

$$\text{rank}_{\text{puri}}(p') \leq O(D \log(1/\epsilon))$$