

$$\mathrm{rank}_{\mathrm{puri}}(\sigma_k) \leq O(D \log(1/\epsilon))$$