UNITARITY ANALYSIS OF THE SCALAR SECTOR OF THE STANDARD MODEL

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Gauge symmetry imposes for vectorial bosons

- massless $\Longrightarrow$ 2 transversal polarizations $\rightarrow$ **2 d.o.f.**
- massive $\Longrightarrow$ $\left.\begin{array}{c}\text{2 transversal polarizations}\\ +\text{1 longitudinal polarization}\end{array}\right\}\rightarrow$ **3 d.o.f.**

but gauge symmetry also forces W^+, W^- and Z to be massless

$\Longrightarrow$ **3 new longitudinal degrees of freedom** are required.

$$\epsilon^\mu(\vec{k},1) = (0,1,0,0)\ , \qquad \epsilon^\mu(\vec{k},2) = (0,0,1,0)\ ,$$

$$\epsilon^\mu_L(\vec{k}) \equiv \epsilon^\mu(\vec{k},3) = \frac{1}{m}(|\vec{k}|,0,0,|k^0|) = \frac{k^\mu}{m} + \ldots$$

ϵ^μ_L grows with energy! $\Rightarrow$ Very sensitive to unitarity violation.

Unitarity is crucial for studying well-behaved theories.

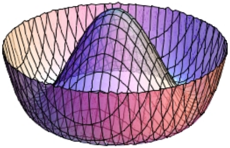
Higgs Mechanism + Spontaneous Symmetry Breaking explain longitudinal polarizations and **preserve gauge symmetry**.

Given $\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix}$, a doublet of complex scalar fields,

$$\mathscr{L}_S = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) \quad \text{and} \quad D^\mu \phi = [\partial^\mu + ig\, \tilde{W}_\mu + ig' y_\phi\, B^\mu]\, \phi\,,$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2\,,$$

$$\lambda > 0 \quad \mu^2 < 0$$



- Subset of minimum energy degenerate states that adquire a v.e.v..
- As we select the vacuum of the theory $\Rightarrow$ **The vacuum breaks the symmetry**.
- $SU(2)_L \otimes U(1)_Y \to U(1)_{QED}$

Goldstone Theorem: *there are as many massless Goldstone bosons as symmetry generators broken by the vacuum.*

The field $\phi(x)$ introduces **4 new degrees of freedom**.

Parametrization:

$$\phi(x) = \exp\left\{i\frac{\sigma_i}{2}\varphi_i(x)\right\}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+H(x)\end{pmatrix}$$

- 1 massive Higgs field,
- 3 massless Goldstone fields.

In the **unitary gauge** (gauge invariance):

$$(D_\mu\phi)^\dagger D^\mu\phi \xrightarrow{\varphi_i=0} \frac{1}{2}\partial_\mu H\,\partial^\mu H+(v+H)^2\left\{\frac{g^2}{4}W_\mu^\dagger W^\mu + \frac{g^2}{8\cos^2\theta_W}Z_\mu Z^\mu\right\}.$$

$W^\pm$ and Z **adquire mass**: 2 d.o.f.$\rightarrow$ 3 d.o.f..

Goldstone bosons are the longitudinal polatizations of these gauge bosons.

Unitarity theory construction. Gauge processes

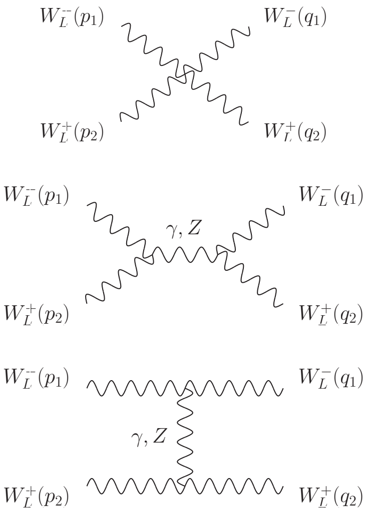
- $W_L^- + W_L^+ \to W_L^- + W_L^+$

$2 \to 2$ amplitudes can't grow with energy $\Longrightarrow$ Unitarity violation,

but any ϵ_L^μ implies an $\sqrt{s}/M_W$ extra term.

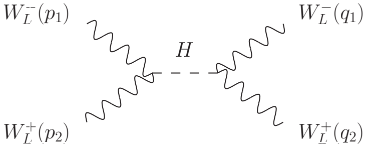
$$\mathcal{M}_0 = \frac{g^2}{8}\,(1+\cos\phi)\,\frac{s}{M_W^2}+\mathcal{O}(1)$$

Gauge symmetry cancellation is not enough.



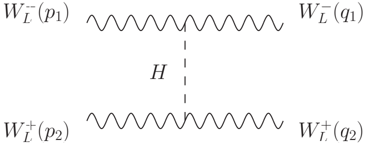
Gauge invariance also fixes the Higgs couplings.

$$\mathcal{M}_H = \mathcal{M}_{sH} + \mathcal{M}_{tH}$$
$$= -\frac{g^2}{8}(1+\cos\phi)\frac{s}{M_W^2}$$



Both contributions:

$$\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_H = \mathcal{O}(1)$$



Unitarity is restored!

Electroweak effective theory. Goldstone processes

We reexpress the scalar Lagrangian

$$\mathscr{L}_S = \frac{1}{2}\mathrm{Tr}[(D_\mu\Sigma)^\dagger D^\mu\Sigma] - \frac{1}{16}\lambda\left(\mathrm{Tr}[\Sigma^\dagger\Sigma] - v^2\right)^2 ,$$

$$\Sigma \equiv \begin{pmatrix} \phi^{(0)*} & \phi^{(+)} \\ -\phi^{(-)} & \phi^{(0)} \end{pmatrix} , \qquad D_\mu\Sigma \equiv \partial_\mu\Sigma - ig\tilde{W}_\mu\Sigma + \frac{i}{2}g' B_\mu\Sigma\,\sigma_3 .$$

If $g' \to 0$, chiral symmetry: $\mathbf{SU(2)_L} \otimes \mathbf{SU(2)_R} \to \mathbf{SU(2)_{L+R}}$

$$\Sigma(x) = \frac{1}{\sqrt{2}}[v+H(x)]\,\overbrace{\exp\left(i\frac{\sigma_i\varphi_i(x)}{v}\right)}^{U(\Phi(x))}, \qquad \Phi(x) = \begin{pmatrix} \varphi^0 & \sqrt{2}\,\varphi^- \\ \sqrt{2}\,\varphi^+ & -\varphi^0 \end{pmatrix} .$$

In the heavy Higgs limit $\Longrightarrow$ Generic Goldstone boson Lagrangian

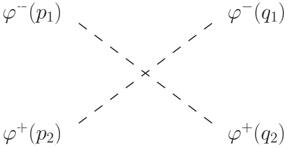
$$\mathscr{L}_0^{(2)} = \frac{v^2}{4}\mathrm{Tr}[(D_\mu U)(D^\mu U)^\dagger]$$

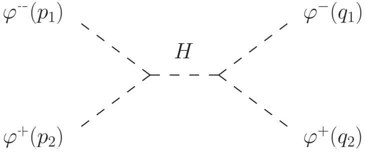
In the unitary gauge, $\mathscr{L}_0^{(2)} \xrightarrow{U=1} M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$,

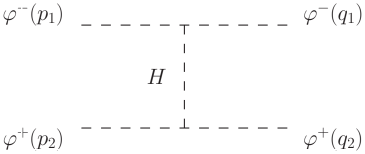
$W^\pm$ and Z adquire mass but **the Higgs doesn't generate them**.

Is the Higgs Mechanism really necessary?

- $\varphi^- + \varphi^+ \to \varphi^- + \varphi^+$







Unitarity requires the Higgs.

$$\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_H = \mathcal{O}(1)$$

Equivalence Theorem:

$$\mathcal{M}\{W_L^a(p_1), W_L^b(p_2), \ldots\} = \mathcal{M}\{\varphi^a(p_1), \varphi^b(p_2), \ldots\} + \mathcal{O}\left(\frac{M_W}{\sqrt{s}}\right)$$

Unitarity analysis

Goldstone bosons are invariant under $SU(2)_{L+R}$
$\Longrightarrow$ **weak isospin** amplitude decomposition ($I = 0, 1, 2$).

An additional expansion in **partial waves** (t_L^I) is performed

$$\sigma_L^I(p_1 p_2 \to q_1 q_2) = \frac{1}{64\pi^2 s} \frac{|\vec{q}_{cm}|}{|\vec{p}_{cm}|} \frac{2L+1}{4\pi} \left| t_L^I(s) \right|^2 \ .$$

Unitarity restriction $\Longrightarrow \quad (\sigma_T)_L^I \leq \frac{4\pi}{|\vec{p}_{cm}|^2}(2L+1)$.

Unitarity bounds for Higgsless SM:

- t_0^0 is the most constraining partial wave.
- $\sqrt{s_{\mathrm{th}}} = \sqrt{8\pi}\, v = 1,23$ TeV

Higgsless SM + 126 GeV scalar coupling $S_1(x)$

$$\mathscr{L}_{S_1} = \frac{1}{2}(D_\mu S_1)^2 + \frac{v^2}{4}\mathrm{Tr}[(D_\mu U)(D^\mu U)^\dagger]\left(1 + \frac{2\omega}{v}S_1\right)$$

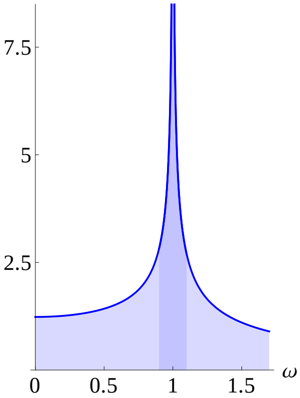
$$\mathcal{M} = \frac{s+t}{v^2}(1-\omega^2) + \mathcal{O}(1)\ .$$

$$\sqrt{s_{\mathrm{th}}}\ [\mathrm{TeV}] = \sqrt{\frac{8\pi}{|1-\omega^2|}}\ v$$

- $\omega = 1$ restores the SM and perturbative unitarity.
- $I = 0$ channel sets the unitarity bound.
- Experimentally: $|1-\omega| < 0{,}1$

$$\sqrt{s} \leq 2{,}69\ \mathrm{TeV}\ .$$

- Higgs couplings are **very sensitive to new physics**.



Conclusions

- Higgs Mechanism and SSB introduce 3 Goldstone bosons which generate the longitudinal polarizations of $W^{\pm}$ and Z.
- A study of dynamical gauge boson scattering is performed. The Higgs boson restores unitarity in the SM.
- Effective field theory allow us to make an equivalent Goldstone boson analysis.
  Unitarity conservation is essencial in order to study fundamental theories.
- The current Higgs couplings to the SM postpone unitarity bounds to higher energies.