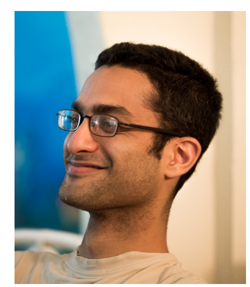
No ψ-epistemic model can fully explain the indistinguishability of quantum states

Eric Cavalcanti
QIP 2014
Barcelona

Jonathan Barrett



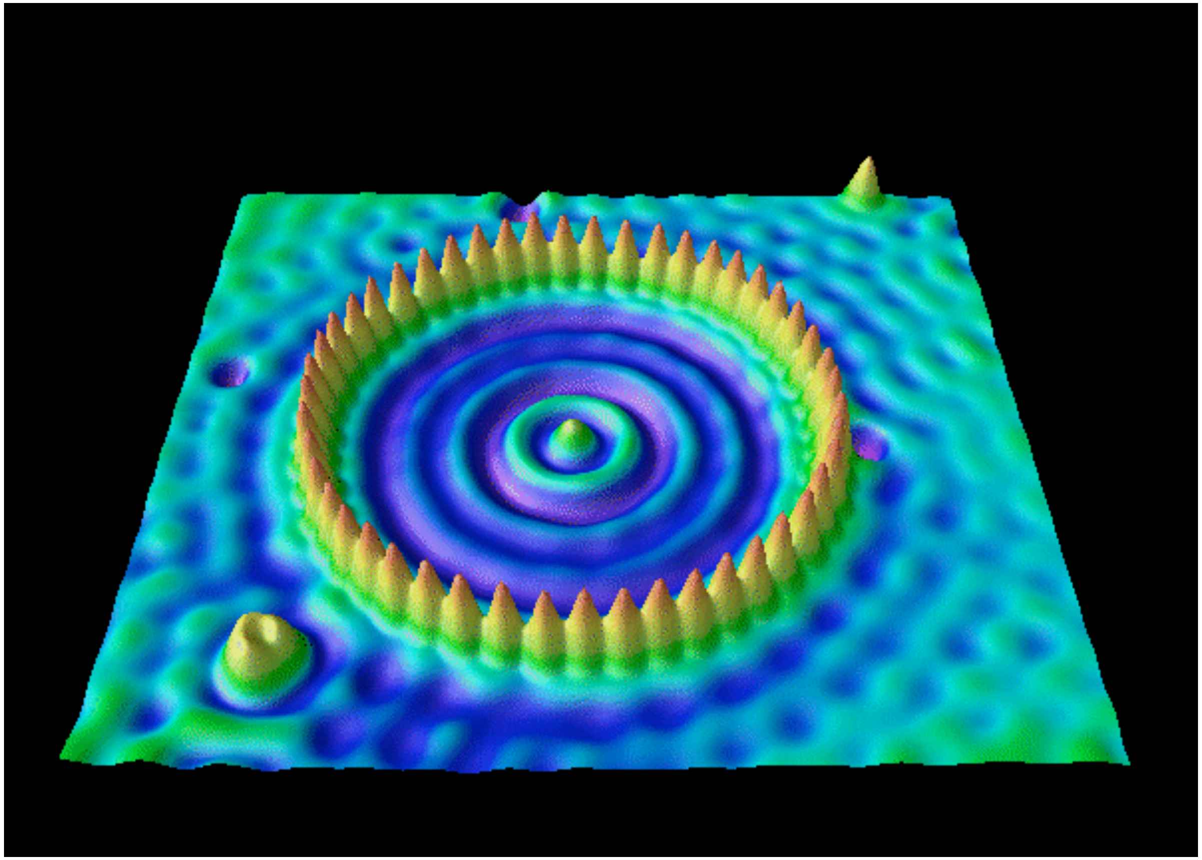
Raymond Lal

Owen Maroney

ψ

$$\psi ?$$

Ontic?



Epistemic?

"There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature..." – Niels Bohr

Arguments for ψ being epistemic

- Non-orthogonal quantum states cannot reliably be distinguished – just like probability distributions.

- The information required to specify a quantum state is exponential in the number of systems – just like probability distributions.

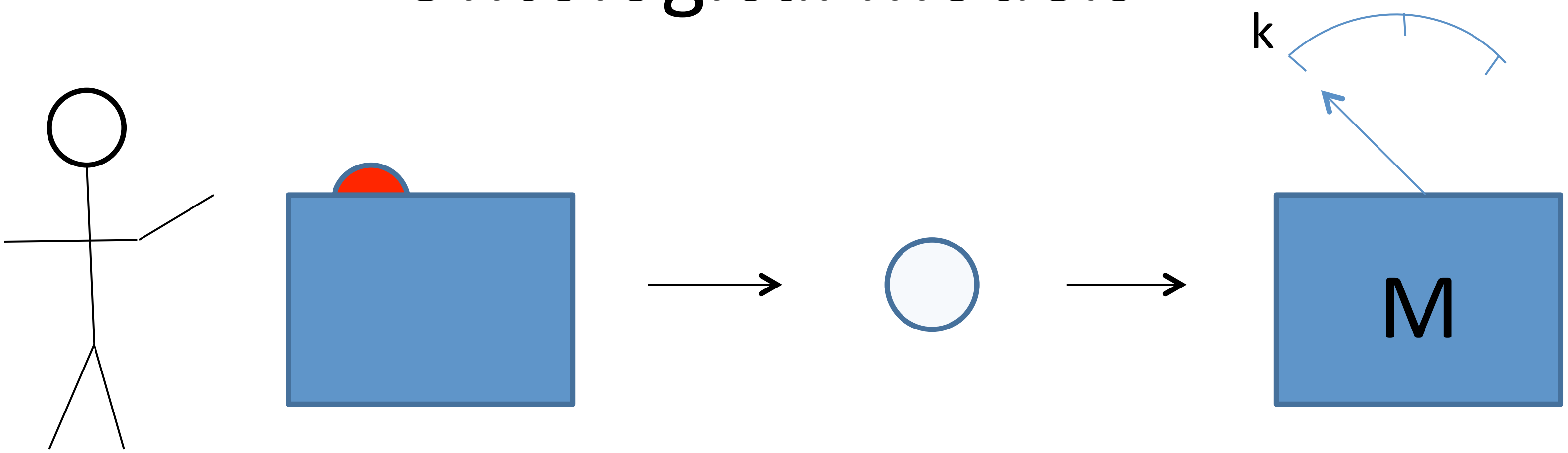
- Quantum states cannot be cloned, can be teleported etc – just like probability distributions.

- R. W. Spekkens, Phys. Rev. A 75, 032110 (2007).

But our present QM formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature --- all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. **For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple.**

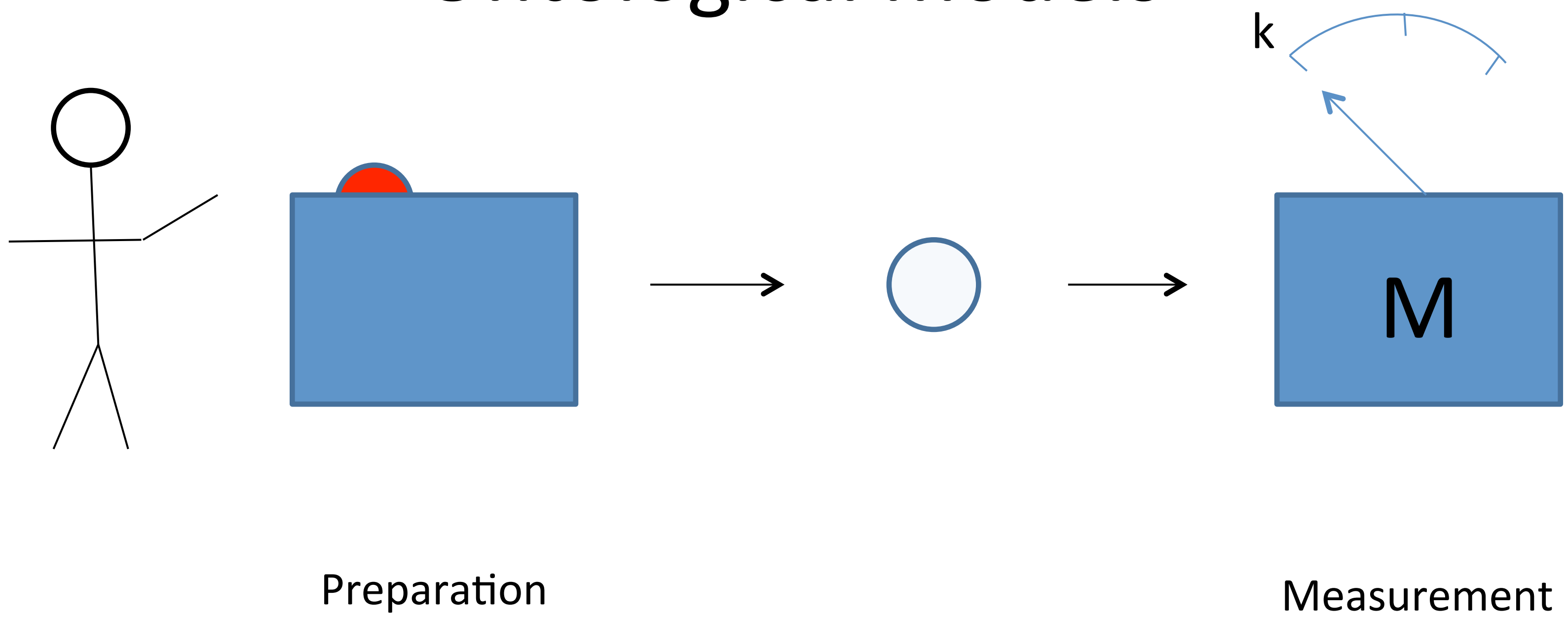
E. T. Jaynes

Ontological models*

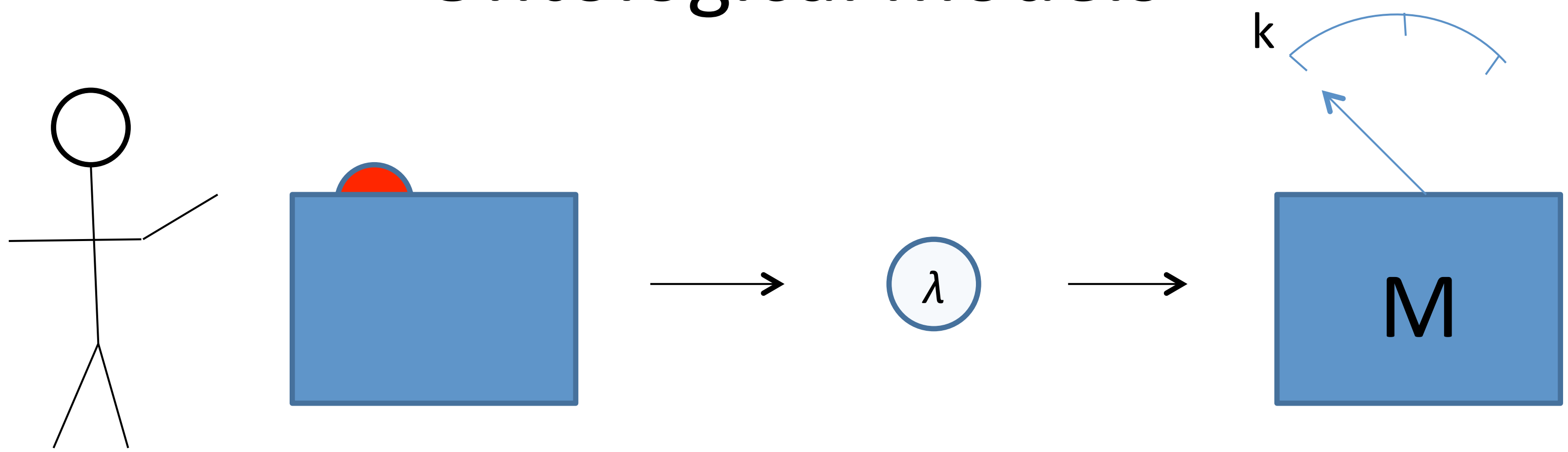


*à la Harrigan, Rudolph, Spekkens

Ontological models

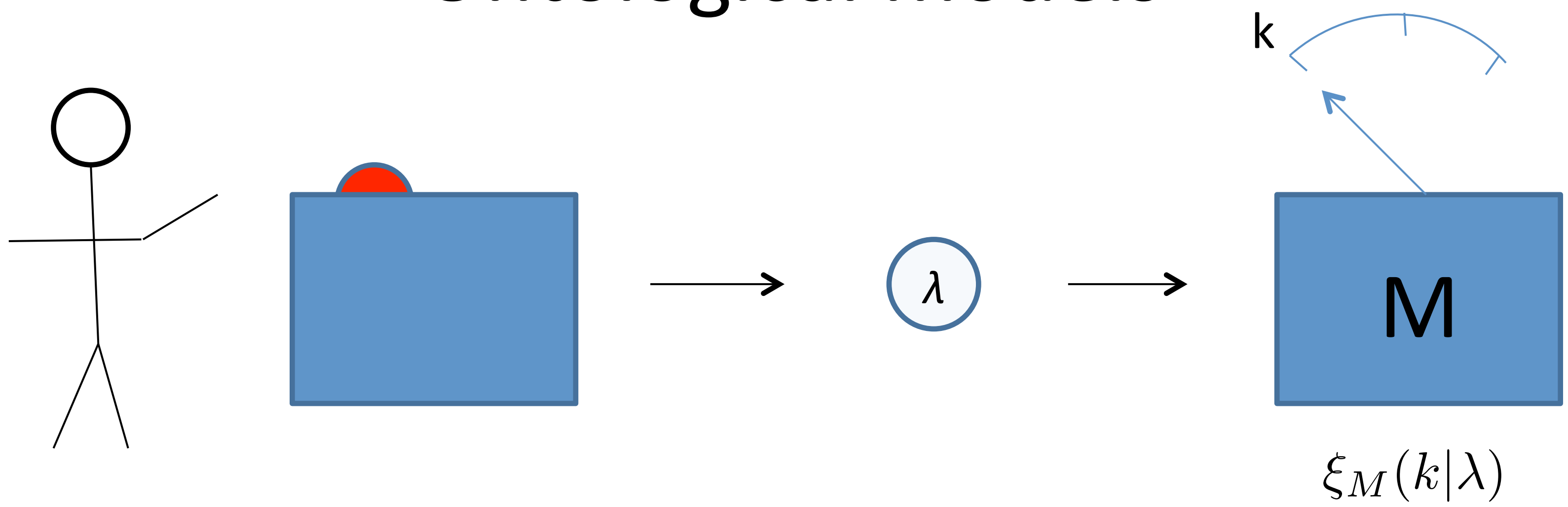


Ontological models



A physical system has an *ontic state* -- an objective physical state, independent of the experimenter, and independent of which measurement is performed. Call this state λ, and the set in which it lives, Λ.

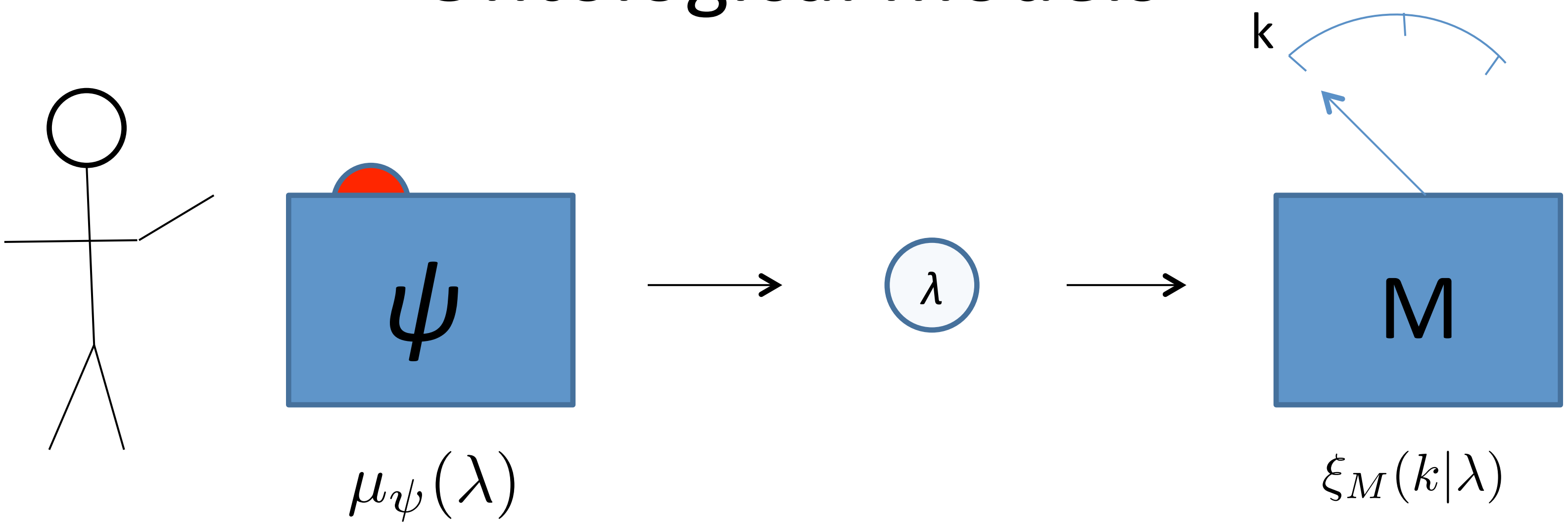
Ontological models



Measurement responds to the physical state. The probability for outcome *k* of a measurement *M* is determined by *λ* through a *response function ξ*.

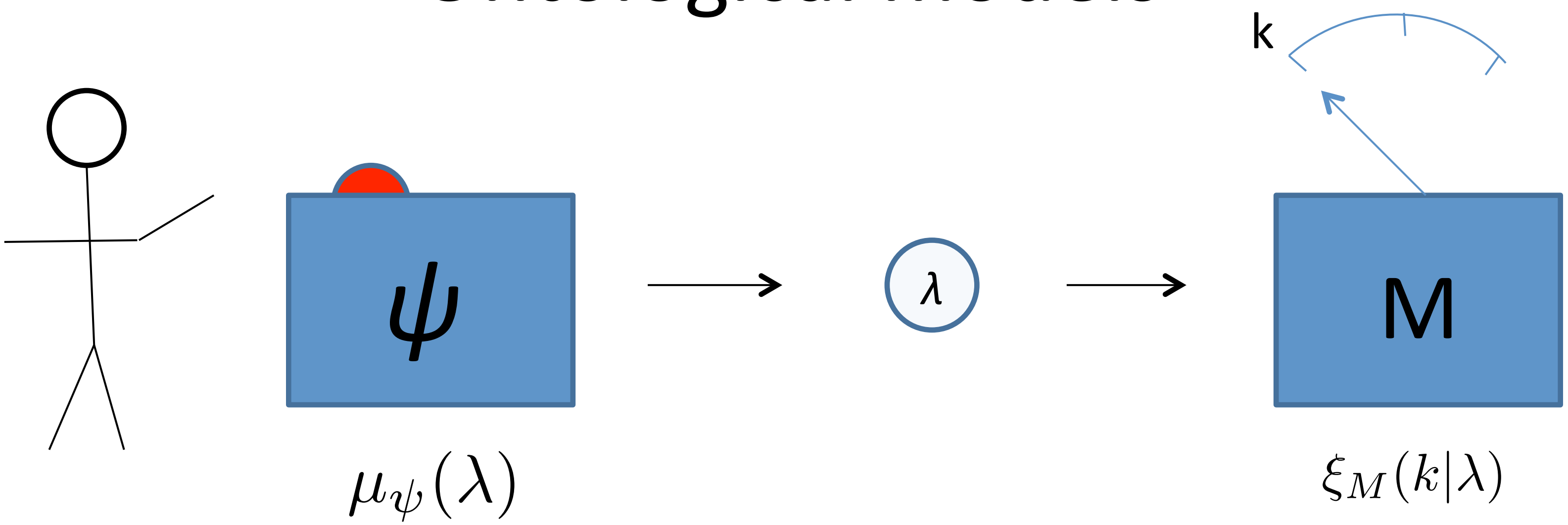
$$Pr(k|\lambda, M) \equiv \xi_M(k|\lambda)$$

Ontological models



A quantum state ψ is associated to a preparation procedure. Given knowledge of ψ, the experimenter's information about the ontic states is represented as a probability distribution μ_ψ.

Ontological models

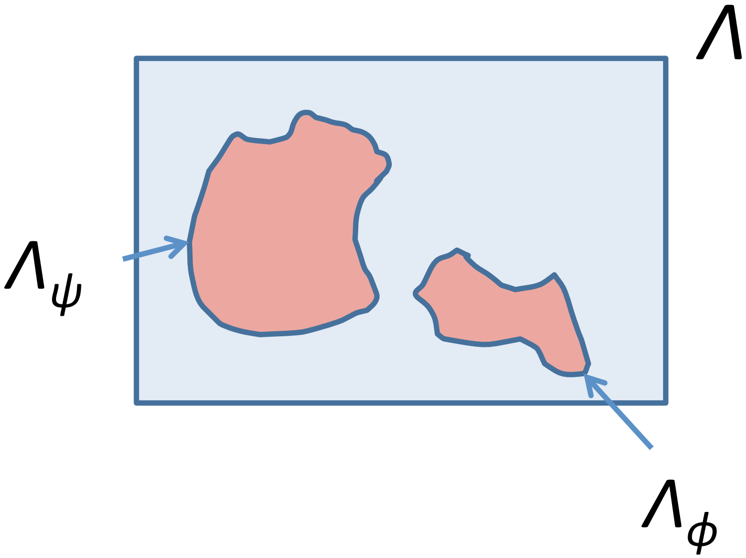


Recover quantum predictions:

$$|\langle k|\psi\rangle|^2 = \int_\Lambda \xi_M(k|\lambda)\mu_\psi(\lambda)d\lambda$$

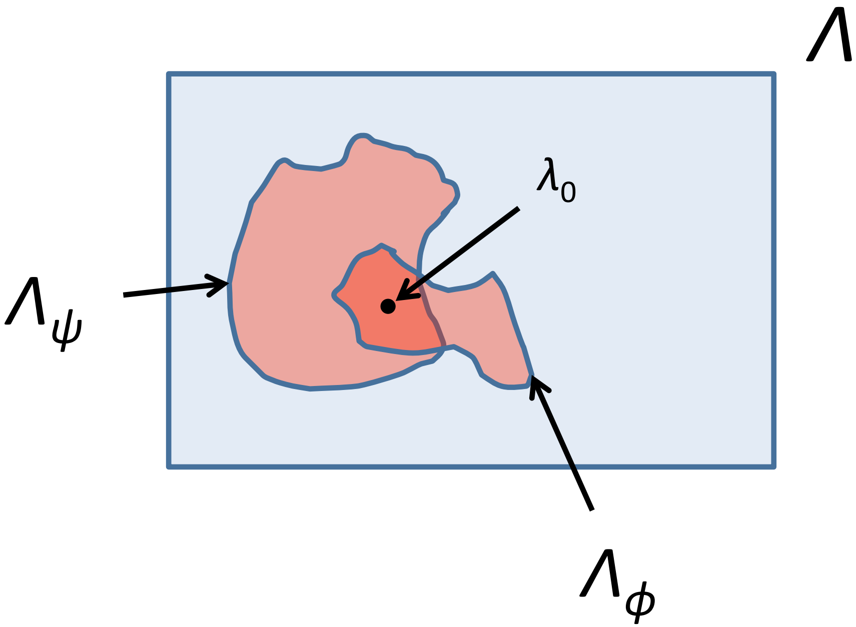
ψ-ontic models

Suppose that for every pair of distinct quantum states ψ and ϕ, the distributions μ_ψ and μ_ϕ do not overlap:



- The quantum state can be inferred from the ontic state.
- The quantum state is a **physical property** of the system, and is not mere information.

ψ-epistemic models



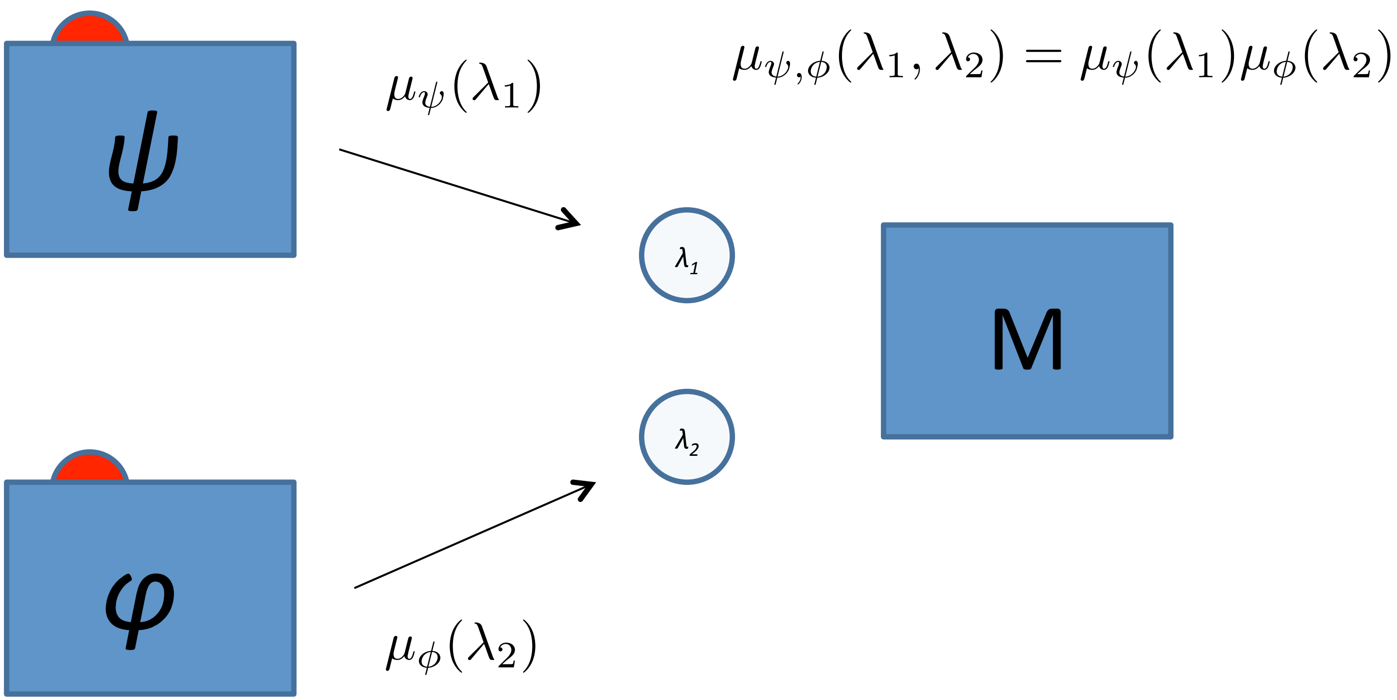
- μ_ψ and μ_ϕ can overlap.
- Given the ontic state λ_0 above, cannot infer whether the quantum state ψ or ϕ was prepared.

The PBR theorem

- Pusey, Barrett, Rudolph, Nature Physics (2012).
- Given an assumption about independent preparations, no psi-epistemic model can reproduce the predictions of quantum theory.

However…

- The theorem requires two systems and requires an assumption that independent preparations are associated with independent distributions.



However…

- Independence ≈ locality?

  Schlosshauer and Fine, arxiv:1306.5805
  Emerson *et al*, arXiv:1312.1345;

  - Since, given Bell's theorem, any ontological model of quantum theory violates (some form of) locality, we cannot dismiss psi-epistemic theories on that basis.

- What can we say about psi-epistemic models without the independence assumption?

Results for single systems

There exist psi-epistemic models

- Explicit psi-epistemic construction: Lewis, Jennings, Barrett and Rudolph, PRL (2012).

- But μ_ψ and μ_ϕ only overlap only for *some* pairs of quantum states. It is a rather trivial psi-epistemic model.

Aaronson *et al.* (arXiv:1303.2834)

- *Maximal non-triviality:* μ_ψ overlaps with μ_φ for *any* pair of non-orthogonal state vectors $|\psi>$ and $|\phi>$.

- *Symmetry:* the ontic states λ are quantum states and $\mu_\psi(\lambda)$ is symmetric under unitaries that fix $|\psi>$, *i.e.* it depends only on $|<\psi|\lambda>|$.

Results:

- Provide an explicit construction of a maximally non-trivial model, but;

- Prove that there are no maximally non-trivial symmetric models for $d \geq 3$.

Hardy, arXiv:1205.1439

- *Ontic indifference:* "Any quantum transformation on a system which leaves unchanged any given pure state, $|\psi>$, can be performed in such a way that it does not affect the underlying ontic states $\lambda \in \Lambda_\psi$ in the ontic support of that pure state."

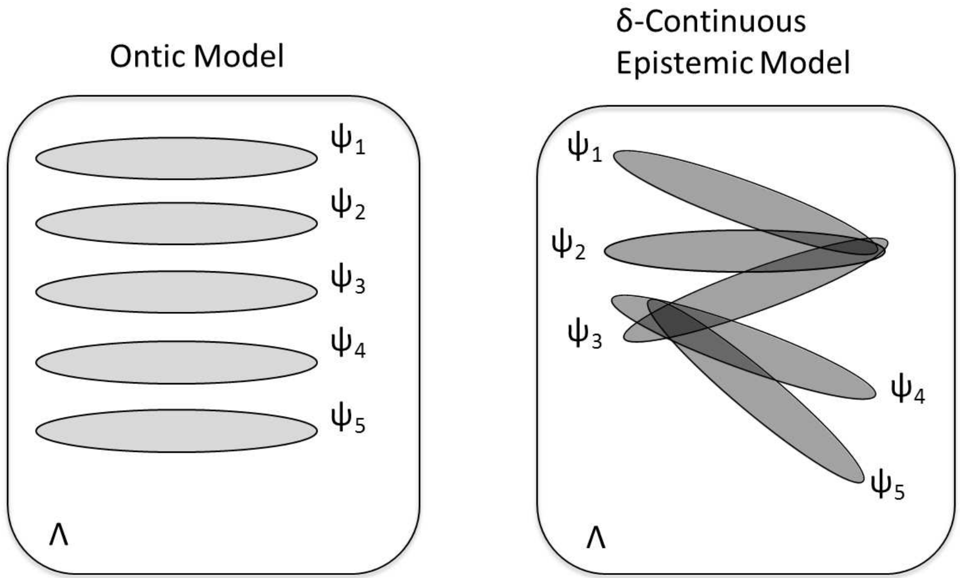
Results:

- Spekkens' toy model violates ontic indifference;

- Theorem: ontic indifference must be violated by any psi-epistemic model of quantum theory.

Patra , Pironio and Massar, PRL 111, 090402 (2013)

- *δ-continuity*: "states that are close to each other ($|\langle\varphi|\psi\rangle| \geq 1 - \delta$) all share common ontic states."



Theorem: "There are no *δ*-continuous models with $\delta \geq 1 - \sqrt{(d-1)/d}$ reproducing the measurement statistics of quantum states in a Hilbert space of dimension *d*."

Patra , Pironio and Massar, PRL 111, 090402 (2013)

Definition **(Continuity)** . A **ψ**-epistemic model is continuous if there exists a non-zero **$\delta > 0$** such that it is **δ**-continuous.

Definition **(Separability)** . Let Q be the preparation of a physical system yielding with non-zero probability $P(\lambda|Q) > 0$ the real state λ. A model is separable if n independent copies $Qn = (Q, \ldots, Q)$ of the preparation devices yield with non-zero probability $P(\lambda = \lambda n|Qn) > 0$ a system in the joint real state λn $= (\lambda, \ldots, \lambda)$, for any positive integer n.

Theorem 2. Separable continuous **ψ**-epistemic models cannot reproduce the measurement statistics of quantum states in a Hilbert space of dimension $d \geq 3$.

What have we learned so far?

- Psi-epistemic models can be constructed, but they must violate a number of different assumptions.
- What can be said about psi-epistemic models *without* any extra assumptions?
- Since psi-epistemic models *can* be constructed, the best one can do is put a *bound* on how much the epistemic distributions overlap.

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- R. W. Spekkens, Phys. Rev. A 75, 032110 (2007).

Distinguishing probability distributions

Consider two preparations of a classical system, corresponding to distributions:

$p = (p_1 , p_2 , \ldots , p_d)$
$q = (q_1 , q_2 , \ldots , q_d)$

A priori probability for each preparation is ½.

With a single-shot measurement on the system, must guess which preparation method was used.

$$\text{Prob(guess correctly)} = \frac{1}{2}(1 + D(p,q))$$

where D(p,q) is the classical trace distance between p and q,

$$D(p,q) = \sum_i |p_i - q_i|$$

Distinguishing quantum states

Consider two preparations of a quantum system, corresponding to state vectors:

$|\psi>$
$|\phi>$

A priori probability for each preparation is ½.

With a single-shot measurement on the system, must guess which preparation method was used. With an optimal measurement:

$$\text{Prob(guess correctly)} = \frac{1}{2}(1 + D_Q(|\psi\rangle, |\phi\rangle))$$

where $D_Q(|\psi> , |\phi>)$ is the quantum trace distance between $|\psi>$ and $|\phi>$,

$$D_Q(|\phi\rangle, |\psi\rangle) = \sqrt{1 - |\langle\phi|\psi\rangle|^2}.$$

A simple Lemma

In any ontological model that reproduces the predictions for a *d* dimensional quantum system:

$$D(\mu_\phi, \mu_\psi) \geq D_Q(|\phi\rangle, |\psi\rangle) \qquad \forall |\phi\rangle, |\psi\rangle$$

Proof sketch

Since the measurement device only has access to λ, distinguishing between $|\psi>$ and $|\phi>$ must be at least as hard as distinguishing between μ_ψ and μ_ϕ .

A simple Lemma

In any ontological model that reproduces the predictions for a *d* dimensional quantum system:

$$D(\mu_\phi, \mu_\psi) \geq D_Q(|\phi\rangle, |\psi\rangle) \qquad \forall |\phi\rangle, |\psi\rangle$$

An ontological model is *maximally psi-epistemic* iff

$$D(\mu_\phi, \mu_\psi) = D_Q(|\phi\rangle, |\psi\rangle) \qquad \forall |\phi\rangle, |\psi\rangle$$

Maximally ψ-epistemic models

An ontological model is *maximally psi-epistemic* iff

$$D(\mu_\phi, \mu_\psi) = D_Q(|\phi\rangle, |\psi\rangle) \qquad \forall |\phi\rangle, |\psi\rangle$$

Why is this natural?

- In a maximally psi-epistemic model, failure to distinguish $|\psi>$ and $|\phi>$ is *entirely due* to the ordinary classical difficulty in distinguishing the probability distributions μ_ψ and μ_ϕ. No other limitations or uniquely quantum effects need be invoked.

Maximally ψ-epistemic models

- Prior work:

OJE Maroney, arXiv:1207.6906
MS Leifer, OJE Maroney, Physical Review Letters (2013)

- The definition of maximally epistemic used in those results, however, suffers from a finite precision problem.

- With our improved definition, maximally epistemic models can be experimentally ruled out for all $d \geq 3$, and the gap between maximally epistemic theories and quantum theory grows with the dimension of the Hilbert space.

This will be useful in the following:

Define the *classical overlap* $\omega(\mu_\psi, \mu_\phi) = 1 - D(\mu_\psi, \mu_\phi)$

Similarly the *quantum overlap* $\omega_Q(|\psi\rangle, |\mu_\rangle) = 1 - D_Q(|\psi\rangle, |\phi\rangle)$

In any ontological model that reproduces the predictions for a *d* dimensional quantum system:

$$\omega(\mu_\psi, \mu_\phi) \leq \omega_Q(|\psi\rangle, |\phi\rangle) \quad \forall\ |\psi\rangle, |\phi\rangle$$

An ontological model is *maximally psi-epistemic* iff

$$\omega(\mu_\psi, \mu_\phi) = \omega_Q(|\psi\rangle, |\phi\rangle) \quad \forall\ |\psi\rangle, |\phi\rangle$$

Our main results

Theorem 1

No maximally psi-epistemic model can recover the quantum predictions in $d \geq 3$.

Theorem 2

Consider an ontological model that reproduces quantum predictions in power prime dimension d and satisfies:

$$\omega(\mu_\phi, \mu_\psi) \geq \alpha \; \omega_Q(|\phi\rangle, |\psi\rangle)) \qquad \forall |\phi\rangle, |\psi\rangle$$

Then $\alpha < 2/d$.

Simple proof of theorem 1 (d≥4)

Suppose for some three states ϕ_a, ϕ_b, ϕ_c, there exists a measurement M:

	M $\lvert q_1\rangle$	$\lvert q_2\rangle$	$\lvert q_3\rangle$
$\lvert\phi_a\rangle$	0	$p(q_2\lvert\phi_a)$	$p(q_3\lvert\phi_a)$
$\lvert\phi_b\rangle$	$p(q_1\lvert\phi_b)$	0	$p(q_3\lvert\phi_a)$
$\lvert\phi_c\rangle$	$p(q_1\lvert\phi_c)$	$p(q_2\lvert\phi_c)$	0

Caves, Fuchs, Schack (2002): such a basis exists whenever

$$x_{ab}+x_{bc}+x_{ca}\leqslant 1 \qquad (x_{ab}+x_{bc}+x_{ca}-1)^2\geqslant 4x_{ab}x_{bc}x_{ca} \qquad x_{ab}=\left|\langle\phi_a|\phi_b\rangle\right|^2$$

If ϕ_a, ϕ_b, ϕ_c, are drawn from three Mutually Unbiased Bases $x=1/d$

$$\frac{3}{d}\leqslant 1 \qquad \left(\frac{d-3}{d}\right)^2\geqslant\frac{4}{d^3}$$

$$\Rightarrow \quad d\geqslant 4$$

In any prime power dimension Hilbert space there are $d+1$ such MUB's

Simple proof of theorem 1 (d≥4)

Suppose ϕ_a, ϕ_b, ϕ_c, are from any three MUBs in a Hilbert space of dimension $d>3$.

There exists a measurement M:

	M		
	$\lvert q_1\rangle$	$\lvert q_2\rangle$	$\lvert q_3\rangle$
$\lvert\phi_a\rangle$	0	$p(q_2\lvert\phi_a)$	$p(q_3\lvert\phi_a)$
$\lvert\phi_b\rangle$	$p(q_1\lvert\phi_b)$	0	$p(q_3\lvert\phi_a)$
$\lvert\phi_c\rangle$	$p(q_1\lvert\phi_c)$	$p(q_2\lvert\phi_c)$	0

$$\forall\lambda \in \Lambda_{\phi_a}, Pr(q_1|M,\lambda) = 0$$
$$\forall\lambda \in \Lambda_{\phi_b}, Pr(q_2|M,\lambda) = 0$$
$$\forall\lambda \in \Lambda_{\phi_c}, Pr(q_3|M,\lambda) = 0$$

$$\forall\lambda, \sum_q Pr(q|M,\lambda) = 1$$

$$\Lambda_{\phi_a} \cap \Lambda_{\phi_b} \cap \Lambda_{\phi_c} = \emptyset$$

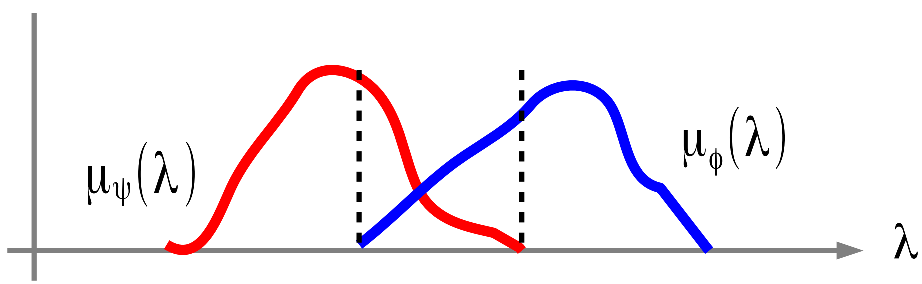
Simple proof of theorem 1 (d≥4)

Suppose ϕ_a, ϕ_b, ϕ_c, are from any three MUBs in a Hilbert space of dimension $d>3$.

$$\Lambda_{\phi_a} \cap \Lambda_{\phi_b} \cap \Lambda_{\phi_c} = \emptyset$$

It is also easy to show that for i≠j, $\Lambda_{\phi_{a_i}} \cap \Lambda_{\phi_{a_j}} = \emptyset$

And for any pair of distributions,



$$\int_{\Lambda_\phi} \mu_\psi(\lambda)\, \mathrm{d}\lambda \geq \int_\Lambda \min\{\mu_\psi(\lambda), \mu_\phi(\lambda)\}\, \mathrm{d}\lambda = \omega(\mu_\psi(\lambda), \mu_\phi(\lambda))$$

Simple proof of theorem 1 (d≥4)

Suppose ϕ_a, ϕ_b, ϕ_c, are from any three MUBs in a Hilbert space of dimension $d>3$.

$$\Lambda_{\phi_a} \cap \Lambda_{\phi_b} \cap \Lambda_{\phi_c} = \emptyset \qquad \Lambda_{\phi_{a_i}} \cap \Lambda_{\phi_{a_j}} = \emptyset$$

$$\int_{\Lambda_\phi} \mu_\psi(\lambda)\, \mathrm{d}\lambda \geq \omega(\mu_\psi(\lambda), \mu_\phi(\lambda))$$

Hence

$$\int_{\Lambda_{\phi_{a_1}} \cup \ldots \cup \Lambda_{\phi_{a_d}}} \mu_c(\lambda)\, \mathrm{d}\lambda \geq \sum_i \omega(\mu_{a_i}(\lambda), \mu_c(\lambda))$$

And assuming maximally epistemic model

$$\omega(\mu_{a_i}, \mu_c) = \omega_Q(|\phi_{a_i}\rangle, |\phi_c\rangle)$$

$$\int_{\Lambda_{\phi_{a_1}} \cup \ldots \cup \Lambda_{\phi_{a_d}}} \mu_c(\lambda)\, \mathrm{d}\lambda \geq d(1 - \sqrt{1 - 1/d}) > 0.5$$

Simple proof of theorem 1 (d≥4)

A similar argument shows that $$\int_{\Lambda_{\phi_{b_1}} \cup \ldots \cup \Lambda_{\phi_{b_d}}} \mu_c(\lambda)\,\mathrm{d}\lambda > 0.5$$

And thus $$\int_{\Lambda_{\phi_{a_1}} \cup \ldots \cup \Lambda_{\phi_{a_d}}} \mu_c(\lambda)\,\mathrm{d}\lambda + \int_{\Lambda_{\phi_{b_1}} \cup \ldots \cup \Lambda_{\phi_{b_d}}} \mu_c(\lambda)\,\mathrm{d}\lambda > 1$$

But since we established that $\forall\, i, j,\ \Lambda_{\phi_{a_i}} \cap \Lambda_{\phi_{b_j}} \cap \Lambda_{\phi_c} = \emptyset$

$$\int_{\Lambda_{\phi_{a_1}} \cup \ldots \cup \Lambda_{\phi_{a_d}}} \mu_c(\lambda)\,\mathrm{d}\lambda + \int_{\Lambda_{\phi_{b_1}} \cup \ldots \cup \Lambda_{\phi_{b_d}}} \mu_c(\lambda)\,\mathrm{d}\lambda \leq \int_{\Lambda} \mu_c(\lambda)\,\mathrm{d}\lambda = 1$$

➔ *contradiction*

Additionally

- A slightly more involved proof gives a noise-tolerant version of both Theorems 1 and 2 for $d \geq 4$. Maximally psi-epistemic models cannot *approximately* recover quantum predictions.

- A (messier) proof covers the d=3 case, and is also noise tolerant.

- An explicit maximally psi-epistemic model exists in d=2 (constructed by Kochen and Specker).

What have we learned?

- No ψ-epistemic model can fully explain the indistinguishability of quantum states – one of the main motivations for those theories.

- In the limit of large Hilbert spaces, the explanation becomes *arbitrarily* bad, as $\alpha < 2/d$.

  (Leifer, arXiv:1401.7996: *exponentially* bad. $\alpha < 2d\, e^{-cd}$)

NOT(psi-epistemic) ≠ psi-ontic

- **An analogy:** Bell's theorem shows that local causality is violated. If there are no hidden variables it is violated *trivially*. However, this does not mean that there is anything going faster than light, and there is an operational notion of locality (signal locality) that is still true.

- In PBR and here, if there are no ontic variables, then also *trivially* the quantum state cannot be a state of knowledge about them.

- But to conclude (with PBR) that the quantum state is *ontic* requires an assumption that the λ's *do* exist (and thus by the theorem, include ψ).

“Whose information? Information about what?”
– John Bell

Two epistemic camps:

1) About measurement outcomes.
2) About ontic variables.

"Whose information? Information about what?"
– John Bell

Two epistemic camps:

But: anti-realism, measurement problem…

1) About measurement outcomes.

2) ~~About ontic variables.~~

A common ground?

- Whether or not the quantum state corresponds to any type of observation-independent reality, it undeniably encodes information about measurement outcomes.

- Thinking operationally has been leading to progress in foundations.
  - e.g. reconstructions, generalised probabilistic theories.

- Hopefully these insights will lead to a better understanding of the type of reality underlying the theory, and progress in going beyond QM.

Why is Spekkens' toy theory (partially) succesfull?

- Our theorem implies that some limitation on the measurability of λ is necessary for any ontological model that reproduces quantum theory.

- Analogue of "knowledge balance principle" of Spekkens' toy theory.

- If the λ is "carried by the system", this limitation is mysterious.

- Operational analogues of this principle have been proposed [Rovelli (1996), Zeilinger (1999), Masanes & Mueller (2012)…].

"We can ask more questions about a quantum system than it can encode".

- Success of toy theory due to knowledge balance rather than psi-epistemicity?

Finally…

- Is this any *use* for anything?
- Apart from the foundational implications, these results are about the (im)possibilities of simulating quantum mechanics with certain classical models.
- Like Bell inequalities, is there any information-theoretic application of this new theorem?
  - Montina (PRL, 2012): quantum communication complexity?

Thank you!