Quantum Simulation of (non-)abelian gauge theories

Enrique Rico Ortega

13 February 2013

The team

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M. Bögli


P. Stebler


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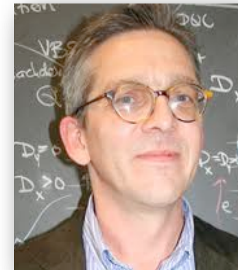
U.-J. Wiese

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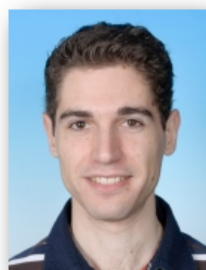
M. Müller

- Technical University Vienna


P. Rabl

- IQOQI - Innsbruck University


M. Dalmonte


D. Marcos


P. Zoller

The references

Atomic Quantum Simulation of Dynamical Gauge Fields Coupled to Fermionic Matter: From String breaking to Evolution after a Quench
Phys. Rev. Lett. 109, 175302 (2012)

Atomic Quantum Simulation of U(N) and SU(N) non-abelian Gauge Theories
arXiv:1211.2242 (2012)

Quantum Simulation of Dynamical Lattice Gauge Field Theories with Superconducting Q-bits

Recent related works on dynamical gauge fields

H. Büchler, M. Hermele, S. Huber, M.P.A. Fisher, P. Zoller, Atomic quantum simulator for lattice gauge theories and ring exchange models, Phys. Rev. Lett. 95, 40402 (2005).

H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, H.P. Büchler, A [digital] open system Rydberg quantum simulator, Nat. Phys. 6, 382 (2010).

J.I. Cirac, P. Maraner, J.K. Pachos, Cold Atom Simulation of Interacting Relativistic Quantum Field Theories, Phys. Rev. Lett. 105, 190403 (2010).

E. Kapit, E. Mueller, Optical-lattice Hamiltonians for relativistic quantum electrodynamics, Phys. Rev. A 83, (2011).

E. Zohar, J.I. Cirac, B. Reznik, Simulating Compact Quantum Electrodynamics with ultracold atoms: Probing confinement and nonperturbative effects, Phys. Rev. Lett. 109, 125302 (2012).

L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Optical Abelian Lattice Gauge Theories, Ann. Phys. 330, 160-191 (2013).

Barcelona, Bern, Innsbruck, Leeds, Madrid, Munich, New York, Stuttgart, Tel Aviv, ...

... and on dynamical non-abelian gauge fields

E. Zohar, J.I. Cirac, B. Reznik, A cold-atom quantum simulator for SU(2) Yang-Mills lattice gauge theory, arXiv:1211.2241 (2012).

L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Simulations of non-Abelian gauge theories with optical lattices, arXiv:1211.2704 (2012).

Barcelona, Bern, Innsbruck, Leeds, Madrid, Munich, New York, Stuttgart, Tel Aviv, ...

Why quantum simulate Gauge theories?

The study of <u>Gauge</u> theories is the study of <u>Nature</u>.

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Gauge symmetry as a fundamental principle

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Gauge symmetry as an emergent phenomenon

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Gauge symmetry as a fundamental principle

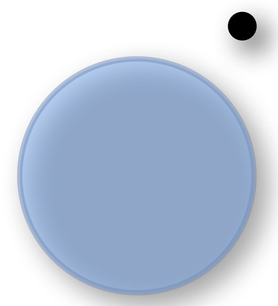
Gauge symmetry as an emergent phenomenon

Gauge symmetry as a resource

Gauge symmetry as a fundamental principle

Standard model: for every force there is a gauge boson,

Gauge symmetry as a fundamental principle



atoms

Standard model: for every force there is a gauge boson,

- The photon is the "carrier" of the electromagnetic force.

Gauge symmetry as a fundamental principle

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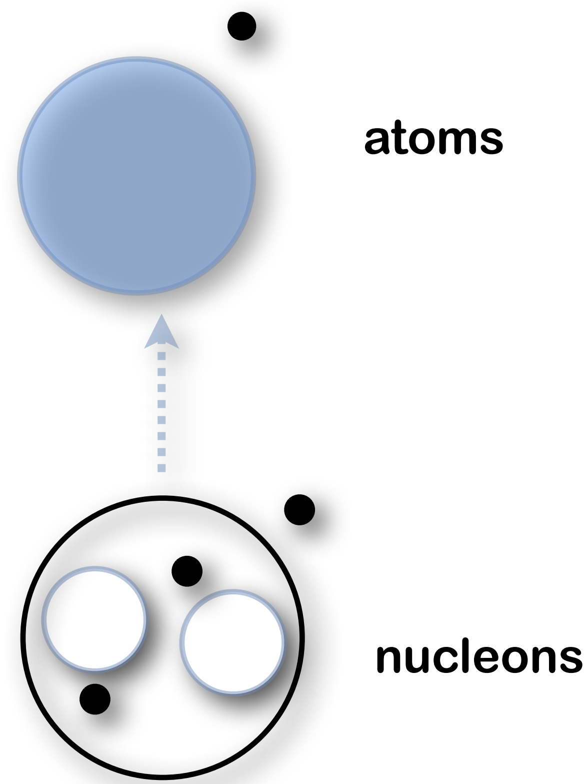
- The photon is the "carrier" of the electromagnetic force.

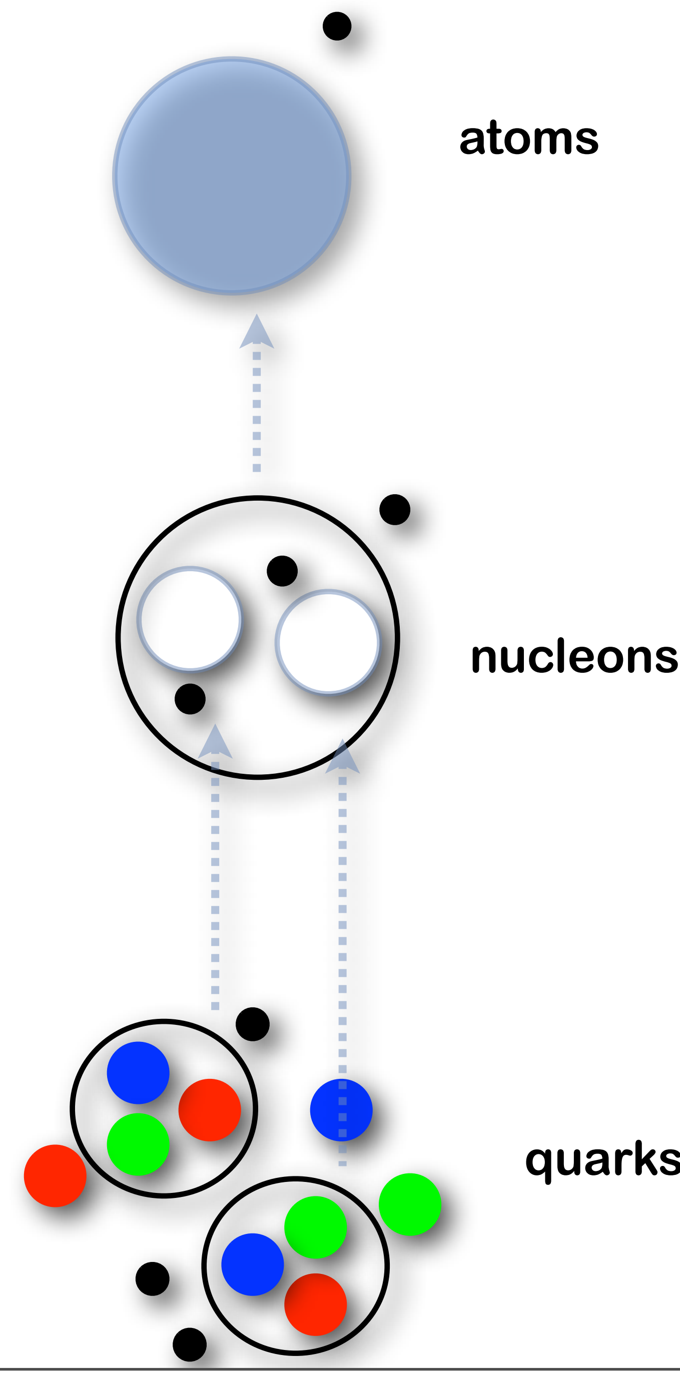
- The W^+, W^- and Z^0 are the "carriers" of the weak force.



Gauge symmetry as a fundamental principle

Standard model: for every force there is a gauge boson,

- The photon is the "carrier" of the electromagnetic force.
- The W^+, W^- and Z^0 are the "carriers" of the weak force.
- The gluons are the "carriers" of the strong force.

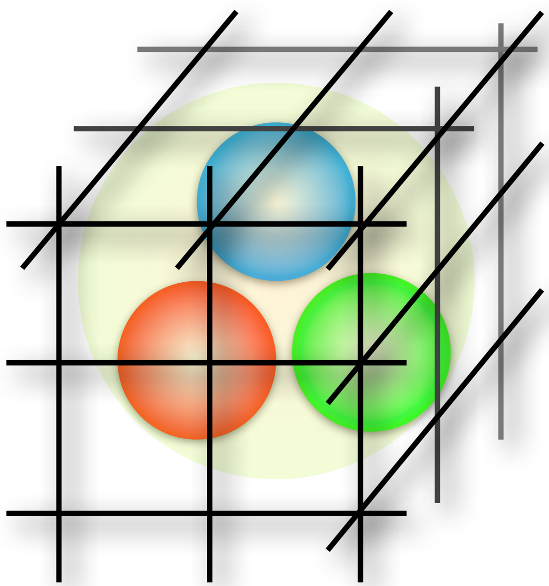


Gauge symmetry as a fundamental principle

Gauge theories on a discrete lattice structure.

Non-perturbative approach to fundamental theories of matter, e.g. Q.C.D.

K. Wilson, Phys. Rev. D (1974)



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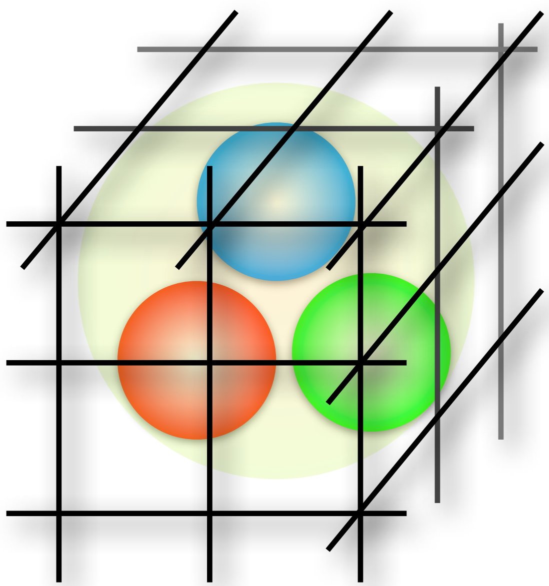
$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}\left[\psi, U\right] e^{-S[\psi,U]} O\left[\psi, U\right]$$



$$\sim \frac{1}{N} \sum_{n=1}^{N} e^{-S[\psi_n,U_n]} O\left[\psi_n, U_n\right]$$

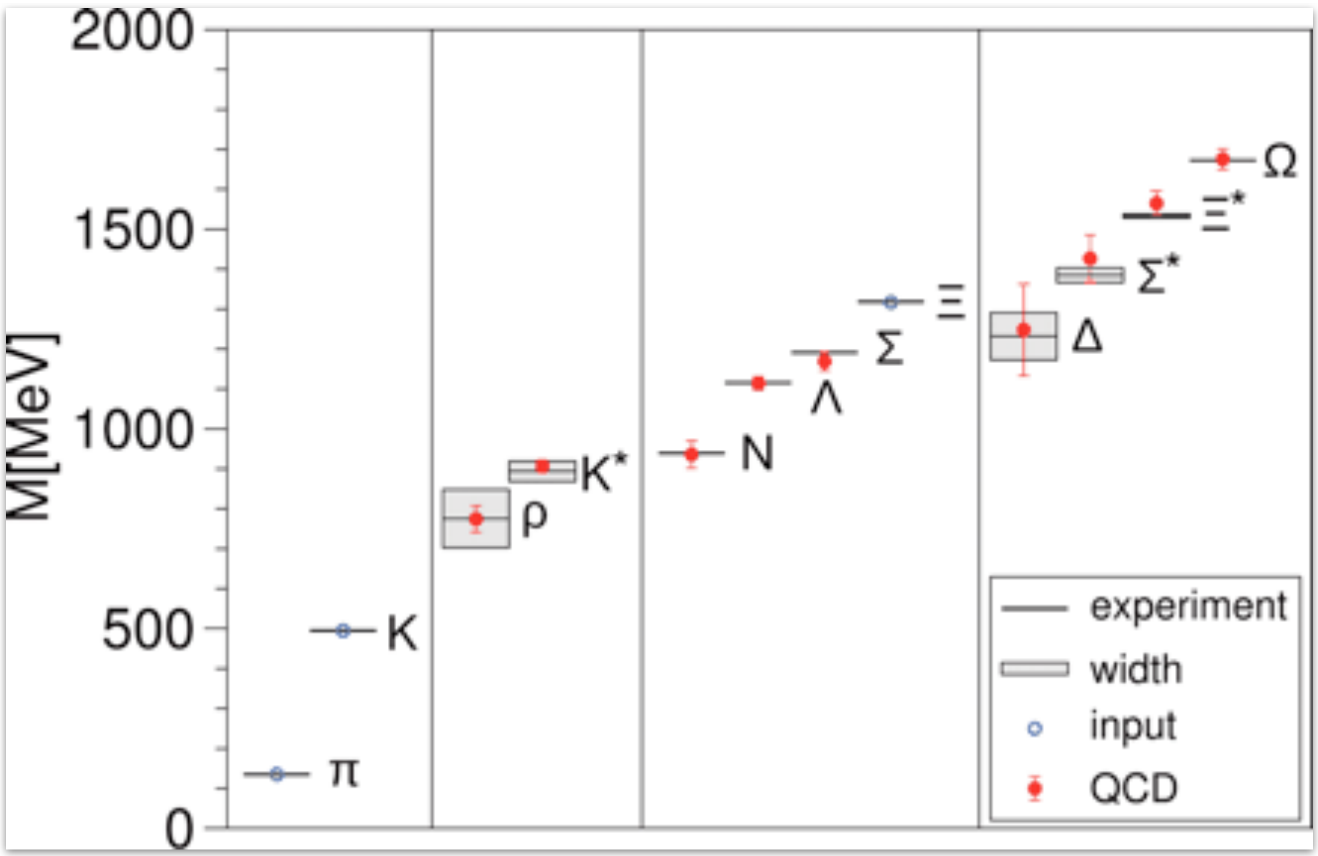
$$\sim \frac{1}{N} \sum_{P[U_n] \propto e^{-S[\psi_n,U_n]}} O\left[\psi_n, U_n\right]$$

Monte Carlo simulation = Classical Statistical Mechanics

Gauge symmetry as a fundamental principle

Achievements by classical Monte-Carlo simulations:

- first evidence of quark-gluon plasma
- ab-initio estimate of the entire hadronic spectrum



S. Dürr, et al., Science (2008)

Gauge symmetry as an emergent phenomenon

Gauge bosons can appear as collective fluctuations of a strongly correlated many-body quantum system

F. Wegner, J. Math. Phys. (1971)
J.B. Kogut, Rev. Mod. Phys. (1979)
A. Kitaev, Ann. Phys. (2003)
P.A. Lee, N. Nagaosa, X.G. Wen, Rev. Mod. Phys. (2006)

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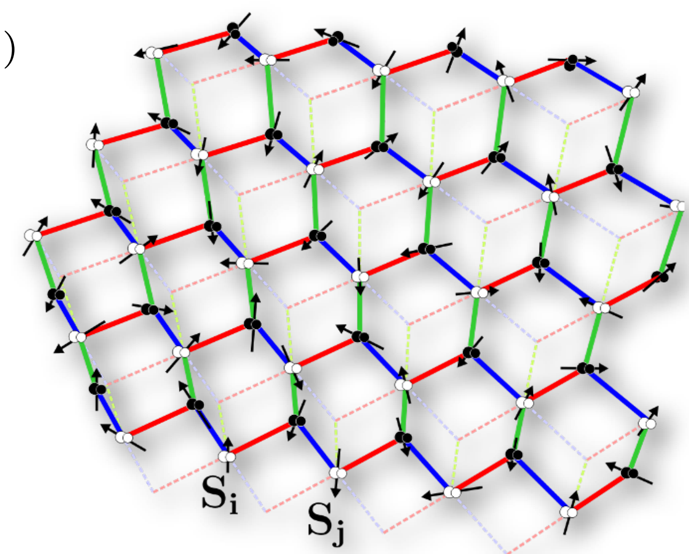
Ex.- Kitaev model (Z_2 gauge theory)

$$H = J_1 \sum_{1\langle i,j\rangle} S_i^{(1)} S_j^{(1)} + J_2 \sum_{2\langle i,j\rangle} S_i^{(2)} S_j^{(2)} + J_3 \sum_{3\langle i,j\rangle} S_i^{(3)} S_j^{(3)}$$

$$\rightarrow \frac{J_1^2 J_2^2}{16|J_3|^3}\left(\sum_{\text{vertex}} XXXX + \sum_{\text{plaq}} ZZZZ\right)$$

$$|J_3| \gg \{|J_1|, |J_2|\}$$



Gauge symmetry as an emergent phenomenon

Some questions:

(i) fractionalization,

(ii) confinement-deconfinement quantum phase transition,

(iii) spin-liquid physics...

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Volume 94B, number 2 PHYSICS LETTERS 28 July 1980

DYNAMICAL STABILITY OF LOCAL GAUGE SYMMETRY

Creation of Light From Chaos

D. FOERSTER H.B. NIELSEN M. NINOMIYA

And God said "Let there be light", and there was light – Genesis 1–3

We show that the large distance behavior of gauge theories is stable, within certain limits, with respect to addition of gauge noninvariant interactions at small distances.

PHYSICAL REVIEW B **68**, 115413 (2003)

Artificial light and quantum order in systems of screened dipoles

Xiao-Gang Wen*

Gauge symmetry as a resource

Topological quantum computation:
Deconfined phases of gauge models may have excitations with non-abelian statistics and degenerate ground states.

A. Kitaev, Ann. Phys. (2003)
M.H. Freedman, A. Kitaev, M.J. Larsen, Z. Wang, Bull. Amer. Math. Soc. (2003)
C. Nayak, S.H. Simon, A. Stern, M. Freedman, S. Das Sarma, Rev. Mod. Phys. (2008)

Some questions:

(i) new materials,
(ii) how to create and manipulate quasi-particles.

Why quantum simulate Gauge theories?

Problems not solvable on a classical machines

Various flavors of sign problems in strongly correlated systems

Why quantum simulate Gauge theories?

Real time evolution:

Heavy ion experiments (collisions)

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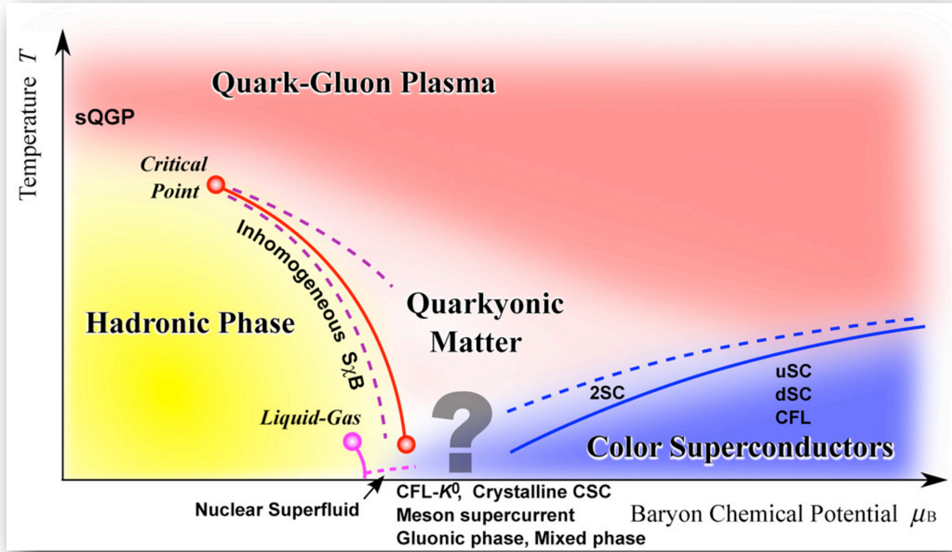
Real time evolution:
Heavy ion experiments (collisions)

QCD with finite density of fermions:
Dense nuclear matter, color superconductivity (phase diagram of QCD)



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M.G. Alford, A. Schmitt, K. Rajagopal, T. Schäfer, Rev. Mod. Phys. (2008)
K. Fukushima, T. Hatsuda, Rep. Prog. Phys. (2011)

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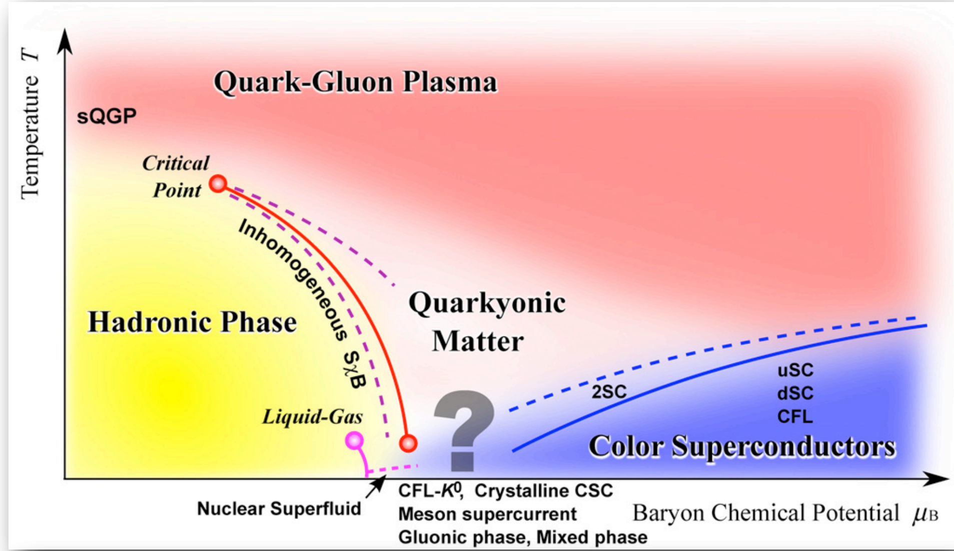
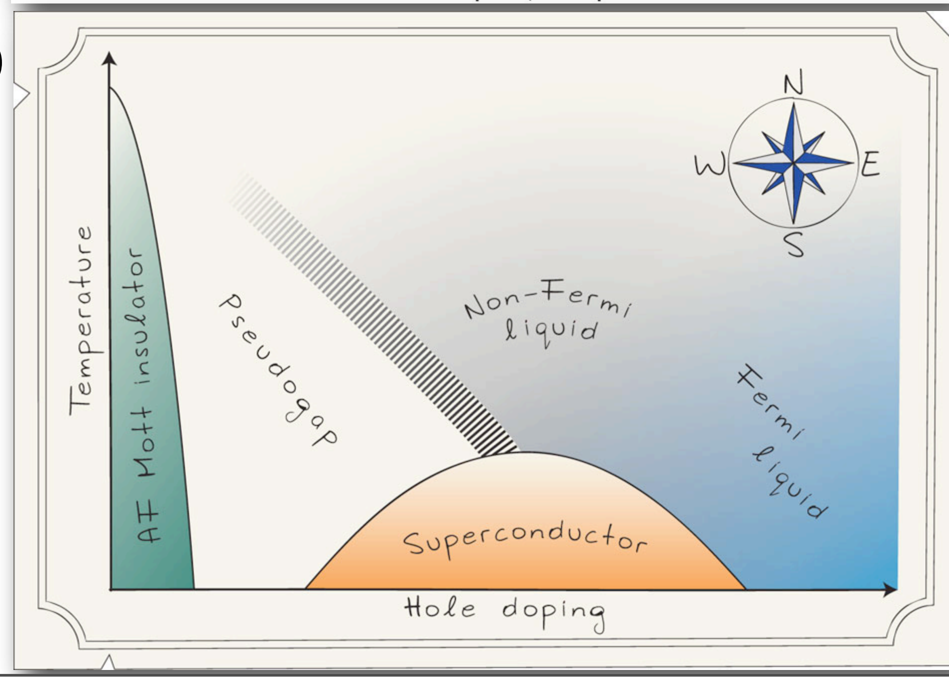


Frustrated spin models:
Spin liquid physics, RVB states (High Tc superconductivity?)

E. Dagotto, Science (2005)
M.R. Norman, D. Pines, C. Kallinl, Adv. Phys. (2005)
P. Wahl, Nat. Phys. (2012)



Why quantum simulate Gauge theories?

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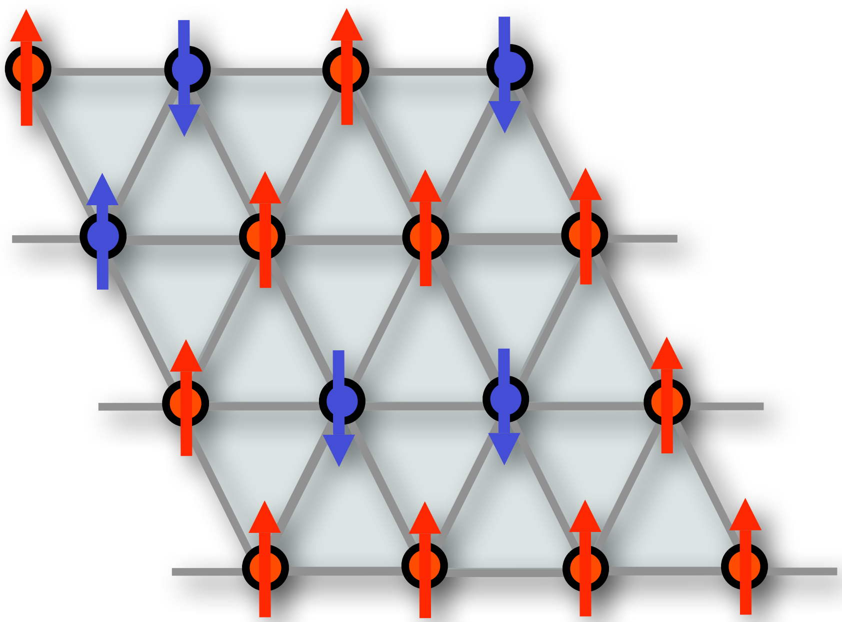
Feynman: "It is difficult to simulate quantum physics on a classical computer"



R.P. Feynman, Int. J. Theor. Phys. (1982)

Entanglement

$$|\psi\rangle = c_1|\uparrow\uparrow\cdots\uparrow\rangle + c_2|\uparrow\uparrow\cdots\downarrow\rangle + \cdots + c_{2^N}|\downarrow\downarrow\cdots\downarrow\rangle$$

Huge

Why quantum simulate Gauge theories?

Feynman's universal quantum simulator: controlled quantum device which efficiently reproduces the dynamics of any other many-particle quantum system.

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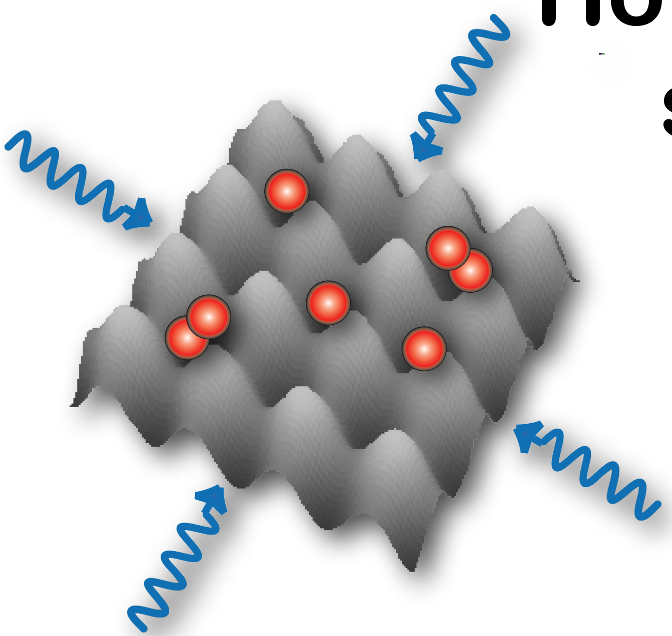
<u>Feynman's universal quantum simulator</u>: controlled quantum device which efficiently reproduces the dynamics of any other many-particle quantum system.

How?... cold atoms, ions, photons, superconducting circuit, etc.



J.I. Cirac, P. Zoller
I. Bloch, J. Dalibard, S. Nascimbène
R. Blatt, C.F. Roos,
A. Aspuru-Guzik, P. Walther
A.A. Hock, H.E. Türeci, J. Koch
Nature Physics Insight -
Quantum Simulation (2012)

Why quantum simulate Gauge theories?

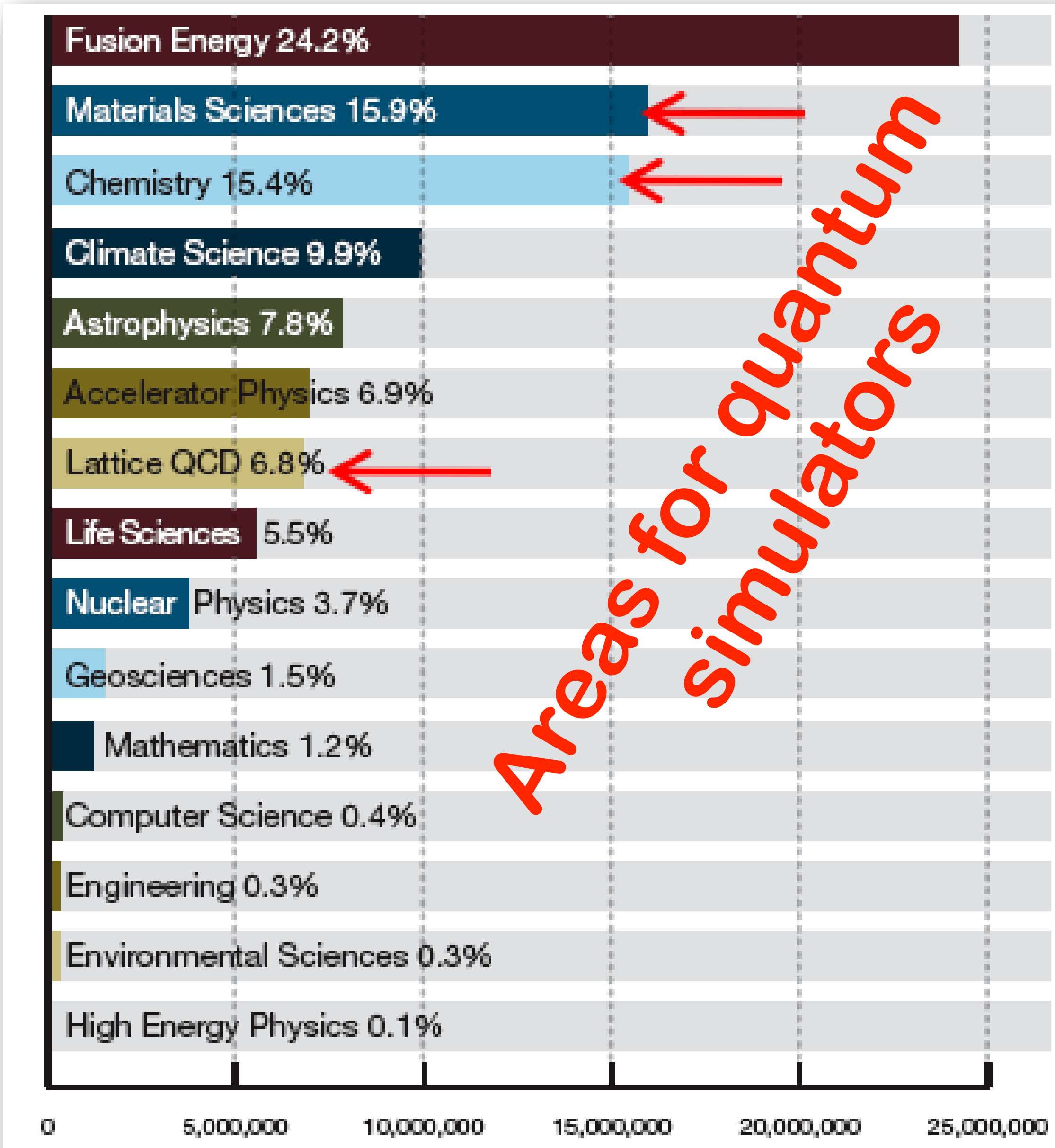
NEED

Design a controlled microscopic quantum simulator for lattice gauge theories.

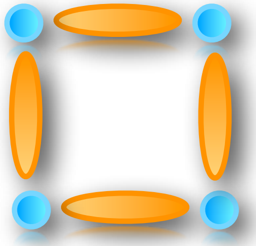
AIM

Investigate relevant phenomena, e.g., characterize the phase diagram and dynamics of strongly coupled lattice gauge models.

Why quantum simulate Gauge theories?



Use of DoE supercomputers by area
(from a talk by Alán Aspuru-Guzik)

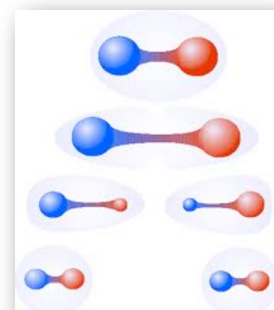


Contents

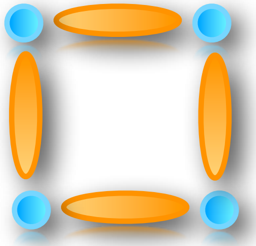
ψ_x : fermion

$U_{x,x+1}$: boson

- **Hamiltonian formulation of lattice gauge theories.** **[degrees of freedom, symmetry generators, dynamics]**



String breaking



Lattice gauge theory

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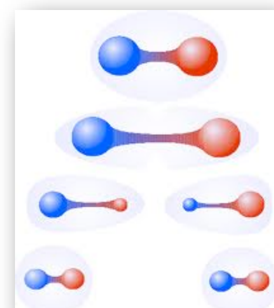

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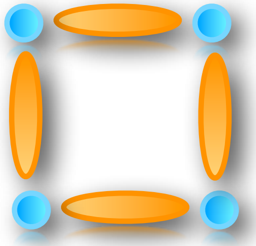

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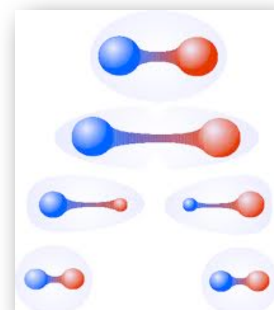

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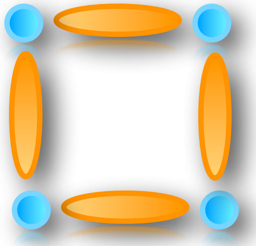

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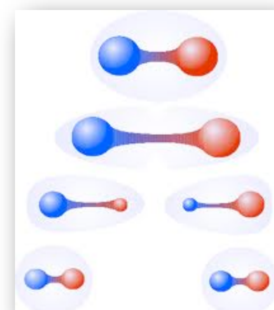

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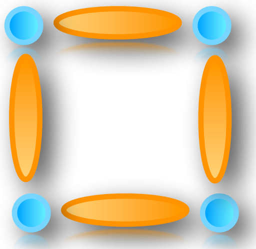

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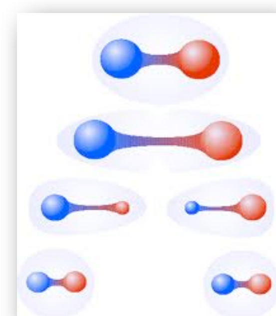
- **Observability of interesting phenomena**



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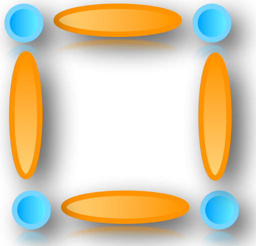

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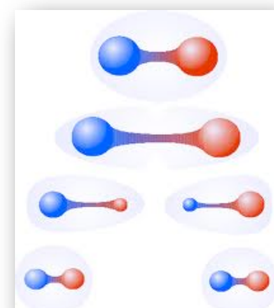
- **Observability of interesting phenomena**
- **Conclusions & Outlook**



String breaking

Hamiltonian formulation of lattice gauge theories

A gauge invariant model is defined by:

Set of local dynamical operators acting on the vertices (matter fields) and on the links (gauge fields)

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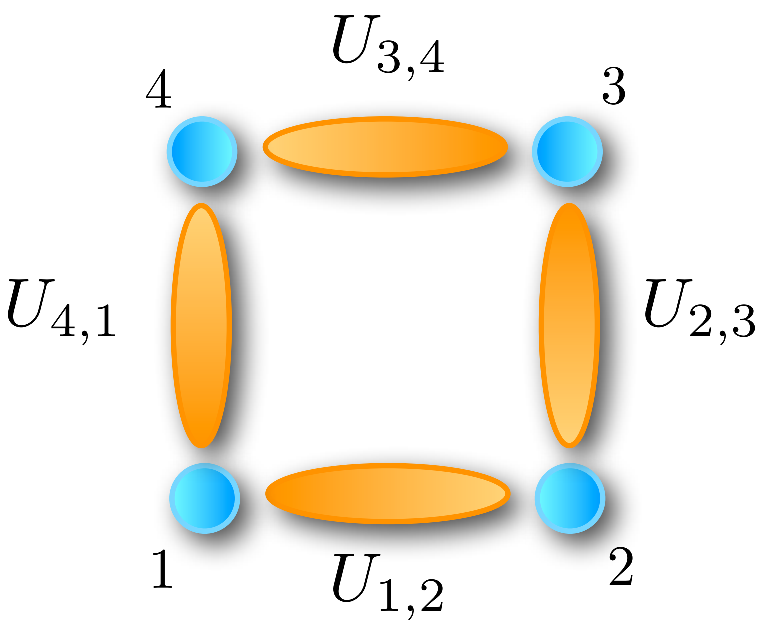
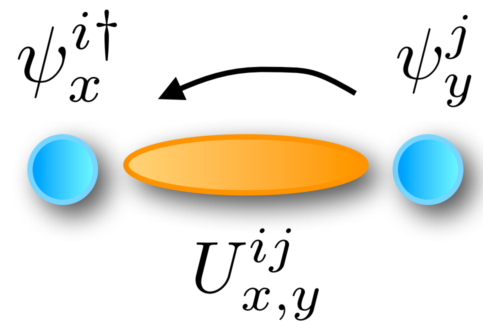
Set of local dynamical operators acting on the vertices (matter fields) and on the links (gauge fields)





(operator acting on a Hilbert space)

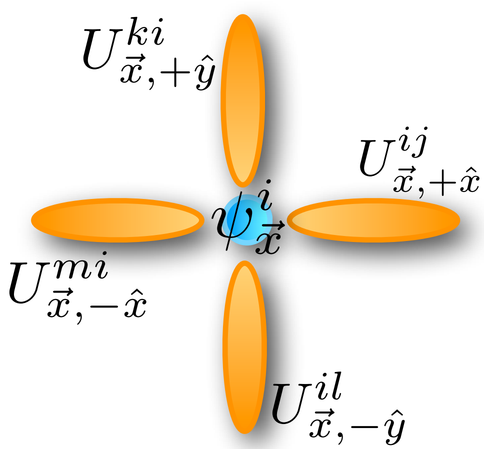
J.B. Kogut, L. Susskind, PRD (1975)

ref. Creutz and Montvay/Muenster books
J.B. Kogut, Rev. Mod. Phys. (1979)

$$i = \begin{cases} 1 & : U(1) \\ \uparrow\downarrow & : U(2) \\ brg & : U(3) \end{cases}$$

Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations



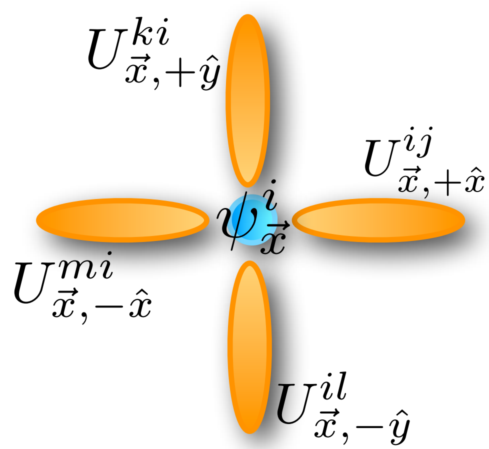
Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations

Generators of the local symmetry:

$$e^{i\sum_z \vec{\theta}_z \vec{G}_z} \psi_x^i e^{-i\sum_z \vec{\theta}_z \vec{G}_z} = \sum_j \Omega_x^{ij} \psi_x^j$$

$$e^{i\sum_z \vec{\theta}_z \vec{G}_z} U_{x,y}^{ij} e^{-i\sum_z \vec{\theta}_z \vec{G}_z} = \sum_{k,l} \Omega_x^{ik} U_{x,y}^{kl} \Omega_y^{jl*}$$



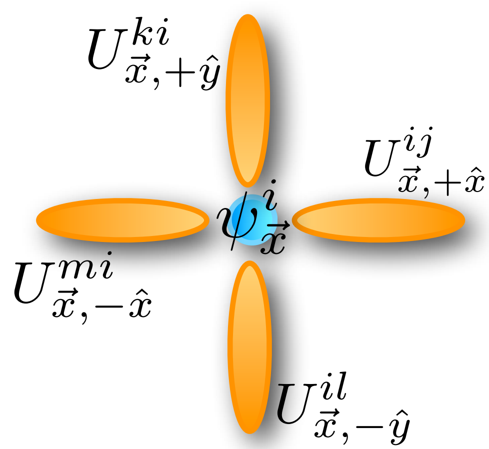
Hamiltonian formulation of lattice gauge theories

Set of local generators of gauge transformations

Define the Hilbert space:

$$\vec{G}_x|\text{physical}\rangle = 0$$

$$\sum_x \left(\vec{G}_x\right)^2 = \begin{pmatrix} \begin{bmatrix} 0 \end{bmatrix} & & \\ & \begin{bmatrix} 1 \end{bmatrix} & \\ & & \ddots \end{pmatrix}$$



Block-diagonal Hilbert space

Hamiltonian formulation of lattice gauge theories

ex.- U(1) group

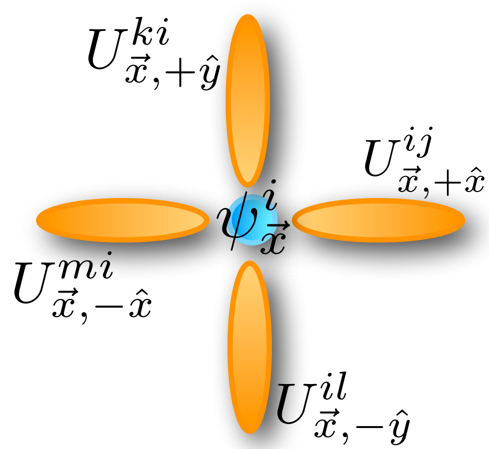
Set of local generators of gauge transformations

$$G_x = \psi_x^\dagger \psi_x - \sum_{\hat{\mu}} E_{x,x+\hat{\mu}} - E_{x-\hat{\mu},x}$$

matter electric field

$$\left[\rho - \vec{\nabla} \cdot \vec{E}\right]_{\text{phys}} = 0$$

: Gauss' law



Hamiltonian formulation of lattice gauge theories

Gauge invariant quantum Hamiltonian:

$$\left[H, \vec{G}\right] = 0 \quad \forall x$$

Local conserved quantities
Gauge (local) symmetries

Wilson formulation:
continuum valued operator, infinite-dimensional local Hilbert space

ex.- U(1) group

$$U_{x,y} \rightarrow e^{i\phi_{x,y}} \qquad\qquad E_{x,y} \rightarrow -i\frac{\partial}{\partial\phi_{x,y}}$$

Implementation in AMO setup very challenging

E. Kapit, E. Mueller, Phys. Rev. A (2011)
E. Zohar, B. Reznik, Phys. Rev. Lett. (2011)

Quantum link formulation:

gauge fields span a finite-dimensional local Hilbert space

D. Horn, Phys. Lett. B (1981)
P. Orland, D. Röhrlich, Nucl. Phys. B (1990)
S. Chandrasekharan, U.-J. Wiese, Nucl. Phys. B (1997)

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S. Chandrasekharan, U.-J. Wiese, Nucl. Phys. B (1997)

Q.C.D. can be formulated as a non-abelian quantum link model

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Quantum Link models

Connections with Quantum Information
(Z_2 gauge theory-Kitaev model)

H. Weimar, M. Müller, I. Lesanovsky, P. Zoller, H.P. Büchler, Nat. Phys. (2010)

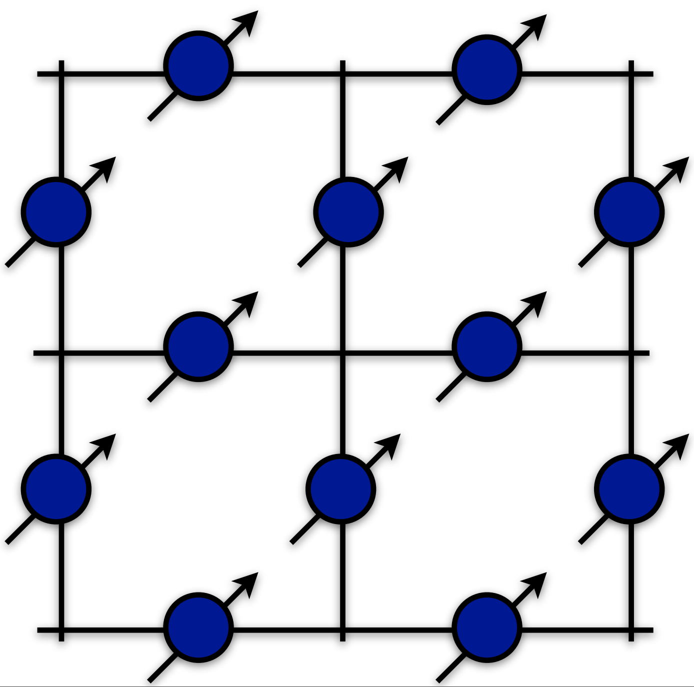
Quantum Link models

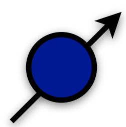
Connections with Quantum Information (Z$_2$ gauge theory-Kitaev model)

H. Weimar, M. Müller, I. Lesanovsky, P. Zoller, H.P. Büchler, Nat. Phys. (2010)

Local degrees of freedom.-

Quantum two level system living on the link





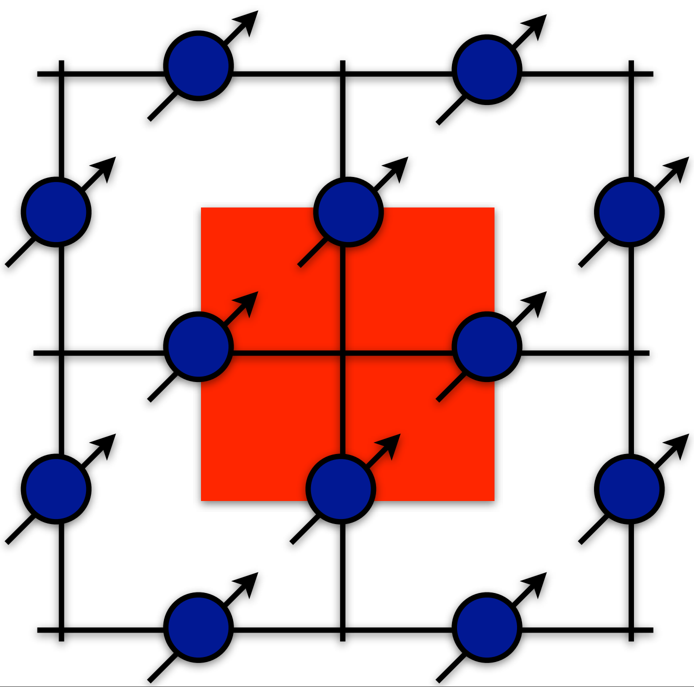
$$\{\sigma^{(3)}_{x,y}, \sigma^{(1)}_{x,y}\}$$

Quantum Link models

Connections with Quantum Information
(Z_2 gauge theory-Kitaev model)

Local generator of gauge transformations.-

Local unitary transformation around every vertex



$$G_{\rm vert} = \sigma^{(1)}_{1,2}\sigma^{(1)}_{2,3}\sigma^{(1)}_{3,4}\sigma^{(1)}_{4,1}$$

$$G_{\rm vert}\ \sigma^{(3)}_{1,2}\ G_{\rm vert} = -\sigma^{(3)}_{1,2}$$

Z_2 gauge transformation

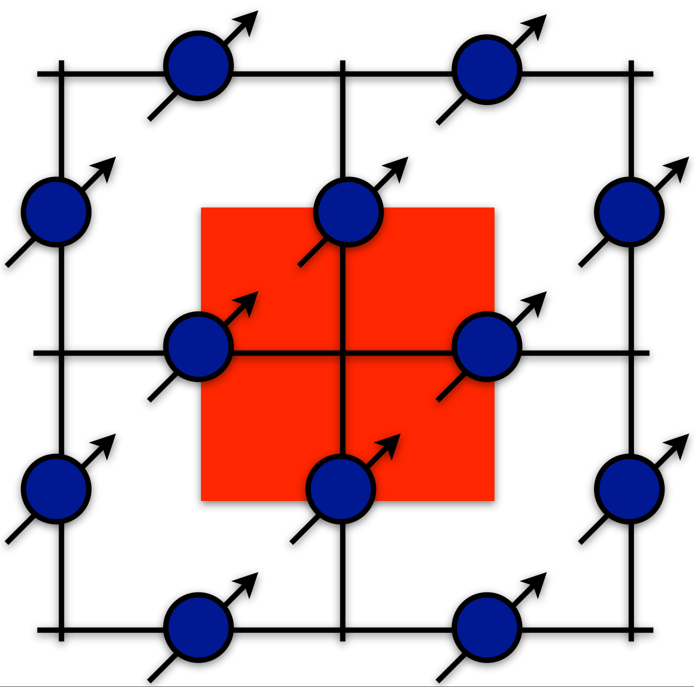
Quantum Link models

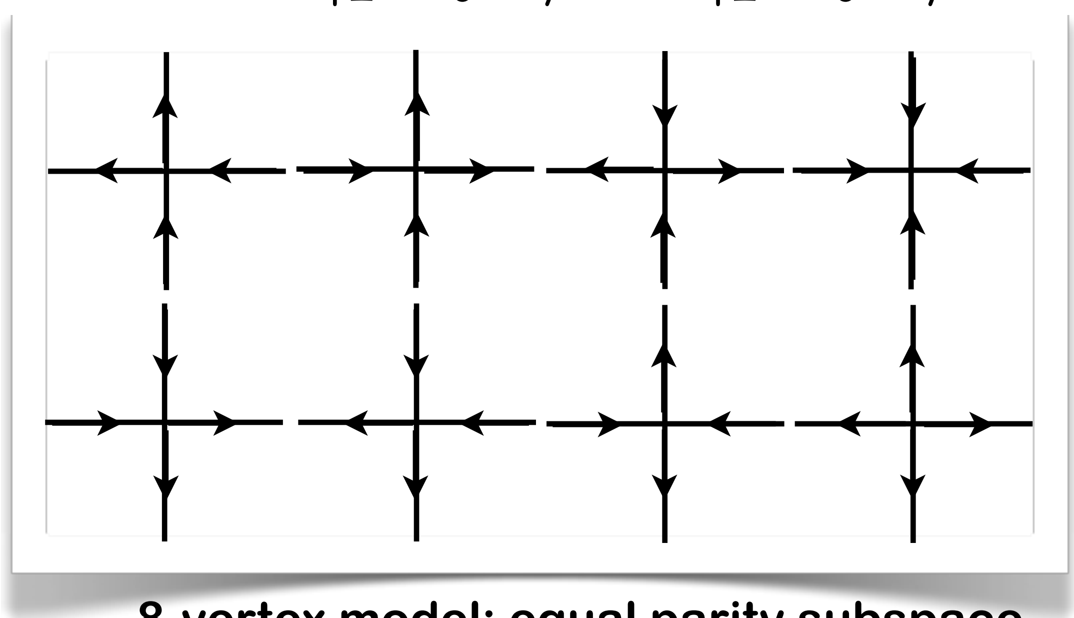
Connections with Quantum Information
(Z_2 gauge theory-Kitaev model)

Local generator of gauge transformations.-

"Physical" Hilbert space (Gauss' law)

$$G_{\text{vert}}|\text{phys}\rangle = |\text{phys}\rangle$$





8-vertex model: equal parity subspace

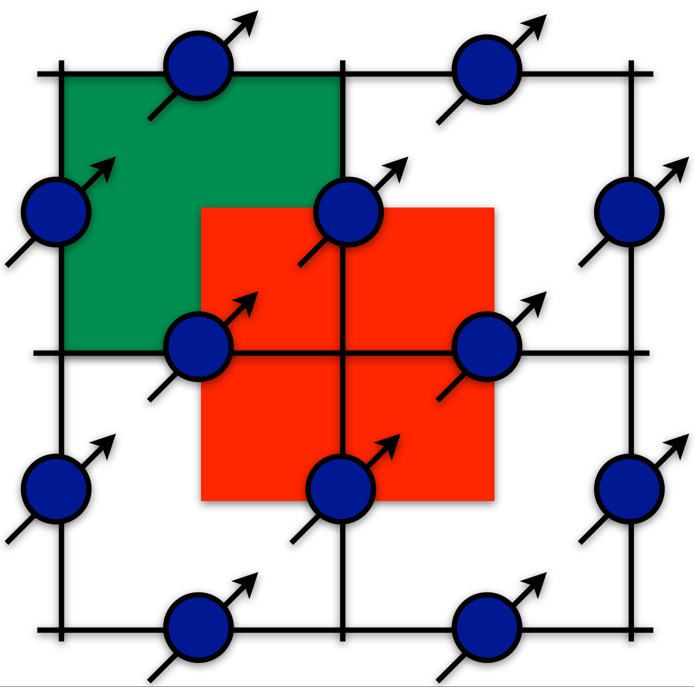
Quantum Link models

Connections with Quantum Information
(Z_2 gauge theory-Kitaev model)

Gauge invariant Hamiltonian.-

$$H = -\sum_{\text{plaq}} \sigma^{(3)}_{1,2}\sigma^{(3)}_{2,3}\sigma^{(3)}_{3,4}\sigma^{(3)}_{4,1} + \lambda \sum_{\langle x,y\rangle} \sigma^{(1)}_{x,y}$$

magnetic term **electric term**



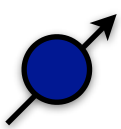
$$[H, G_{\text{vert}}] = 0, \quad \forall \text{ vertex}$$

Quantum Link models

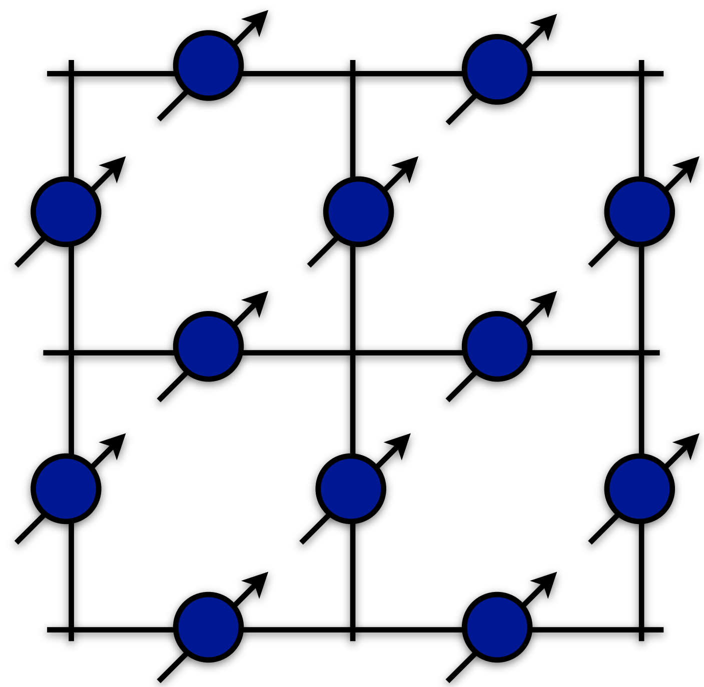
Connections with Condensed Matter (U(1) gauge theory-Quantum Spin Ice model)

Local degrees of freedom.-

Quantum two level system living on the link



$$\{\sigma^{(3)}_{x,y}, \sigma^{(+)}_{x,y}, \sigma^{(-)}_{x,y}\}$$



L. Balents, Nature (2010)
C. L. Henley, Ann. Rev. Cond. Matt. Phys. (2010)
C. Castelnovo, R. Moessner, and S.L. Sondhi, Ann. Rev. Cond. Matt. Phys. (2012)

Quantum Link models

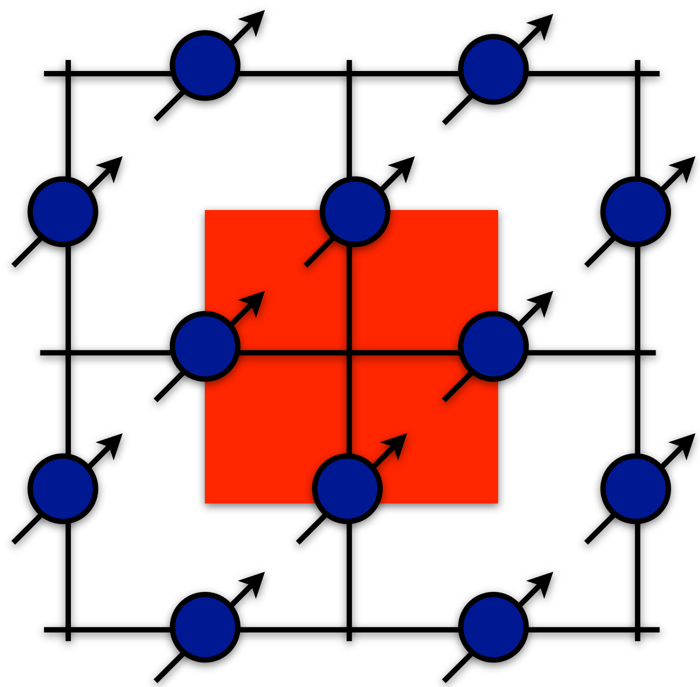
Connections with Condensed Matter
(U(1) gauge theory-Quantum Spin Ice model)

Local generator of gauge transformations.-

Local generator around every vertex

$$\exp\left[i\frac{\theta_{\rm vert}}{2}G_{\rm vert}\right]\sigma^{(+)}_{1,2}\exp\left[-i\frac{\theta_{\rm vert}}{2}G_{\rm vert}\right]=e^{i\theta_{\rm vert}}\sigma^{(+)}_{1,2}$$

$$G_{\rm vert}=\sigma^{(3)}_{1,2}+\sigma^{(3)}_{2,3}+\sigma^{(3)}_{3,4}+\sigma^{(3)}_{4,1}$$



U(1) gauge transformation

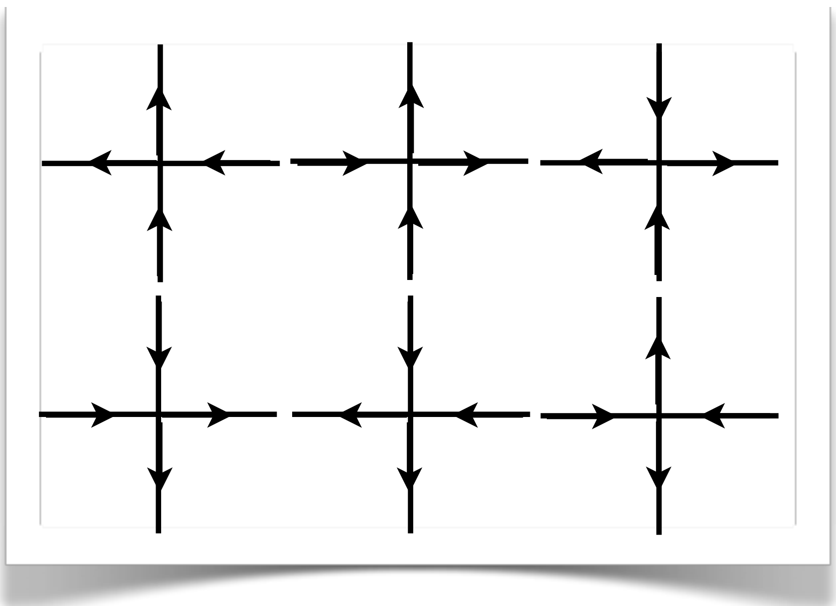
Quantum Link models

Connections with Condensed Matter
(U(1) gauge theory-Quantum Spin Ice model)

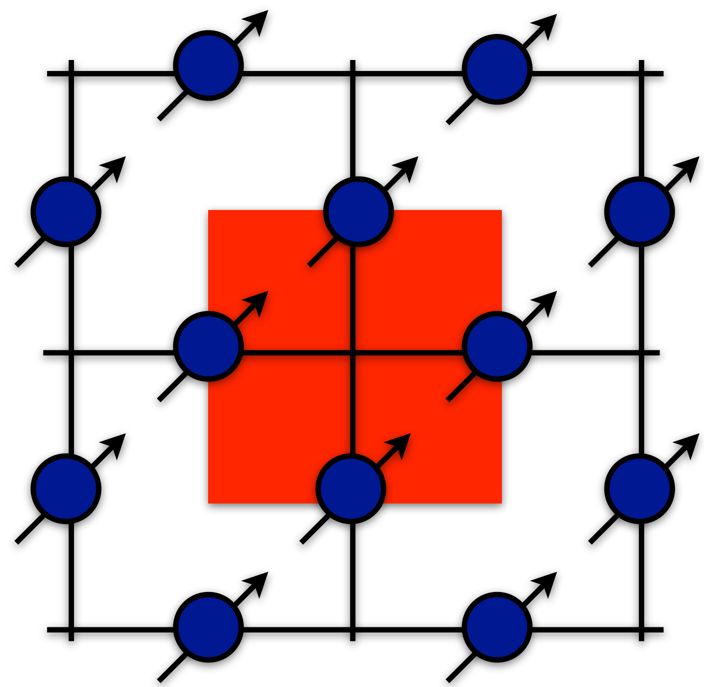
Local generator of gauge transformations.-

"Physical" Hilbert space (Gauss' law)

$$G_{\text{vert}}|\text{phys}\rangle = 0$$



6-vertex model:
zero magnetization subspace



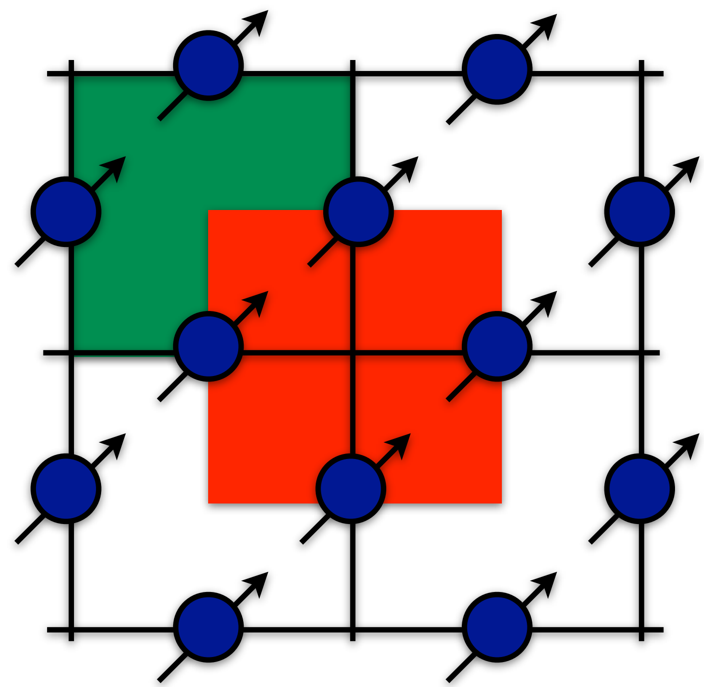
Quantum Link models

Connections with Condensed Matter (U(1) gauge theory-Quantum Spin Ice model)

Gauge invariant Hamiltonian.-

$$H = -\sum_{\text{plaq}} \left[\sigma^+_{1,2}\sigma^-_{2,3}\sigma^+_{3,4}\sigma^-_{4,1} + \sigma^-_{1,2}\sigma^+_{2,3}\sigma^-_{3,4}\sigma^+_{4,1}\right]$$

magnetic term



$$[H, G_{\text{vert}}] = 0, \quad \forall \text{ vertex}$$

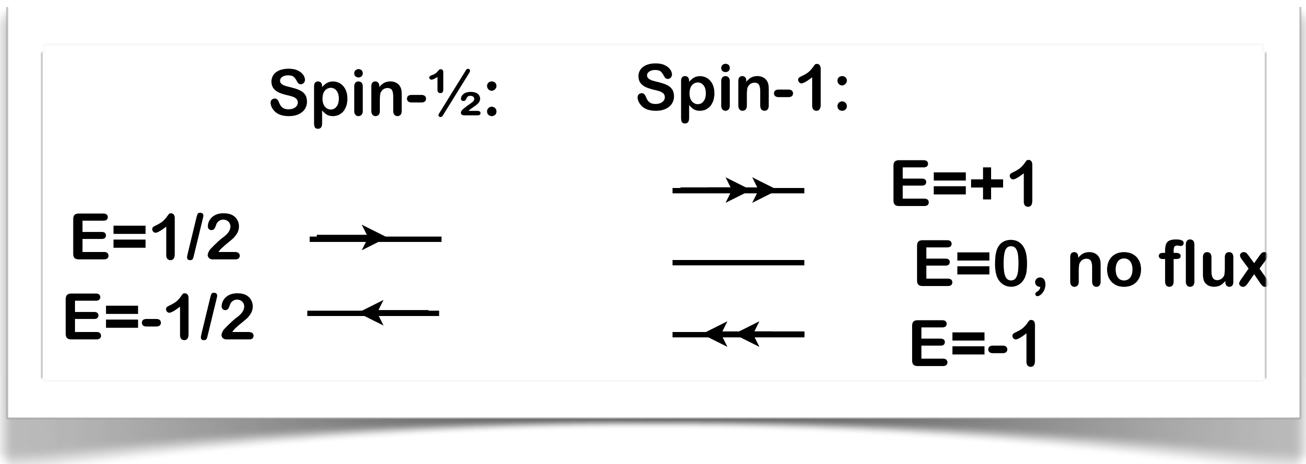
Quantum Link models

Local degrees of freedom.-

Quantum link carrying an electric flux

$$U_{x,y} \equiv S^{+}_{x,y}$$

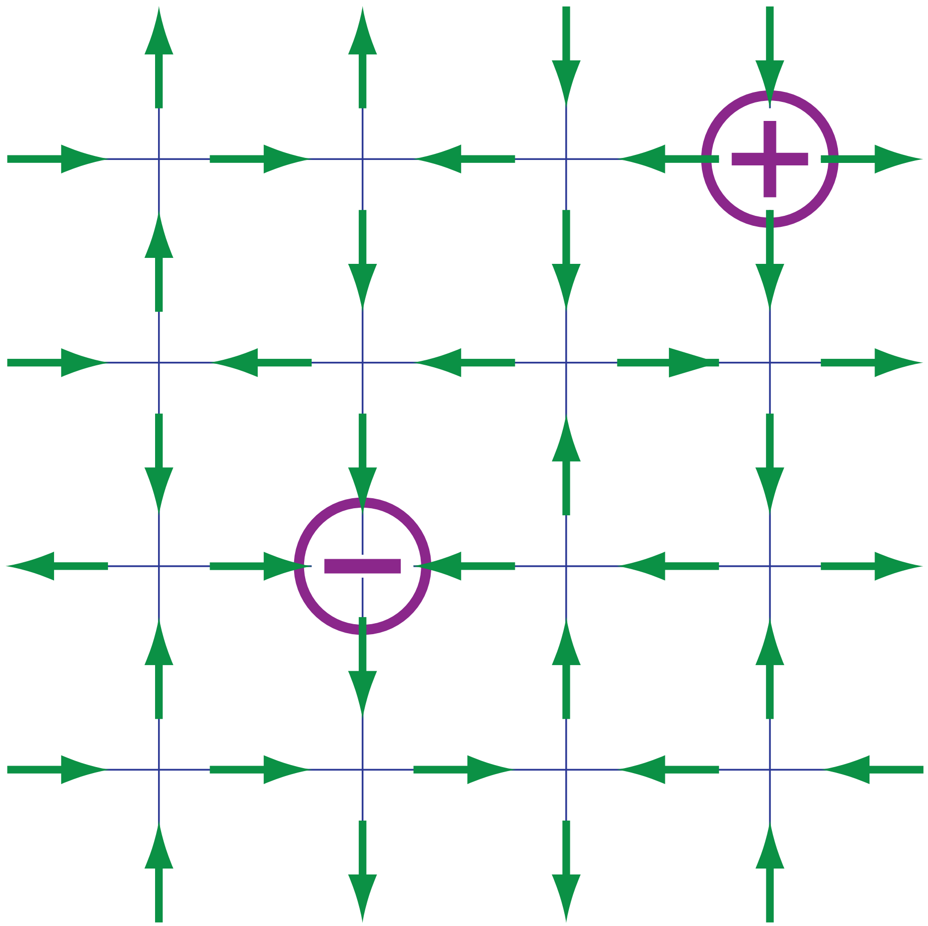
$$E_{x,y} \equiv S^{(3)}_{x,y}$$



Quantum Link models

Gauss' law.-

$$G_{\text{vert}}|\text{phys}\rangle = 0 \Leftrightarrow \vec{\nabla}\cdot\vec{E} = 0$$



configuration obeying ice rules, except for defects (charge or monopoles)

Quantum Link models

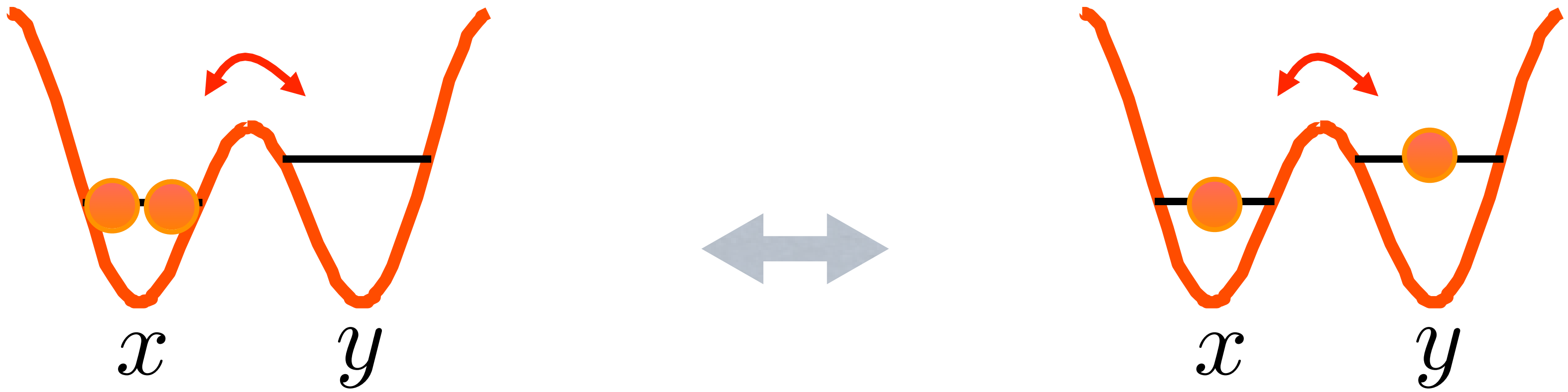
Gauge invariant Hamiltonian.-

$$H = \frac{g^2}{2} \sum_{\langle x,y \rangle} [E_{x,y}]^2 - \frac{1}{4g^2} \sum_{\text{plaq}} \left[U^\dagger_{1,2} U_{2,3} U^\dagger_{3,4} U_{4,1} + U_{1,2} U^\dagger_{2,3} U_{3,4} U^\dagger_{4,1} \right]$$

Electric term **Magnetic term**

Quantum Link models

Rishon (Schwinger) representation



Link operator $$U_{x,y} \equiv S^{+}_{x,y} = c^{\dagger}_{y} c_{x}$$

Electric field [U(1) generator] $$E_{x,y} \equiv S^{(3)}_{x,y} = \frac{1}{2}\left[c^{\dagger}_{y} c_{y} - c^{\dagger}_{x} c_{x}\right]$$

$$\{c_x , c^{\dagger}_{y}\} = \delta_{x,y}$$ **Schwinger fermions (rishons)**

$$\left[c_x , c^{\dagger}_{y}\right] = \delta_{x,y}$$ **Schwinger bosons**

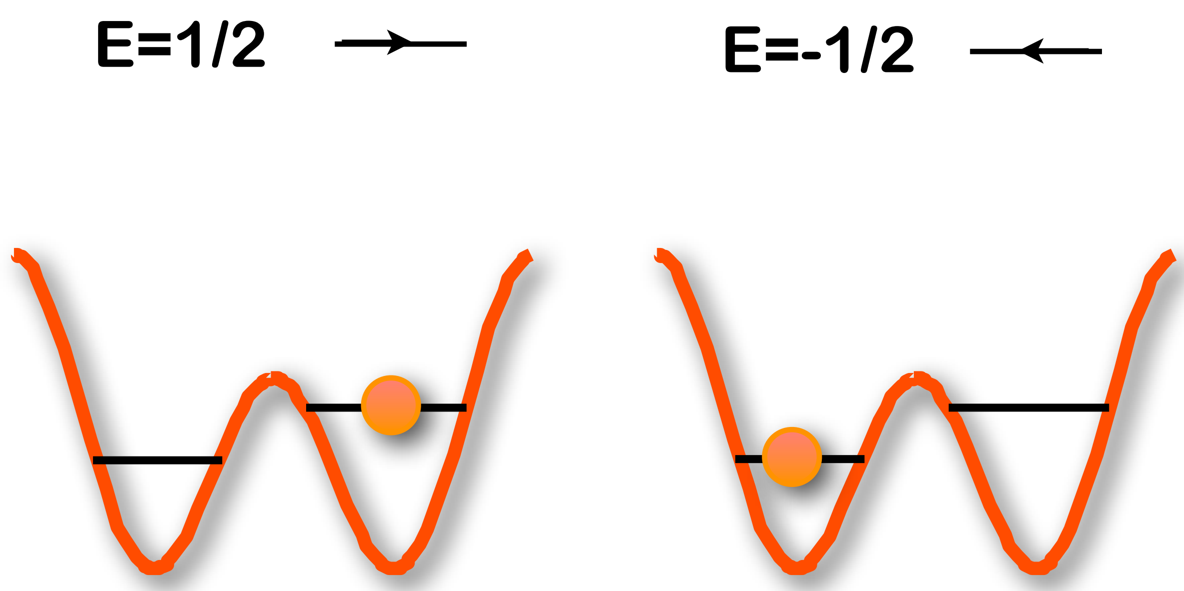
Quantum Link models

Rishon (Schwinger) representation

Spin representation:

$$N_{x,y} = c_y^\dagger c_y + c_x^\dagger c_x \qquad \left[\vec{S}_{x,y}\right]^2 \equiv \frac{N_{x,y}}{2}\left[\frac{N_{x,y}}{2}+1\right]$$

Spin-½:



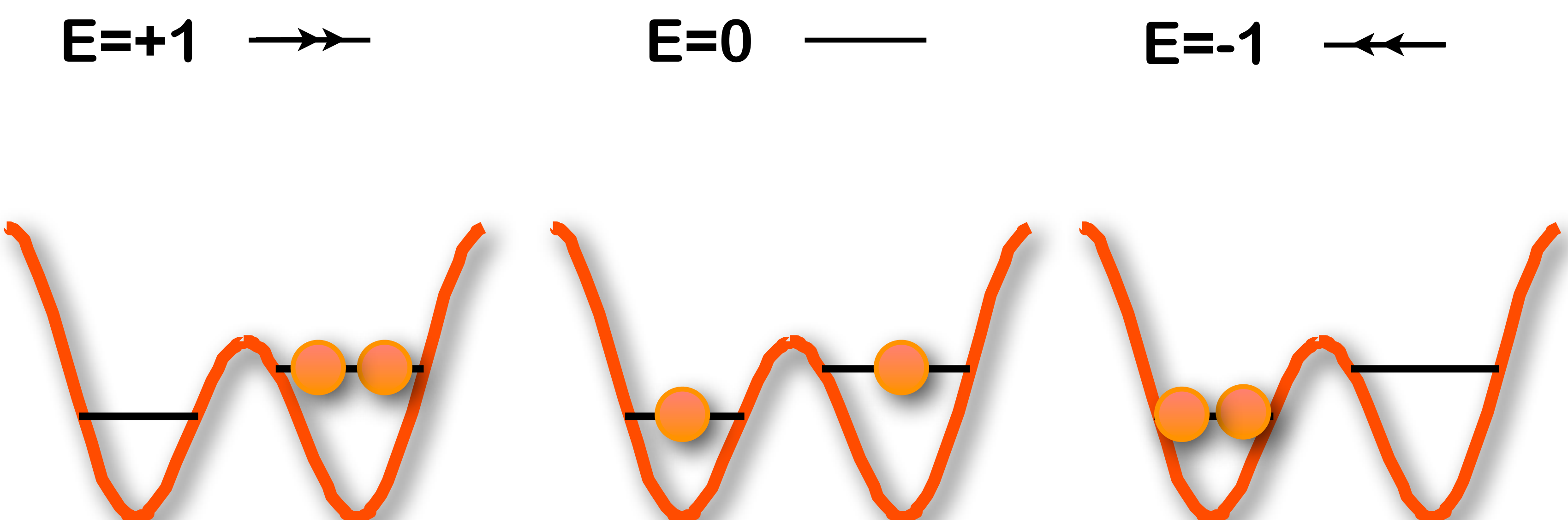
Quantum Link models

Rishon (Schwinger) representation

Spin representation:

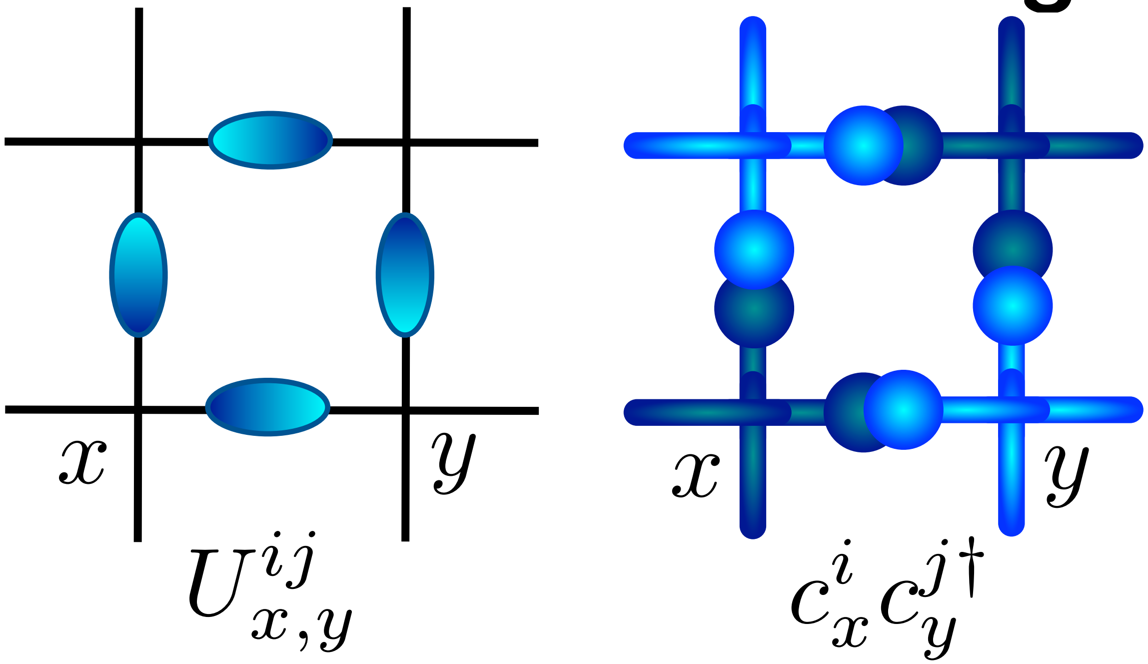
$$N_{x,y} = c_y^\dagger c_y + c_x^\dagger c_x \qquad \left[\vec{S}_{x,y}\right]^2 \equiv \frac{N_{x,y}}{2}\left[\frac{N_{x,y}}{2}+1\right]$$

Spin-1:



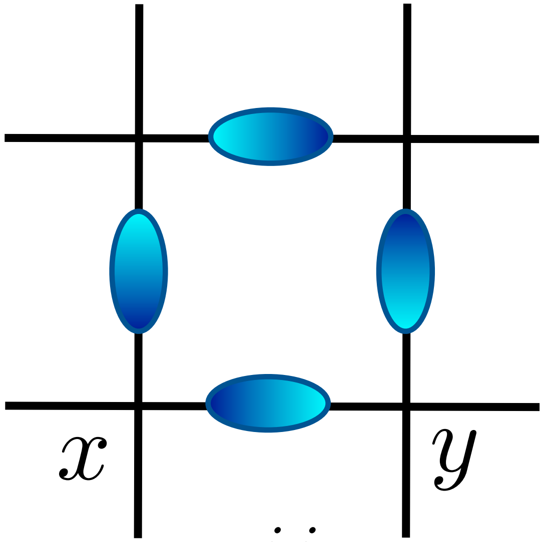
Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

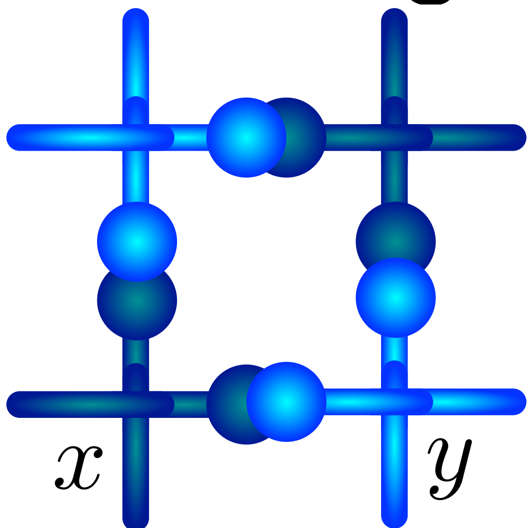


Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

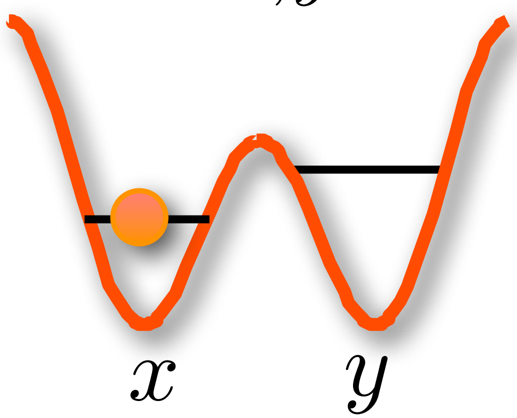


$U^{ij}_{x,y}$

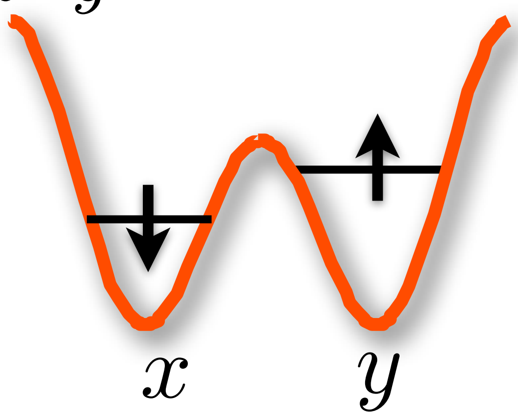


$c^i_x c^{j\dagger}_y$

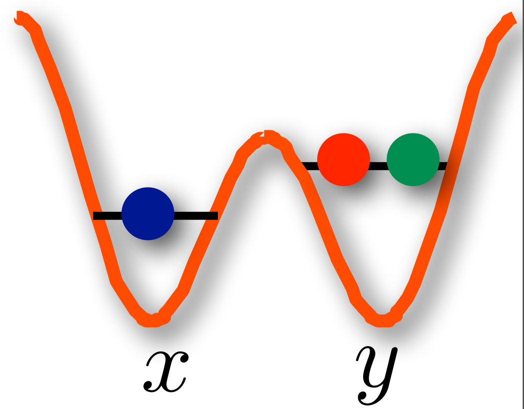
$$i = \begin{cases} 1 & : U(1) \\ \uparrow\downarrow & : U(2) \\ brg & : U(3) \end{cases}$$



U(1) group



U(2) group



U(3) group

Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

Local degrees of freedom.-

Link operator $$U^{ij}_{x,y} \equiv c^i_x c^{j\dagger}_y$$

Electric field [U(1) generator] $$E_{x,y} \equiv \frac{1}{2}\left[c^{i\dagger}_y c^i_y - c^{i\dagger}_x c^i_x\right]$$

Representation [occupation] $$N_{x,y} = c^{i\dagger}_y c^i_y + c^{i\dagger}_x c^i_x$$

Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

Local degrees of freedom.-

Non-abelian electric fields [SU(N) generators]

Left generators

$$L^a_{x,y} = c^{i\dagger}_x \lambda^a_{ij} c^i_x$$

Right generators

$$R^a_{x,y} = c^{i\dagger}_y \lambda^a_{ij} c^i_y$$

$$\exp\left[i\theta^a_x L^a_{x,y}\right] U_{x,y} \exp\left[-i\theta^a_x L^a_{x,y}\right] = \exp\left[-i\theta^a_x \lambda^a\right] U_{x,y}$$

$$\exp\left[i\theta^a_y R^a_{x,y}\right] U_{x,y} \exp\left[-i\theta^a_y R^a_{x,y}\right] = U_{x,y} \exp\left[i\theta^a_y \lambda^a\right]$$

Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

Local generators.-

$$G_x = -\sum_k \left(E_{x,x+\hat{k}} - E_{x-\hat{k},x} \right)$$

$$G_x^a = \sum_k \left(L^a_{x,x+\hat{k}} + R^a_{x-\hat{k},x} \right)$$

Non-abelian quantum link models

Rishon (Schwinger) representation with internal degrees of freedom

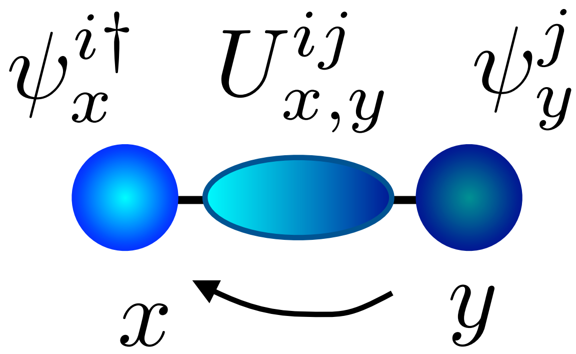
Hamiltonian.-

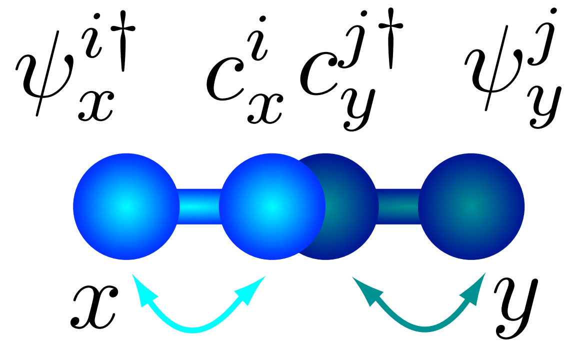
$$H = \frac{g'^2}{2} \sum_{\langle x,y \rangle} \left(E_{x,y}\right)^2 + \frac{g^2}{2} \sum_{\langle x,y \rangle} \left[\left(\vec{L}_{x,y} \right)^2 + \left(\vec{R}_{x,y} \right)^2 \right]$$

$$- \frac{1}{4g^2} \sum_{\text{plaq}} \left[U^\dagger_{1,2} U_{2,3} U^\dagger_{3,4} U_{4,1} + U_{1,2} U^\dagger_{2,3} U_{3,4} U^\dagger_{4,1} \right]$$

Non-abelian quantum link models

with matter





$$H = -t \sum_{\langle x,y\rangle, i,j} \left(\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j + \text{h.c.}\right) + \cdots = -t \sum_{\langle x,y\rangle} \left[\left(\sum_i \psi_x^{i\dagger} c_x^i \right) \left(\sum_j c_y^{j\dagger} \psi_y^j \right) + \text{h.c.} \right] + \cdots$$

Matter - gauge interaction
= hopping of fermions mediated by a quantum link
= correlated hopping of fermions and rishons

Non-abelian quantum link models

with matter

Local generators.-

U(1) generator

$$G_x = \psi_x^{i\dagger}\psi_x^i - \sum_k \left(E_{x,x+\hat{k}} - E_{x-\hat{k},x}\right)$$

SU(N) generator

$$G_x^a = \psi_x^{i\dagger}\lambda_{ij}^a\psi_x^j + \sum_k \left(L_{x,x+\hat{k}}^a + R_{x-\hat{k},x}^a\right)$$

“Physical” Hilbert space

$$\vec{G}_x|\mathrm{phys}\rangle = 0 \qquad \forall x$$

Non-abelian quantum link models

with matter

Hamiltonian.-

$$[H, G_x] = [H, G_x^a] = 0 \qquad \forall x$$

Non-abelian quantum link models

Strong coupling Hamiltonian with staggered fermions

$$H = \frac{g'^2}{2}\sum_{\langle x,y\rangle}(E_{x,y})^2 + \frac{g^2}{2}\sum_{\langle x,y\rangle}\left[\left(\vec{L}_{x,y}\right)^2 + \left(\vec{R}_{x,y}\right)^2\right] - t\sum_{\langle x,y\rangle,i,j}\left(\psi_x^{i\dagger}U_{x,y}^{ij}\psi_y^j + \text{h.c.}\right) + m\sum_{x,i}(-1)^x\,\psi_x^{i\dagger}\psi_x^i$$

Staggered mass

Non-abelian electric field

Electric field

Matter-gauge interaction

Non-abelian quantum link models

Strong coupling Hamiltonian with staggered fermions

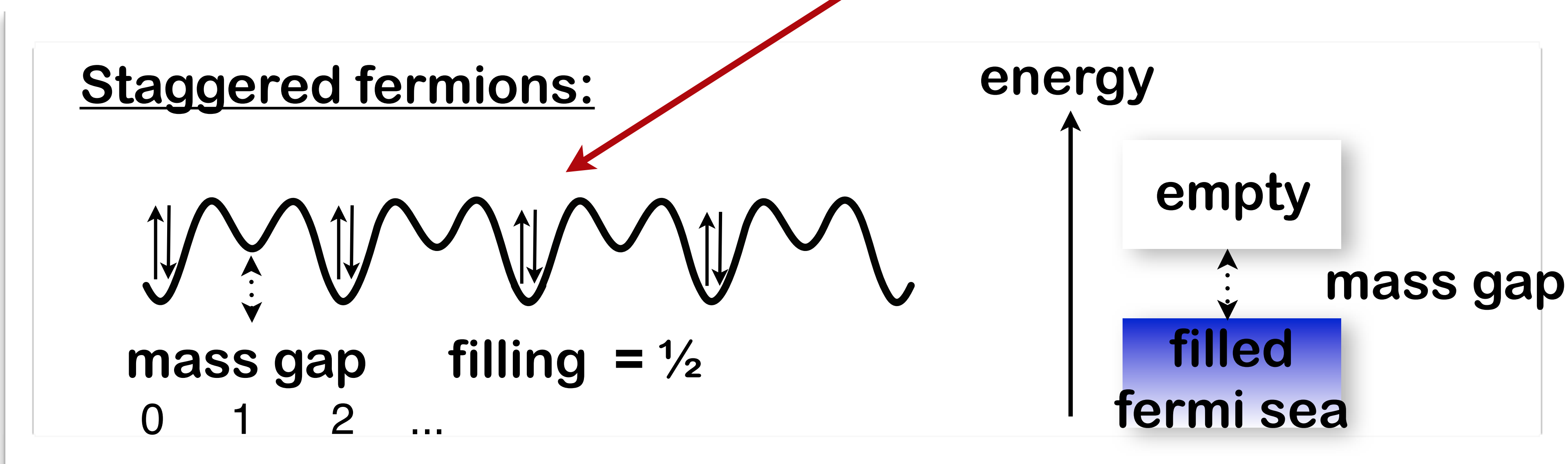
$$H = \frac{g'^2}{2}\sum_{\langle x,y\rangle}(E_{x,y})^2 + \frac{g^2}{2}\sum_{\langle x,y\rangle}\left[\left(\vec{L}_{x,y}\right)^2 + \left(\vec{R}_{x,y}\right)^2\right] - t\sum_{\langle x,y\rangle,i,j}\left(\psi_x^{i\dagger}U_{x,y}^{ij}\psi_y^j + \mathrm{h.c.}\right) + m\sum_{x,i}(-1)^x\,\psi_x^{i\dagger}\psi_x^i$$

Staggered mass

Non-abelian electric field

Electric field

Matter-gauge interaction



Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

Gauss' law

$$G_x = \psi_x^\dagger \psi_x + \frac{(-1)^x - 1}{2} - (E_{x,x+1} - E_{x-1,x})$$

$$G_x|\text{phys}\rangle = 0 \Leftrightarrow \rho - \vec{\nabla} \cdot \vec{E} = 0$$

Spin-1 representation

$|0\rangle$  $|1\rangle$ 

$|-1\rangle$ $|0\rangle$ $|+1\rangle$

Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

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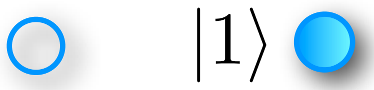
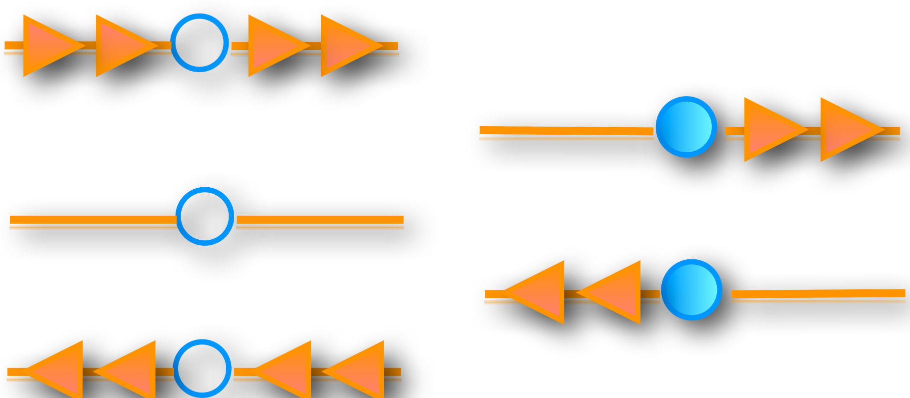


$|-1\rangle$ $|0\rangle$ $|+1\rangle$

Even sites



Phenomenology

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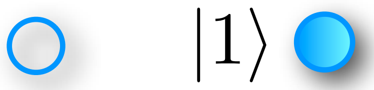
Spin-1 representation

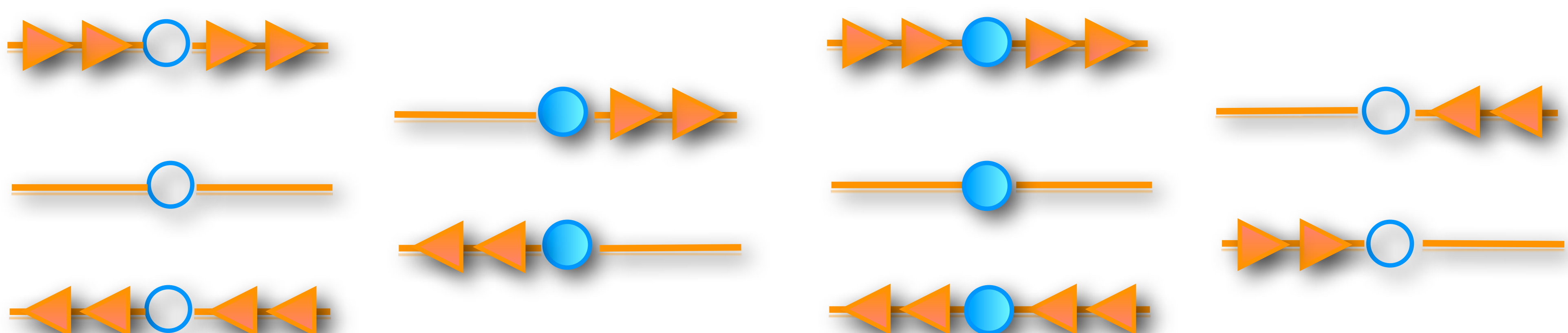
$|0\rangle$ $|1\rangle$



Even sites

$|-1\rangle$ $|0\rangle$ $|+1\rangle$

Odd sites

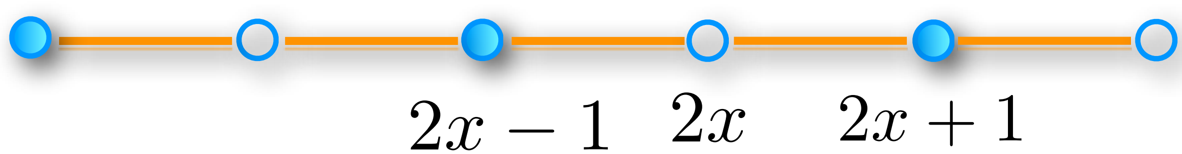


Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

Vacuum state

$$H = \frac{g^2}{2} \sum_{\langle x,y \rangle} \left(S^{(3)}_{x,y} \right)^2 + m \sum_x (-1)^x \, \psi^\dagger_x \psi_x$$



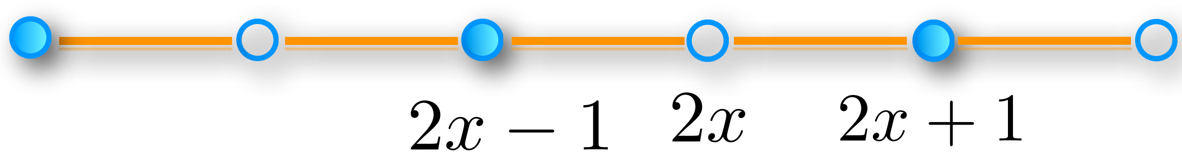
Creating a quark - antiquark pair: $\psi^\dagger_{2x} S^{+}_{2x,2x+1} \psi_{2x+1}$

Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

Vacuum state

$$H = \frac{g^2}{2} \sum_{\langle x,y \rangle} \left(S^{(3)}_{x,y} \right)^2 + m \sum_{x} (-1)^x \, \psi^\dagger_x \psi_x$$



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Phenomenology

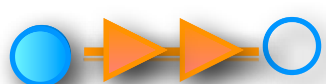
Confinement and string breaking: QED in 1+1D (Schwinger model)

Vacuum state

$$H = \frac{g^2}{2}\sum_{\langle x,y\rangle}\left(S^{(3)}_{x,y}\right)^2 + m\sum_x (-1)^x\,\psi^\dagger_x\psi_x$$

$2x-1$ $\quad 2x \quad$ $2x+1$

Creating a quark - antiquark pair: $\psi^\dagger_{2x} S^{+}_{2x,2x+1}\psi_{2x+1}$

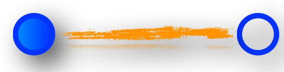


Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)



Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

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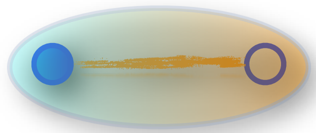
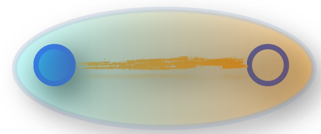
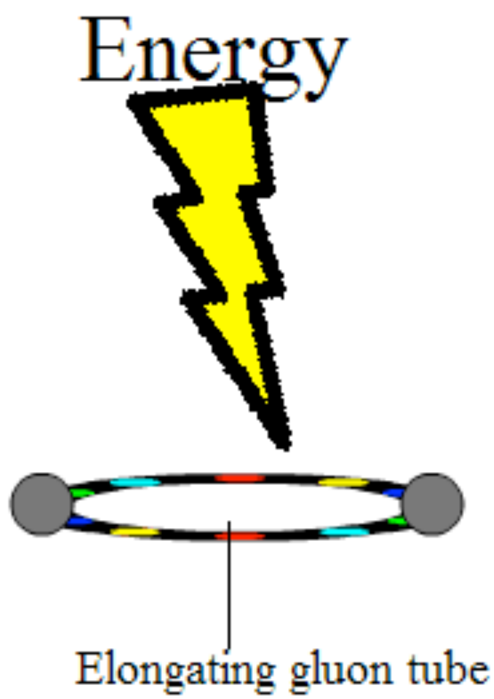


meson



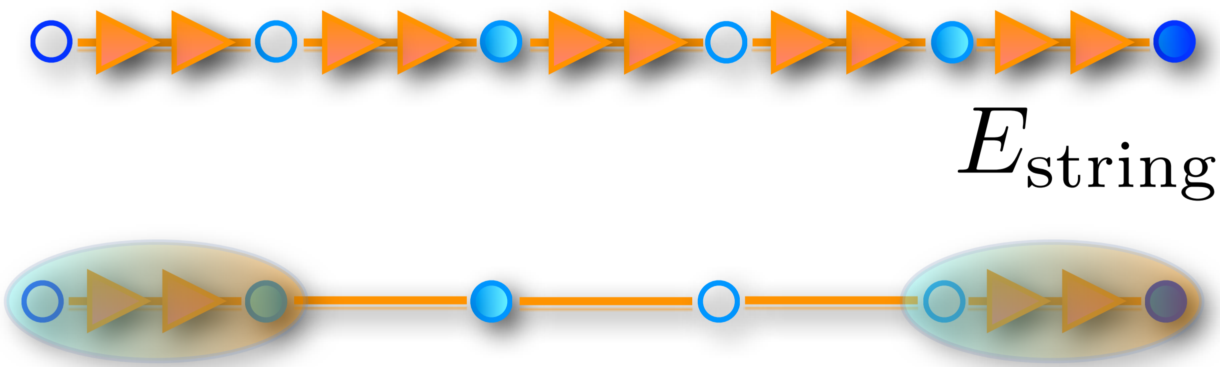
meson



Phenomenology

Confinement and string breaking: QED in 1+1D (Schwinger model)

Microscopic picture:

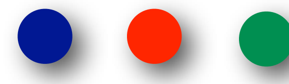


$$E_{\text{string}} = \frac{g^2}{2}(L-1) - \frac{Lm}{2}$$

$$E_{\text{meson}} = g^2 - \frac{(L-2)\,m}{2}$$

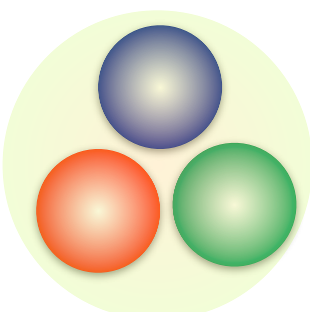
$$L_{\text{c}} = 2 + \frac{2m}{g^2}$$

Quantum Chromodynamics: Confinement under normal conditions

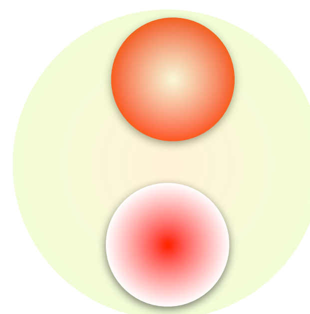
Quarks and gluons carry a color charge $\psi_x^i \quad i =$ 

Quarks are confined into color-neutral (color singlet) bound states (hadrons)

qqq baryons: proton, neutron, ...



$q\bar{q}$ mesons: pions (lightest), kaon, rho, ...

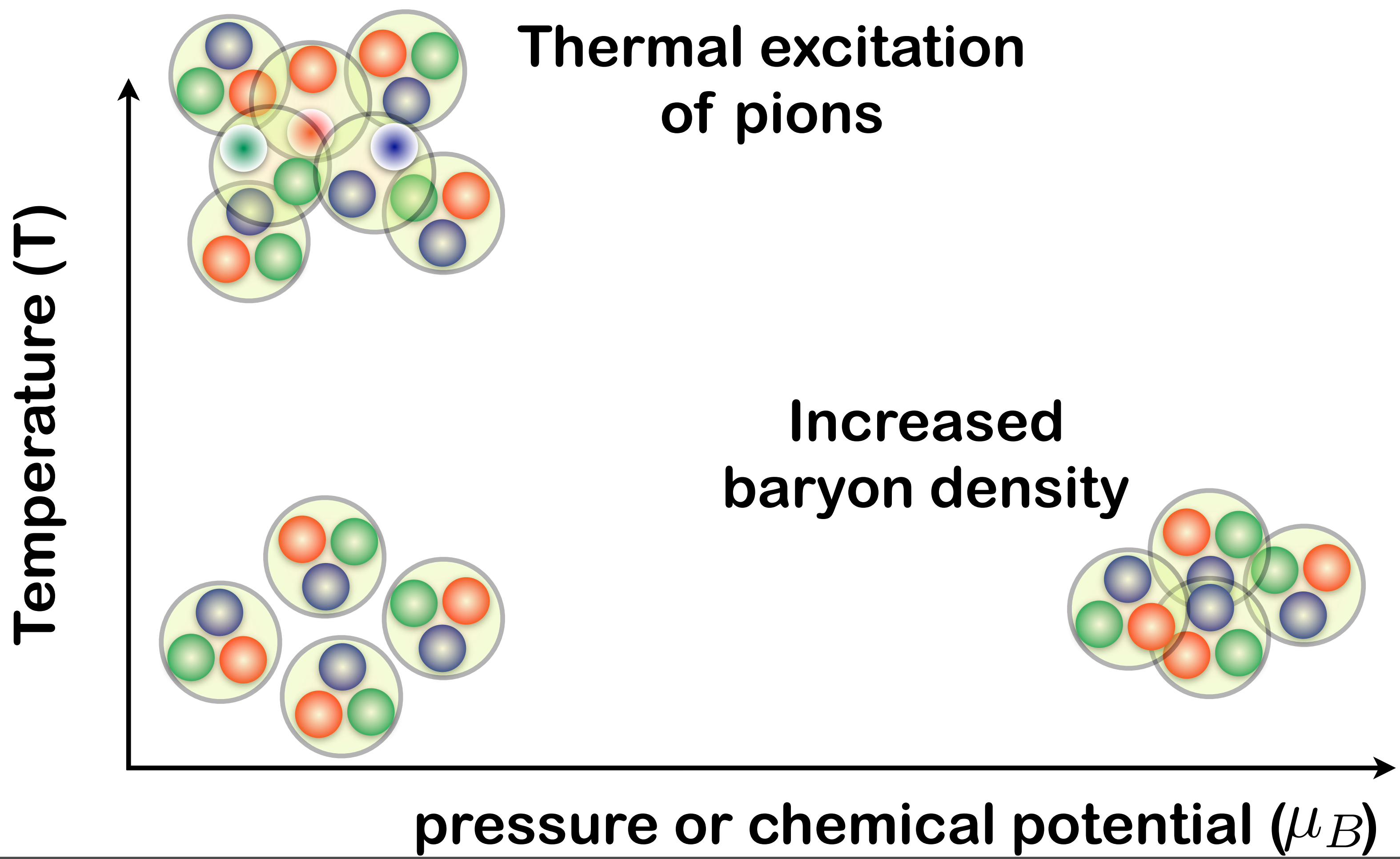


Quarks interact by exchanging gluons $\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j$

QCD under extreme conditions

Compress or heat baryons → Hadrons overlap → Confinement is “lost”

Expect interesting/unsual behaviour



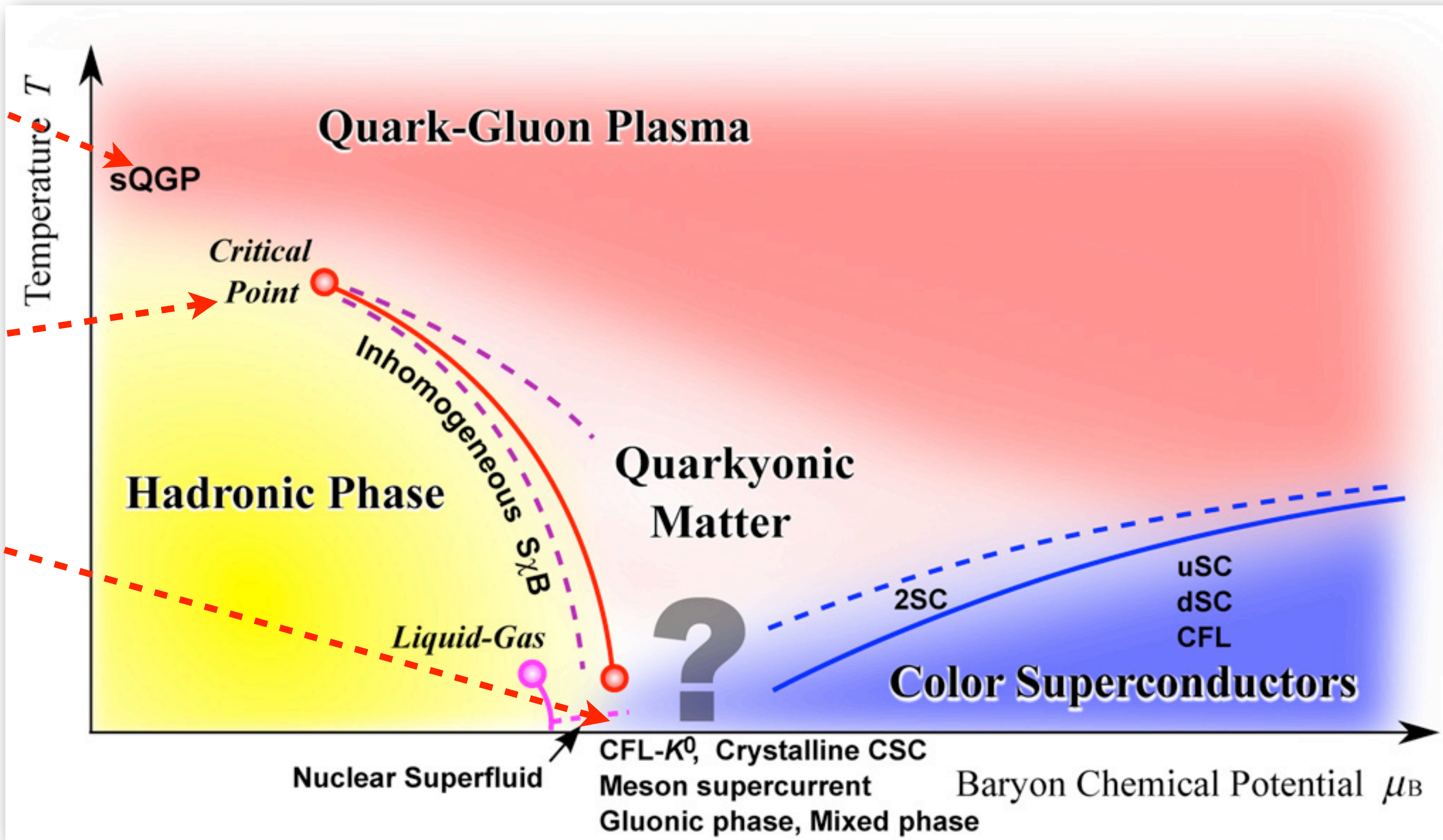
QCD under extreme conditions

Compress or heat baryons → Hadrons overlap → Confinement is “lost”

Early universe

Heavy-ion collisions

Neutron stars



pressure or chemical potential (μ_B)

Implementation of (non-)abelian quantum link models

Strong coupling Hamiltonian with staggered fermions

$$H = \frac{g'^2}{2} \sum_{\langle x,y \rangle} (E_{x,y})^2 + \frac{g^2}{2} \sum_{\langle x,y \rangle} \left[\left(\vec{L}_{x,y} \right)^2 + \left(\vec{R}_{x,y} \right)^2 \right] - t \sum_{\langle x,y \rangle, i,j} \left(\psi_x^{i\dagger} U_{x,y}^{ij} \psi_y^j + \text{h.c.} \right) + m \sum_{x,i} (-1)^x \psi_x^{i\dagger} \psi_x^i$$

Staggered mass

Non-abelian electric field

Electric field

Matter-gauge interaction

Implementation of (non-)abelian quantum link models

Strong coupling Hamiltonian with staggered fermions

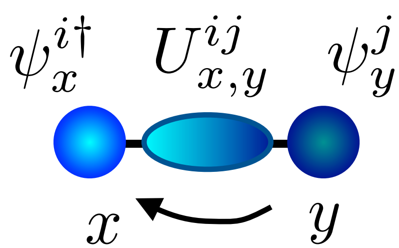
$$H=\frac{g'^2}{2}\sum_{\langle x,y\rangle}\left(E_{x,y}\right)^2+\frac{g^2}{2}\sum_{\langle x,y\rangle}\left[\left(\vec{L}_{x,y}\right)^2+\left(\vec{R}_{x,y}\right)^2\right]-t\sum_{\langle x,y\rangle,i,j}\left(\psi_x^{i\dagger}U_{x,y}^{ij}\psi_y^j+\mathrm{h.c.}\right)+m\sum_{x,i}(-1)^x\,\psi_x^{i\dagger}\psi_x^i$$

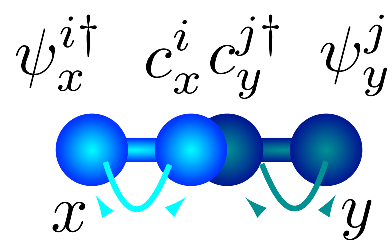
Staggered mass

Non-abelian electric field

Electric field

Matter-gauge interaction

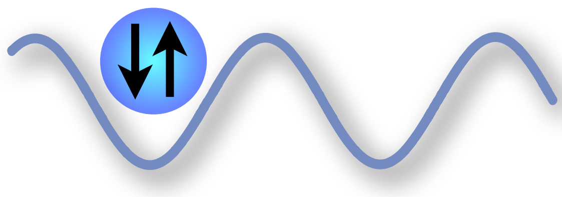




$$H=-t\sum_{\langle x,y\rangle,i,j}\left(\psi_x^{i\dagger}U_{x,y}^{ij}\psi_y^j+\mathrm{h.c.}\right)+\cdots=-t\sum_{\langle x,y\rangle}\left[\left(\sum_i\psi_x^{i\dagger}c_x^i\right)\left(\sum_j c_y^{j\dagger}\psi_y^j\right)+\mathrm{h.c.}\right]+\cdots$$

Matter - gauge interaction
= hopping of fermions mediated by a quantum link
= correlated hopping of fermions and rishons

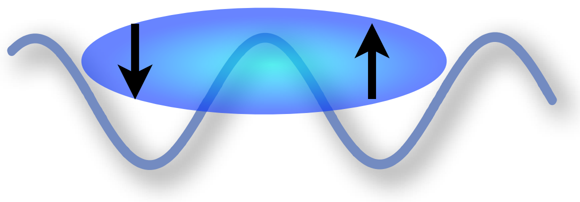
Implementation of (non-)abelian quantum link models



$$H_{\text{hop}} = -\tilde{t} \sum_i \psi_x^{i\dagger} c_x^i + \text{h.c.}$$

(Color singlet) hopping fermion-rishon

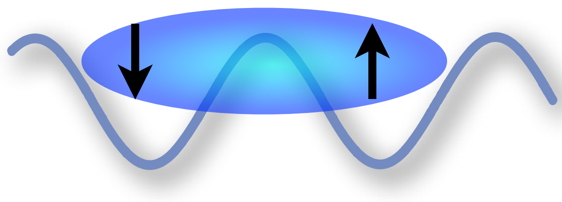
Implementation of (non-)abelian quantum link models



$$H_{\text{hop}} = -\tilde{t} \sum_i \psi_x^{i\dagger} c_x^i + \text{h.c.}$$

(Color singlet) hopping fermion-rishon

Implementation of (non-)abelian quantum link models



$$H_{\text{hop}} = -\tilde{t}\sum_i \psi_x^{i\dagger} c_x^i + \text{h.c.}$$

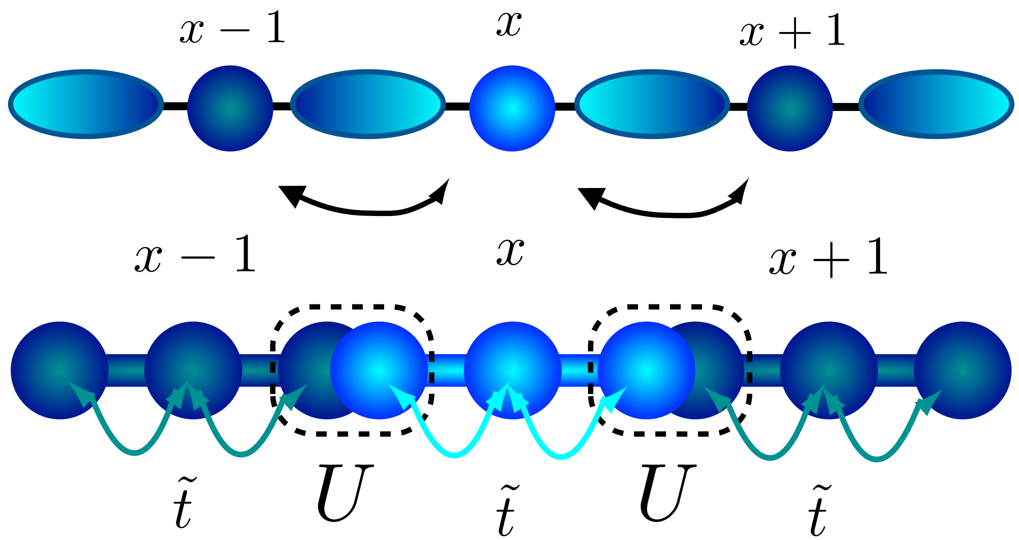
$$\frac{1}{\sqrt{2}}\left(|\uparrow_\psi \downarrow_c\rangle - |\downarrow_\psi \uparrow_c\rangle\right)$$

(Color singlet) hopping fermion-rishon

The building block is already gauge invariant (summation over internal degrees of freedom)

Action of the hopping fermion-rishon swaps the local singlet to nearest-neighbor ones

Implementation of (non-)abelian quantum link models



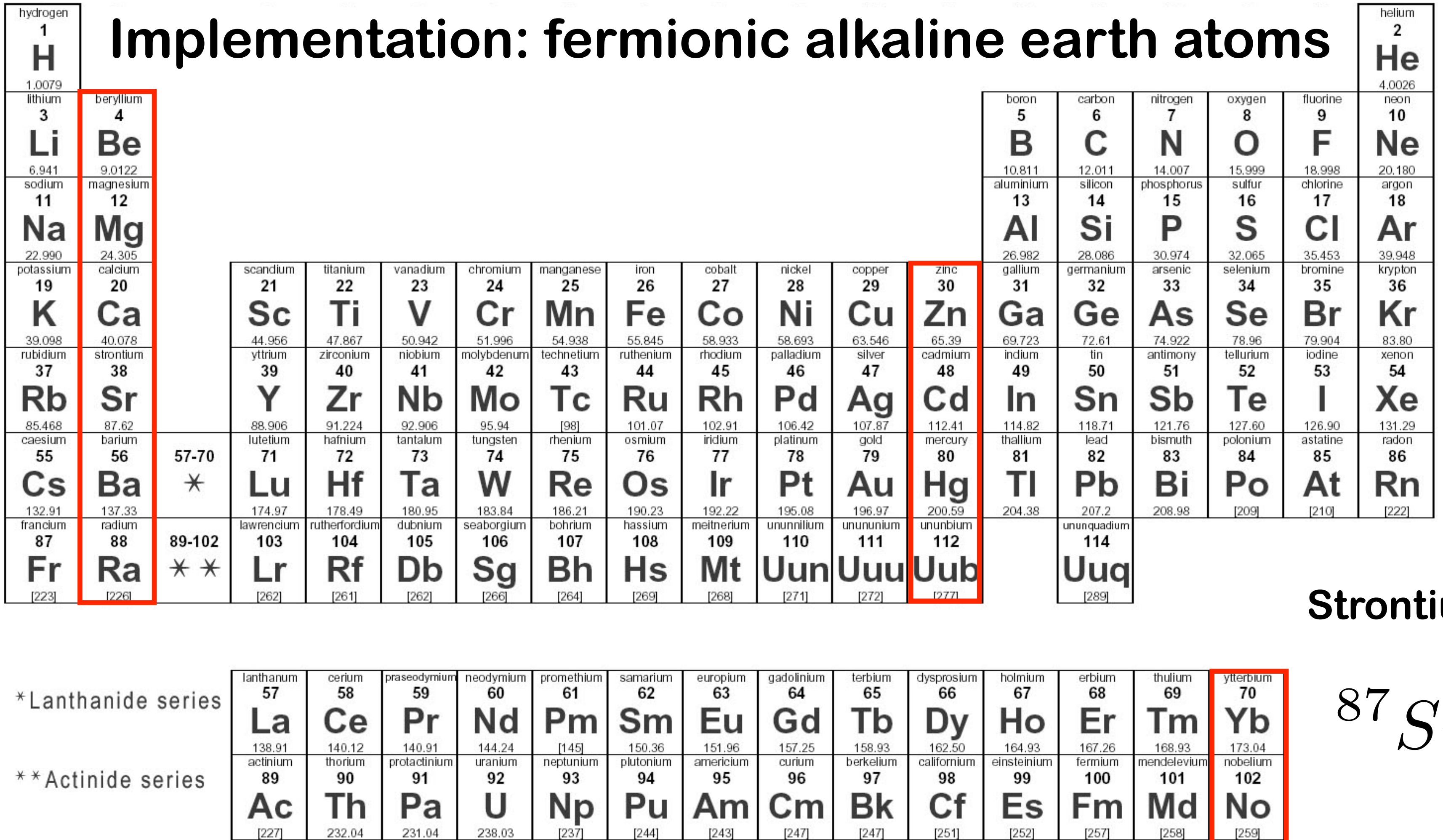
(Color-singlet interaction/constraint) number of rishon per link

$$H_U = U\left[N_{x,y} - n\right]^2 = U\left[\sum_i \left(c_x^{i\dagger} c_x^i + c_y^{i\dagger} c_y^i\right) - n\right]^2$$

Constraint: on-site SU(2N) interaction

Implementation of (non-)abelian quantum link models

Implementation: fermionic alkaline earth atoms



Strontium / Ytterbium

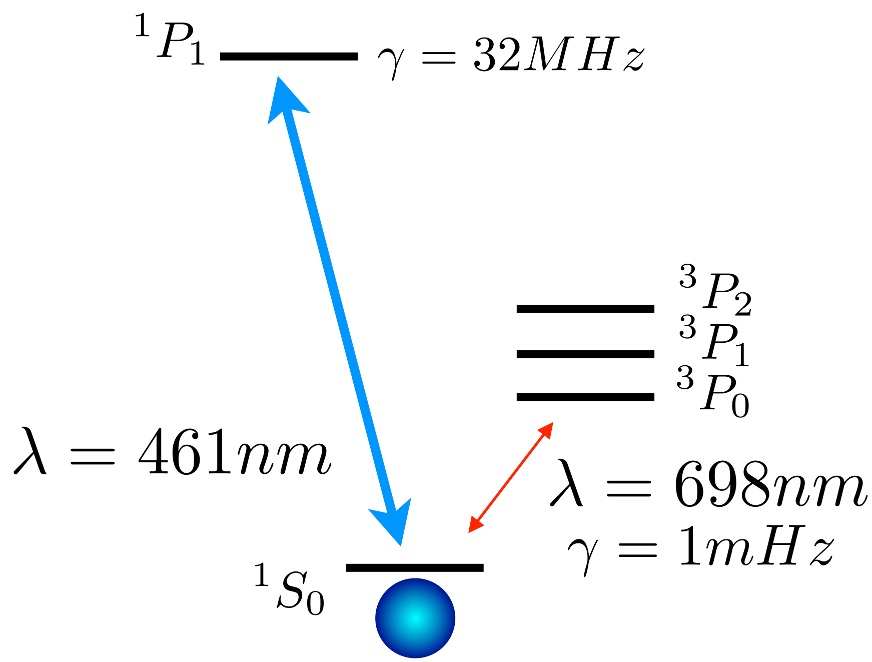
$$^{87}Sr(I = 9/2)$$

$$^{173}Yb(I = 5/2)$$

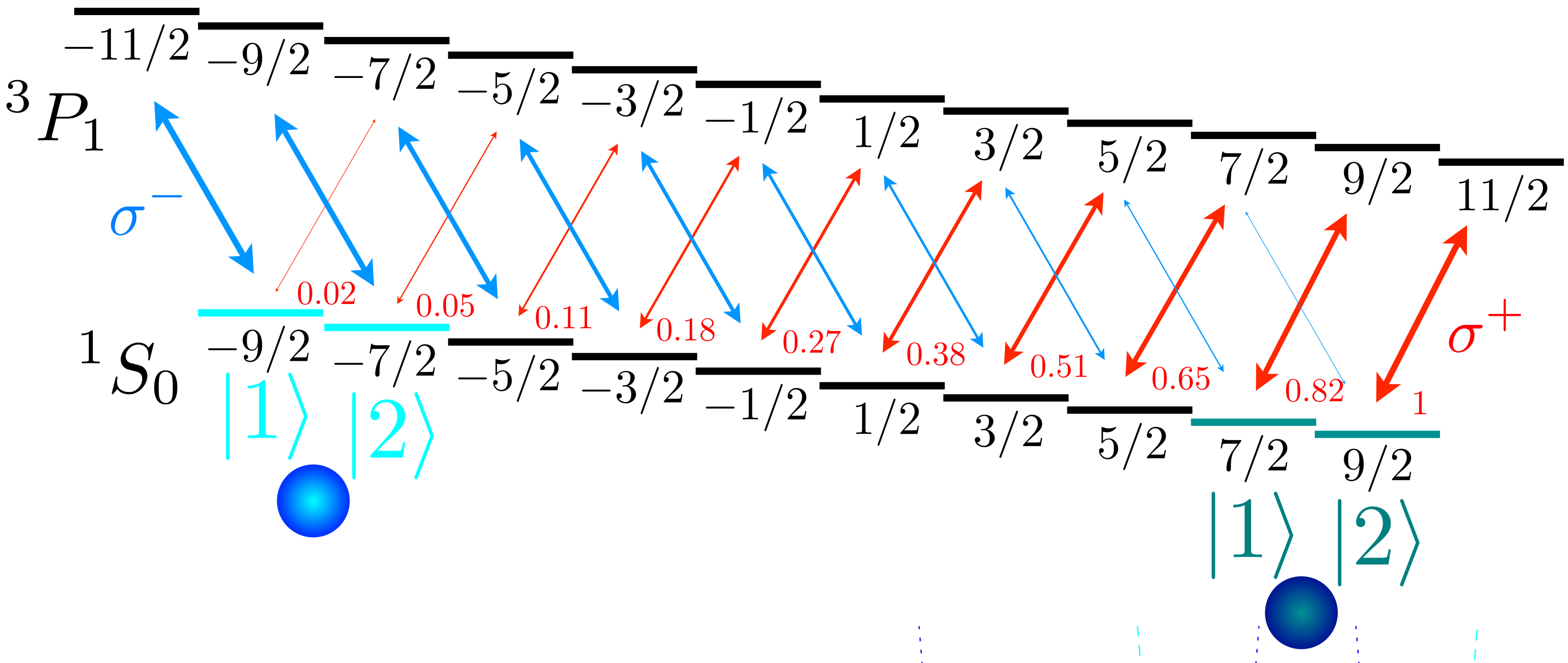
i) fermionic alkaline earths have nuclear spin I>0
ii) scattering independent of the nuclear spin

Implementation of (non-)abelian quantum link models

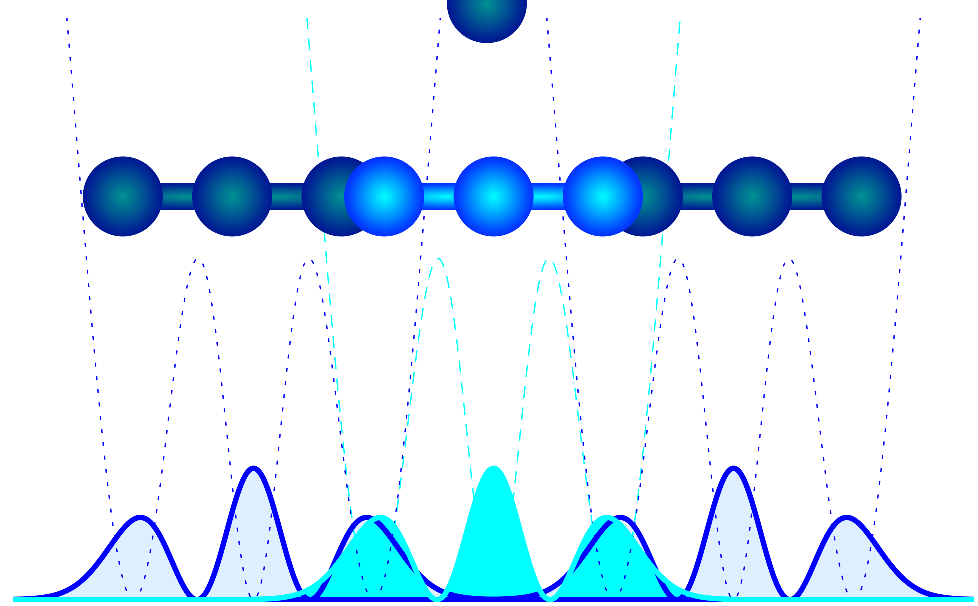
Ground state hyperfine Zeeman levels encode the color degrees of freedom



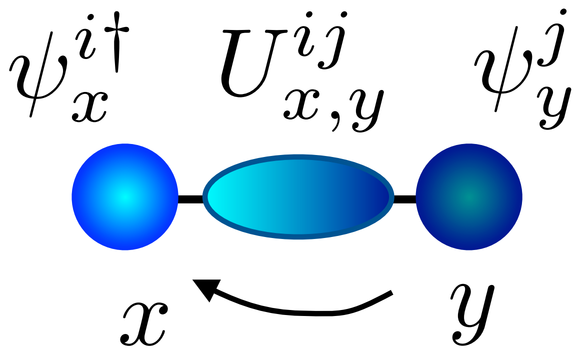
Implementation of (non-)abelian quantum link models

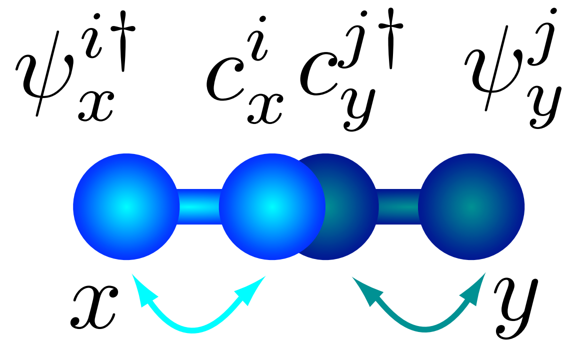


Different polarizations are used to trapped different internal levels



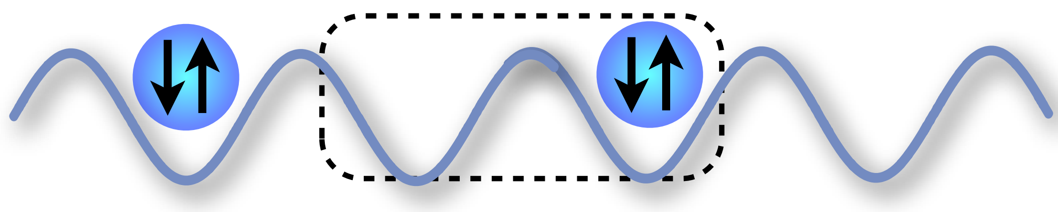
Implementation of (non-)abelian quantum link models





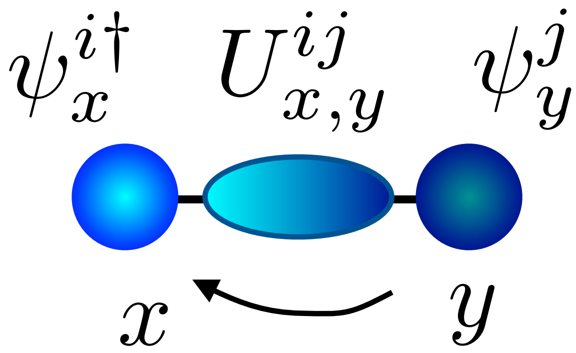
At second order in perturbation theory (t/U):

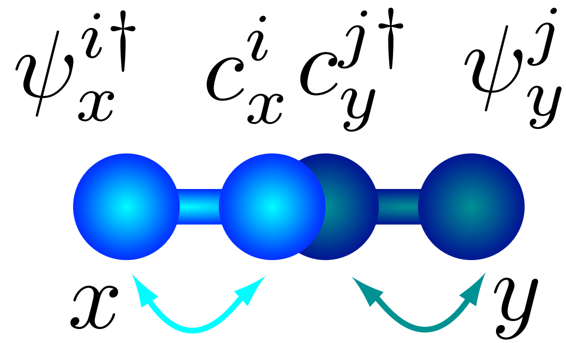
$$H_{\rm micro} = U \left[\sum_i \left(c_x^{i\dagger} c_x^i + c_y^{i\dagger} c_y^i \right) - n \right]^2 - \tilde{t} \sum_i \left(\psi_x^{i\dagger} c_x^i + c_y^{i\dagger} \psi_y^i \right) + \text{h.c.}$$



$$H_{\rm eff} = -t \sum_{\langle x,y \rangle} \left[\left(\sum_i \psi_x^{i\dagger} c_x^i \right) \left(\sum_j c_y^{j\dagger} \psi_y^j \right) + \text{h.c.} \right] + \cdots$$

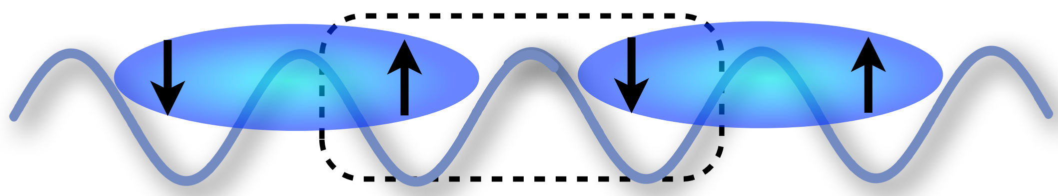
Implementation of (non-)abelian quantum link models





At second order in perturbation theory (t/U):

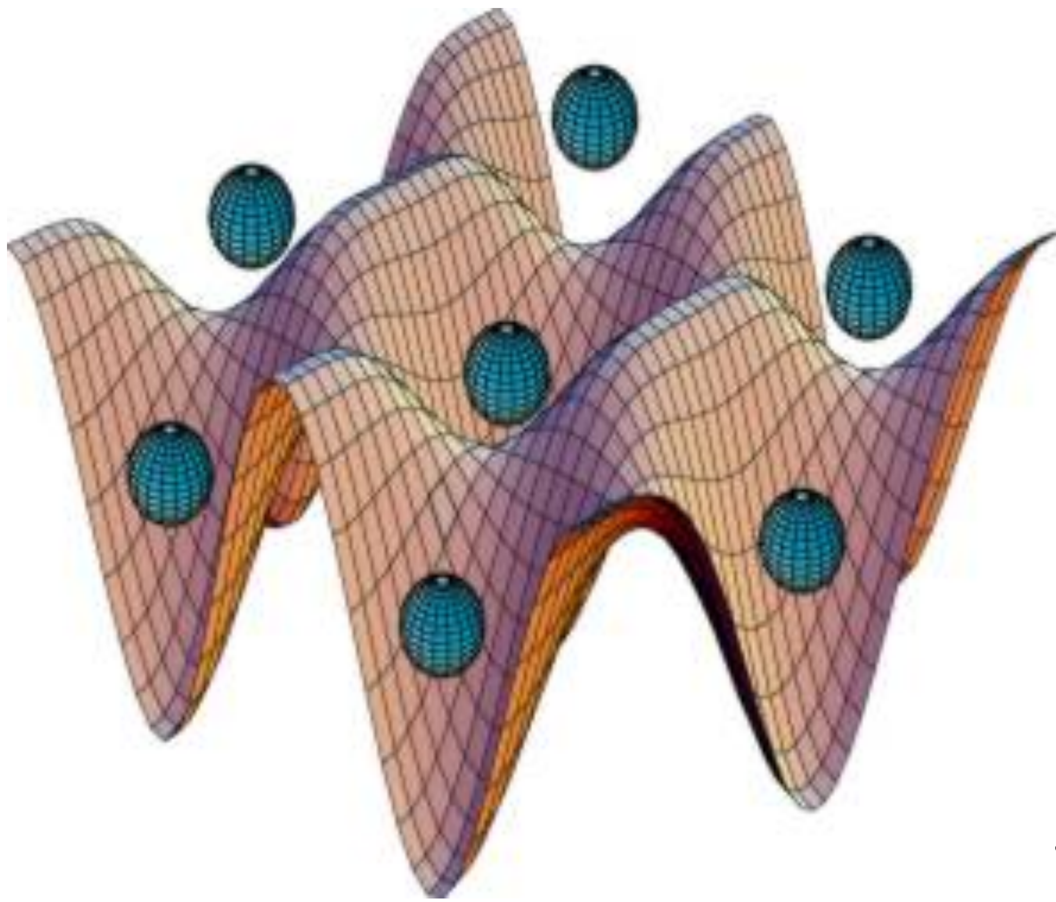
$$H_{\text{micro}} = U\left[\sum_i \left(c_x^{i\dagger} c_x^i + c_y^{i\dagger} c_y^i\right) - n\right]^2 - \tilde{t}\sum_i \left(\psi_x^{i\dagger} c_x^i + c_y^{i\dagger}\psi_y^i\right) + \text{h.c.}$$



$$H_{\text{eff}} = -t\sum_{\langle x,y\rangle}\left[\left(\sum_i \psi_x^{i\dagger} c_x^i\right)\left(\sum_j c_y^{j\dagger}\psi_y^j\right) + \text{h.c.}\right] + \cdots$$

Observability of phenomena

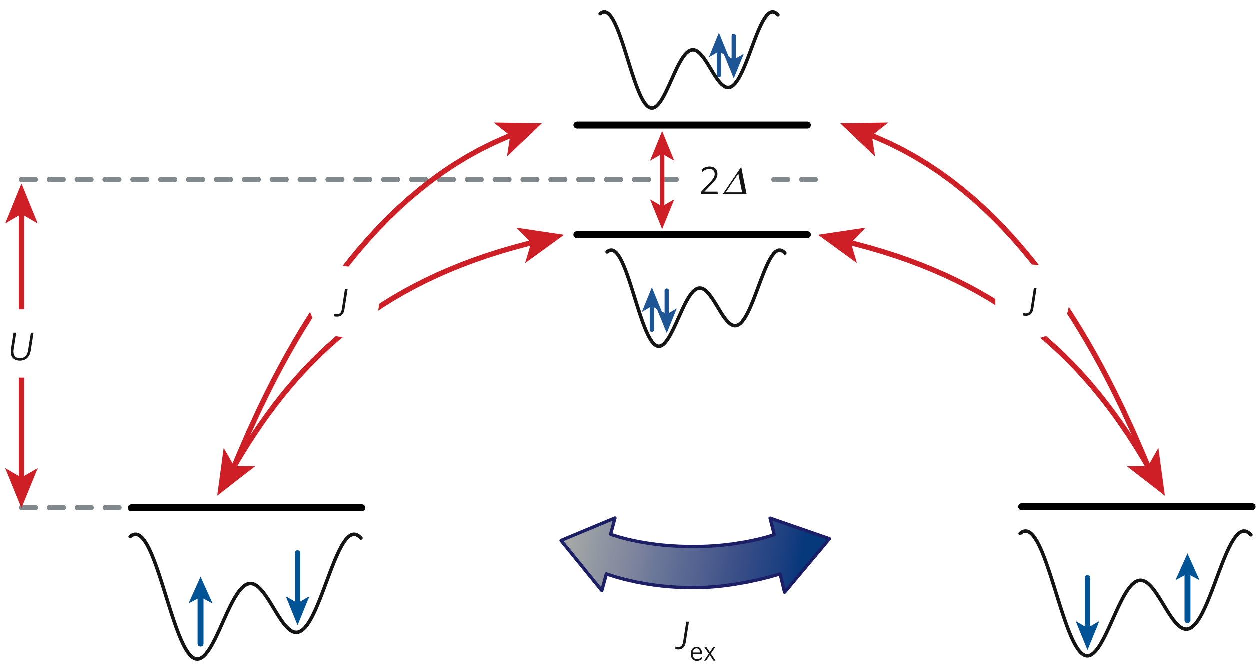
Preparation of many body states (Mott phase)



Greiner et al. (2002)
Joerdens et al. (2008)
Schneider et al. (2008)

Observability of phenomena

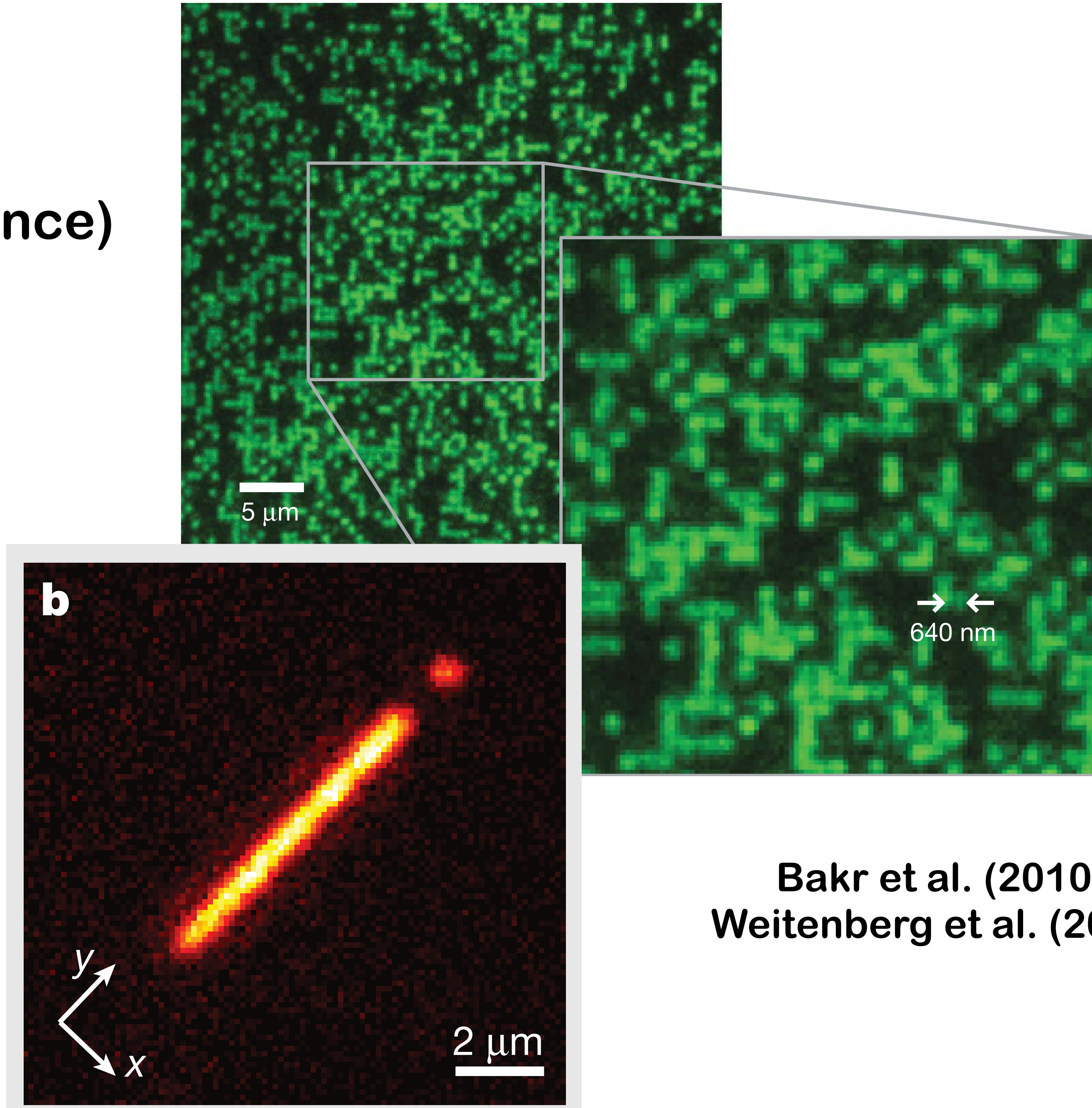
Evolution
(Super-exchange)



Anderlini et al. (2007)
Trotzky et al. (2008)

Observability of phenomena

Detection
(Single-site fluorescence)



Bakr et al. (2010)
Weitenberg et al. (2011)

Conclusions

Simpler atomic/molecular/solid state implementations (not in the talk: QLM with magnetic atoms/polar molecules!)?

Superconducting Qubits (D. Marcos,...), Dipoles and Rydberg (A. Glaetzle, ...)

Connection with gauge magnets and spin liquids (in principle accessible within this toolbox)

Finite-temperature confinement/deconfinement phase transition, deconfined criticality in 'feasible quantum link'?

Still very far away from QCD (even with the SU(3)): next steps?

The team

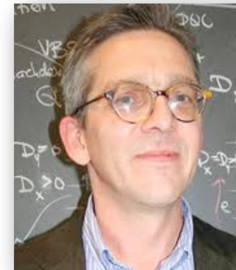
- **Albert Einstein Center - Bern University**

D. Banerjee

M. Bögli

P. Stebler

P. Widmer



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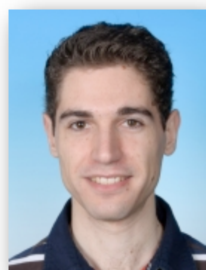
M. Müller

- **Technical University Vienna**

P. Rabl

- **IQOQI - Innsbruck University**

M. Dalmonte



D. Marcos

P. Zoller