Mapping out Scattering Amplitudes and Resonances using Lattice QCD

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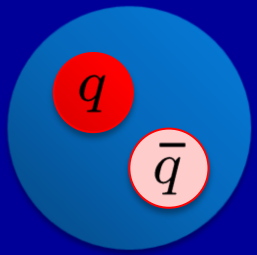
UNIVERSITY OF CAMBRIDGE

Hadron Spectrum Collaboration

Outline

- Introduction
- Resonances etc on the lattice
  - The ρ in isospin-1 $\pi\pi$ scattering
  - πK, ηK coupled-channel scattering
- Summary and outlook

Meson Spectroscopy



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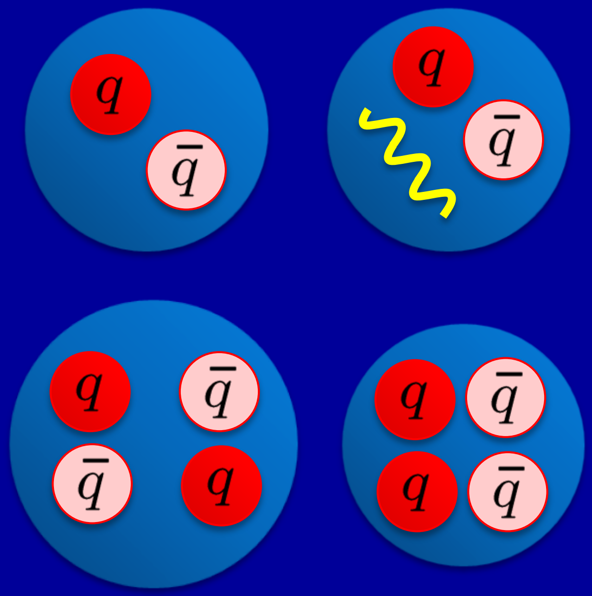

Exotic J^{PC} (**0^{--}, 0^{+-}, 1^{-+}, 2^{+-}**, ...) or flavour quantum numbers – can't just be a $q\bar{q}$ pair

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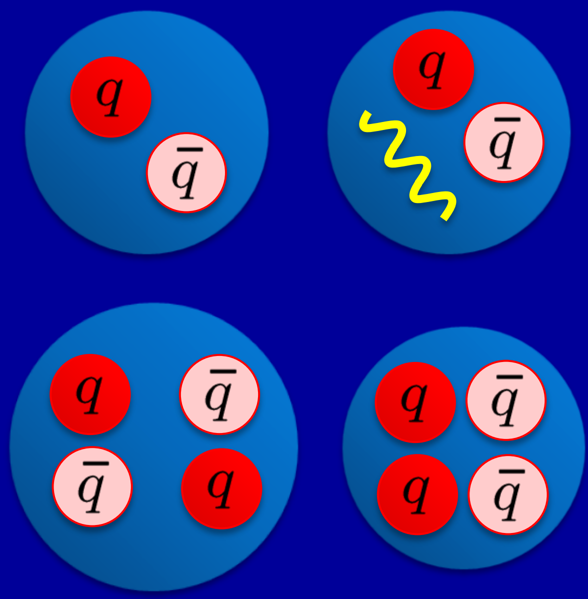


Exotic J^{PC} (**0**$^{--}$**, 0**$^{+-}$**, 1**$^{-+}$**, 2**$^{+-}$, ...)
or flavour quantum numbers
– can't just be a $q\bar{q}$ pair

BESIII

LHC

JLab @ 12 GeV

COMPASS

KLOE

CLAS12

GLUEX

$\bar{\text{p}}$anda

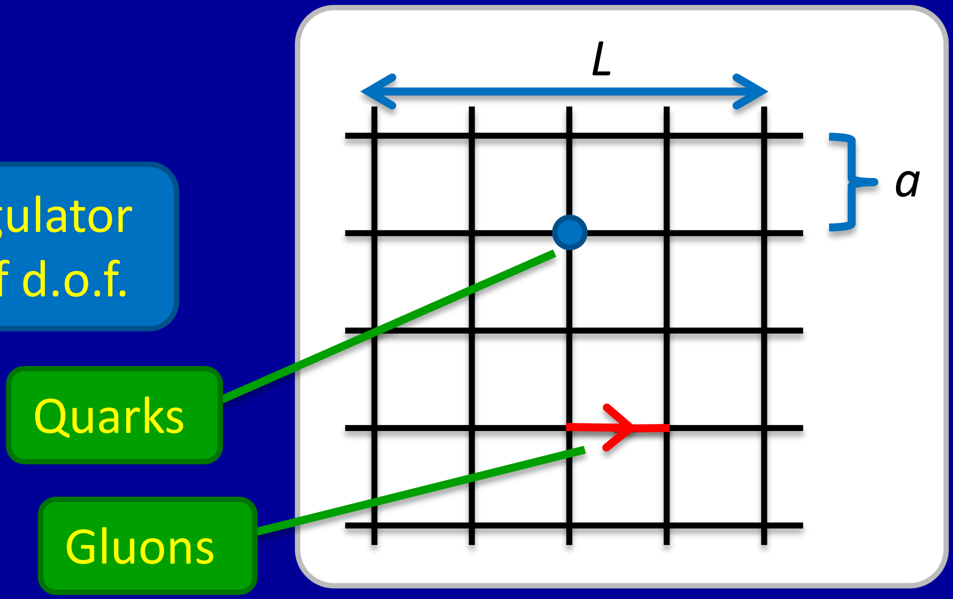
X(3872), *Y*(4260), Z^+(4430),
Z_c^+(3900), Z_b^+, D_s(2317),
light scalars, ... (also baryons)
Gluonic excitations?

Resonances or near threshold

Can we understand these within QCD? → lattice QCD

QCD on a lattice

Discretise (spacing = a) – regulator
Finite volume → finite no. of d.o.f.



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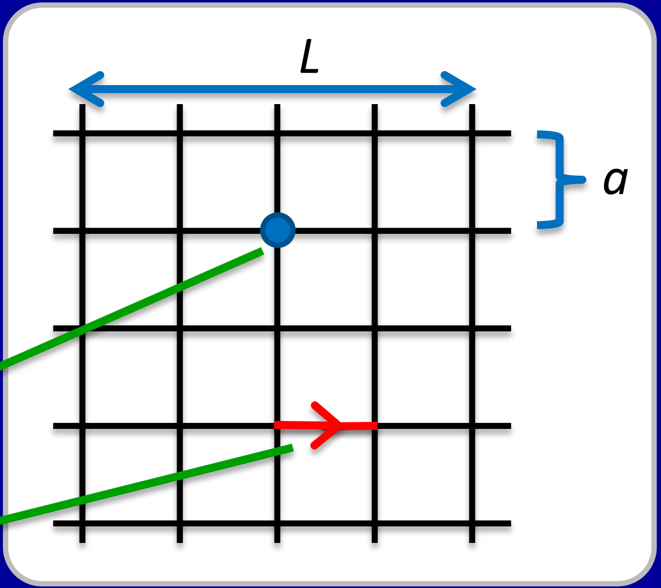

Path integral formulation
Euclidean time $t \rightarrow i\,t$

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \, f(\psi, \bar{\psi}, U) e^{-\tilde{S}[\psi,\bar{\psi},U]}$$

Numerical methods (Monte Carlo)

- Finite *a* and *L* (and reduced sym.)
- Unphysical m_π

Spectroscopy on the lattice

Energy eigenstates from 2-pt corrs.

$$C_{ij}(t) =< 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

Interpolating operators

$$\bar{\psi}\Gamma\psi$$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2E_n} < 0|\mathcal{O}_i(0)|n >< n|\mathcal{O}_j^\dagger(0)|0 >$$

Spectroscopy on the lattice

(our approach)

Energy eigenstates from 2-pt corrs.

$$C_{ij}(t) =< 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

$$O(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \; \bar{\psi}(x) \left[\Gamma \overleftrightarrow{D} \overleftrightarrow{D} \ldots\right] \psi(x)$$

Large no. of ops. with different structures

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Large basis of ops → matrix of corrs. – generalised eigenvalue problem

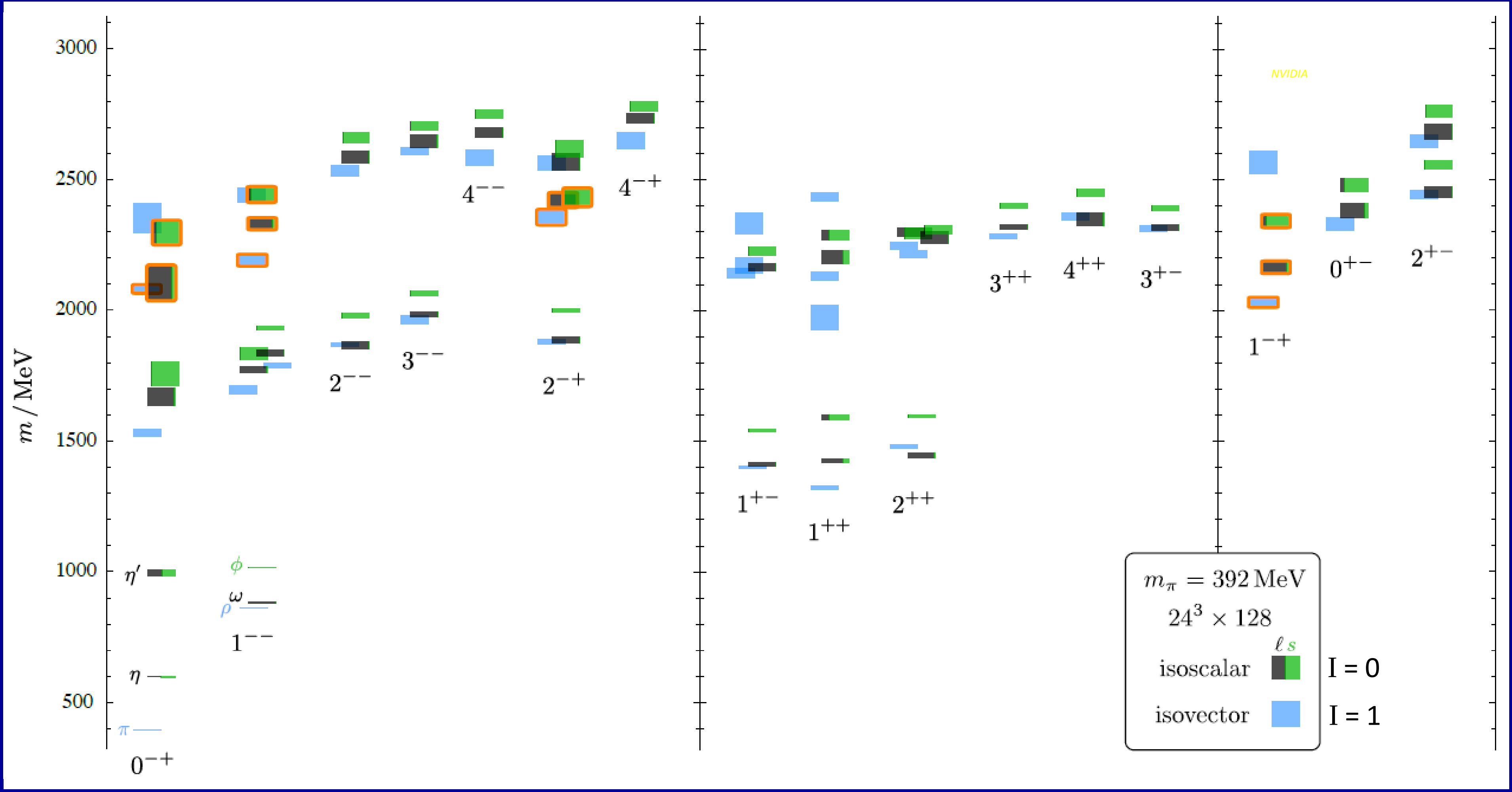
$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$

$$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)}$$

$$v_i^{(n)} \rightarrow Z_i^{(n)} \equiv < 0|\mathcal{O}_i|n >$$

($t >> t_0$)

Light mesons (isospin = 0 and 1)



[Dudek, Edwards, Guo, CT, PR D88, 094505 (2013); update of PR D83, 111502 (2011)]
Anisotropic Clover [N_f = 2+1], $a_s \approx$ 0.12 fm, $a_s/a_t \approx$ 3.5;   **+ other volumes and m_π**

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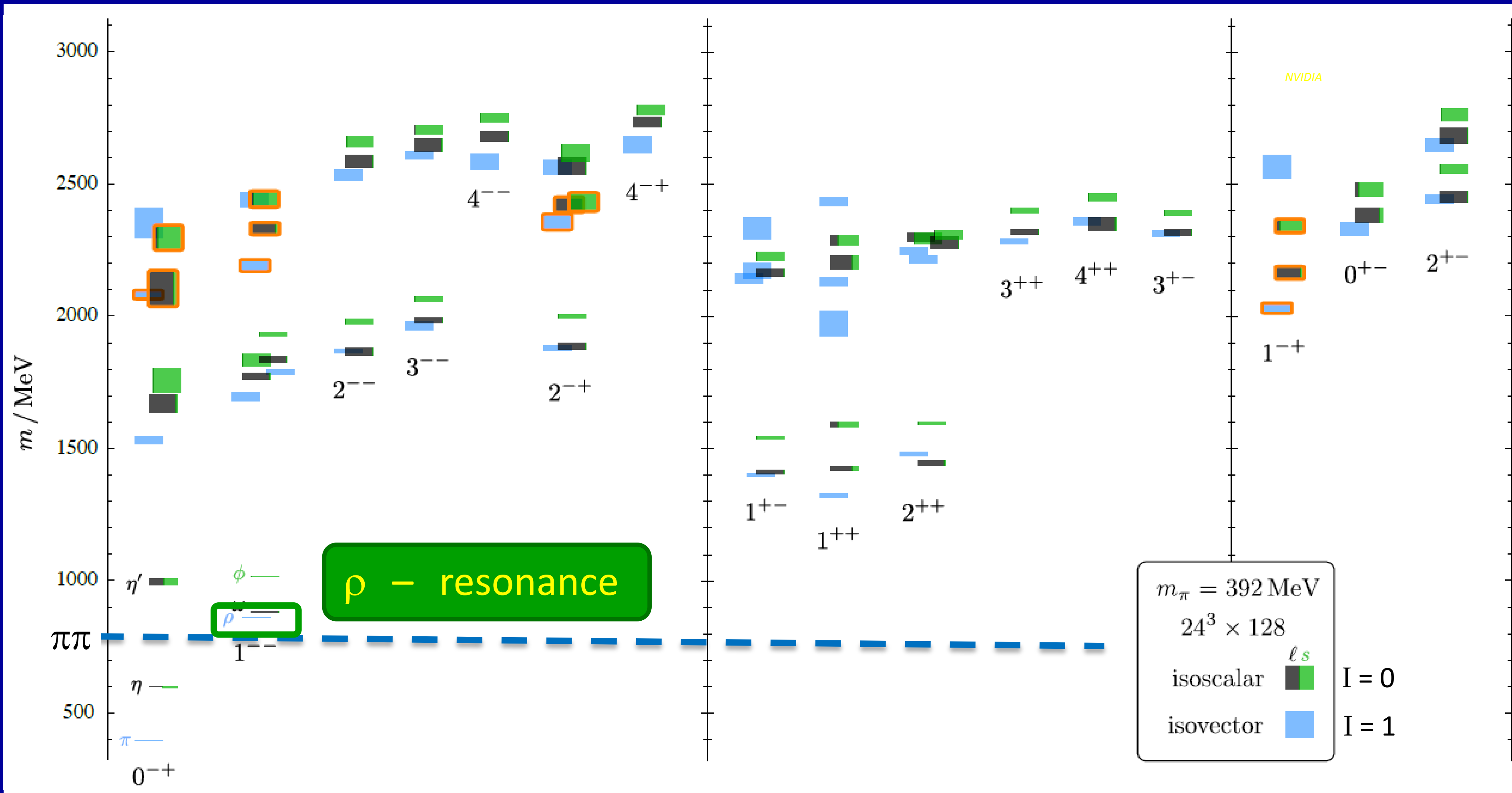

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Scattering on the lattice

Imaginary time – can’t study dynamics (e.g. scattering) directly

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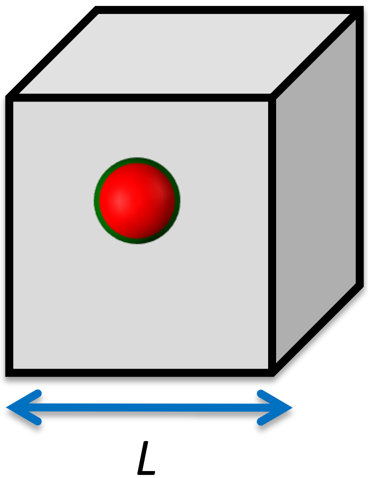
Single hadron in a finite volume



periodic b.c.s
(torus)

$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Scattering on the lattice

Two hadrons: **non-interacting**

$$E_{AB} = \sqrt{m_A^2 + \vec{k}_A^2} + \sqrt{m_B^2 + \vec{k}_B^2}$$

Infinite volume

Continuous spectrum

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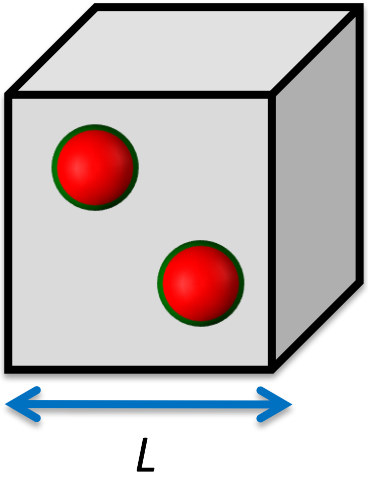
Finite volume

Discrete spectrum



$$\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

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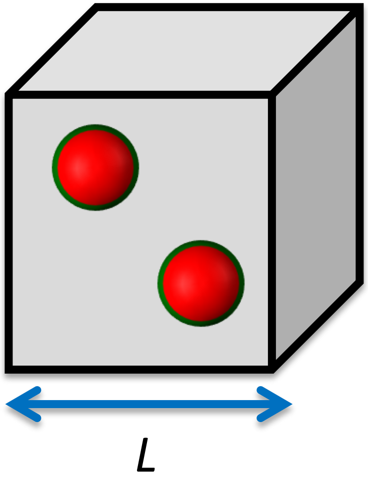

$$\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$$

c.f. 1-dim: $k = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$

Scattering on the lattice – 'Lüscher method'

Lüscher, Nucl. Phys. B354, 531 (1991); extended by many others
[see refs in PR D87, 034505 and arXiv:1406.4158 and other talks]

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{mm'} + i\rho_i\ t_{ij}^{(l)}\ \left(\delta_{\ell\ell'}\delta_{mm'} + i\mathcal{M}^{\vec{P}}_{\ell m;\ell' m'}(q_i^2)\right)\right] = 0$$

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i, *j* label channels
e.g. Kπ, Kη

$$\rho_i = \frac{2k_{\text{cm},i}}{E_{\text{cm}}}$$

$$\vec{q} \equiv \vec{k}_{\text{cm}} L/2\pi$$

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Scattering *t*-matrix: $S_{ij} = \delta_{ij} + 2i\sqrt{\rho_i \rho_k}\; t_{ij}$

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~ gen. zeta fns. – effect of finite vol.

Reduced symmetry → ℓ mix

Subduce to lattice irrep (Λ) →

$$\mathcal{M}^{\vec{d},\Lambda}_{\ell n;\ell' n'}\delta_{\Lambda\Lambda'}\delta_{\mu\mu'}$$

(ℓ that subduce to Λ mix)

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$\vec{P}$ = overall mom.

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{mm'} + i\rho_i\, t^{(l)}_{ij}\left(\delta_{\ell\ell'}\delta_{mm'} + i\mathcal{M}^{\vec{P}}_{\ell m;\ell' m'}(q_i^2)\right)\right] = 0$$

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Given *t*: solns → finite-vol. spec. $\{E_{\mathrm{cm}}\}$

We need: spectrum → *t*-matrix

Elastic scattering

$$t^{(\ell)} = \frac{1}{\rho} e^{i\delta_\ell} \sin \delta_\ell$$

$$\det \left[\delta_{\ell\ell'}\delta_{nn'} + i\rho_i \, t^{(l)} \left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}^{\vec{P},\Lambda}_{\ell n;\ell' n'}(q_i^2) \right) \right] = 0$$

If assume only lowest ℓ relevant [near threshold $t \sim k^{2\ell}$]
→ can solve equ. for each E_{cm} → phase shift $\delta(E_{cm})$

Alternatively parameterise $t(s)$ and fit $\{E_{lat}\}$ to $\{E_{param}\}$

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Need many (multi-hadron) energy levels

Single and multi-hadron ops

Non-zero P_{cm}, different box sizes and shapes, twisted b.c.s, ...

Map out phase shift → resonance parameters etc

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Resonance: Breit-Wigner param.

$$t^{(\ell)} = \frac{1}{\rho(s)} \frac{\sqrt{s}\Gamma_\ell(s)}{m_R^2 - s - i\sqrt{s}\Gamma_\ell(s)}$$

$$\Gamma_\ell(s) = \frac{g_R^2}{6\pi} \frac{k_{\text{cm}}^{2\ell+1}}{s\ m_R^{2(\ell-1)}}$$

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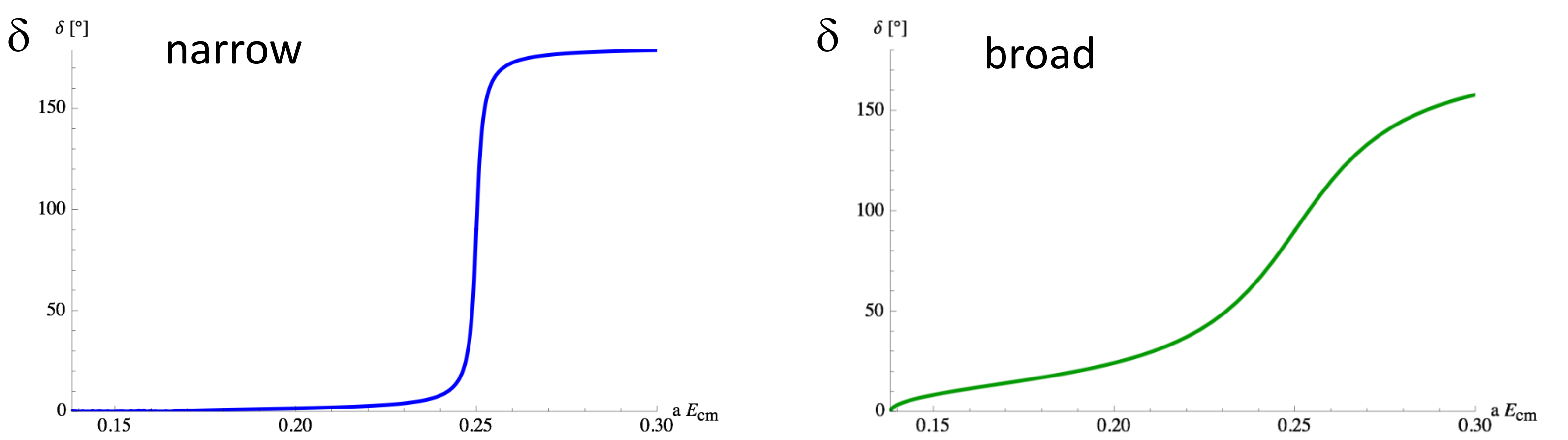


D. Wilson

The ρ resonance in ππ I=1 scattering

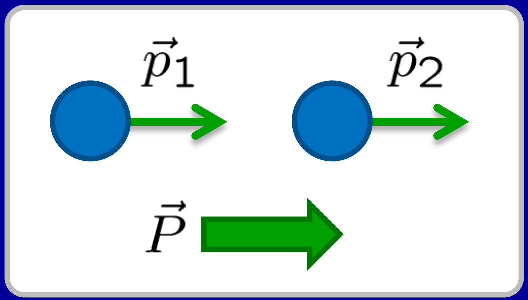
$\pi\pi \rightarrow \rho \rightarrow \pi\pi$

3 volumes ($L \approx 2 - 3$ fm), $a_s \approx 0.12$ fm, $M_\pi \approx 400$ MeV

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

single-meson $\sim \bar{\psi}\Gamma D \ldots \psi$

and ππ ops. $\mathcal{O}(\vec{P}) = \sum_{\hat{p}_1,\, \hat{p}_2} \mathcal{C}_\Lambda(\vec{P}, \vec{p}_1, \vec{p}_2)\ \mathcal{O}_\pi(\vec{p}_1)\ \mathcal{O}_\pi(\vec{p}_2)$



Dudek, Edwards, CT [PR D87, 034505 (2013)]

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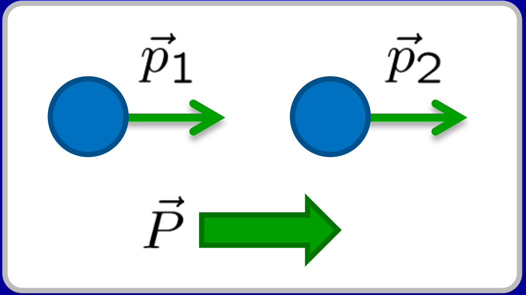
Variationally optimised π ops



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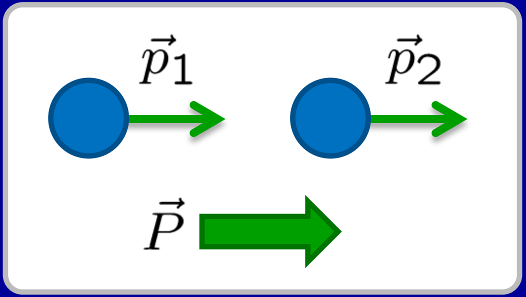

Variationally optimised π ops

Assume $\delta_{l>3} \approx 0$ in this energy range – find no significant signal for $\delta_{l=3}$

Dudek, Edwards, CT [PR D87, 034505 (2013)]

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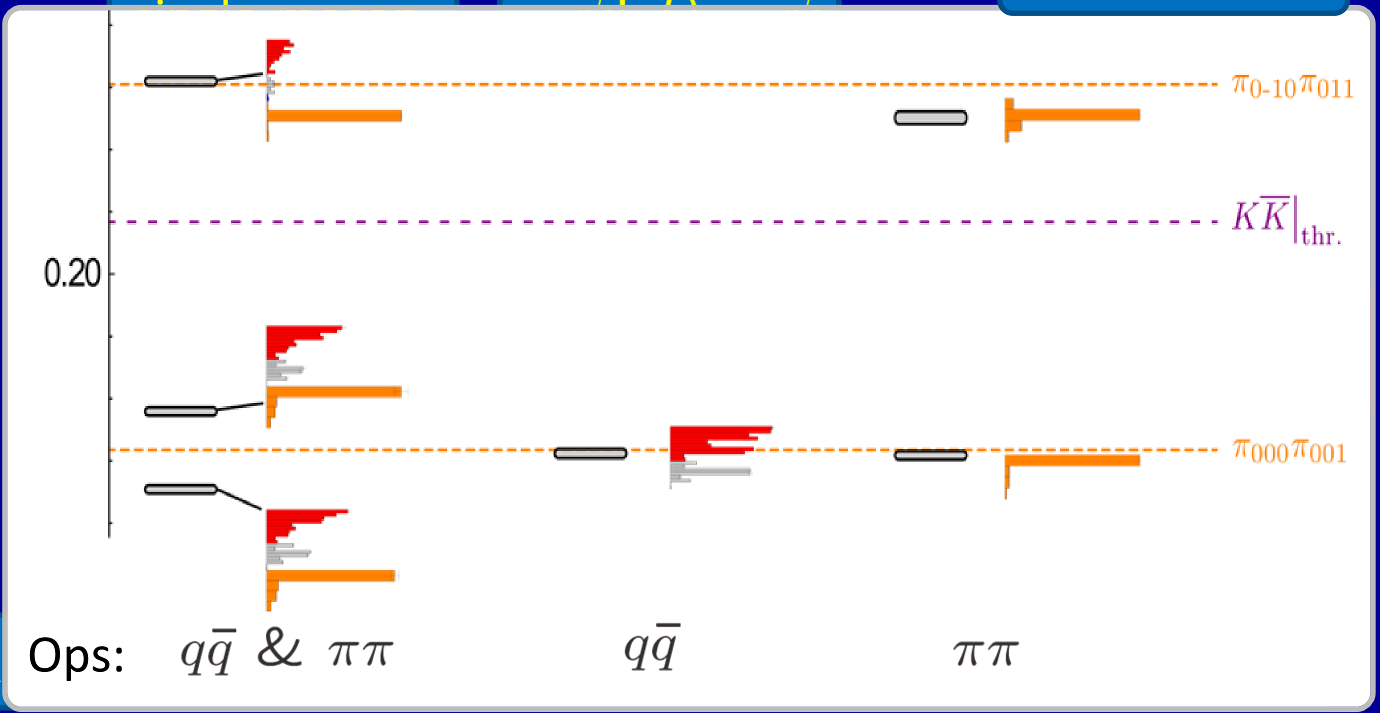
24^3, P = [0,0,1] A_1



Assume $\delta_{l>}$

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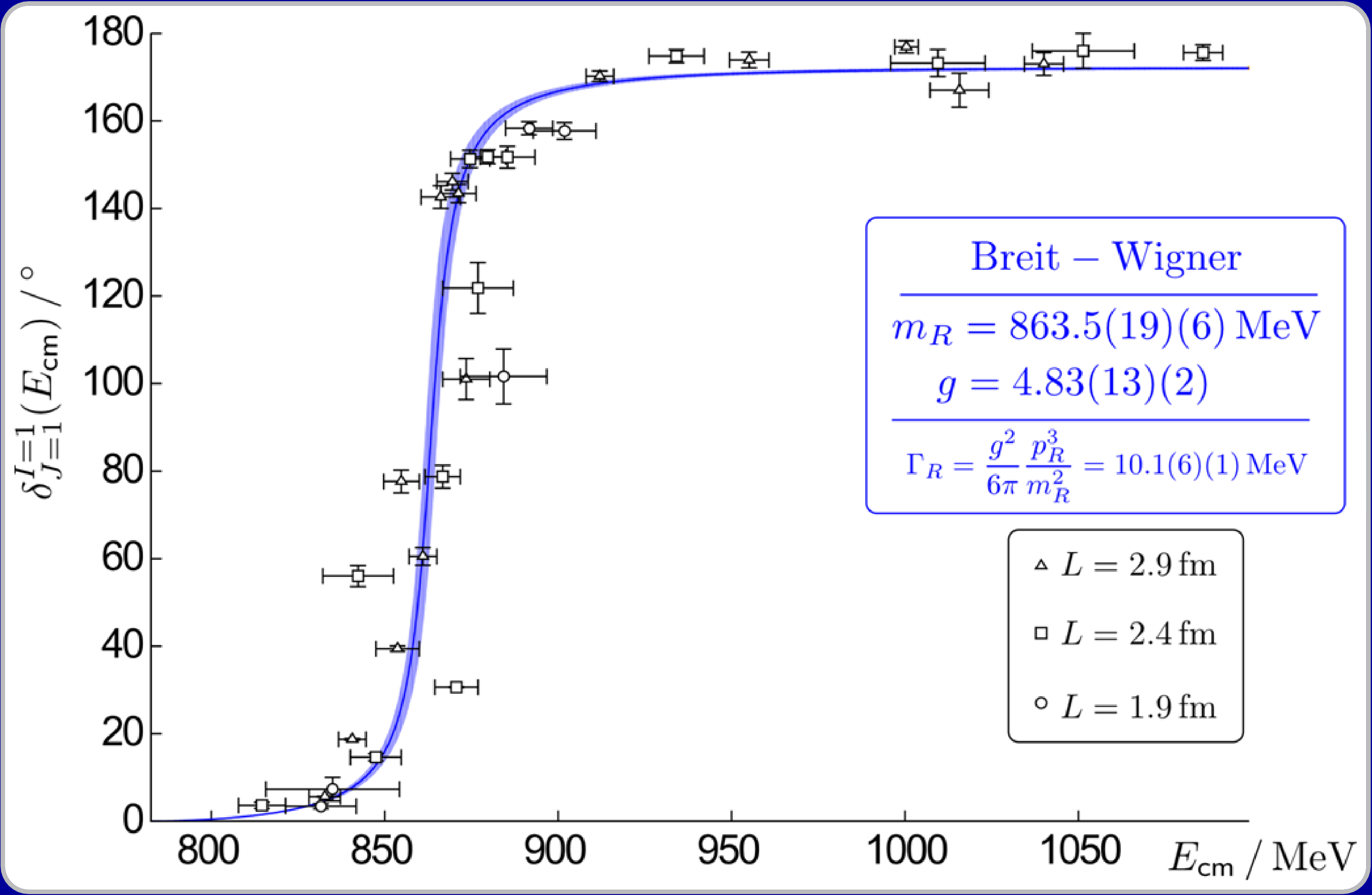

Mapped out in detail

Dudek, Edwards, CT [PR D87, 034505 (2013)]

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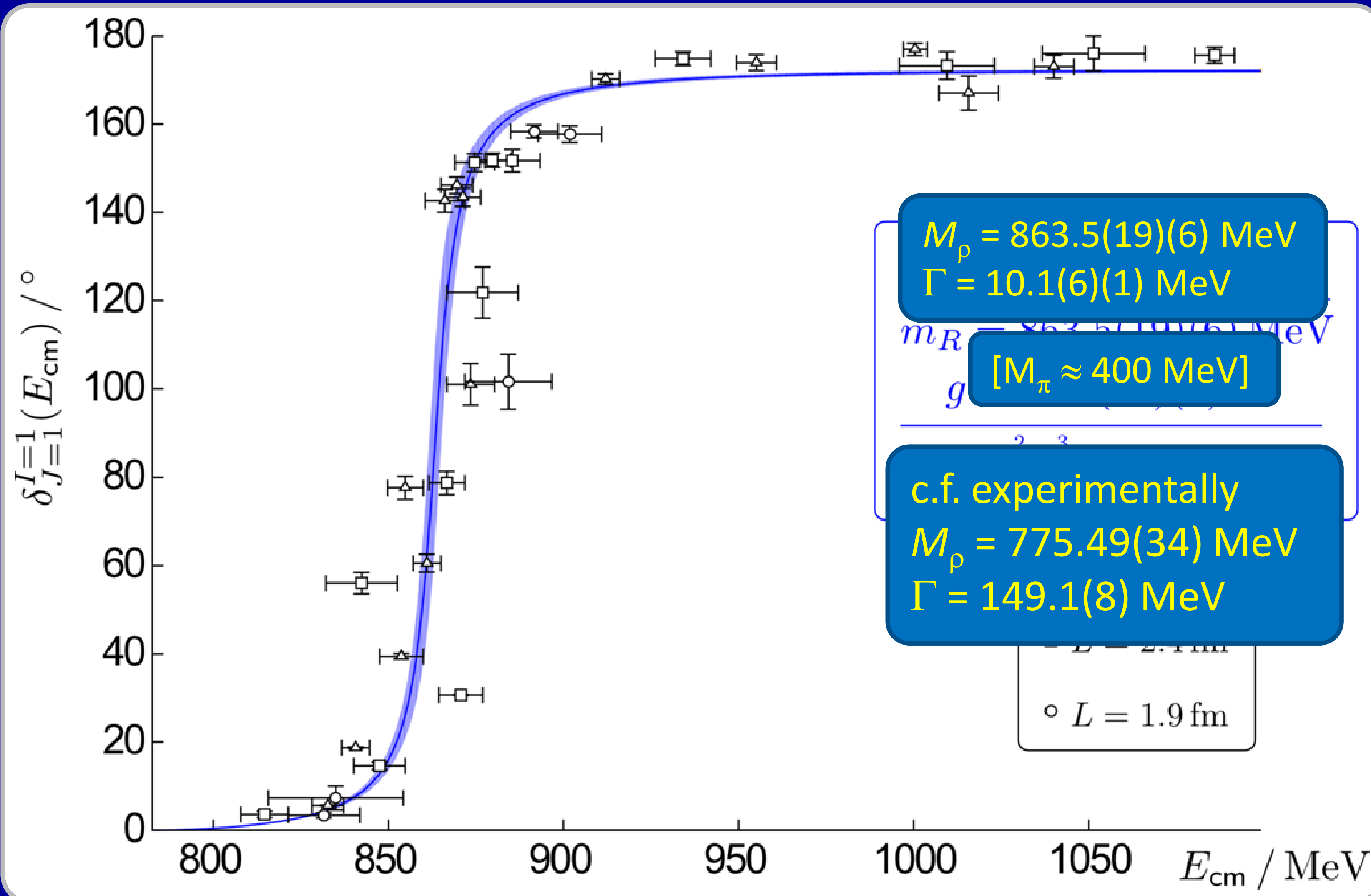

M_ρ = 863.5(19)(6) MeV
Γ = 10.1(6)(1) MeV

[$M_\pi \approx$ 400 MeV]

c.f. experimentally
M_ρ = 775.49(34) MeV
Γ = 149.1(8) MeV

Mapped out in detail

Dudek, Edwards, CT [PR D87, 034505 (2013)]

πK, ηK (I=1/2) coupled-channel scattering

$J^P = 0^+$	κ, $K_0^*(1430)$, ...
$J^P = 1^-$	$K^*(892)$, ...
$J^P = 2^+$	$K_2^*(1430)$, ...

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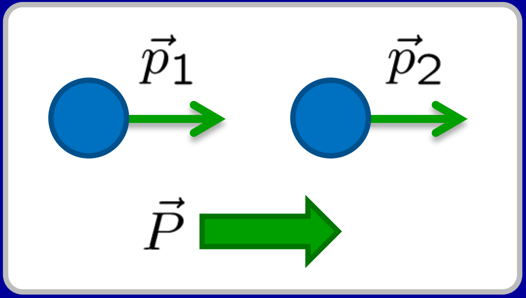
Jo Dudek, Robert Edwards, David Wilson, CT [arXiv:1406.4158]



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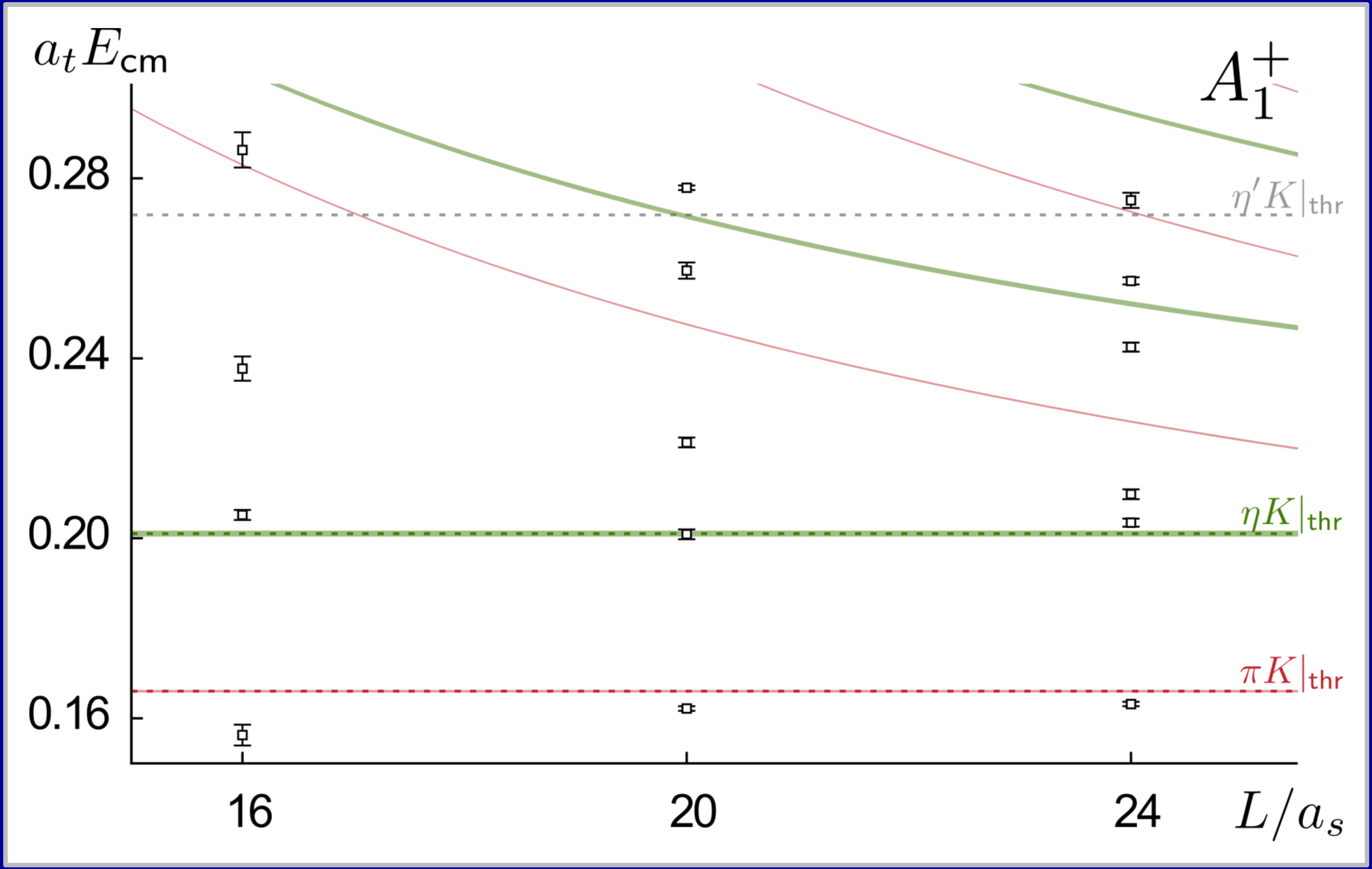
M_π = 391 MeV, M_K = 549 MeV, M_η = 589 MeV; 3 volumes ($L \approx 2 - 3$ fm), $a_s \approx 0.12$ fm

πK, ηK (I=1/2) spectra

P = [0,0,0] A_1^+



$J^P = 0^+, 4^+, \ldots$ [ℓ = 0, 4, ...]

[arXiv:1406.4158]

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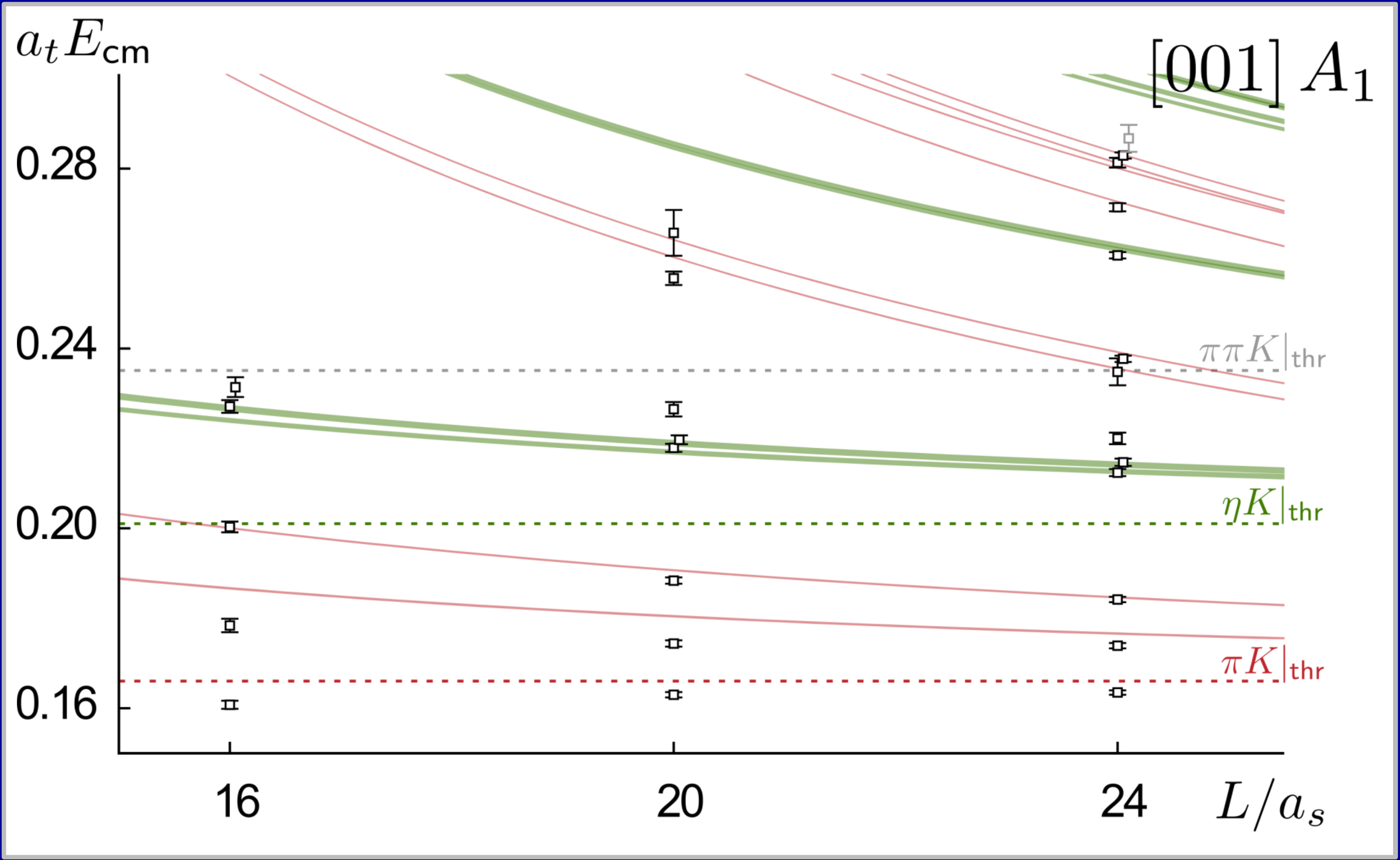

|λ| = 0^+, 4, … [ℓ = 0, 1, 2, 3, 4^2, …]

[arXiv:1406.4158]

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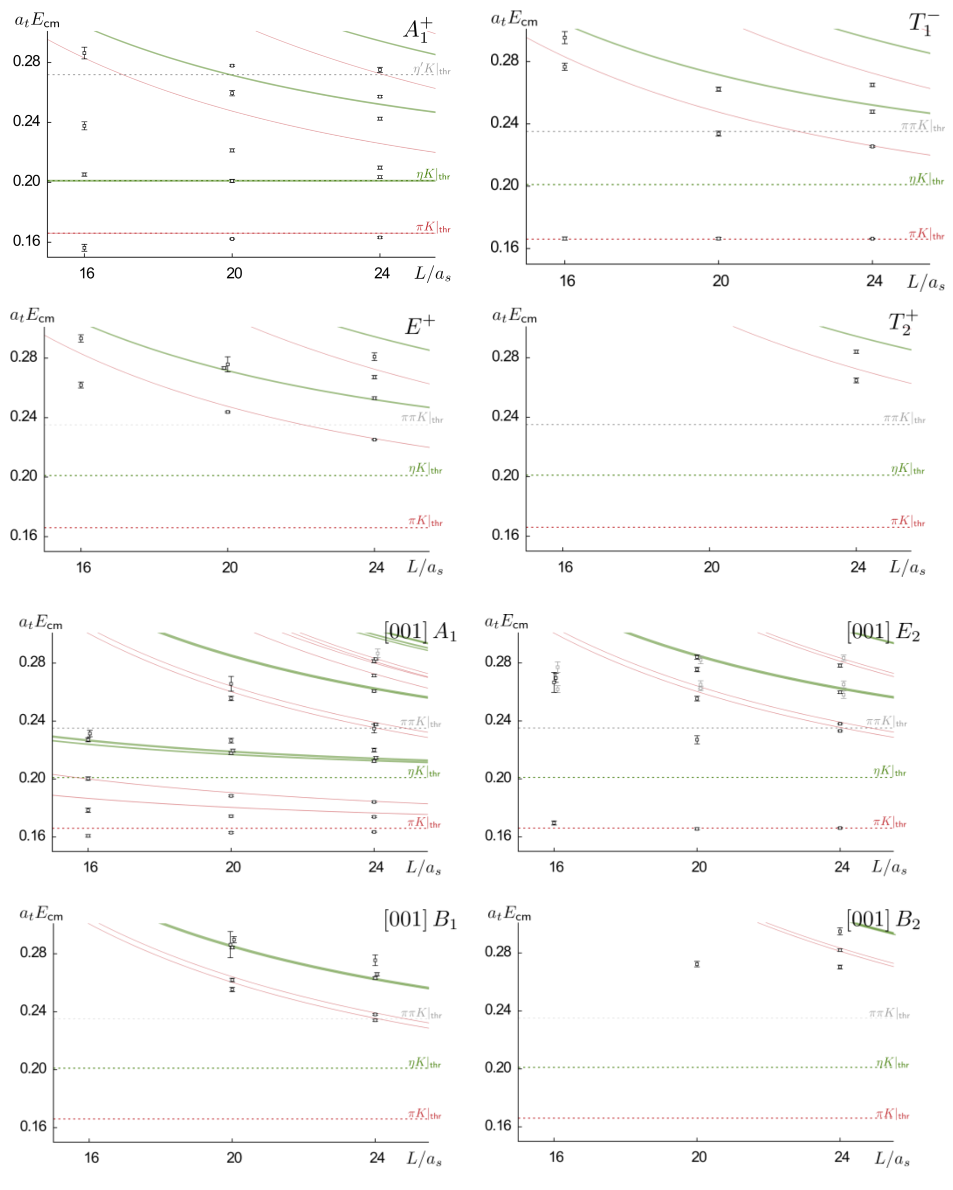
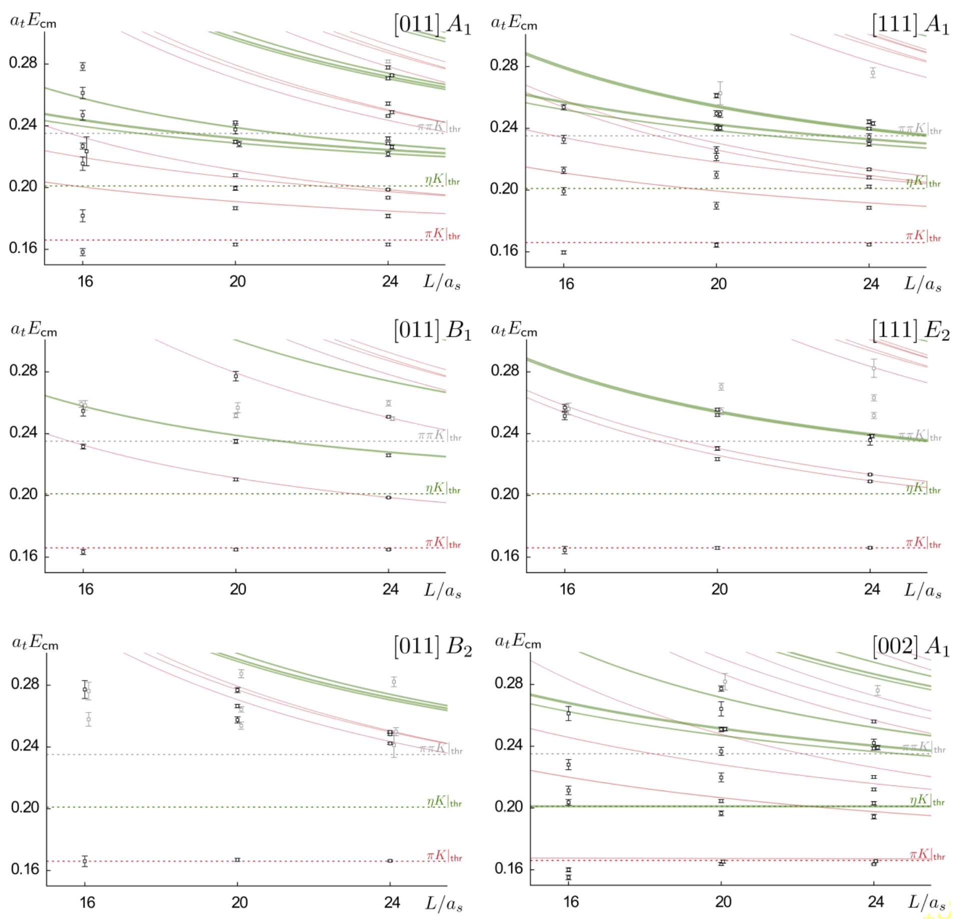


> 100 energy levels in total



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Under-constrained equation → parameterise $t_{ij}(s)$ and fit E_{lat} to E_{param}

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Under-constrained equation → parameterise $t_{ij}(s)$ and fit E_{lat} to E_{param}

K-matrix param – respects unitarity (conserve prob.) and flexible:

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s)$$

$$\text{Im} I_{ij} = -\delta_{ij}\rho_i(s)$$

[arXiv:1406.4158]

πK, ηK (I=1/2) coupled-channel scattering

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{nn'} + i\rho_i\ t^{(l)}_{ij}\ \left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}^{\vec{P},\Lambda}_{\ell n;\ell' n'}(q_i^2)\right)\right] = 0$$

Under-constrained equation → parameterise $t_{ij}(s)$ and fit E_{lat} to E_{param}

K-matrix param – respects unitarity (conserve prob.) and flexible:

$$t^{-1}_{ij}(s) = \frac{1}{(2k_i)^\ell} K^{-1}_{ij}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s)$$

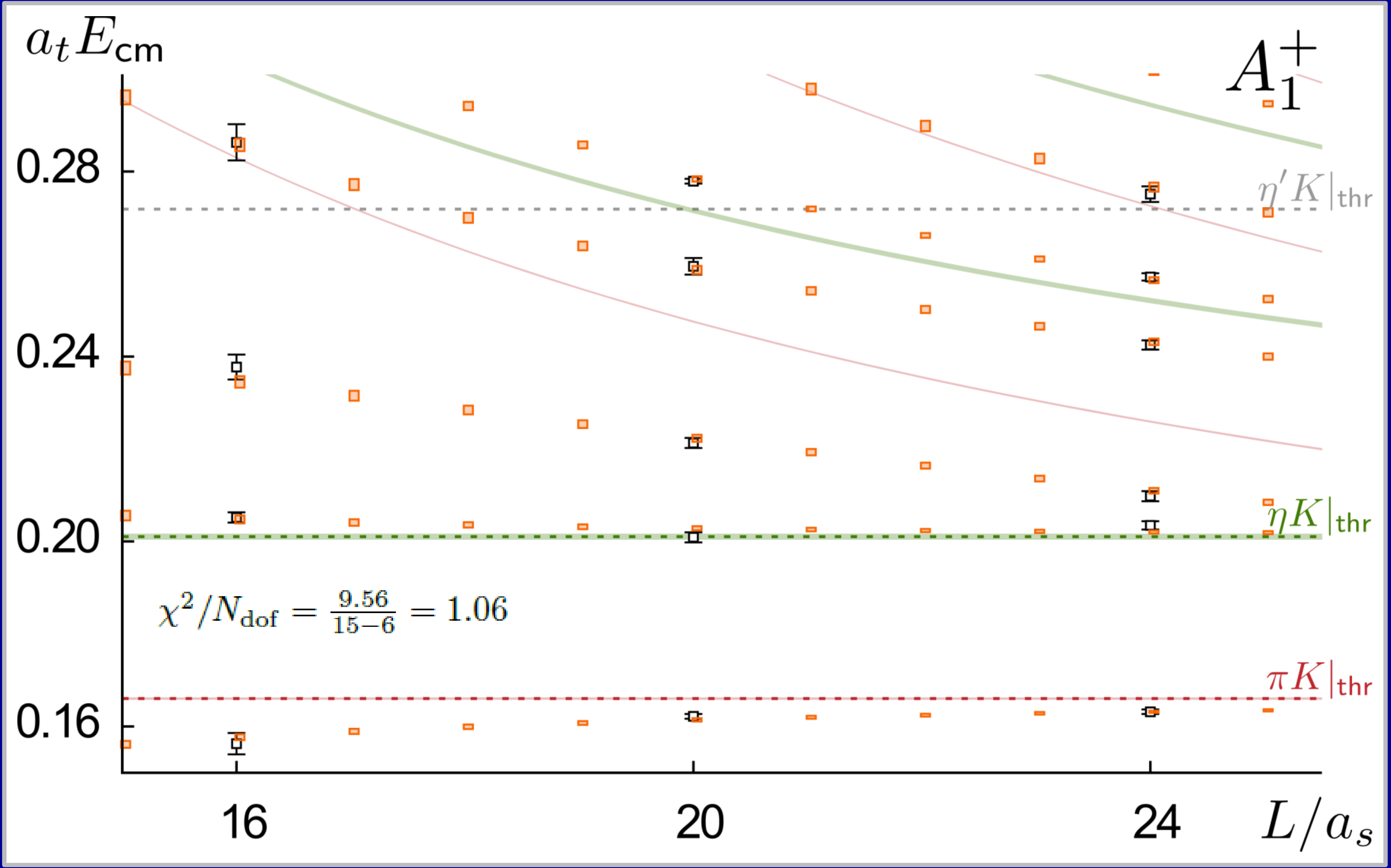
$$\mathrm{Im} I_{ij} = -\delta_{ij}\rho_i(s)$$

In current study:

$$K = \frac{1}{m^2 - s}\begin{bmatrix} g^2_{\pi K} & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g^2_{\eta K} \end{bmatrix} + \begin{bmatrix} \gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K} \end{bmatrix}$$

[arXiv:1406.4158]

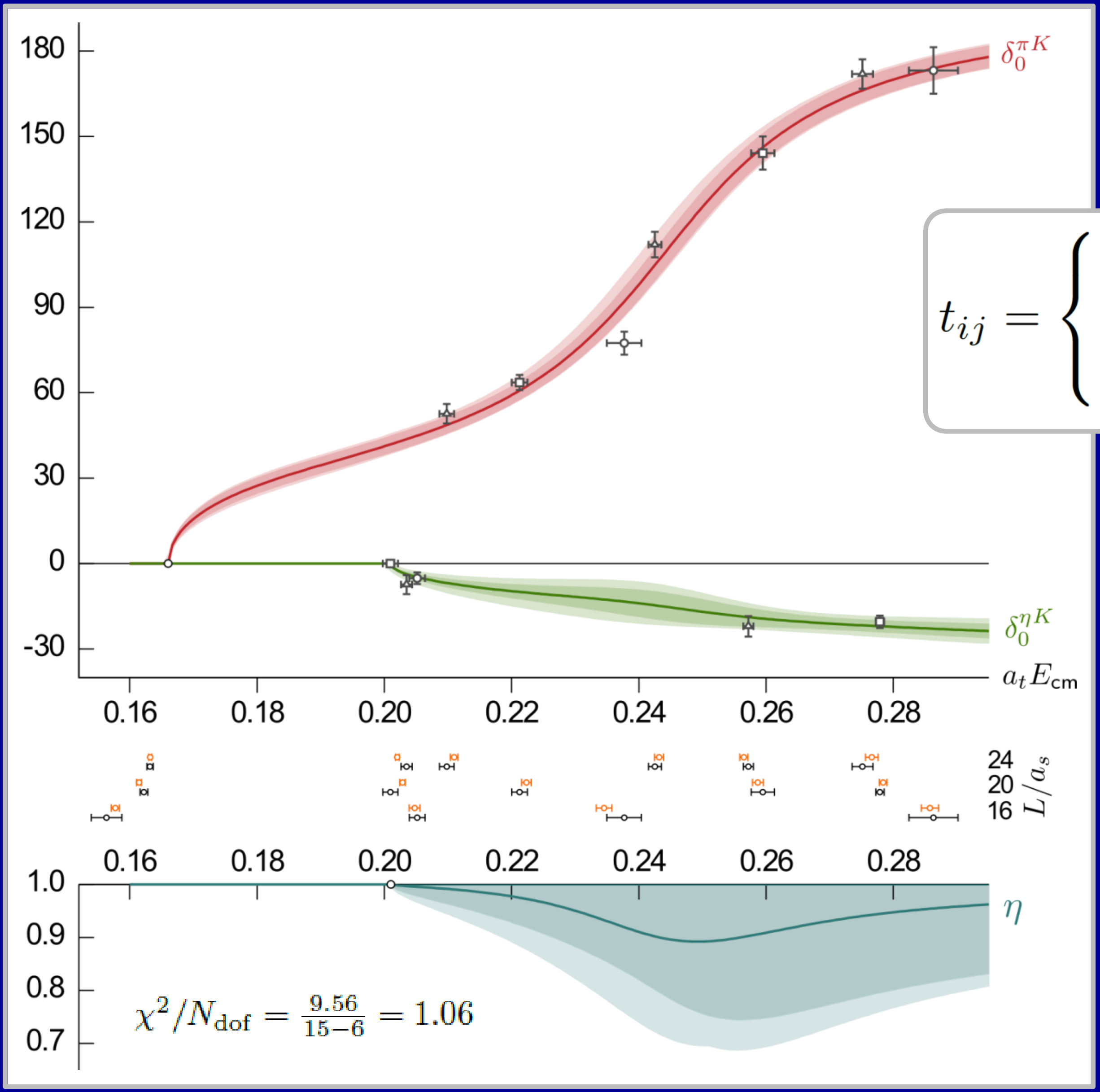
πK, ηK (I=1/2): S-wave – only using P = [0,0,0] A_1^+



Neglect ℓ ≥ 4: only ℓ = 0 contributes

[arXiv:1406.4158]

πK, ηK (I=1/2): S-wave – only using P = [0,0,0] A_1^+



$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\,\rho_i} & (i = j) \\ \dfrac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}} & (i \neq j) \end{cases}$$

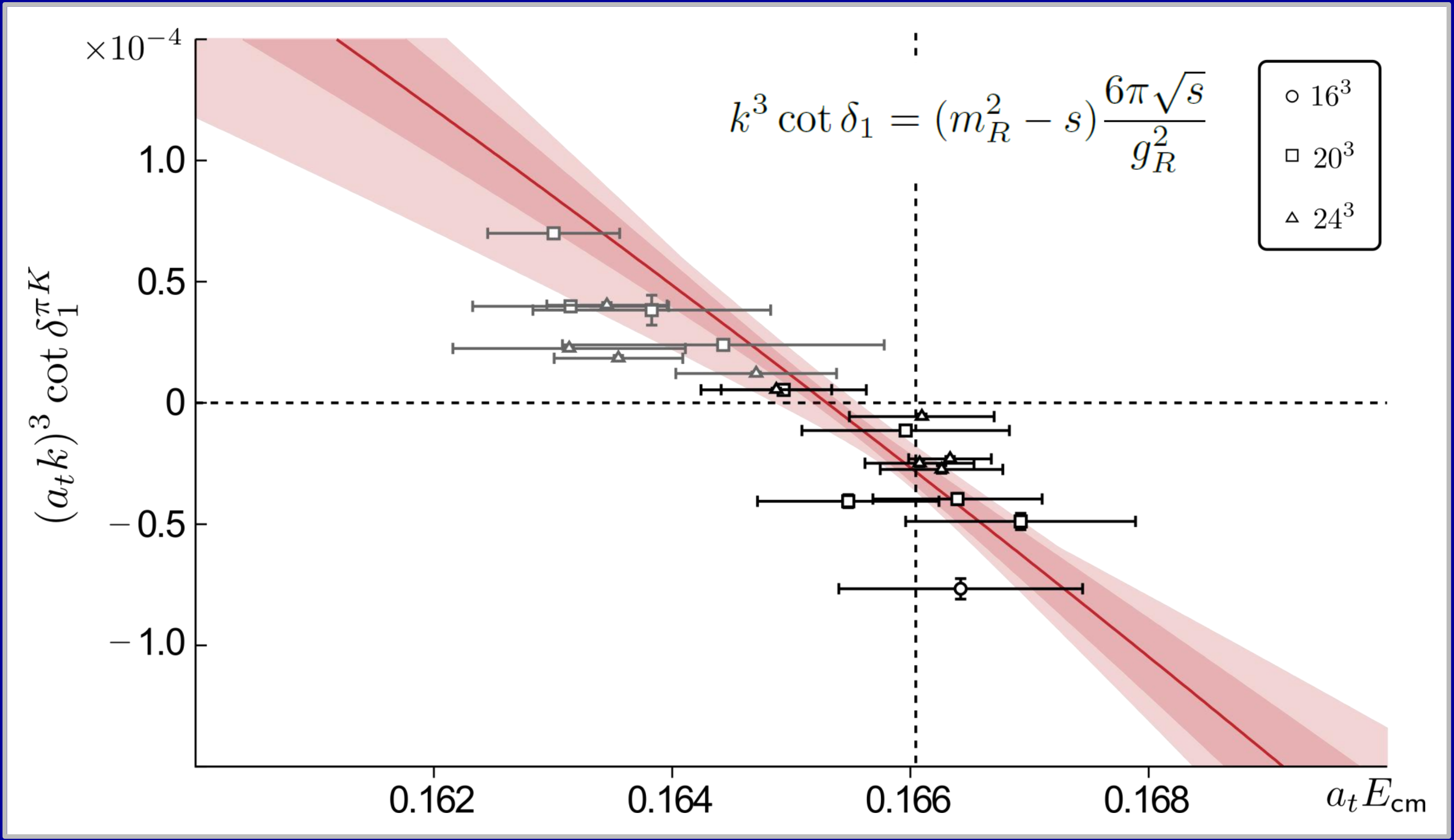
Up to πππK threshold

N.B. points assume πK, ηK decouple (good approx.)

[arXiv:1406.4158]

πK (I=1/2): P-wave near threshold

(well below ηK threshold)

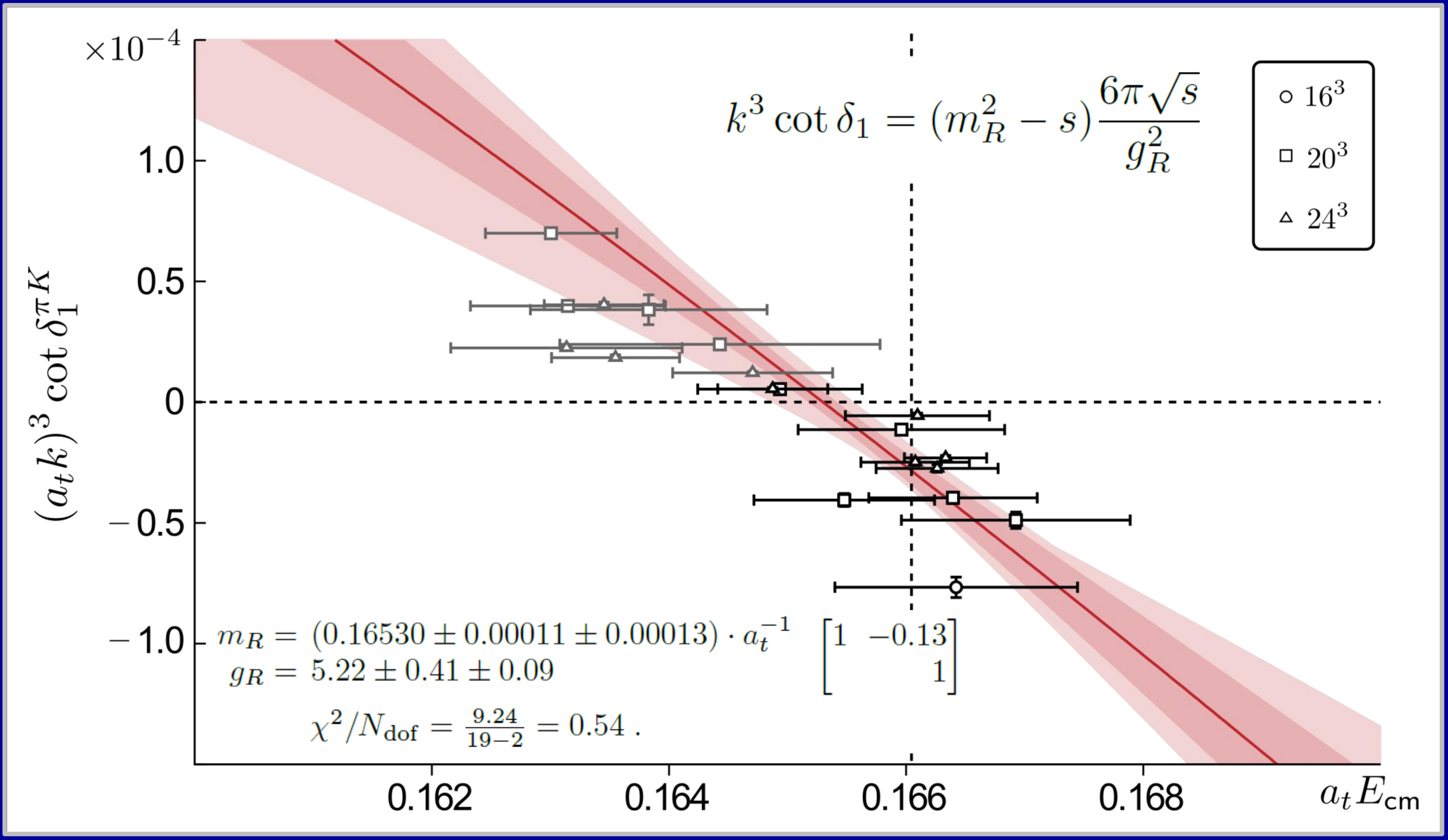


Relativistic Breit-Wigner; use S-wave from prev. slide

[arXiv:1406.4158]

πK (I=1/2): P-wave near threshold

(well below ηK threshold)



Relativistic Breit-Wigner; use S-wave from prev. slide

[arXiv:1406.4158]

πK (I=1/2): P-wave near threshold

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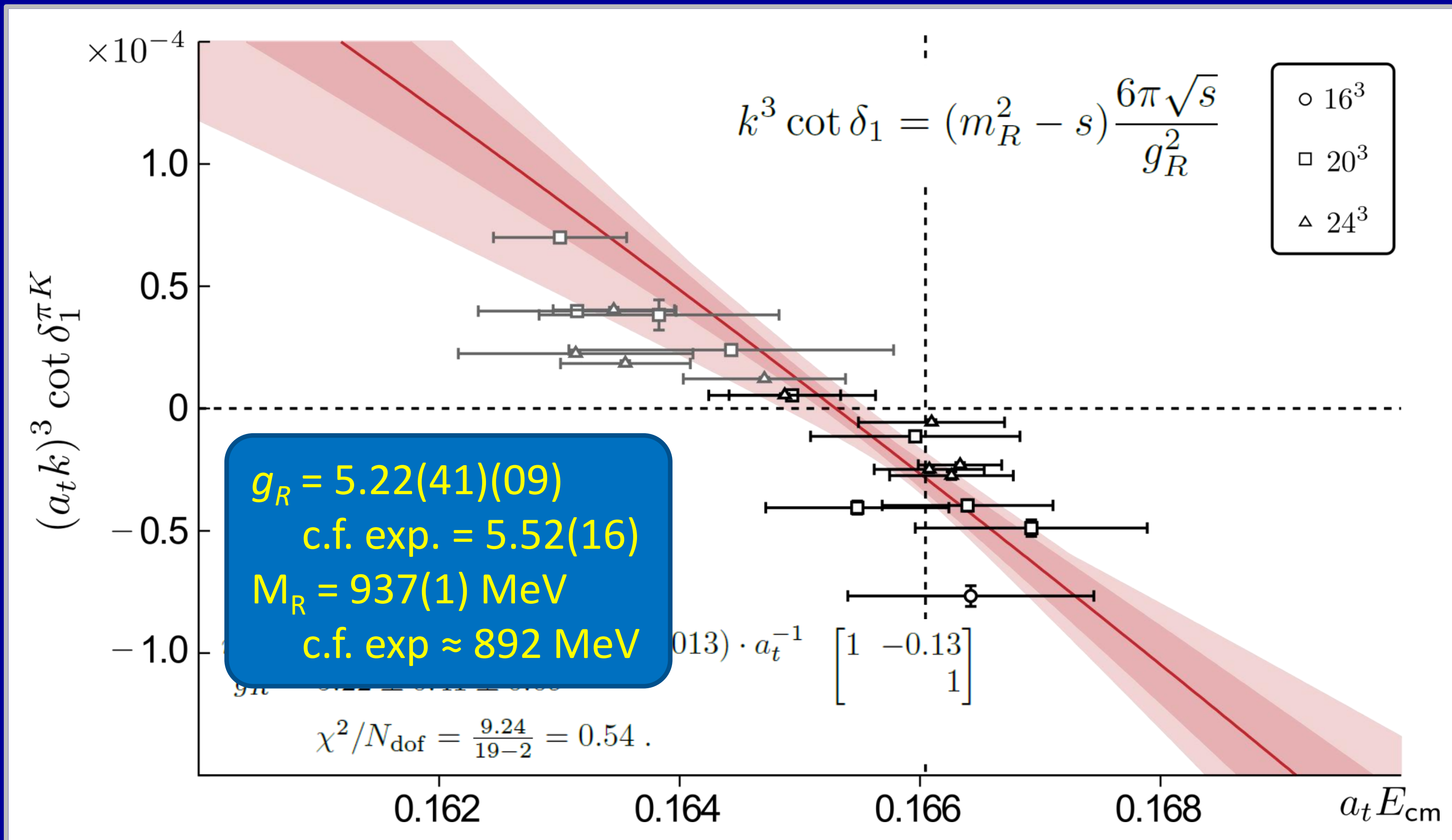


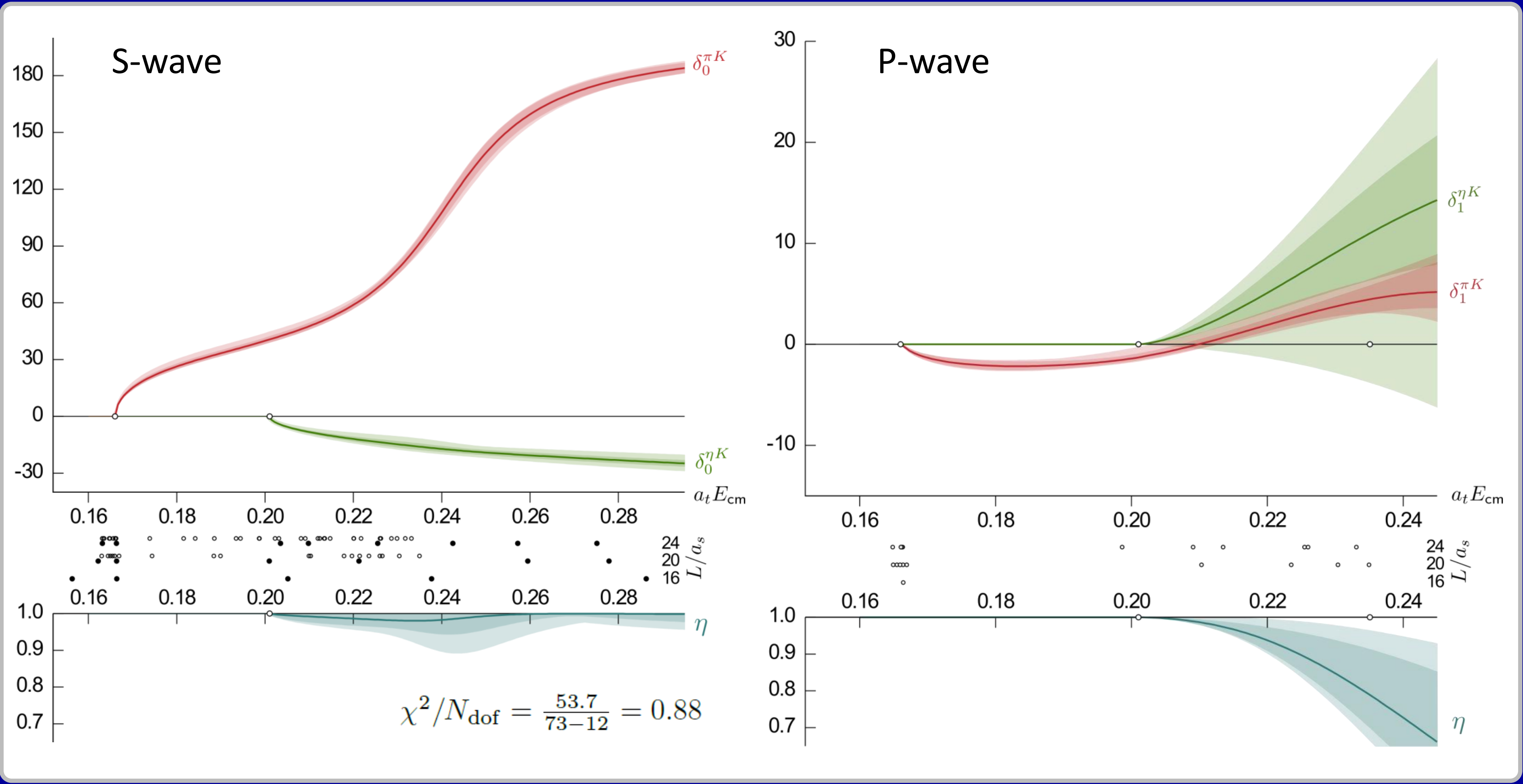
g_R = 5.22(41)(09)
c.f. exp. = 5.52(16)
M_R = 937(1) MeV
c.f. exp ≈ 892 MeV

Relativistic Breit-Wigner; use S-wave from prev. slide

[arXiv:1406.4158]

πK, ηK (I=1/2): S & P-waves

(73 energy levels)



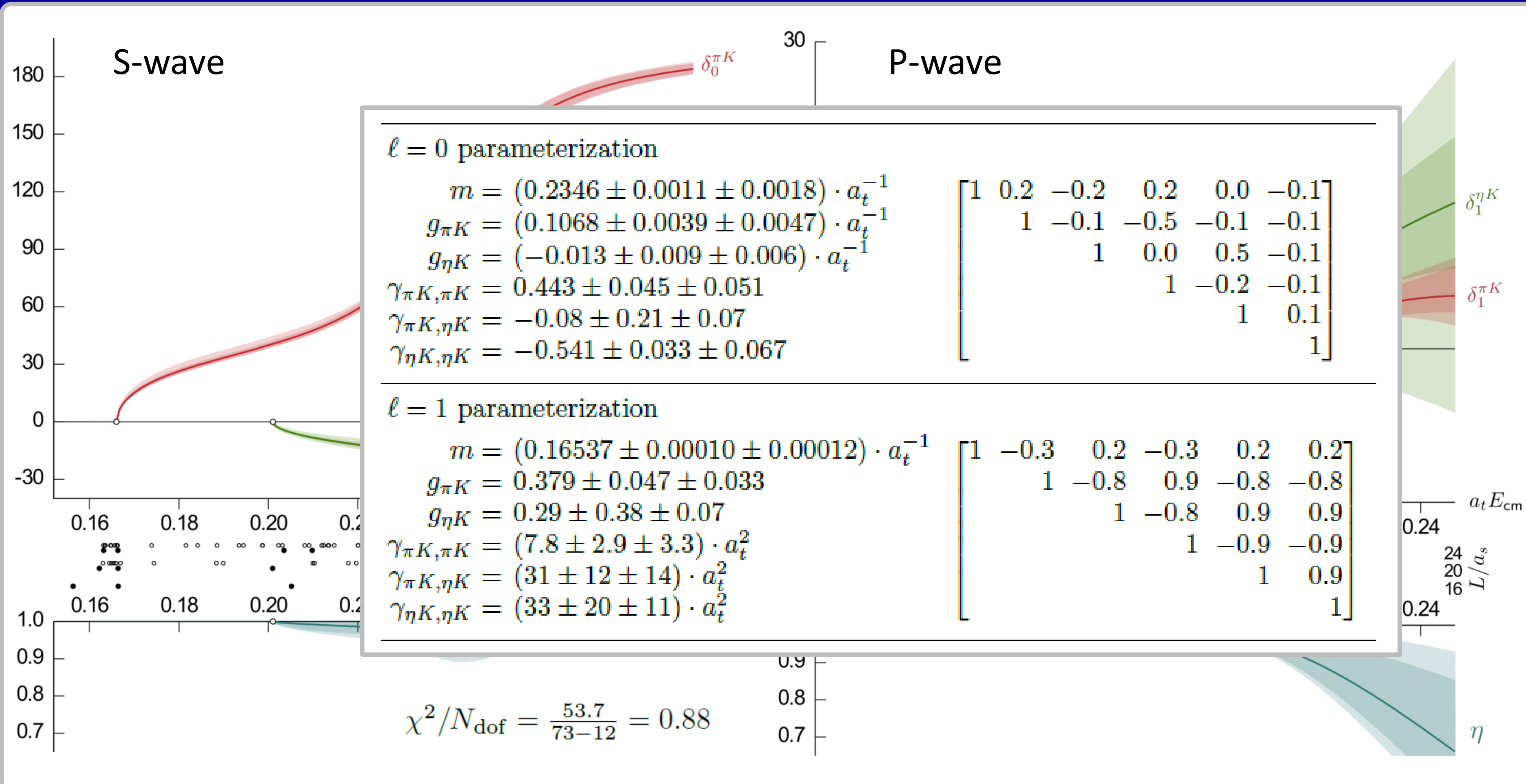
Up to ππK threshold, except [0,0,0] A_1^+ up to πππK

Assume ℓ ≥ 2 negligible in this region (see later)

[arXiv:1406.4158]

πK, ηK (I=1/2): S & P-waves

(73 energy levels)



$\ell = 0$ parameterization

$$
\begin{aligned}
m &= (0.2346 \pm 0.0011 \pm 0.0018) \cdot a_t^{-1} \\
g_{\pi K} &= (0.1068 \pm 0.0039 \pm 0.0047) \cdot a_t^{-1} \\
g_{\eta K} &= (-0.013 \pm 0.009 \pm 0.006) \cdot a_t^{-1} \\
\gamma_{\pi K,\pi K} &= 0.443 \pm 0.045 \pm 0.051 \\
\gamma_{\pi K,\eta K} &= -0.08 \pm 0.21 \pm 0.07 \\
\gamma_{\eta K,\eta K} &= -0.541 \pm 0.033 \pm 0.067
\end{aligned}
\qquad
\begin{bmatrix}
1 & 0.2 & -0.2 & 0.2 & 0.0 & -0.1 \\
 & 1 & -0.1 & -0.5 & -0.1 & -0.1 \\
 & & 1 & 0.0 & 0.5 & -0.1 \\
 & & & 1 & -0.2 & -0.1 \\
 & & & & 1 & 0.1 \\
 & & & & & 1
\end{bmatrix}
$$

$\ell = 1$ parameterization

$$
\begin{aligned}
m &= (0.16537 \pm 0.00010 \pm 0.00012) \cdot a_t^{-1} \\
g_{\pi K} &= 0.379 \pm 0.047 \pm 0.033 \\
g_{\eta K} &= 0.29 \pm 0.38 \pm 0.07 \\
\gamma_{\pi K,\pi K} &= (7.8 \pm 2.9 \pm 3.3) \cdot a_t^2 \\
\gamma_{\pi K,\eta K} &= (31 \pm 12 \pm 14) \cdot a_t^2 \\
\gamma_{\eta K,\eta K} &= (33 \pm 20 \pm 11) \cdot a_t^2
\end{aligned}
\qquad
\begin{bmatrix}
1 & -0.3 & 0.2 & -0.3 & 0.2 & 0.2 \\
 & 1 & -0.8 & 0.9 & -0.8 & -0.8 \\
 & & 1 & -0.8 & 0.9 & 0.9 \\
 & & & 1 & -0.9 & -0.9 \\
 & & & & 1 & 0.9 \\
 & & & & & 1
\end{bmatrix}
$$

$$\chi^2/N_{\mathrm{dof}} = \frac{53.7}{73-12} = 0.88$$

Up to ππK threshold, except [0,0,0] A_1^+ up to πππK
Assume ℓ ≥ 2 negligible in this region (see later)

[arXiv:1406.4158]

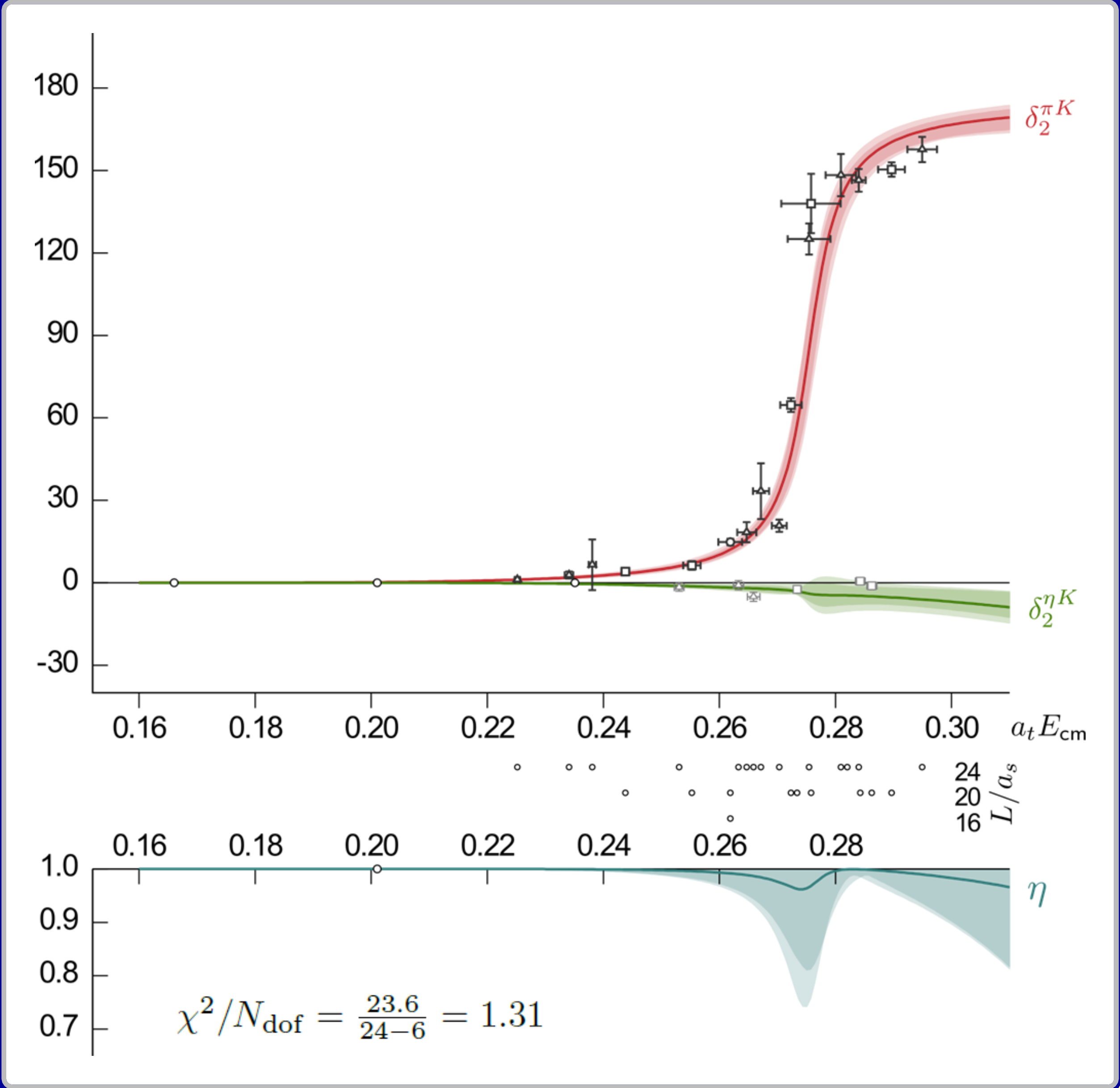
πK, ηK (I=1/2): D-wave

Only irreps where J = 2 is lowest → 24 levels

Assume ℓ ≥ 3 negligible

Up to πππK threshold; neglect coupling to ππK

Points assume πK, ηK decouple (good approx)



[arXiv:1406.4158]

πK, ηK (I=1/2): D-wave

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Assume ℓ ≥ 3 negligible

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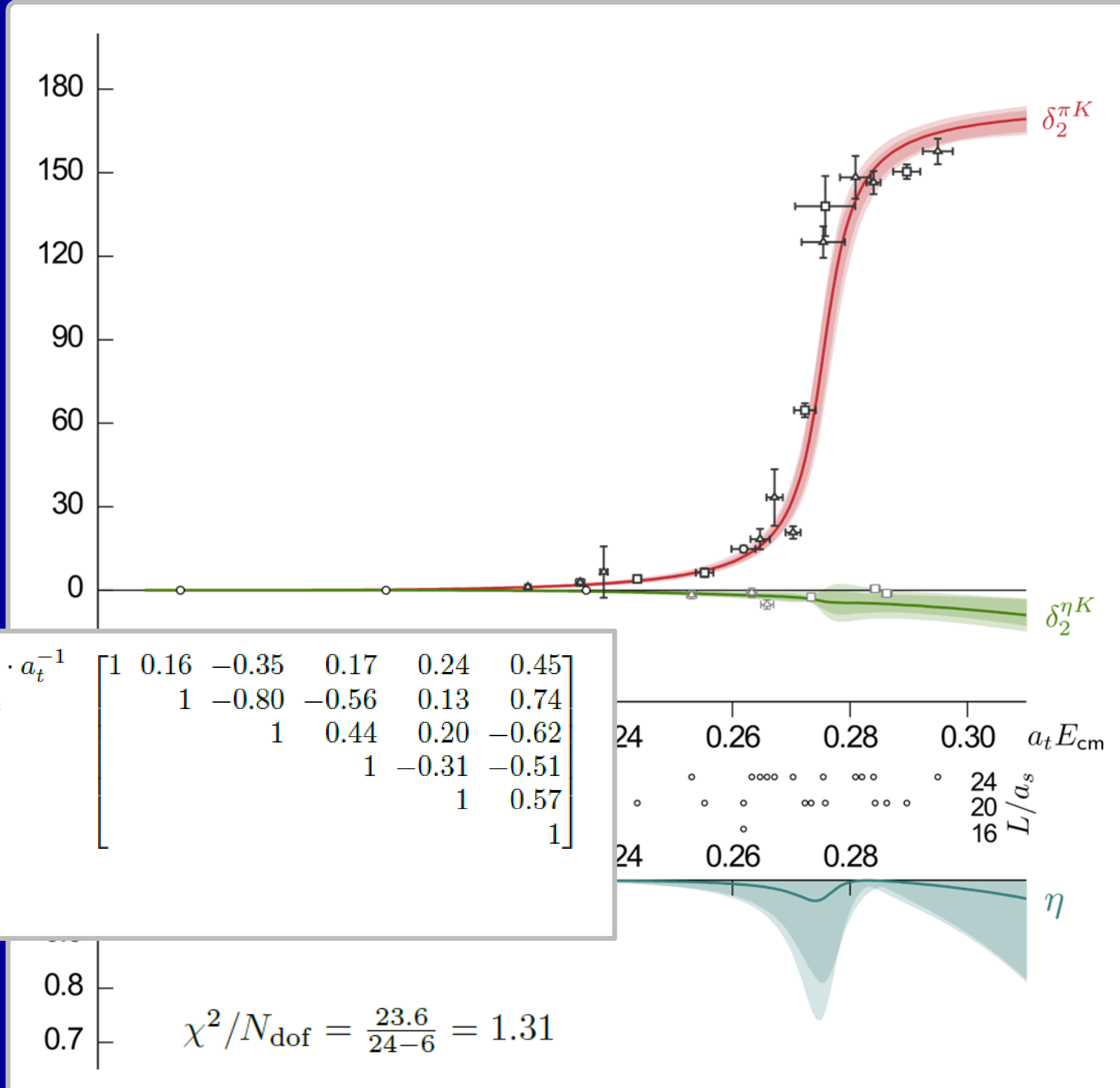
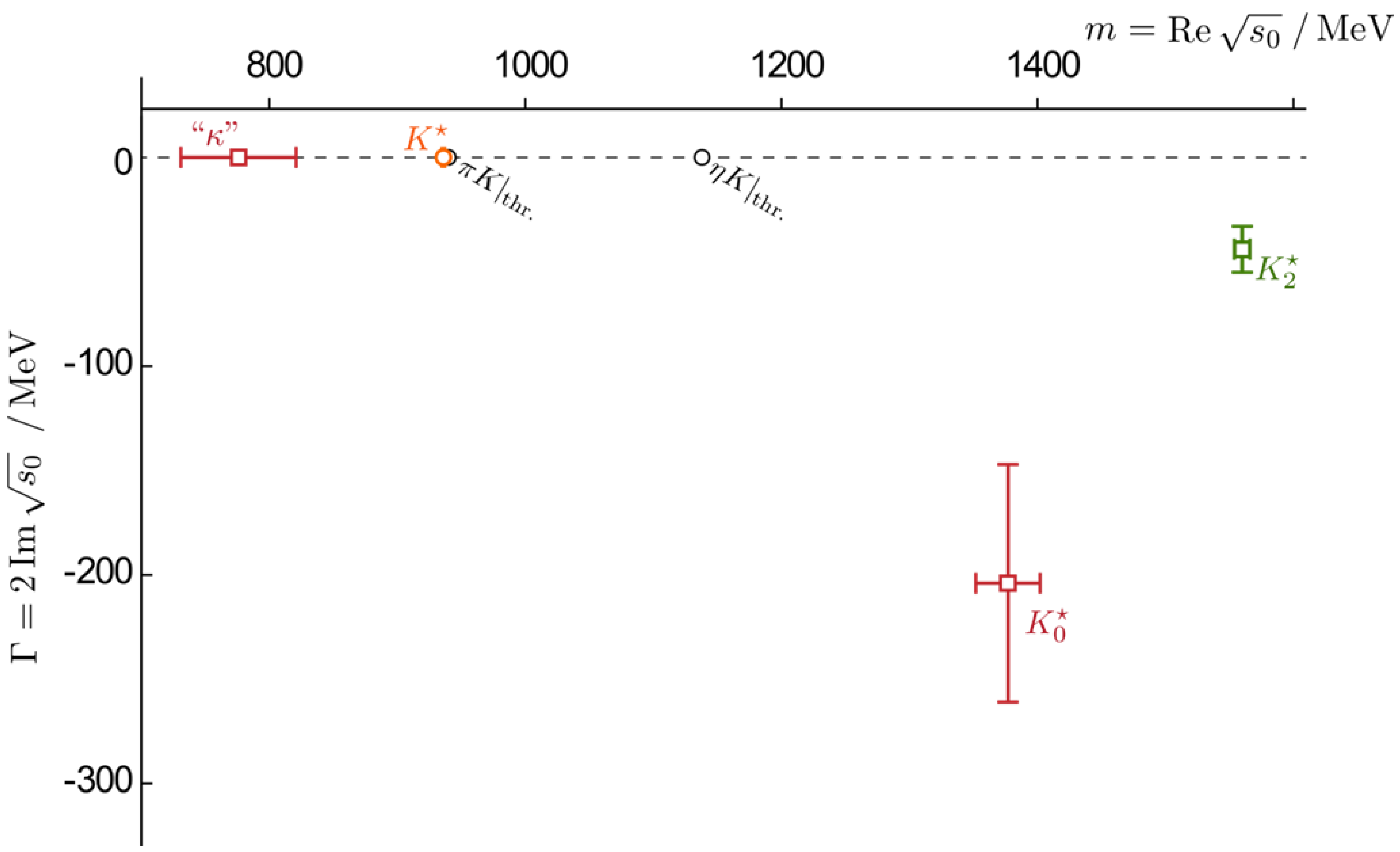


$$
\begin{aligned}
m &= (0.2758 \pm 0.0010 \pm 0.0003) \cdot a_t^{-1} \\
g_{\pi K} &= (1.080 \pm 0.080 \pm 0.057) \cdot a_t \\
g_{\eta K} &= (0.19 \pm 0.63 \pm 0.40) \cdot a_t \\
\gamma_{\pi K,\,\pi K} &= (11 \pm 15 \pm 10) \cdot a_t^4 \\
\gamma_{\pi K,\,\eta K} &= (53 \pm 104 \pm 95) \cdot a_t^4 \\
\gamma_{\eta K,\,\eta K} &= (-63 \pm 30 \pm 24) \cdot a_t^4
\end{aligned}
\qquad
\begin{bmatrix}
1 & 0.16 & -0.35 & 0.17 & 0.24 & 0.45 \\
 & 1 & -0.80 & -0.56 & 0.13 & 0.74 \\
 & & 1 & 0.44 & 0.20 & -0.62 \\
 & & & 1 & -0.31 & -0.51 \\
 & & & & 1 & 0.57 \\
 & & & & & 1
\end{bmatrix}
$$

$$\chi^2/N_{\mathrm{dof}} = \frac{23.6}{24-6} = 1.31\,.$$

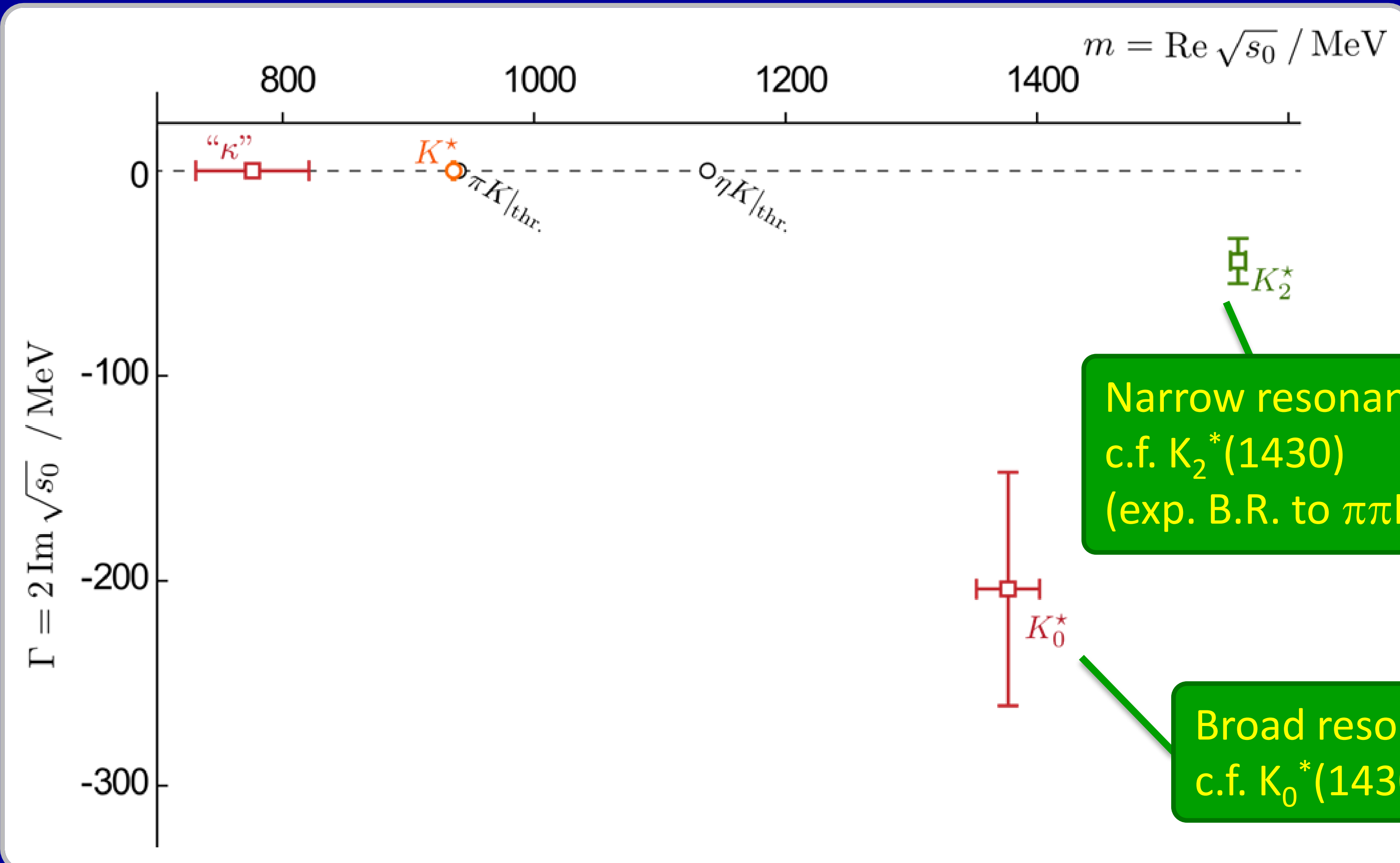
[arXiv:1406.4158]

πK, ηK (I=1/2): t-matrix poles



[arXiv:1406.4158]

πK, ηK (I=1/2): t-matrix poles

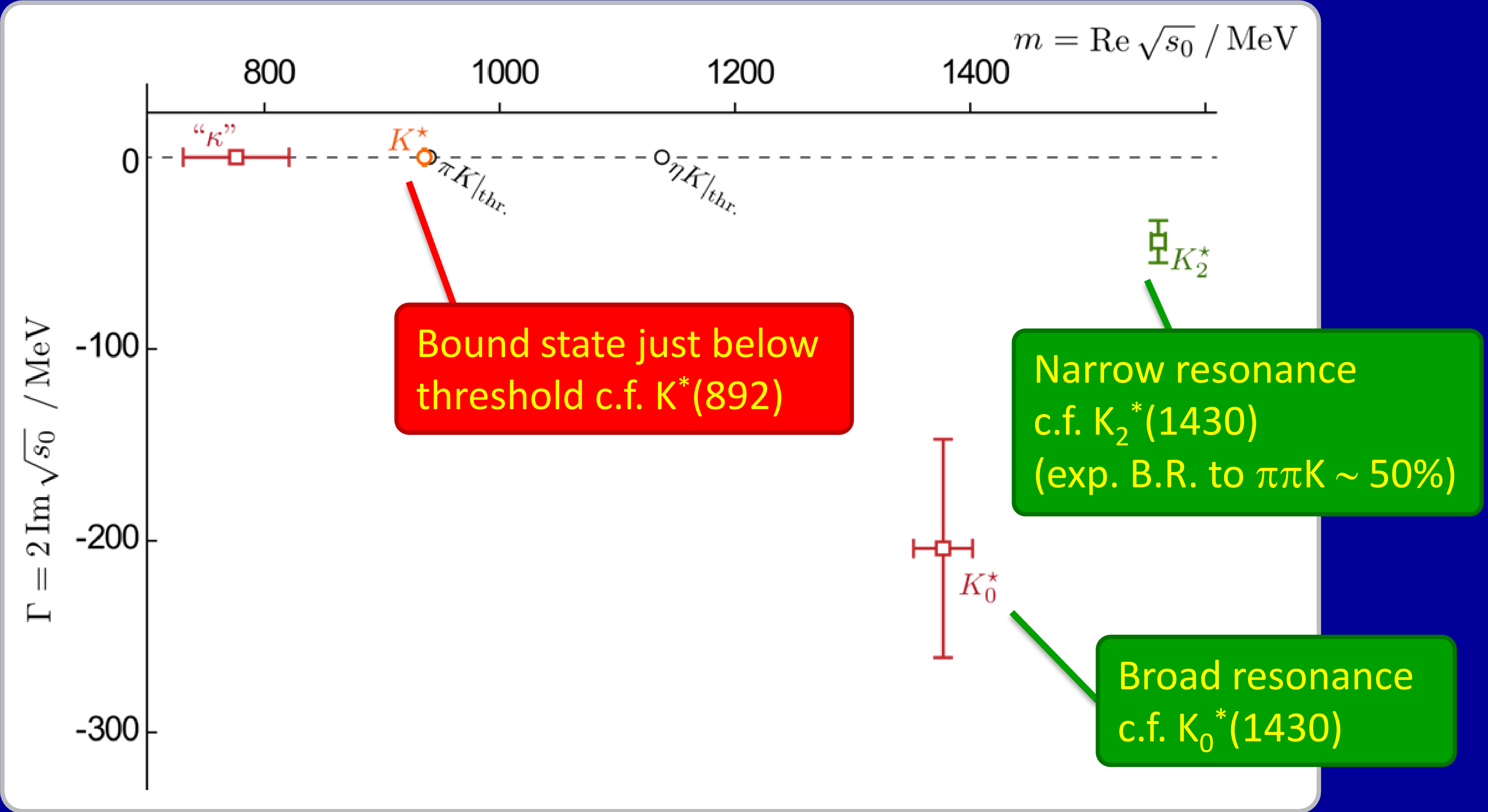


Narrow resonance
c.f. $K_2^*(1430)$
(exp. B.R. to ππK ~ 50%)

Broad resonance
c.f. $K_0^*(1430)$

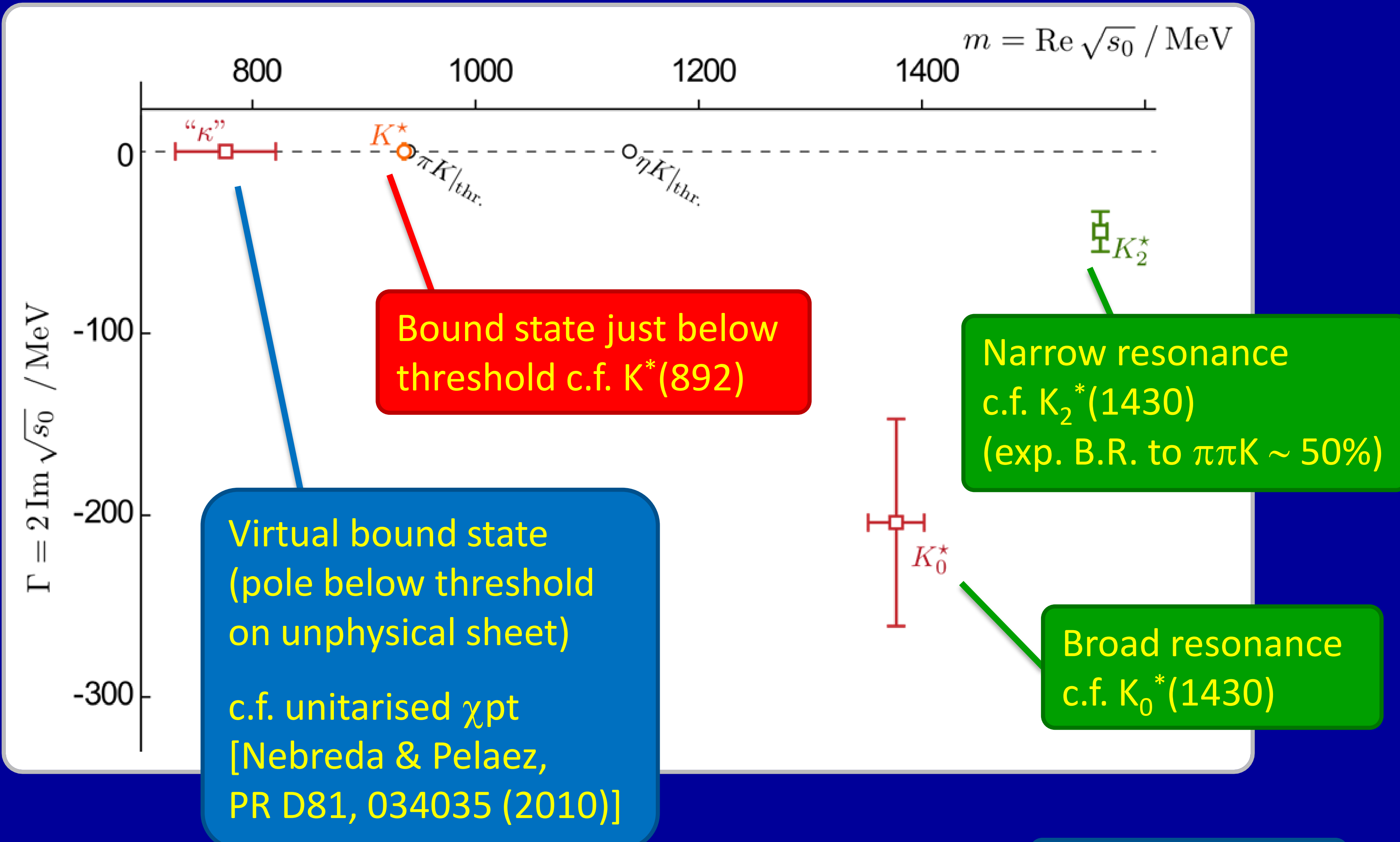
[arXiv:1406.4158]

πK, ηK (I=1/2): t-matrix poles



[arXiv:1406.4158]

πK, ηK (I=1/2): t-matrix poles



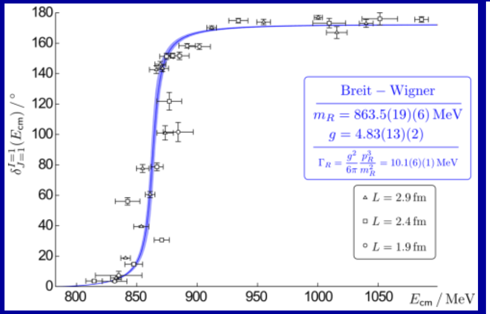
[arXiv:1406.4158]

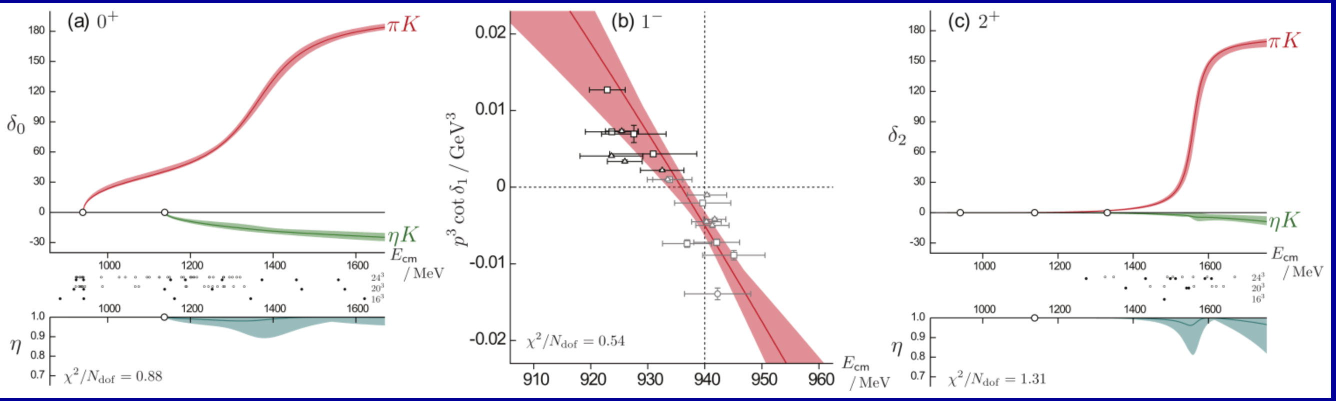
Summary and outlook

Summary

- **Map out** energy dependence of **scattering in detail**
  - ππ I=1: ρ resonance (also ππ I=2 in S and D-wave)
  - πK, ηK I=1 **– first coupled-channel scattering from LQCD**

  → broad & narrow resonances, bound state, v.b.s.





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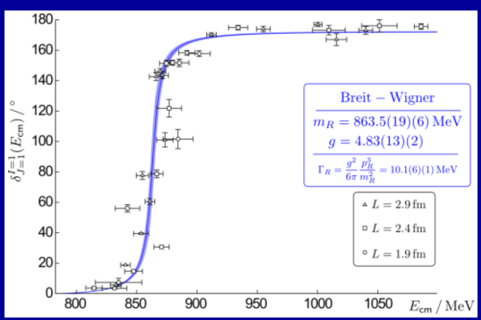

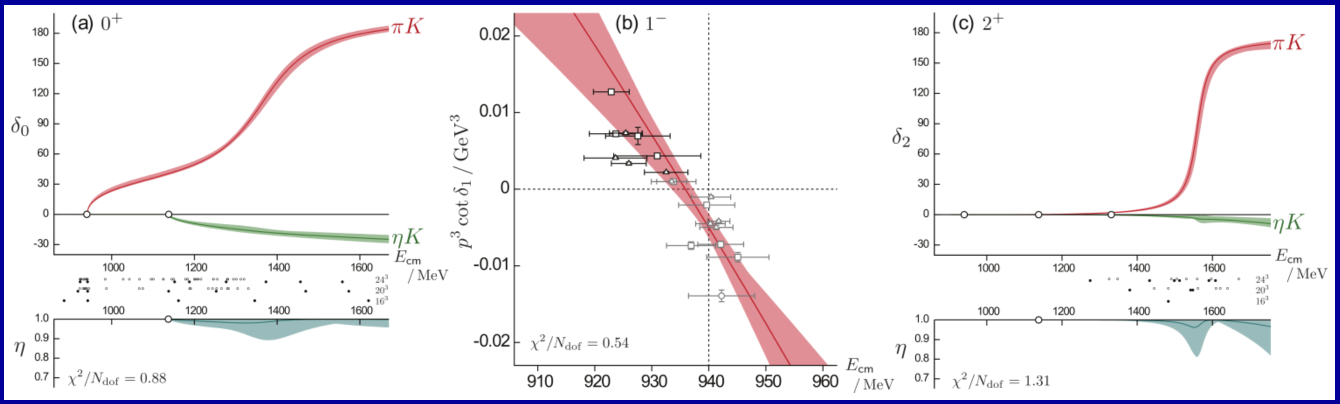

Outlook

- **Many other interesting cases to consider using methodology**
- **>2 hadrons is challenge**. Lighter π → lower 3-hadron thresh.