Problems

1. The SM Higgs-potential has the form:

$$V(h) = \frac{1}{2} m^2 h^2 + \frac{1}{4} \lambda h^4$$

This leads to the tree-level expression: $m_h^2 = 2\lambda v^2$.

Show that if m_h were substantially larger than 126 GeV, then $\lambda(Q) \to \infty$ at a certain scale $Q = \Lambda_L$ (Landau pole).

Use the 1-loop RGE for λ:

$$16\pi^2 \frac{d\lambda}{d \ln Q} = 24\lambda^2 - 6\lambda_t^4 + \frac{9}{8} g^4 + \dots$$

keeping just the dominant term. How do you interpret this result?

2. The MSSM Higgs-potential has the form:

$$V(H_1, H_2) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \left(-m_{12}^2 H_1 \cdot H_2 + h.c.\right) + \lambda \left(|H_1|^2 - |H_2|^2\right)^2$$

where $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$, $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$,

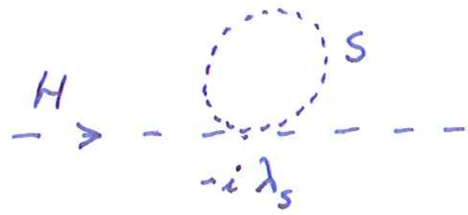
$$H_1 \cdot H_2 \equiv H_1^0 H_2^0 - H_1^- H_2^+ , \quad \lambda = \frac{1}{8}(g^2 + g'^2)$$

Show that the minimum of this potential does not break $U(1)_{e.m.}$

3. An interaction term between the Higgs field, H, and a complex scalar field, S,

$$\mathcal{L} \supset -\lambda_S |H|^2 |S|^2$$

induces quadratically (and logarithmically) divergent contributions to the Higgs mass from the diagram



This auto-energy reads: $\Sigma = -i\lambda_S \int \frac{d^4q}{(2\pi)^4} \frac{i}{q^2 - m_S^2 + i\epsilon}$

Compute Σ using a cut-off for the integral, and extract the contribution to m_H^2.

Hint: perform a Wick rotation to go to Euclidean space