons,Wendt,Vawter,Reno'01,..

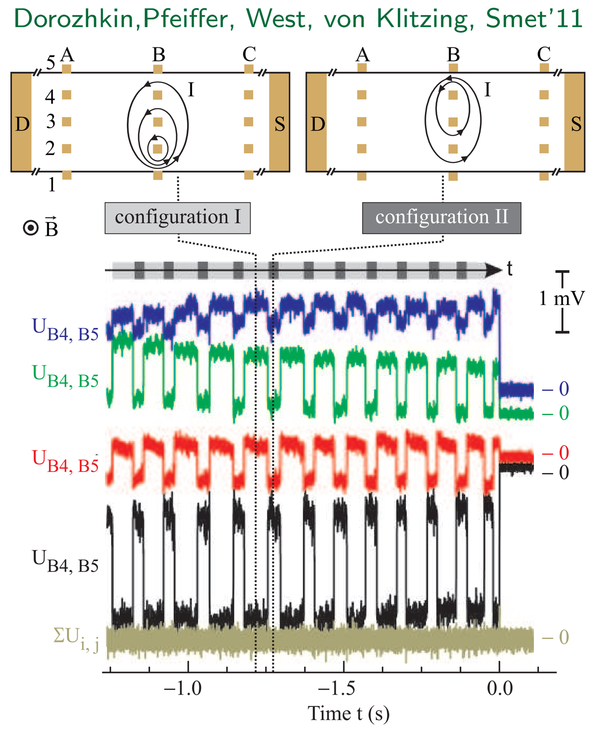
Direct evidence for static domains in ZRS?

Time-resolved measurement of photovoltage signals between internal probes under continuous illumination

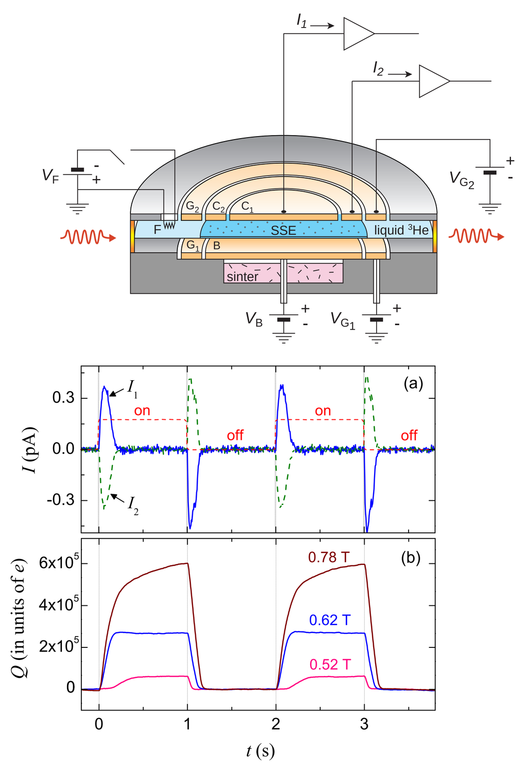
Random telegraph signals for Hall voltages

Interpreted as spontaneous switching between two nearly degenerate configurations of domains

Dorozhkin,Pfeiffer, West, von Klitzing, Smet'11



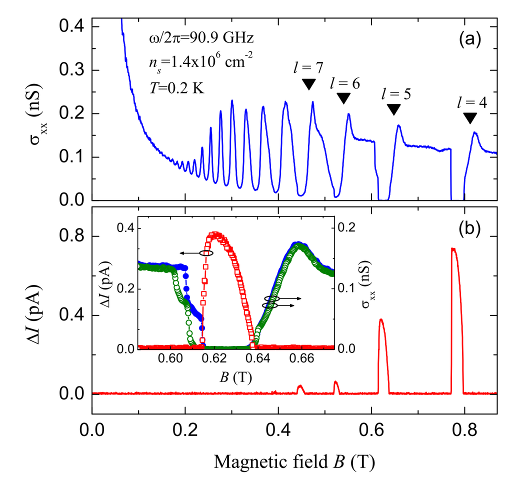
Wigner electron liquid on surface of liquid He



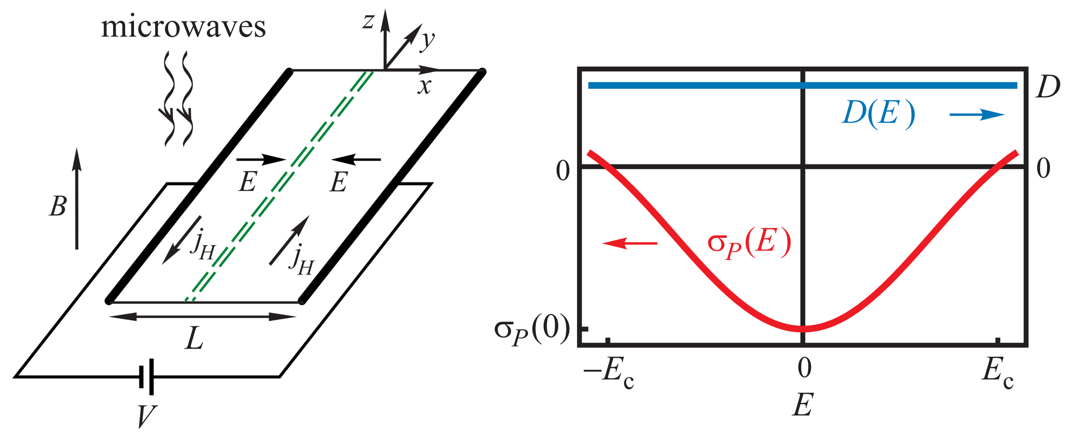
Konstantinov and Kono'09,10, Konstantinov,Chepelianskii,Kono'12

- Oscillations and vanishing dissipation under microwave illumination.
- Time-resolved capacitive measurement for microwaves switching on and off:

Charge transfer due to domain formation



1D model of the domain state in ZRS



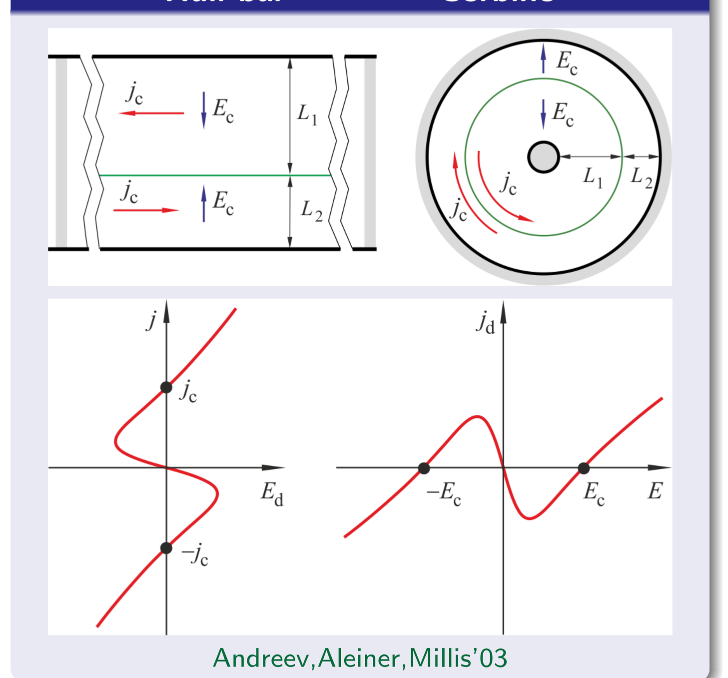
Negative absolute conductivity of homogeneous state, $\mathbf{j}\cdot\mathbf{E}<0$

$\Rightarrow$ Electrical instability: Translational symmetry spontaneously broken

$\Rightarrow$ Electric domains $\equiv$ Spontaneously formed inhomogeneous state

Domains $\Rightarrow$ zero resistance/conductance

Hall bar **Corbino**



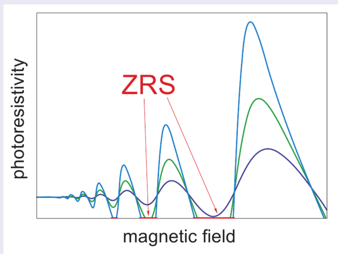
Andreev,Aleiner,Millis'03

Domains with $j_c \perp E_c = \rho_H j_c$, where $\sigma(E_c)=0$ and $\rho_H = B/enc$.

External bias moves the domain wall

Hall bar – zero resistance state: $V=0$ for any $I=j_c(L_1-L_2)$

Corbino – zero conductance state: $I=0$ for any $V=E_c(L_1-L_2)$



Overview of the model: Key ingredients

2D charge density $\rho(x)$ & in-plane electric field $E(x)$

Poisson equation: $-\epsilon\Delta\phi = 4\pi\rho\delta(z) \quad\Rightarrow\quad \rho(x) = -\int \frac{\epsilon dx'}{2\pi^2}\frac{E(x')}{x-x'}$

Continuity equation: $\partial_t\rho + \nabla\cdot j = 0$ (Static domains $\Rightarrow$ $\partial_t\rho = 0$)

$j = \sigma(E)E - D\nabla\rho$: conductivity $\sigma(0)<0$, diffusion coefficient $D>0$

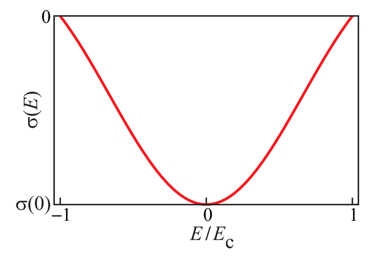
Einstein relation doesn't hold: $e^2\nu_0 D \neq \sigma$!

Resulting self-consistent nonlinear integral equation

$$j = \sigma(E)\,E(x) + D\,\partial_x \int \frac{\epsilon dx'}{2\pi^2}\frac{E(x')}{x-x'} \quad + \quad \text{boundary conditions}$$

Step 1: Domain solution in infinite system

- Solve $j=0$ with boundary conditions $E|_{x\to\pm\infty} = \pm E_c$
- Take $\sigma(E) = \sigma(0)\frac{\sin \pi E/E_c}{\pi E/E_c}$, $\sigma(0)<0$

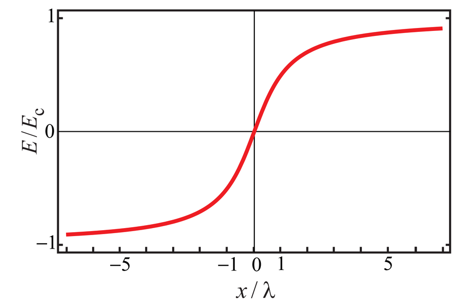


$$\sigma(E)\,E(x) + D\,\partial_x \int \frac{\epsilon dx'}{2\pi^2}\frac{E(x')}{x-x'} = 0 \quad\Rightarrow\quad -\sin\frac{\pi E}{E_c} + \lambda\,\partial_x \int \frac{dx'}{x-x'}\frac{E(x')}{E_c} = 0$$

Solution: $\frac{E}{E_c} = \frac{2}{\pi}\arctan\frac{x}{\lambda}$, $\lambda = \frac{\epsilon D}{2\pi|\sigma(0)|}$

Nonequilibrium screening length λ:

- The only spatial scale, width of domain wall
- Diverges at $\sigma(0)\to 0$ $\Rightarrow$ critical parameter
- reduces to $\lambda_{TF} = \frac{\epsilon}{2me^2}$ in equilibrium



Step 2: Domain solution in unbiased finite system

Spatio-temporal fluctuations $\delta\rho(t)e^{iq_y y}\sin(q_x x)$: Stability $\leftrightarrow$ $\partial_t \ln\delta\rho(t) < 0$

$\rho(0)=\rho(L)=0 \quad\Rightarrow\quad q_x = n\pi/L \;\Rightarrow q = \sqrt{q_x^2+q_y^2} > \pi/L$

- Finite L: uniform state $\rho(x)\equiv 0$ is stable for $\sigma > -\epsilon D/2L$ $\Leftrightarrow$ $l = L/\pi\lambda < 1$

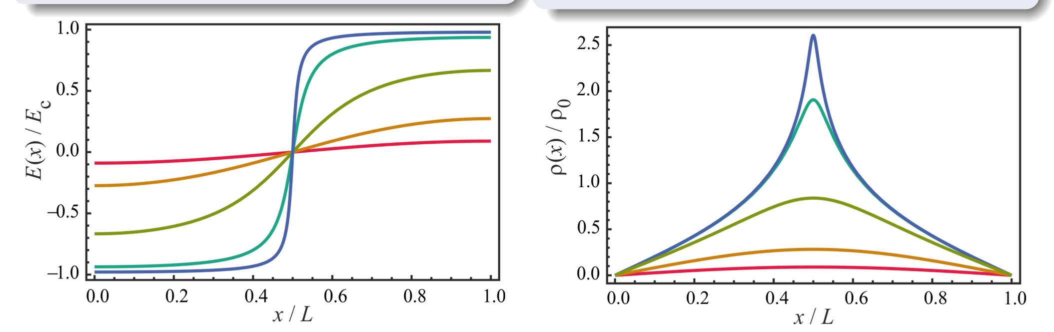
Size-dependent instability threshold! • $L\to\infty$: usual condition $\sigma>0$

- Phenomena at $-\epsilon D/2L < \sigma < 0$ $(0<l<1)$ $\rightarrow$ Dorozhkin,Dmitriev,Mirlin,PRB'11

$l = L/\pi\lambda$

$\frac{E(x)}{E_c} = -\frac{2}{\pi}\arctan\left(\sqrt{l^2-1}\cos\frac{\pi x}{L}\right)$ $\qquad$ $\frac{2\pi\rho(x)}{\epsilon E_c} = \frac{2}{\pi}\text{artanh}\left(\sqrt{1-l^{-2}}\sin\frac{\pi x}{L}\right)$

$l=30$, $l=10$, $l=2$, $l=1.1$, $l=1.01$



General approach to stability: Lyapunov functional

Lyapunov functional $\Phi = K - G$: Stable solution $\equiv$ Global minimum of Φ

Application to ZRS (3D, limit $\lambda \ll L$): A. Auerbach, I. Finkler, B. I. Halperin, A. Yacoby'05

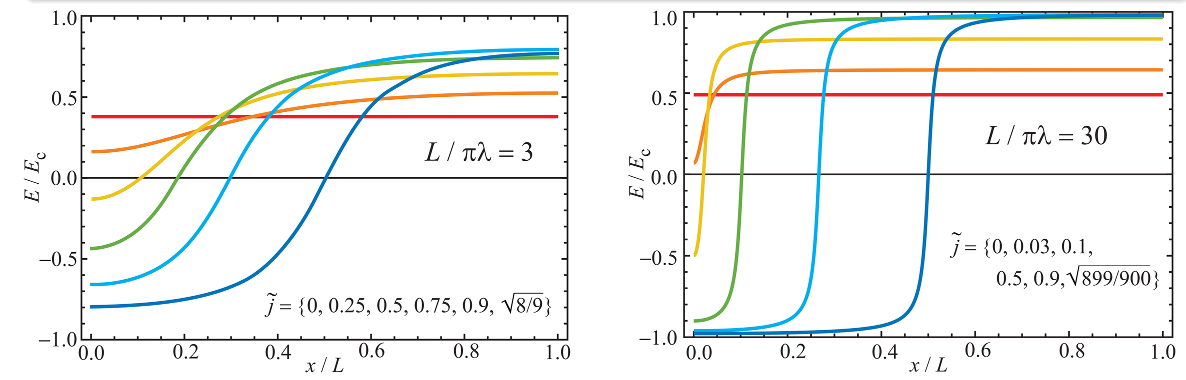
- Gain $G = -\int dx \int_0^{E(x)} \sigma(E')E'dE'$: $\sigma<0$ $\Rightarrow$ Maximized for $|E(x)| = E_c$
- Domain walls $K = \frac{D}{2}\int dx\, E(x)\,\hat{C}\,E(x) \geqslant 0$ $\Rightarrow$ Minimized for $\partial_x E = 0$
- capacitance C: $\rho(x) = \hat{C}\phi(x)$, $[\hat{C},\partial_x]=0$ $\Rightarrow$ $\partial_x\rho = -\hat{C}E$

- Allow $E(x,t)$: $\dot\Phi = \int dx\,[\sigma(E)\,E\,\dot E + D\,\dot E\,\hat C\,E] = \int dx\,[\sigma(E)\,E - D\,\partial_x\rho]\,\dot E = \int dx\, j\,\dot E$
- Poisson $E = -\hat{C}^{-1}\partial_x\rho$ & Continuity $\dot\rho = -\partial_x j$ $\Rightarrow$ $\dot E = \hat C^{-1}\partial_x^2 j$

$$\Longrightarrow \quad \dot\Phi = \int dx\, j\,\dot E = \int dx\, j\,\hat C^{-1}\partial_x^2 j = -\int dx\,(\partial_x j)\,\hat C^{-1}(\partial_x j) \leqslant 0$$

Step 3: Domain solution in a biased 2D stripe

- Solve $\sin\frac{\pi E}{E_c} - \lambda\,\partial_x\int\frac{dx'}{E_c}\frac{E(x')}{x-x'} = \tilde j$; (Anti)dissipative current $\tilde j = \frac{\pi j}{\sigma(0)E_c}$

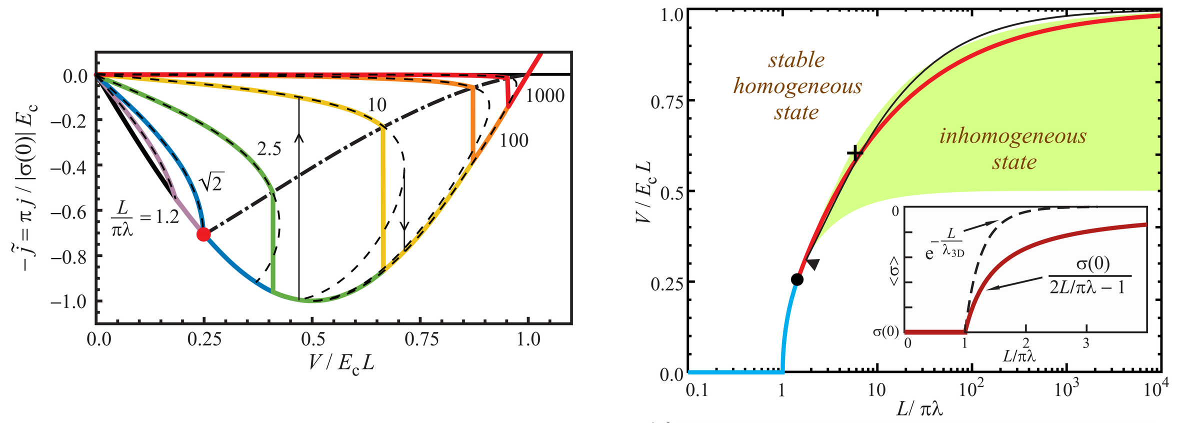


$$\frac{E}{E_c} = \frac{2}{\pi}\arctan\left(l^2\tilde j\sqrt{1-\tilde j^2} - l\sqrt{1-l^{-2}-\tilde j^2}\sqrt{1+l^2\tilde j^2}\cos\frac{\pi x}{L}\right) - \frac{1}{\pi}\arcsin\tilde j, \quad l = \frac{L}{\pi\lambda}$$

Full solution: $\Psi \equiv \frac{\pi}{2E_c}E + i\frac{\pi^2}{\epsilon E_c}\rho = i\ln\frac{\cosh(\xi - iw)}{\sinh\xi} - \frac{1}{2}\arcsin\tilde j$, $\quad |\tilde j| \leqslant \sqrt{1-l^{-2}}$,

where $2\xi = i\pi(x/L - 1/2) + iw + \beta$, $w = \arctan(\tilde j l)$, $\beta = \text{arcoth}(l\sqrt{1-\tilde j^2})$

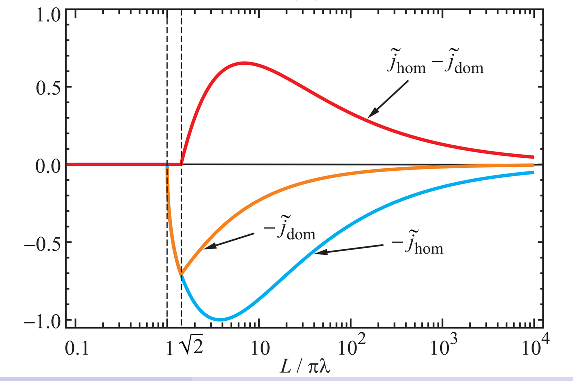
Full CVC, phase diagram, and current discontinuity



I– and II–order phase transitions:

- $l \leqslant \sqrt{2}$: continuous j across the PB
- $l > \sqrt{2}$: Discontinuity $\Delta\tilde j$ across the PB

- Frozen transverse fluctuations $\Rightarrow$ hysteresis
- Allow transverse fluctuations $\Rightarrow$ suppressed hysteresis, voltage-driven transverse instability!



Summary and Outlook

Summary: Analytical model of the domain state in ZRS

- 2D electrostatics
- Evolution with system size L and external bias V
- Transverse instability at large V and L

Outlook

- Mean field: Analysis of transverse fluctuations, inclusion of contact potentials, macroscopic inhomogeneities, periodic spatial modulation etc.
- Critical behavior at the transition to ZRS: Influence of noise, nature of transition, dynamics at "zero" and finite temperature
- Domain structure in zero *differential* resistance states in Hall and Corbino geomeries
- Domain structure in "zero admittance states" in electron liquid on surface of liquid He

- Experimental evidence of the domain formation: S. I. Dorozhkin et al., *Nature Phys.* **7**, 336 (2011); Konstantinov et al., *J. Phys. Soc. Jpn.* **81**, 093601 (2012).
- Review: I. A. Dmitriev, A. D. Mirlin, D. G. Polyakov, M. A. Zudov, *Rev. Mod. Phys. 84, 1709 (2012)*
- Domains in "3D": A. Auerbach et al., *Phys. Rev. Lett.* **94**, 196801 (2005); I. G. Finkler and B. I. Halperin, *Phys. Rev. B* **79**, 085315 (2009); J. Alicea et al., *Phys. Rev. B* **71**, 235322 (2005); A. F. Volkov and V. V. Pavlovskii, *Phys. Rev. B* **69**, 125305 (2004)

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