CONTROL OF THE SONIC BOOM: NUMERICAL DIFFICULTIES

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Partial Differential Equations, Optimal Design and Numerics

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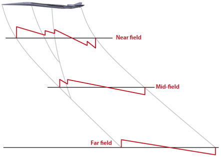
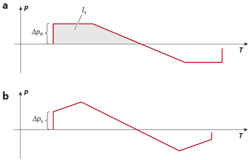
Outline

Motivation

- Nowadays, one of the main goals in aeronautics is to handle the sonic boom phenomenon produced by supersonic flights.
- Historically, linear theory has been used to model this situation, following the works by Witham (1952) and Hayes (1967).
- More recently, new nonlinear models have been developed (Rallabhandi, 2011).
- CFD techniques can accurately approximate the state in the near field, but it needs new tools for the propagation from that near field to the ground level.





J. J. Alonso and M. R. Colonno, *Multidisciplinary Optimization with Applications to Sonic-Boom Minimization*, Annual Review of Fluid Mechanics 44 (2012), 505-526.

The sonic boom model

The sonic boom can be modeled by the Augmented Burgers Equation:

$$\frac{\partial P}{\partial \sigma} = P\frac{\partial P}{\partial \tau} + \underbrace{\frac{1}{\Gamma}\frac{\partial^2 P}{\partial \tau^2}}_{\text{absorption}} + \underbrace{\sum_{\nu} C_\nu \frac{\frac{\partial^2}{\partial \tau^2}}{1+\theta_\nu \frac{\partial}{\partial \tau}} P}_{\text{molecular relaxations}} - \underbrace{\frac{1}{2G}\frac{\partial G}{\partial \sigma}P}_{\text{ray tube spreading}} + \underbrace{\frac{1}{2\rho_0 c_0}\frac{\partial(\rho_0 c_0)}{\partial \sigma}P}_{\text{atmospheric stratification}} \quad \text{(ABE)}$$

where:

- The operator in the molecular relaxation term is given by:

$$\frac{\theta_\nu}{1+\theta_\nu \frac{\partial}{\partial \tau}} f(\tau) = \int_{-\infty}^{\tau} e^{(s-\tau)/\tau_\nu} f(s)ds \qquad (1)$$

- $P = P(\sigma, \tau)$ is the dimensionless pressure, σ is the non-dimensional distance and τ is the dimensionless time.
- Ambient density ρ_0, ambient speed of sound c_0, thermo-viscous parameter Γ, dimensionless relaxation time parameter θ_ν, non-dimensional dispersion parameter C_ν, dimensionless time for each relaxation mode τ_ν and ray tube area G.

S. K. Rallabhandi, *Advanced sonic boom prediction using augmented Burgers equation*, Journal of Aircraft 48, 4 (2011), 1245-1253.

Constant parameters model

Assumptions:

- Only one molecular relaxation phenomena.
- Constant $\rho_0 c_0$, Γ, θ_ν, C_ν, τ_ν and G.

Notation:

- $t := \sigma$, $x := \tau$ and $u(x,t) := P(\tau, \sigma)$
- $\nu := \frac{1}{\Gamma}$, $c := C_\nu$ and $\theta := \theta_\nu$.

Hence, we obtain a model in which an integral kernel is added to the classical viscous Burgers Equation:

$$\begin{cases} u_t = uu_x + \nu u_{xx} + \frac{c}{\theta}\int_{-\infty}^{x} e^{(y-x)/\theta} u_{xx}(y,t)dy, & (x,t) \in \mathbb{R} \times (0,\infty) \\ u(x,0) = u_0(x), & x \in \mathbb{R} \end{cases}$$

or, equivalently:

$$\begin{cases} u_t = uu_x + \nu u_{xx} + \frac{c}{\theta^3}\left(K_\theta * u - \theta u + \theta^2 u_x\right), & (x,t) \in \mathbb{R} \times (0,\infty) \\ u(x,0) = u_0(x), & x \in \mathbb{R} \end{cases} \qquad \text{(CABE)}$$

where $K_\theta(z) := e^{-z/\theta}\chi_{\{z>0\}}(z)$.

Optimal control problem

We want to solve this optimization problem:

$$\mathcal{J}(u^0) = \min_{u^0 \in \mathcal{U}_{ad}} \frac{1}{2} \int_{\mathbb{R}} \left[u(x,T) - u^d(x) \right]^2 dx$$

subject to

$$\begin{cases} u_t = uu_x + \nu u_{xx} + \frac{c}{\theta^3} \left(K_\theta * u - \theta u + \theta^2 u_x \right), & (x,t) \in \mathbb{R} \times (0,T) \\ u(x,0) = u_0(x), & x \in \mathbb{R} \end{cases}$$

Which are the main difficulties we have to deal with?

S. K. Rallabhandi, *Sonic Boom Adjoint Methodology and its Applications*, AIAA 2011-3497, 29th AIAA Applied Aerodynamics Conference (2011).

Open problems and numerical issues

We need:

1. An accurate numerical scheme to solve (CABE)
2. An efficient optimization strategy to solve the optimal control problem

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$$\int_{-\infty}^{x} e^{(y-x)/\theta} \left(u(y,t) - u(x,t) + \theta u_x(x,t)\right) dy$$

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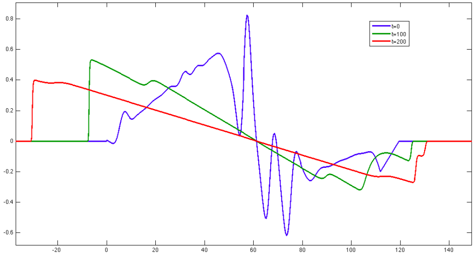

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[1] L. I. Ignat, A. Pozo and E. Zuazua, *Large-time asymptotics, vanishing viscosity and numerics for 1-D scalar conservation laws*, submitted.

[2] A. Porretta and E. Zuazua, *Long time versus steady state optimal control*, submitted.

[3] M. Ersoy, E. Feireisl and E. Zuazua, *Sensitivity analysis of 1-d steady forced scalar conservation laws*, Journal of Differential Equations 254 (2013), 3817-3834.

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2. Large time behavior: N-wave [1,2,3]
3. Similarity to hyperbolic scalar conservation laws: quasi-shocks [4,5]

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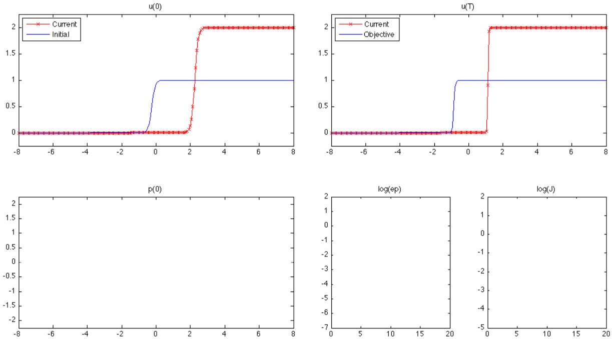
[4] C. Castro, F. Palacios and E. Zuazua, *An alternating descent method for the optimal control of the inviscid Burgers equation in the presence of shocks*, Mathematical Models and Methods in Applied Sciences 18, 3 (2008), 369-416.

[5] C. Castro, F. Palacios and E. Zuazua, *Optimal control and vanishing viscosity for the Burgers equation*, Integral Methods in Science and Engineering 2 (7), Birkhäuser Verlag (2010), 65-90.

Gradient descent method: continuous approach



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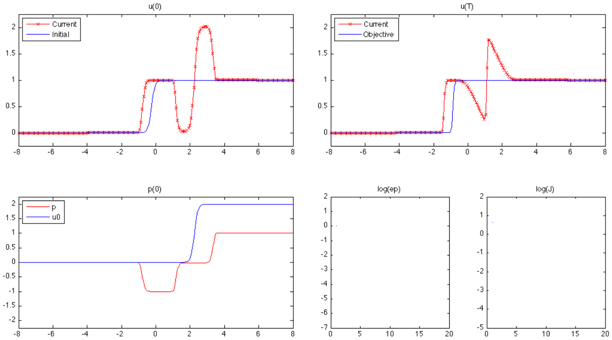

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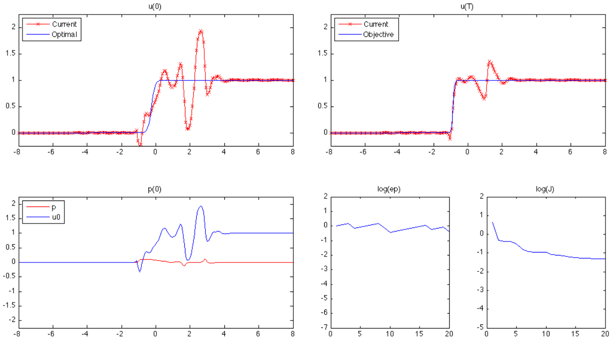
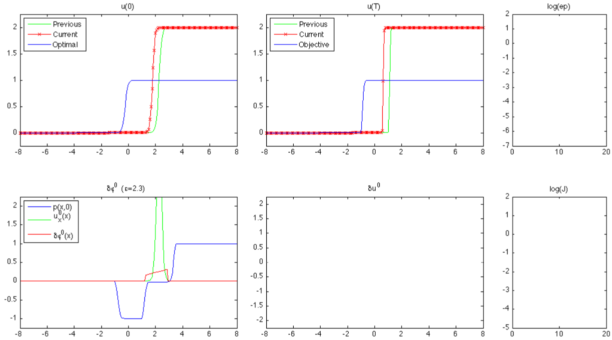


Gradient descent method: alternating descent method



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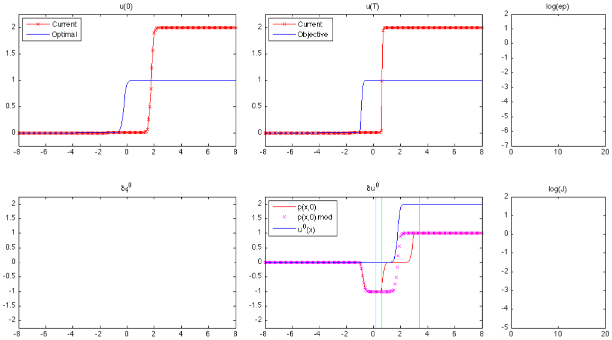

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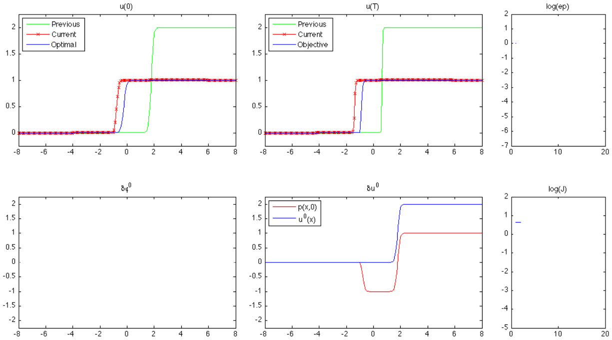

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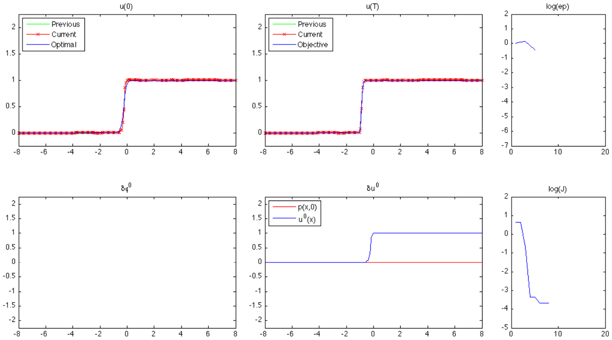

Thank you very much!