Control of underwater vehicles in potential fluids

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Control of underwater vehicles in potential fluids

We consider a rigid body $S \subset \mathbb{R}^3$ with two planes of symmetry, surrounded by a fluid, and which is controlled by controls fluid flows, which represent turbines or thrusters.

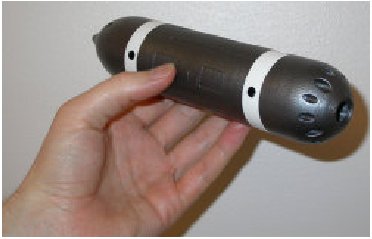
Bow thruster

Longitudinal propeller

Figure : Example of the a bow thruster and longitudinal propeller

Control of underwater vehicles in potential fluids

Tiny Submersible Could Search for Life in Europa's Ocean



Movie sequence of a miniature submarine exploring under the ice. Credit: Jonas Jonsson, Angstrom Space Technology Centre of Uppsala University

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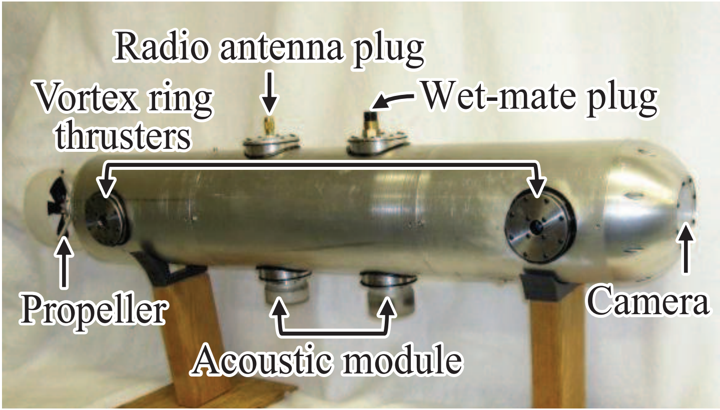
Prototype miniature submersible



Prototype of a miniature submarine. Credit: Yiming Xu, Zheng Ren, and Kamran Mohseni, University of Florida

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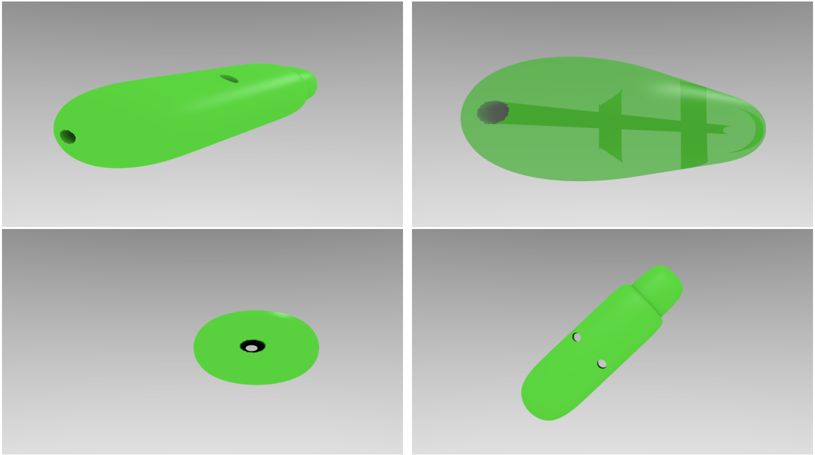
Example of submarine



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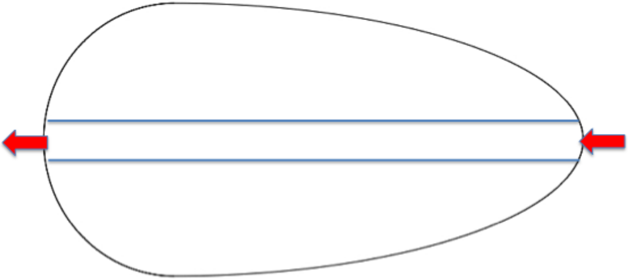
Example of controls



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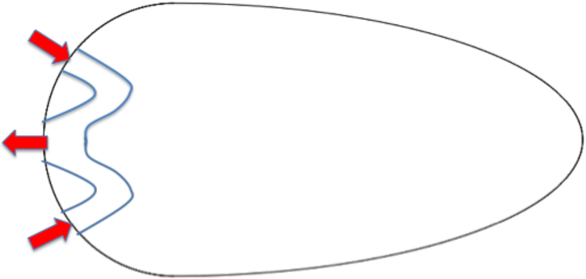
Example of controls



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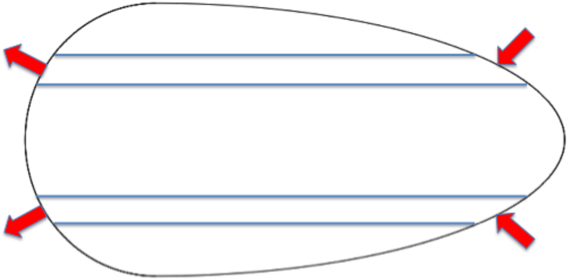
Example of controls



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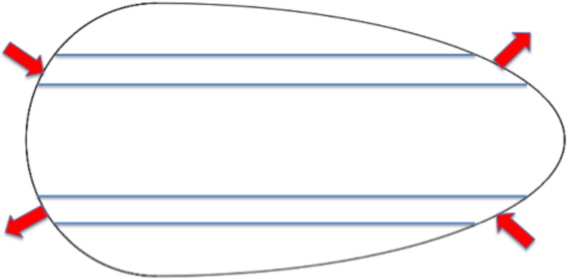
Example of controls



System under investigation, $\Omega(t) = \mathbb{R}^3 \setminus S(t)$

$$\text{Euler}\begin{cases} \dfrac{\partial u}{\partial t} + (u \cdot \nabla) u + \nabla p = 0, & x \in \Omega(t) \\ \mathrm{div}\ u = 0, & x \in \Omega(t) \\ u \cdot \hat{n} = (h' + \omega \times (x - h)) \cdot \hat{n} + w(t, x), & x \in \partial\Omega(t) \\ \lim\limits_{|x| \to +\infty} u(t, x) = 0, & \end{cases}$$

$$\text{Newton}\begin{cases} m h'' = \displaystyle\int_{\partial\Omega(t)} p \hat{n}\, d\sigma, \\ \dfrac{d}{dt}(Q J_0 Q^* \omega) = \displaystyle\int_{\partial\Omega(t)} (x - h) \times p \hat{n}\, d\sigma, \\ Q' = S(\omega) Q, \qquad S(\omega) y = \omega \times y \ \ \forall y \in \mathbb{R}^3, \end{cases}$$

System supplemented with Initial Conditions, and with the value of the vorticity at the incoming flow (in $\partial\Omega(t)$) for the uniqueness

System under investigation

Main difficulties

1. The systems describing the motions of the fluid and the solid are nonlinear and strongly coupled.
2. The fluid domain $\mathbb{R}^3 \setminus S(t)$ is an unknown function of time

We follow in this work the same approach as in:

L. ROSIER and O. GLASS.
On the control of the motion of a boat.
Mathematical Models and Methods in Applied Sciences, 23(04):617–670, 2013.

System in a frame linked to the solid

After a change of variables we obtain in $\Omega := \mathbb{R}^3 \setminus S(0)$

$$\text{Fluid} \begin{cases} \dfrac{\partial v}{\partial t} + ((v - l - r \times y) \cdot \nabla) v + r \times v + \nabla q = 0, & y \in \Omega, \\ \operatorname{div} v = 0, & y \in \Omega, \\ v \cdot \hat{n} = (l + r \times y) \cdot \hat{n} + \sum_{1 \le j \le n} w_j(t) \chi_j(y), & y \in \partial\Omega, \\ \lim_{|y| \to +\infty} v(t, y) = 0, & \end{cases}$$

$$\text{Body} \begin{cases} m\dot{l} = \displaystyle\int_{\partial\Omega} q \hat{n} \, d\sigma - mr \times l, \\ J_0 \dot{r} = \displaystyle\int_{\partial\Omega} q (y \times \hat{n}) \, d\sigma - r \times J_0 r, \end{cases}$$

and initial conditions $(l(0), r(0)) = (h_1, r_0), \; v(0, y) = u_0(y)$.

Relation with Kirchhoff laws

We consider $\mathcal{J} = \begin{pmatrix} mI_d & 0 \\ 0 & J_0 \end{pmatrix} + \begin{pmatrix} M & D \\ D^* & J \end{pmatrix}$ and $(P, \Pi) \in \mathbb{R}^3 \times \mathbb{R}^3$ defined by $\mathcal{J} \begin{pmatrix} l \\ r \end{pmatrix} = \begin{pmatrix} P \\ \Pi \end{pmatrix}$
Then the dynamics of the system are governed by the following Kirchhoff equations

$$\frac{dP}{dt} + C^M \dot{w} = (P + C^M w) \times r - \sum_{1 \le p \le n} w_p \left\{ L_p^M l + R_p^M r + W_p^M w \right\},$$

$$\frac{d\Pi}{dt} + C^J \dot{w} = (\Pi + C^J w) \times r + P \times l - \sum_{1 \le p \le n} w_p \left\{ L_p^J l + R_p^J r + W_p^J w \right\}$$

where $w(t) := (w_1(t), ..., w_n(t)) \in \mathbb{R}^n$ denotes the control input.

The dynamics of the full system (position and attitude)

Then, the dynamics of $(h, q, l, r, w) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^n$ is given by

$$\begin{cases} \begin{pmatrix} h' \\ q' \end{pmatrix} & = & g(q, l, r) \\ \begin{pmatrix} l' \\ r' \end{pmatrix} & = & \mathcal{J}^{-1}(C w' + F(l, r, w)) \end{cases}$$

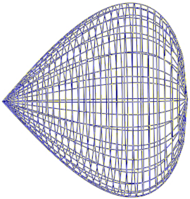
$$F(l, r, w) = -\begin{pmatrix} S(r) & 0 \\ S(l) & S(r) \end{pmatrix} \left(\mathcal{J} \begin{pmatrix} l \\ r \end{pmatrix} - C w \right) - \sum_{p=1}^{n} w_p \begin{pmatrix} L_p^M l + R_p^M r + W_p^M w \\ L_p^J l + R_p^J r + W_p^J w \end{pmatrix}$$

Rigid bodies with symmetries

Uncontrollability

Solid of revolution

$$\partial\Omega = \Big\{ \Big(y_1, f(y_1)\cos(\beta), f(y_1)\sin(\beta) \Big) \; : \; y_1 \in [a,b],\ \beta \in [0, 2\pi) \Big\}$$



- Equation for angular velocity

$$J_0 \dot{r} = \int_{\partial\Omega} q(y \times \hat{n})\, d\sigma - r \times J_0 r$$

- $J_0 \cdot \hat{e}_1 = (J_1, 0, 0)^*$
- $(y \times \hat{n}) e_1 = 0$

$$J_1 \dot{r}_1 \equiv 0$$

Linearize the system

The linearization of the system around $(h, q, l, r, w) = (0, 1, 0, 0, 0)$ reads

$$\left\{ \begin{array}{ccc} h' & = & l \\ 2q' & = & r \\ \begin{pmatrix} l' \\ r' \end{pmatrix} & = & \mathcal{J}^{-1} C w' \end{array} \right.$$

Taking $w' \in \mathbb{R}^n$ as control, it is controllable if, and only if, $\mathrm{rank}(C) = 6$.

Ellipsoidal vehicle

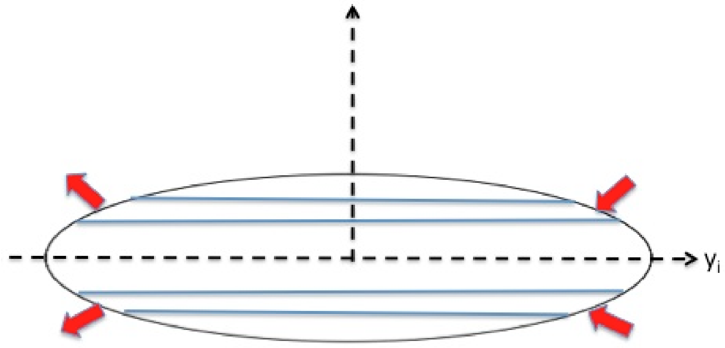
We assume here that the vehicle fills the ellipsoid

$$S = \left\{ y \in \mathbb{R}^3; \;\; \left(\frac{y_1}{c_1}\right)^2 + \left(\frac{y_2}{c_2}\right)^2 + \left(\frac{y_3}{c_3}\right)^2 \leq 1 \right\}$$

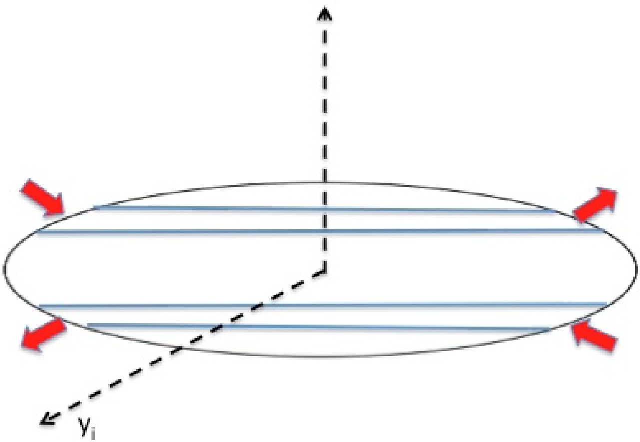
where $c_1 > c_2 > c_3 > 0$. Our first aim is to compute explicitly the functions ϕ_i and φ_i for $i = 1, 2, 3$ for

$$\Omega = \left\{ y \in \mathbb{R}^3; \;\; \left(\frac{y_1}{c_1}\right)^2 + \left(\frac{y_2}{c_2}\right)^2 + \left(\frac{y_3}{c_3}\right)^2 > 1 \right\}$$

Controllability of the ellipsoid with six controls

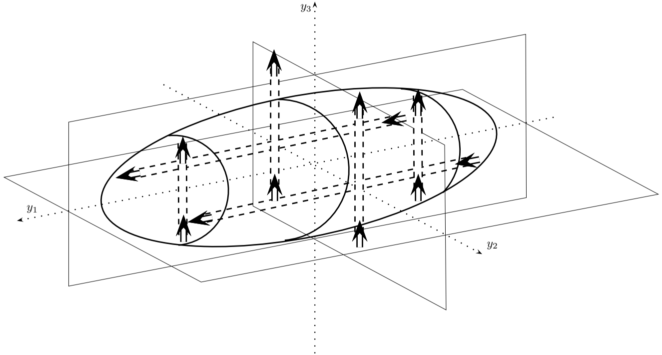


Controllability of the ellipsoid with six controls



Controllability of the ellipsoid with six controls

$C = \mathrm{diag}(C_1, C_2, C_3, C_4, C_5, C_6)$ and hence $\mathrm{rank}(C) = 6$



Rigid bodies with symmetries

Let us introduce the operators $S_i(y) = y - 2y_i e_i$, i.e.

$$S_1(y) = (-y_1, y_2, y_3),$$
$$S_2(y) = (y_1, -y_2, y_3),$$
$$S_3(y) = (y_1, y_2, -y_3).$$

Definition

- Let $i \in \{1, 2, 3\}$. We say that Ω is symmetric with respect to the plane $\{y_i = 0\}$ if $S_i(\Omega) = \Omega$.
- Let $f : \Omega \subset \mathbb{R}^3 \to \mathbb{R}$. If $f(S_i(y)) = \varepsilon_f^i f(y)$ for any $y \in \Omega$ and some number $\varepsilon_f^i \in \{-1, 1\}$, then f is said to be even (resp. odd) with respect to S_i if $\varepsilon_f^i = 1$ (resp. $\varepsilon_f^i = -1$).

Symmetric domain

We assume that

1. Ω is invariant under the operators S_2 and S_3, i.e.

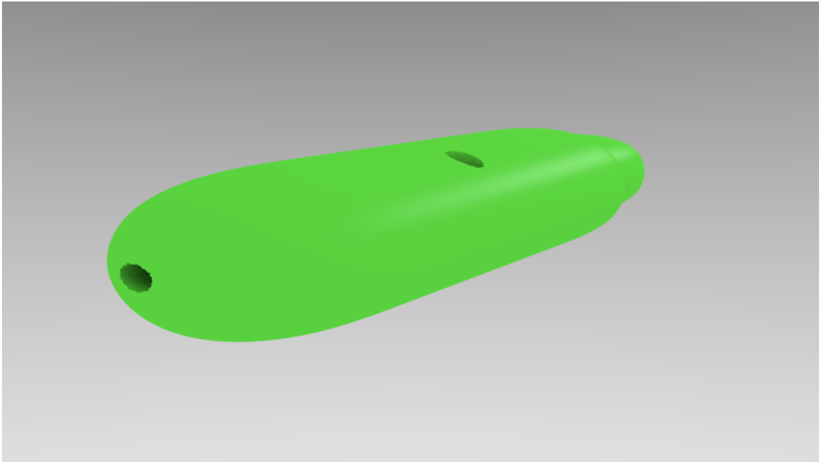
$$S_p(\Omega) = \Omega, \quad \forall p \in \{2, 3\},$$

2. $\varepsilon^p_{\chi_1} = 1$, i.e.

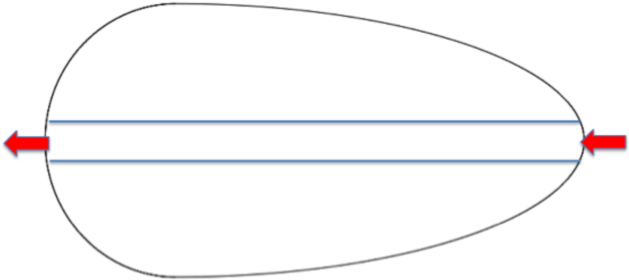
$$\chi_1(S_p(y)) = \chi_1(y) \quad \forall y \in \partial\Omega, \forall p \in \{2, 3\}$$

In other words, the set S and the control χ_1 are symmetric with respect to the two planes $\{y_2 = 0\}$ and $\{y_3 = 0\}$.

Symmetric domain, $S_p(\Omega) = \Omega$ and $\varepsilon^p_{\chi_1} = 1 \ \forall p \in \{2, 3\}$



Symmetric domain, $S_p(\Omega) = \Omega$ and $\varepsilon^p_{\chi_1} = 1 \ \forall p \in \{2, 3\}$



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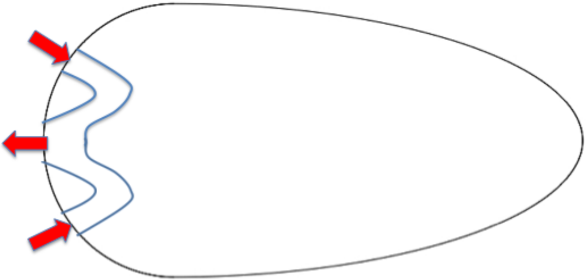
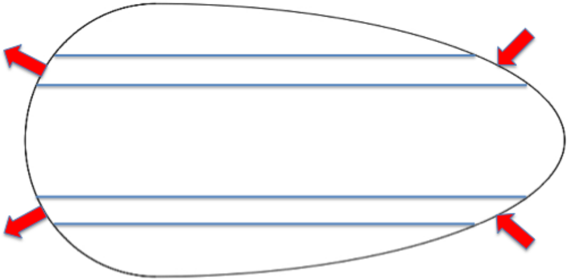


Symmetric domain, $S_p(\Omega) = \Omega$ and $\varepsilon^p_{\chi_1} = 1 \; \forall p \in \{2, 3\}$



Loop-shaped trajectory

We consider a special trajectory of the toy problem, constructed as in the flatness approach due to M. Fliess, J. Levine, P. Martin, P. Rouchon

- We first define the trajectory

$$\overline{h}_1(t) = \lambda(1 - \cos(2\pi t/T))$$

$$\bar{l}_1(t) = \lambda(2\pi/T)\sin(2\pi t/T)$$

- We next solve the Cauchy problem

$$\begin{cases} \dot{\overline{w}}_1 = \alpha^{-1}(\dot{\bar{l}}_1 - \beta\bar{l}_1\overline{w}_1 - \gamma\overline{w}_1^2) \\ \overline{w}_1(0) = 0 \end{cases}$$

  to design the control input.

- Then $\overline{w}_1$ exists on $[0, T]$ for $0 < \lambda << 1$. $(\overline{h}_1, \bar{l}_1) = 0$ at $t = 0, T$. Nothing can be said about $\overline{w}_1(T)$.

Return Method

We linearize along the above (non trivial) reference trajectory to use the nonlinear terms. We obtain a system of the form

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} D(t) & Id \\ 0 & A(t) \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{bmatrix} 0 \\ B(t) \end{bmatrix} w + \begin{bmatrix} 0 \\ \mathcal{J}^{-1} C \end{bmatrix} w'$$

where $x = (h, \vec{q})^*$ and $y = (l, r)^*$

$$\begin{aligned} D(t) &= f(t)\mathbf{D} + o(\lambda) \\ A(t) &= f(t)\mathbf{A} + o(\lambda) \\ B(t) &= f(t)\mathbf{B} + o(\lambda) \end{aligned}$$

where

$$f(t) = \frac{1}{\alpha} \bar{l}_1(t)$$

Control result for potential flows

Theorem

- Assume that $S_p(\Omega) = \Omega$ and $\varepsilon^p_{\chi_1} = 1 \; \forall p \in \{2, 3\}$ hold
- $\alpha = \frac{-(C^M)_{11}}{m + M_{11}} \neq 0$
- $\mathrm{rank}\,(C, \mathbf{B} + \mathbf{A}\mathcal{J}^{-1}C) = 6$
- $\mathrm{rank}\,(C, \frac{1}{2}\mathcal{J}\mathbf{D}\mathcal{J}^{-1}C + \mathbf{B} + \mathbf{A}\mathcal{J}^{-1}C) = 6$

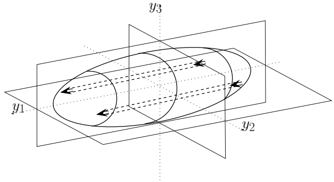
Then for any $T > 0$ the system with state $(h, q, l, r) \in \mathbb{R}^3 \times S^3 \times \mathbb{R}^6$ and control $w \in \mathbb{R}^n$, is locally controllable around the origin in time T.

In the case when $J_0, m >> 1$, the rank conditions are satisfied

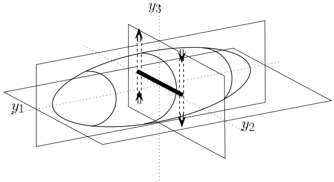
$$\mathrm{rank}\,(C, \mathbf{B}_\infty) = 6$$

Controllability of the ellipsoid with four controls

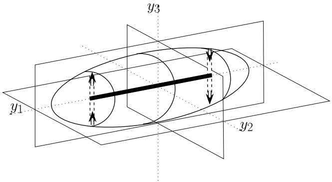
Control χ_1



Control χ_4



Control χ_5



Control χ_6

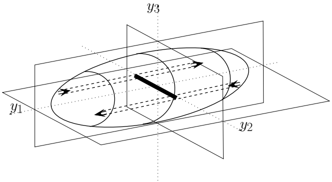


Figure : Ellipsoid with four controls.

Controllability of the ellipsoid with four controls

We consider the same controllers χ_1, χ_4, χ_5 and χ_6 Then the matrices $B_\infty = \lim_{m, J_0 \to +\infty} B$ and C are given by

$$C = -\begin{pmatrix} C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & C_4 & 0 & 0 \\ 0 & 0 & C_5 & 0 \\ 0 & 0 & 0 & C_6 \end{pmatrix}, \quad B_\infty = -\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_6 \\ 0 & 0 & B_5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

with

$$B_5 = \int_{\partial\Omega} (\nabla\psi_1 \cdot \nabla\psi_5)\, \hat{n}_3, \quad B_6 = \int_{\partial\Omega} (\nabla\psi_1 \cdot \nabla\psi_6)\, \hat{n}_2,$$

where $\Delta\psi_j = 0$ in Ω and $\partial_n \psi_j = \chi_j$ on $\partial\Omega$.

Thus, if $B_5 \neq 0$ and $B_6 \neq 0$, we see that the rank conditions are fulfilled, so that the local controllability of the system is ensured for m and J_0 large enough.

Path Forward

- Global controllability result.
- Numerics.
- Flow displays some vorticity.
- Stabilization.
- Disturbance rejection.

Thank you!