Noisy Interactive Quantum Communication

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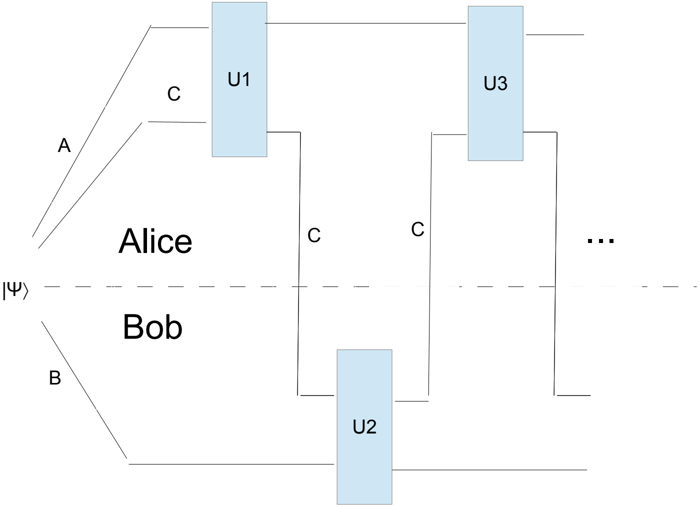
Problem

- Simulate *highly interactive* quantum protocols over *noisy* channels
  - Positive communication rate
  - Positive adversarial error rate



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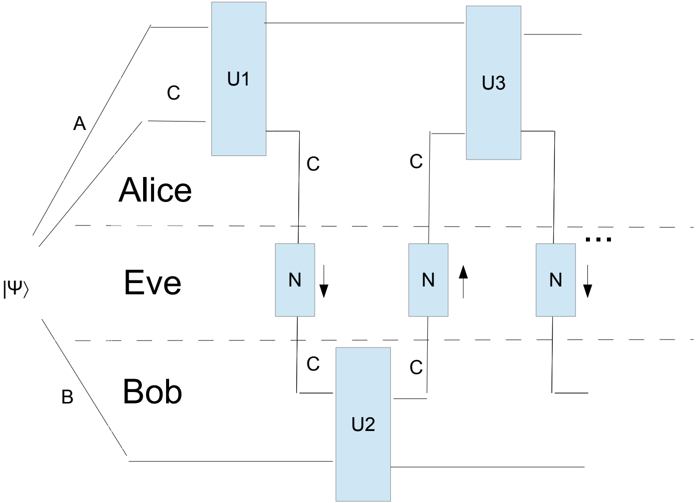
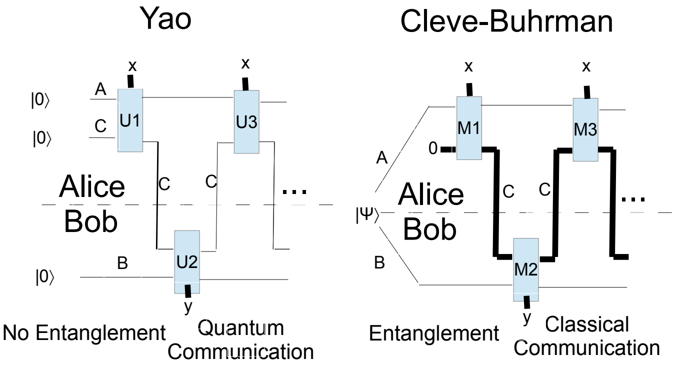


Noiseless Interactive Quantum Protocols

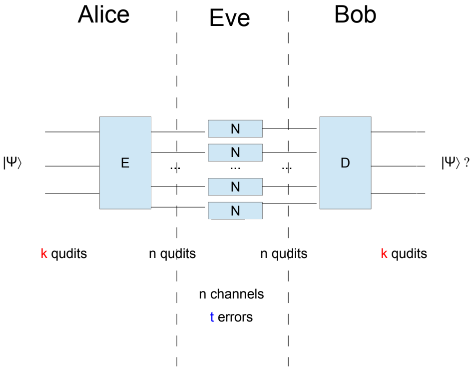
- Well-studied: Quantum communication complexity
  - 2 Models for computing classical $f : \mathcal{X} \times \mathcal{Y} \to \mathcal{Z}$



- Exponential separations in communication complexity
  - Classical vs. quantum
  - N-rounds vs. N+1-rounds

Noisy Quantum Communication

- Well-studied for unidirectional data transmission



- Quantum information theory: Random noise, à la Shannon
- Communication rate $R = k/n$
- Quantum coding theory: Adversarial noise, à la Hamming
- Error rate $\delta = t/n$

Noisy Interactive Quantum Communication

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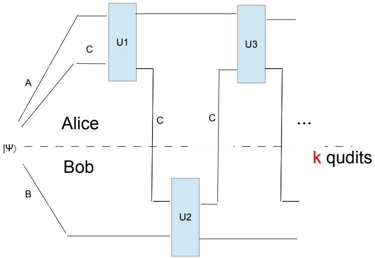
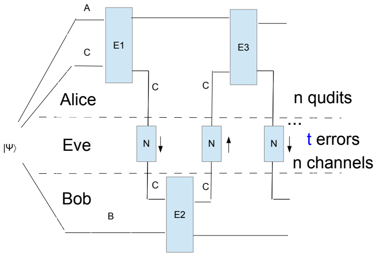
- Error rate $\delta = t/n$



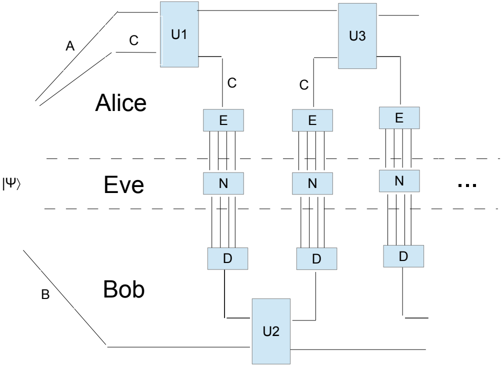
- Noiseless protocol



- Simulation protocol

Naive Strategy

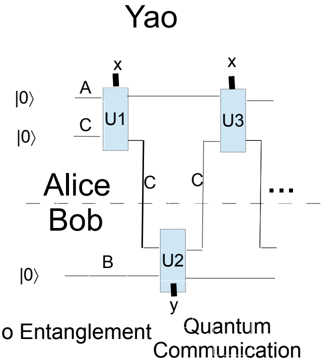
- Encode each transmission into a QECC

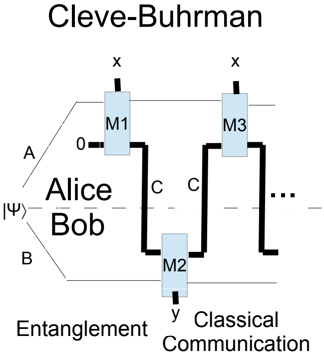


- Worst case interaction: 1 qubit communication
  - Random noise: communication rate → 0
  - Adversarial noise: tolerable error rate → 0
- Classical protocols: same problems but... [Schulman'96]
  Simulation protocols with *positive* communication and error *rates*

Problems for Quantum Simulation

- Classical information can be copied and resent if destroyed by noise
    - Yao model problem: no-cloning theorem
- Cleve-Buhrman model: communication is classical
    - Problem: quantum measurements are irreversible
- Can we do better than naive (block coding) strategy?





Noisy Communication Models

- Consider 3 distinct noisy communication models
- Noisy quantum communication, no shared entanglement
  - Noisy analogue to the Yao model
- Noisy classical communication, perfect shared entanglement
  - Noisy analogue to the Cleve-Buhrman model
- Noisy classical communication, noisy shared entanglement
  - Noisy EPR pairs (Werner states)

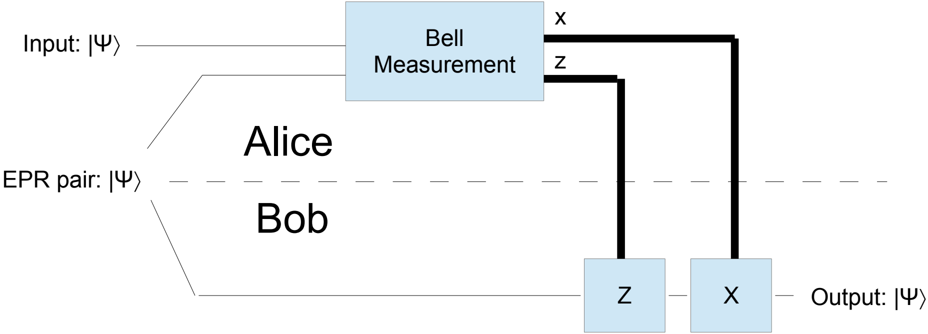
Results

- Simulations in all 3 models
- Positive communication rates
  - Yao model: $O(\frac{1}{Q})$ overhead over depolarizing channel
  - Cleve-Buhrman: $O(\frac{1}{C})$ overhead over binary symmetric channel
- Tolerate positive adversarial error rates
  - Yao model: $\frac{1}{6} - \epsilon$
  - Cleve-Buhrman model: $\frac{1}{2} - \epsilon$, *optimal*
- *First* interactive analogue of good quantum code

Results

- Noisy entanglement: Simulation for *any* non-separable Werner state
- Cleve-Buhrman model: $O(\frac{1}{C})$ overhead is *optimal*
- Yao model: $O(\frac{1}{Q})$ overhead is *not*
    - Simulation for some $Q = 0$ depolarizing channel!

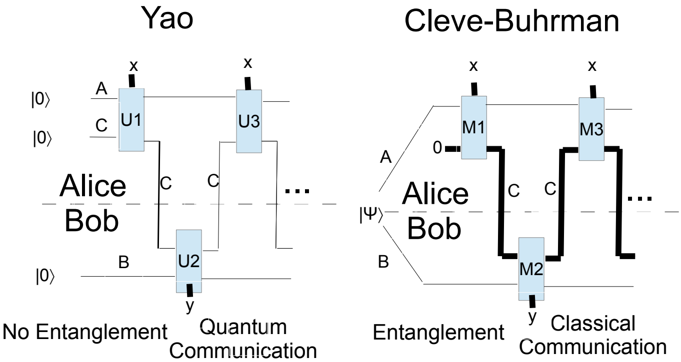
Main Ingredient : Teleportation Protocol



- State after Bell measurement: $X^x Z^z |\psi\rangle$
- Bob decodes with $Z^{z'} X^{x'}$
- Obtains $\pm X^{x+x'} Z^{z+z'} |\psi\rangle$
- Noisy classical communication → Pauli error on $|\psi\rangle$

Solutions to Quantum Simulation Problems

- Cleve-Buhrman model: Make everything coherent
  - Measurements → pseudo-measurements
- Yao model: Use teleportation to avoid losing quantum information
- Evolve sequence of noiseless unitaries
- Everything on joint register is a sequence of reversible operations



Yao

Cleve-Buhrman

No Entanglement

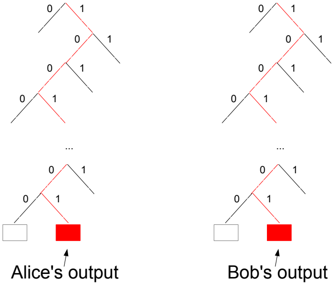
Quantum Communication

Entanglement

Classical Communication

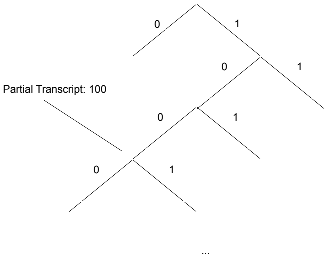
Quantum Simulation Protocol

- Yao: To distribute EPR pairs, use tools from quantum coding theory
- For interaction, use tools from classical interactive coding
- Can we use classical simulation protocols: No!
  - Classical goal: Alice and Bob agree on transcript
  - Here: Contains mostly random teleportation outcomes



Tools for Classical Simulation Protocols

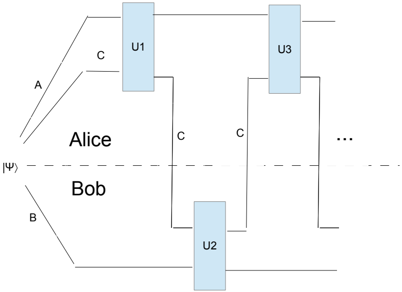
- Tree representation for communication protocols



- Tree codes
  - Online codes
  - Self-healing property
- Blueberry codes
  - Randomized error detection codes
- Classical strategy: Simulate evolution in protocol tree
  - Error → go back to last agreement point

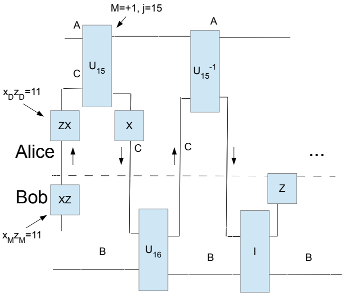
Further Problems for Quantum Simulation

- For quantum protocols, no protocol tree to synchronize on
    - Can still synchronize on sequential structure of quantum protocol
- Cannot restart with a copy of previous state (no-cloning)
    - Need to rewind unitaries, leading to more errors

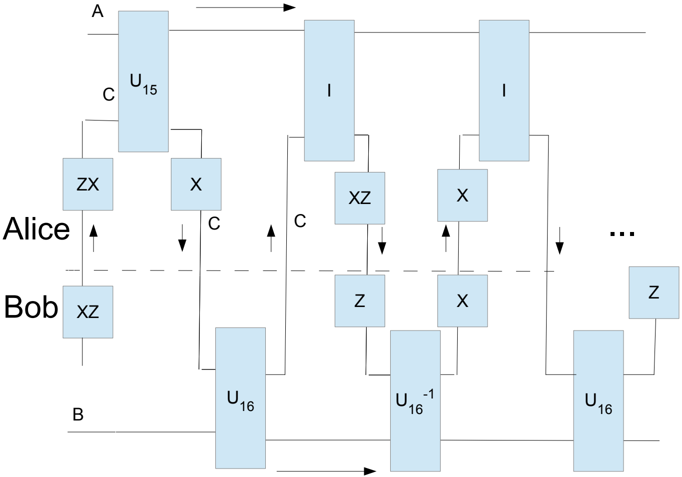


Classical Information Sent over Noisy Channel

- Teleportation measurement outcome : $x_M, z_M \in \{0,1\}$
- Teleportation decoding operation : $x_D, z_D \in \{0,1\}$
- Direction for evolution of noiseless protocol : $M \in \{-1,0,+1\}$
- Index of noiseless protocol unitary : $j \in [n+1]$
    - Implicit: $j_\ell = \sum_{i<\ell} 2M_i + M_\ell$ (+1 for Bob)



Example Run of Simulation Protocol



Conclusion : Summary

- Communication complexity robust under noisy communication
- Tolerate maximal error in perfect shared entanglement model
    - Requires new bound on tree codes
- Positive communication rates for some $Q = 0$ depolarizing channel
    - Separation between standard and interactive quantum capacity

Further Research Directions

- Adaptation of classical results to quantum realm
  - Computationally efficient protocols against adversarial noise
  - High communication rates for low random noise
- Upper bound on interactive quantum capacity
- Improve tolerable error rate in quantum model
  - Possibly by developing a fully quantum approach
  - Construction of quantum tree codes?
- Integration into larger fault-tolerant framework