Emulation of relativistic effects in electromagnetism on an optical table

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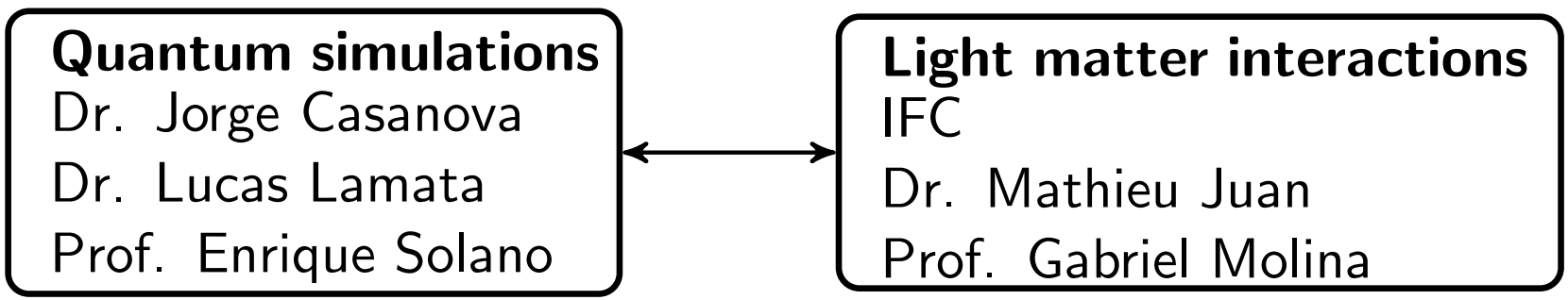
MACQUARIE UNIVERSITY

ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

Australian Government
Australian Research Council

The Bilbao-Sydney connection in this project

Work in progress



Relativistic effects in (quantum) electromagnetism

High speeds, high accelerations, space-time curvature

- Effects on entanglement
- Polarization rotation
- Gravitational phase
- Unruh effect
- ...

A path to emulation on Earth

Macroscopic Maxwell's equations

$$\nabla \cdot \mathbf{D} = 0, \ \nabla \cdot \mathbf{B} = 0,$$
$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E},$$
$$\partial_t \mathbf{D} = \nabla \times \mathbf{H}.$$

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$M = I$ in an empty a flat spacetime

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Space-time geometry induces certain constitutive relations[1]:

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The constitutive relations depend on the space-time metric g.

$$\underline{\underline{\epsilon}} = \underline{\underline{\mu}} = -\frac{g^{ij}}{g_{00}},\ \underline{\underline{\chi}} = -\underline{\underline{\gamma}} = \frac{1}{c}[ijk]\frac{g_{0i}}{g_{00}}$$

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Main idea[2]

Empty space Maxwell's equations in a general gravitational field
are equivalent to
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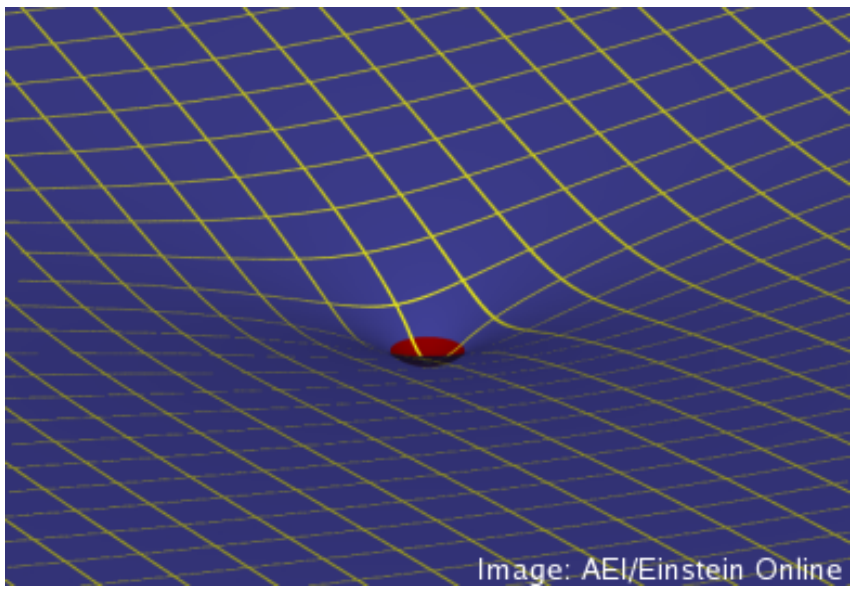

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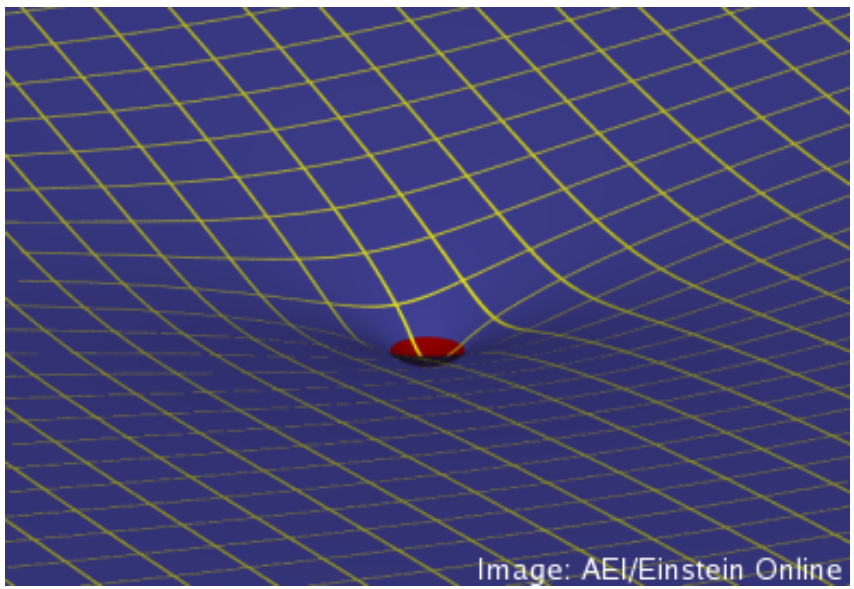

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A way to emulate them on Earth: Metamaterials

- **If** we can fabricate a medium $M(g)$
- EM inside the medium emulates EM in a space-time g

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- **If** we can fabricate a medium $M(g)$
- EM inside the medium emulates EM in a space-time g
- Previous related work[3,4]. White holes in optical fibers.
- Nature does not provide arbitrary $M(g)$: Metamaterials

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Metamaterials

- Assemblies of multiple individual elements (meta atoms)
- Arranged in periodic patterns
- Features much smaller than the wavelength

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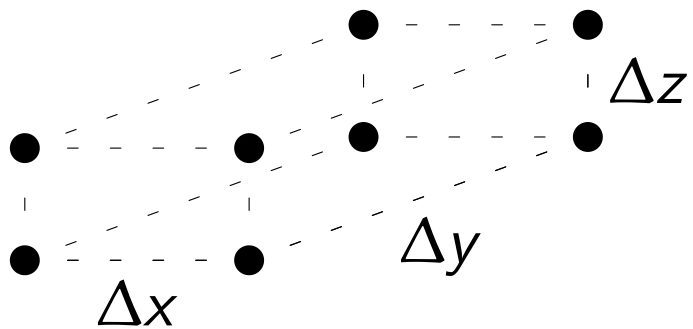
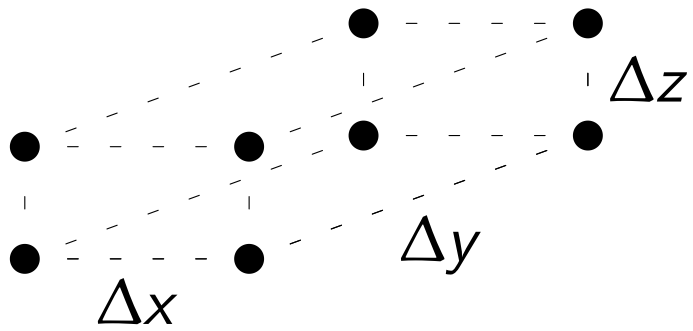


Design and fabrication of $M(g)$



Challenges:

1. Given a general M, lattice ? inclusion polarizability ?
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A conservation law inherent in general relativity

- Helicity is preserved in a gravitational field[5]

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- $\Lambda = \frac{\mathbf{J}\cdot\mathbf{P}}{|\mathbf{P}|},\ \Lambda\left(\mathbf{E} \pm i\mathbf{H}\right) = \pm\left(\mathbf{E} \pm i\mathbf{H}\right) = \pm\mathbf{G}_{\pm}$

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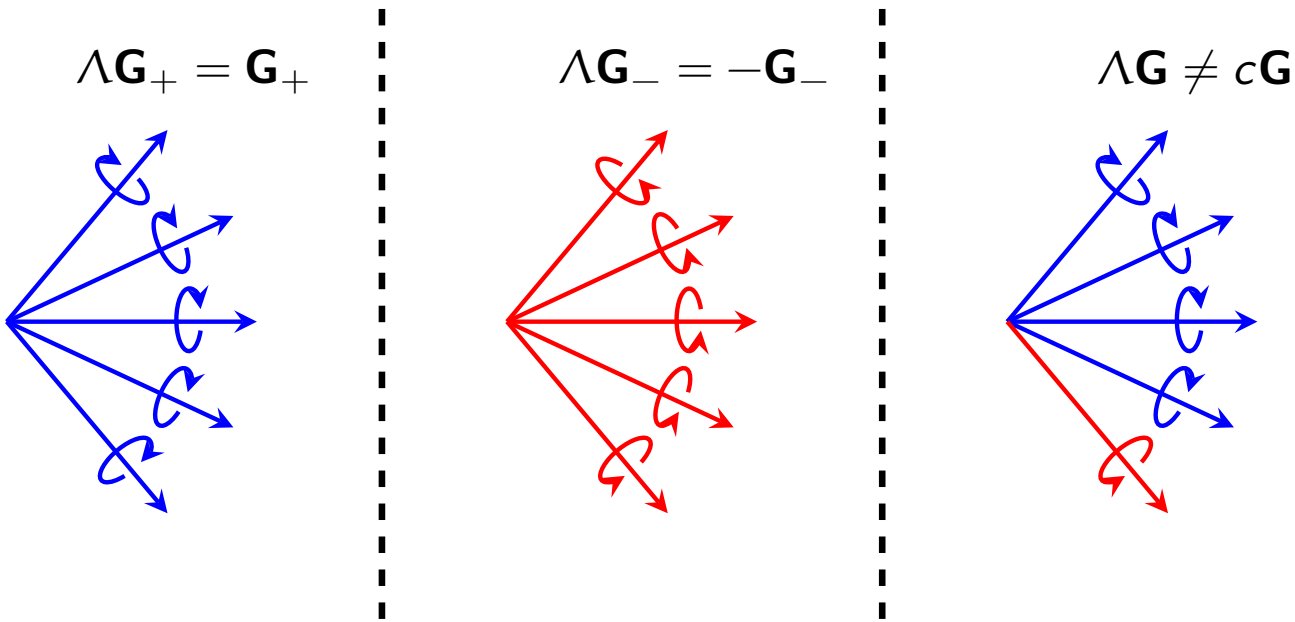
- Interpretation in the plane wave decomposition



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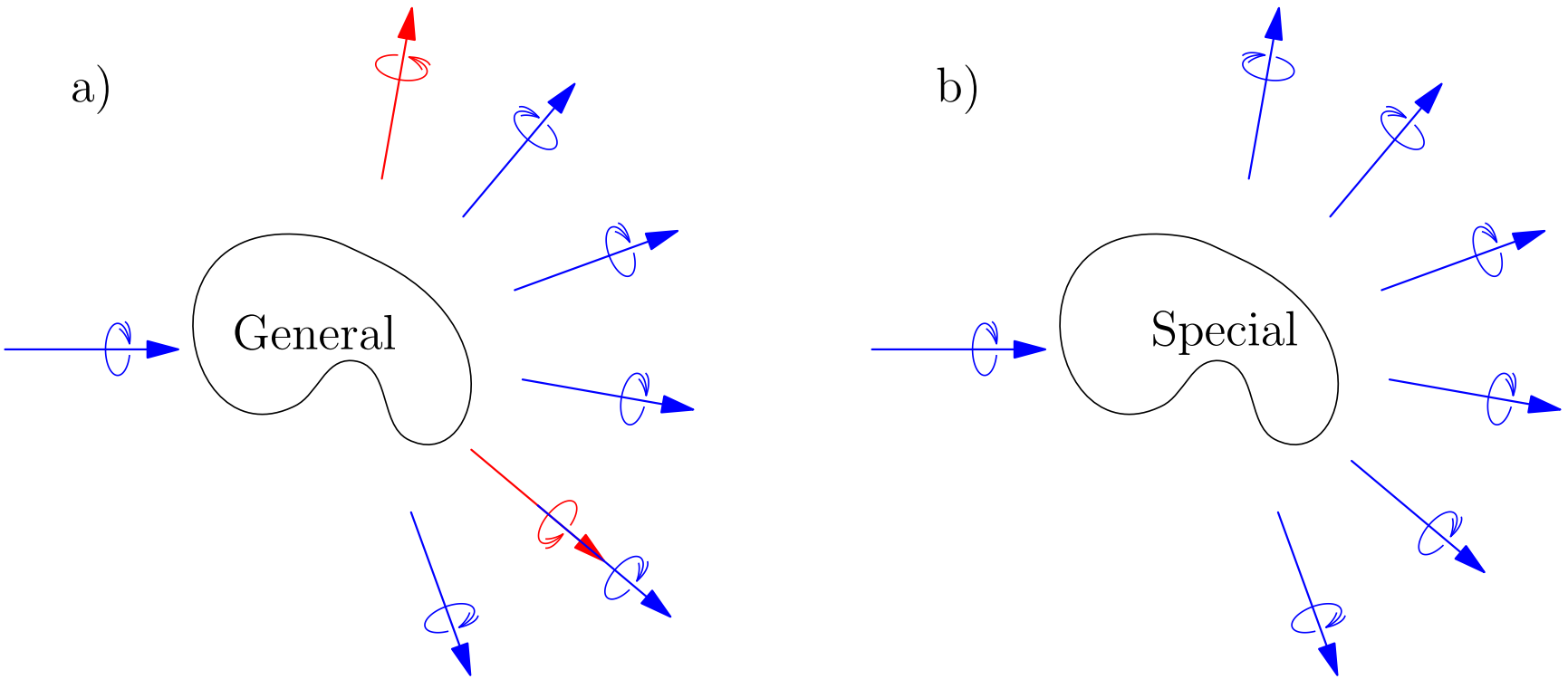
Helicity preserving vs. non-preserving



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Emulation of space time geometry $\Longrightarrow$ helicity preserving object

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Angular momentum generates rotations:

$R_z(\alpha) = \exp(-i\alpha J_z)$

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Emulation of space time geometry $\Longrightarrow$ helicity preserving object

How to fabricate helicity preserving objects ?

How to fabricate angular momentum preserving objects ?

We make them rotationally symmetric

Angular momentum generates rotations:

$R_z(\alpha) = \exp(-i\alpha J_z)$

Helicity generates duality transformations: $D(\theta) = \exp(-i\theta\Lambda)$:

$$\mathbf{E} \to \mathbf{E}_\theta = \mathbf{E}\cos\theta - \mathbf{H}\sin\theta,$$
$$\mathbf{H} \to \mathbf{H}_\theta = \mathbf{E}\sin\theta + \mathbf{H}\cos\theta,$$

Helicity conservation $\equiv$ duality symmetry

Helicity preservation

- Helicity is preserved in free-space propagation

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It turns out[6,7] that decoupled evolution equations for each helicity can be obtained in :

Macroscopic equations

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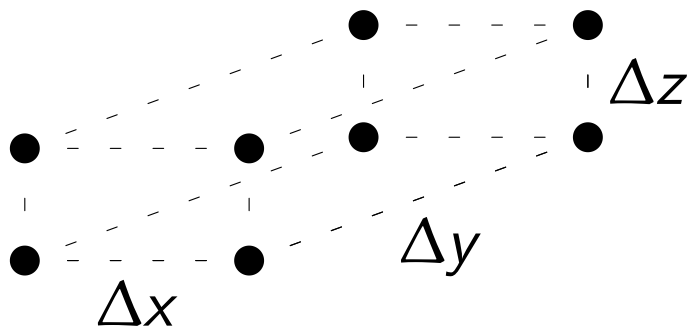
Helicity preserving meta-atoms

One (only ?) way of making an helicity preserving media is by using helicity preserving meta-atoms[8].



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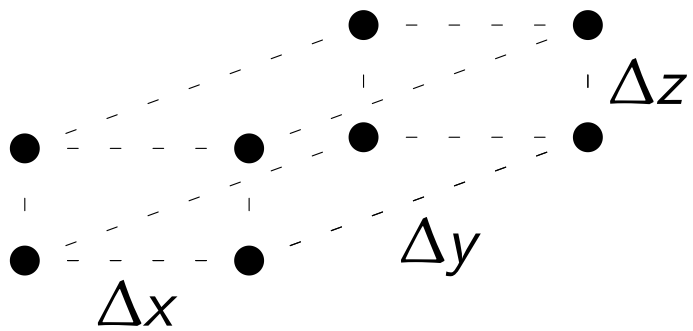

Do they exist ? Yes

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Helicity preserving meta-atoms

In the dipolar approximation

- Individual small dielectric spheres and
- Individual conducting helices

Can be engineered to be helicity preserving

Helicity preserving meta-atoms

In the dipolar approximation

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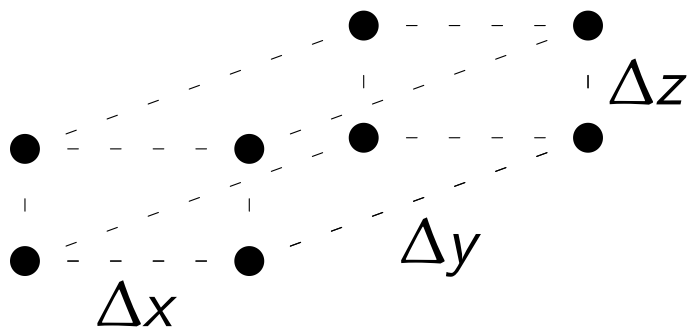
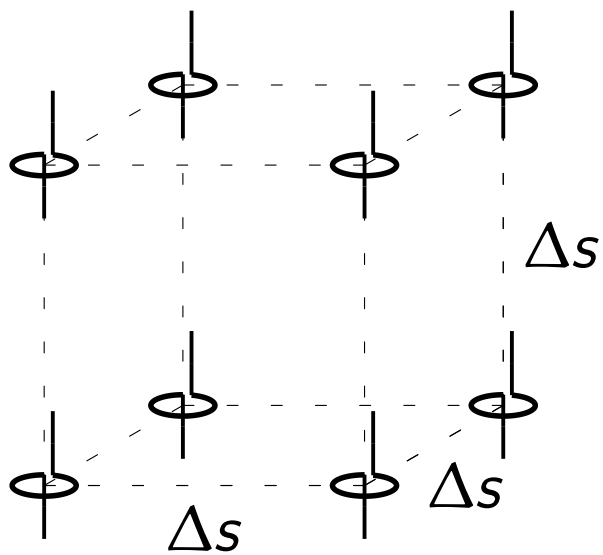
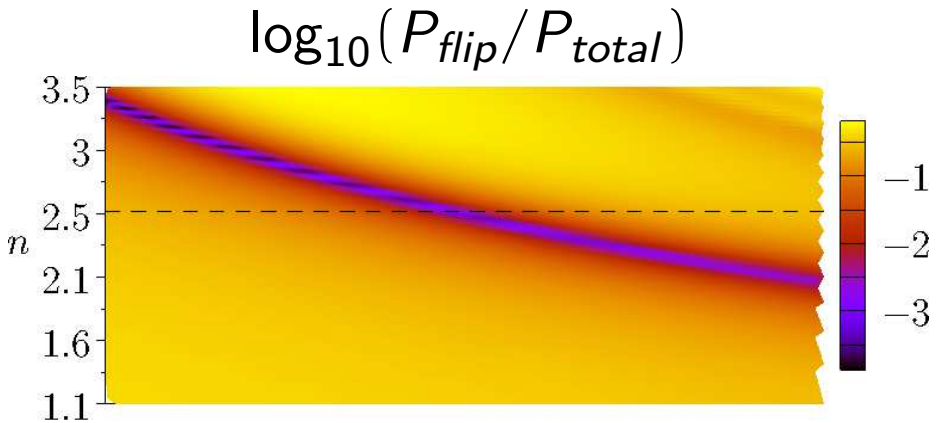
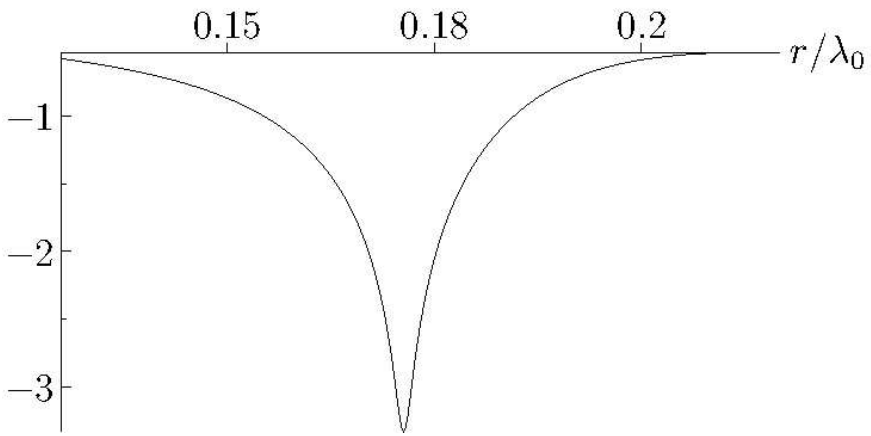
Can be engineered to be helicity preserving
Their collective response will be helicity preserving





Dielectric spheres[9]





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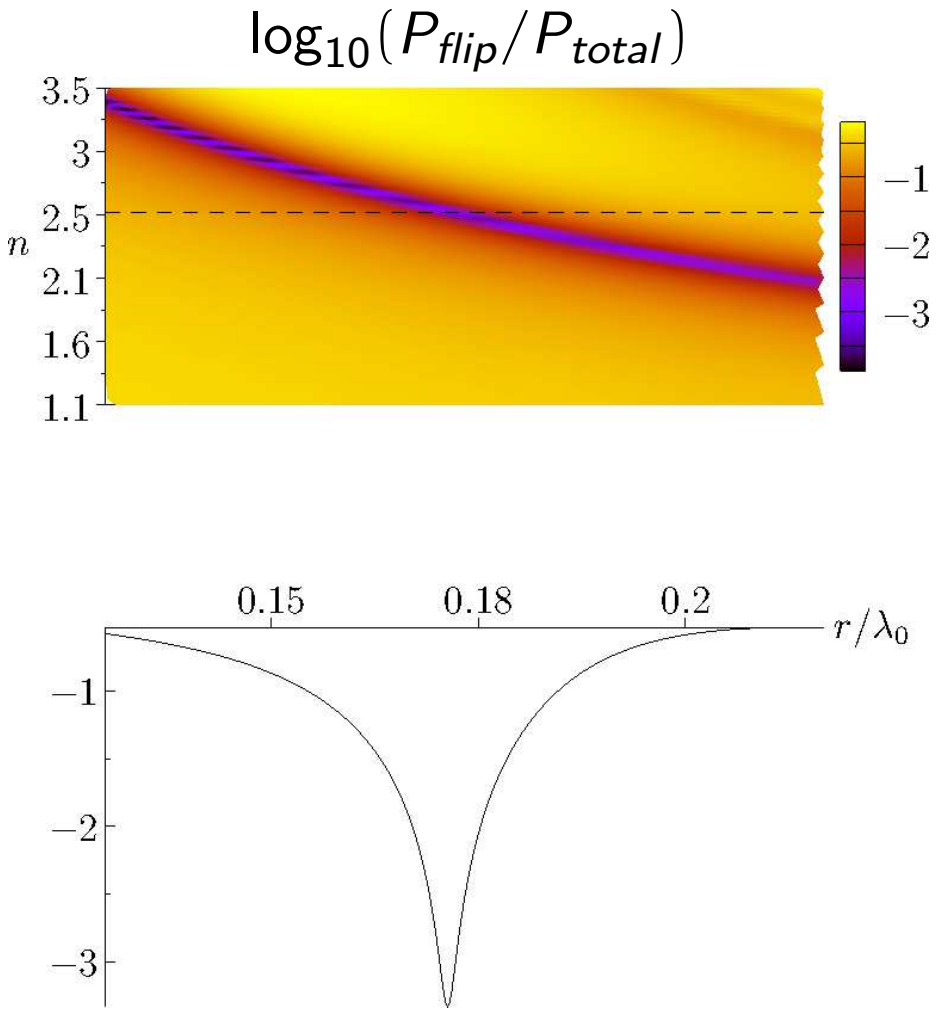

- $\lambda_0 = 780\text{nm}$,
- radius $r = 130\text{nm}$
- $n = \sqrt{\epsilon}$ =2.55
- $10^{-3.5}$ flip

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Design and fabrication of $M(g)$

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For the first one,

One piece of the puzzle

Helicity preserving meta-atoms: Restricted polarizability

Assume the challenges are solved

- Test relativistic effects in electromagnetism
  - Table top platform
  - Test bed for satellite based experiments[10]

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  - Perturb $M \implies$ Perturb g
- Emulate exotic g's
  - non-physical stress energy tensors
  - negative pressure (cosmology)

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Conclusions

- Metamaterials capable of emulating relativistic effects in EM
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- Challenges remain in design and fabrication
- Future work: Lots of it
- **Thank you for your time**

Helicity and duality in light matter interactions

Promote use of helicity/duality in light matter interactions
[11,12,13,14,15,16,17]

---

[11] I. Fernandez-Corbaton et al., "Helicity and angular momentum: a symmetry-based framework for the study of light-matter interactions", Physical Review A **86**, 042103 (2012).

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[13] I. Fernandez-Corbaton et al., "Necessary symmetry conditions for the rotation of light", The Journal of Chemical Physics **138**, 214311–214311–7 (2013).

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[15] X. Zambrana-Puyalto et al., "Duality symmetry and kerker conditions", Optics Letters **38**, 1857–1859 (2013).

[16] X. Zambrana-Puyalto et al., "Dual and anti-dual modes in dielectric spheres", Optics Express **21**, 17520–17530 (2013).

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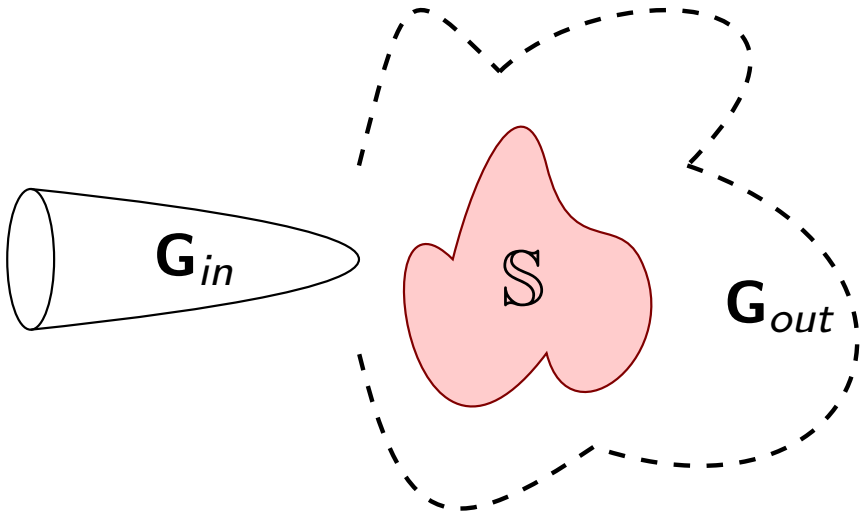
Emulation of gravitons with photons

A related but different topic

Can we emulate graviton scattering off classical massive objects using electromagnetic scattering ?



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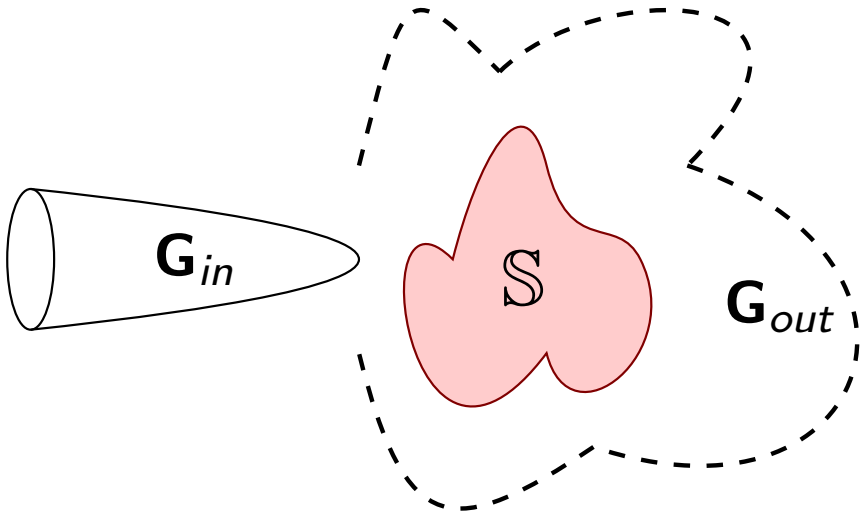

- Graviton: massless, spin-2, helicity ± 2
- Photon: massless, spin-1, helicity ± 1
  - Two entangled photons, helicity $\pm 2, 0$

Proposals for experiments using satellites

IOP PUBLISHING CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **29** (2012) 224011 (44pp) doi:10.1088/0264-9381/29/22/224011

Fundamental quantum optics experiments conceivable with satellites—reaching relativistic distances and velocities

David Rideout[1,2,3], Thomas Jennewein[2,4], Giovanni Amelino-Camelia[6], Tommaso F Demarie[7], Brendon L Higgins[2,4], Achim Kempf[2,3,4,5], Adrian Kent[3,8], Raymond Laflamme[2,3,4], Xian Ma[2,4], Robert B Mann[2,4], Eduardo Martín-Martínez[2,4,5], Nicolas C Menicucci[3,9], John Moffat[3], Christoph Simon[10], Rafael Sorkin[3], Lee Smolin[3] and Daniel R Terno[7]

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