Resonances and QCD bound states in the charmed-meson and charmonium spectrum from Lattice QCD

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Outline

1. Introduction and lattice basics
   - Motivation
   - Extracting and exploring excited energy levels
   - Scattering phase-shifts and Lüscher's finite volume method
   - Lattices used
   - Heavy quarks with the Fermilab method
2. Charmed and charmed-strange mesons
   - $D\pi$ and $D^*\pi$ scattering and D meson resonances
   - DK scattering and $D_{s0}^*(2317)$, $D_{s1}(2460)$
3. Search for a charged charmonium-like Z_c
4. Conclusions & outlook

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Many interesting issues

Renewed interest in hadron spectroscopy (in experiment and theory)

- X, Y and Z Charmonium-like states
- light scalar mesons
- D_s spectrum: $D^*_{s0}(2317)$ (0^+), $D_{s1}(2460)$
- Highly excited light-quark mesons and baryons

In addition puzzling lattice data for

- Roper resonance
- Λ baryons

Methods used interesting with regard to

- Radiative decays, hadronic transitions
- Puzzles observed in semileptonic B decays
- What kind of hadron resonances/bound states do exist beyond $\bar{q}q$ mesons and qqq baryons?

Example operators

Need: *Interpolating field operator* that creates states with correct quantum numbers.

- Example I: Pseudoscalar mesons with $IJ^{PC} = 10^{-+}$

$$O_\pi^{(1)} = \bar{u}\gamma_5 d$$
$$O_\pi^{(2)} = \bar{u}\overleftrightarrow{D}\gamma_i\gamma_t\gamma_5 d$$

- Example II: Nucleon

$$O_N = \epsilon_{abc}\,\Gamma_1\, u_a \left(u_b^T\,\Gamma_2\, d_c - d_b^T\,\Gamma_2\, u_c\right)$$

- In practice: Many (slightly different) constructions possible!
- In a QFT they should all be OK; Overlap?

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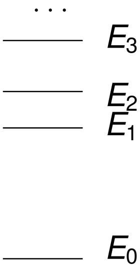
The problem with excited states

From the analysis of Euclidean correlators:

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle_T \propto \sum_n e^{-tE_n} < 0|\hat{O}_2|n >< n|\hat{O}_1|0 >$$

- The whole tower of states contributes
- Ground state is dominant at large t
- Exited states appear as sub-leading exponentials
- Noisy background from limited statistics



- For a single correlator, fit to several exponentials leads to poor results
  $\rightarrow$ Advanced methods needed for excited states!

(My) Method of choice: The variational method

Matrix of correlators projected to fixed momentum (will assume 0)

$$C(t)_{ij} = \sum_n \mathrm{e}^{-tE_n} \langle 0|O_i|n\rangle \left\langle n|O_j^\dagger|0\right\rangle$$

Solve the generalized eigenvalue problem:

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t)C(t_0)\vec{\psi}^{(k)}$$
$$\lambda^{(k)}(t) \propto \mathrm{e}^{-tE_k}\left(1 + \mathcal{O}\left(\mathrm{e}^{-t\Delta E_k}\right)\right)$$

At large time separation: only a single state in each eigenvalue.
Eigenvectors can serve as a fingerprint.

`Michael Nucl. Phys. B259, 58 (1985)`

`Lüscher and Wolff Nucl. Phys. B339, 222 (1990)`

`Blossier et al. JHEP 04, 094 (2009)`

Using single hadron interpolators, what do we see?

- In practical calculations $\bar{q}q$ and qqq interpolators couple very weakly to multi-hadron states

  McNeile & Michael, Phys. Lett. B 556, 177 (2003); Engel et al. PRD 82, 034505 (2010); Bulava et al. PRD 82, 014507(2010); Dudek et al. PRD 82, 034508(2010);

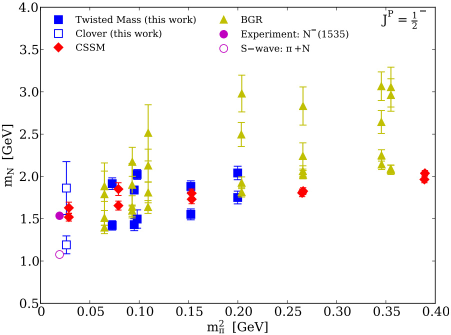
- This is not unlike observations in string breaking studies

  Pennanen & Michael hep-lat/0001015;Bernard et al. PRD 64 074509 2001;

- This necessitates the inclusion of hadron-hadron interpolators
- We know: Energy levels $\neq$ resonance masses
  Naïve expectation: Correct up to $\mathcal{O}(\Gamma_R(m_\pi))$
- Was good enough for heavy pion masses where one would deal with bound states or very narrow resonances.

An example: Negative parity Nucleons

Alexandrou, Korzec, Koutsou, Leontiou, PRD 89 034502 (2014)

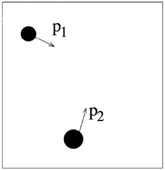


- Beware: different scale setting schemes
- Suggests considerable dependence on interpolator construction
- Should be remedied by including multi-hadron interpolators explicitly!

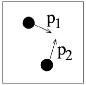
The Lüscher method for elastic scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

$E = E(p_1) + E(p_2)$



L >> few fm



L= few fm

$E = E(p_1) + E(p_2) + \Delta_E$

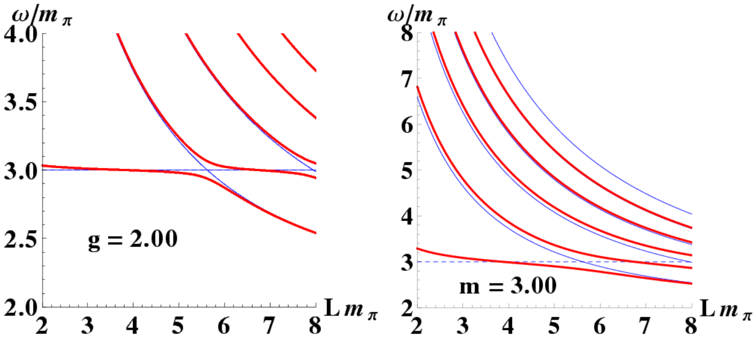
$$E_n(L) \quad \xrightarrow{(2)} \quad \delta_l \quad \xrightarrow{(3)} \quad m_R; \quad \Gamma_R \text{ or coupling } g$$

(1) Extract energy levels $E_n(L)$ in a finite box

(2) The Lüscher formula relates this spectrum to the phase shift of the continuum scattering amplitude

(3) Extract resonance parameters with some degree of modeling/approximation

Energy levels in a box - an illustration



animations by C. B. Lang and DM

- **Left:** Expectations for ρ-like resonance at varying coupling $g_{\rho\pi\pi}$
- **Right:** Expectations for ρ-like resonance with physical $g_{\rho\pi\pi}$ and varying mass

Lüscher method and extensions (selected papers)

- Rest-frame calculation in multiple spatial volumes L^3

M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531;
Nucl. Phys. B 364 (1991) 237.

- Moving frames for equal mass hadrons $m_{h1} = m_{h2}$

Rummukainen, Gottlieb, Nucl. Phys. B 450, 397 (1995);
Kim, Sachrajda, Sharpe, Nucl. Phys. B 727, 218 (2005);
Feng, Jansen, Renner, PoS LAT2010 104 (2010);
Dudek, Edwards, Thomas, PRD 86 034031 (2012).

- Moving frames for $m_{h1} \neq m_{h2}$: Even and odd l mix

Fu, PRD 85 014506 (2012); Döring *et al.* EPJ A48 114 (2012);
Göckeler *et al.* PRDD 86 094513 (2012); Leskovec, Prelovsek, PRD 85 114507 (2012);

- Calculations in multiple asymmetric boxes i.e. $L^2 \times L_z$
- 3-particle scattering

Hansen, Sharpe 1311.4848; Polejaeva, Rusetsky, EPJ A48 67 (2012);
Briceno, Davoudi, PRD 87 094507 (2013)

- Twisted boundary conditions

Briceno, Davoudi, Luu, PRD 88, 034502 (2013);
Briceno, Davoudi, Luu and Savage, PRD 89, 074509 (2014);
Briceno, PRD 89, 074507 (2014)

For more see Raul's talk!

Alternative approaches

- Transition amplitude method

  McNeile, Michael, Pennanen, PRD 65 094505 (2002)

- Histogram method

  Bernard, Lage, Meißner, Rusetsky, JHEP 0808 (2008) 024

- Correlator method

  Meißner, Polejaeva, Rusetsky, Nucl. Phys. B 846,1 (2011)

- Finite-volume Hamiltonian EFT

  Hall *et al.*, PRD 87 094510 (2013)

- HALQCD method: Extract a potential

  Ishii *et al.*, PLB 712, 437 (2012)

Technicalities: Lattices used

ID	$N_L^3 \times N_T$	N_f	a[fm]	L[fm]	#configs	m_π[MeV]	m_K[MeV]
(1)	$16^3 \times 32$	2	0.1239(13)	1.98	280/279	266(3)(3)	552(2)(6)
(2)	$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

- Ensemble (1) has 2 flavors of nHYP-smeared quarks

  Gauge ensemble from Hasenfratz et al. PRD 78 054511 (2008)

  Hasenfratz et al. PRD 78 014515 (2008)

- Ensemble (2) has 2+1 flavors of Wilson-Clover quarks

  PACS-CS, Aoki et al. PRD 79 034503 (2009)

- On the small volume we use distillation
  On the larger volume we use stochastic distillation

  Peardon et al. PRD 80, 054506 (2009);

  Morningstar et al. PRD 83, 114505 (2011)

Heavy quarks using the *Fermilab method*

El-Khadra et al., PRD 55,3933

- We tune κ for the spin averaged kinetic mass $(M_{\eta_c} + 3M_{J/\Psi})/4$ to assume its physical value
- General form for the dispersion relation

Bernard et al. PRD83:034503,2011

$$E(p) = M_1 + \frac{p^2}{2M_2} - \frac{a^3 W_4}{6} \sum_i p_i^4 - \frac{(p^2)^2}{8M_4^3} + \dots$$

- We compare results from three different fit strategies
- Energy splittings are expected to be close to physical
- For MeV values of masses

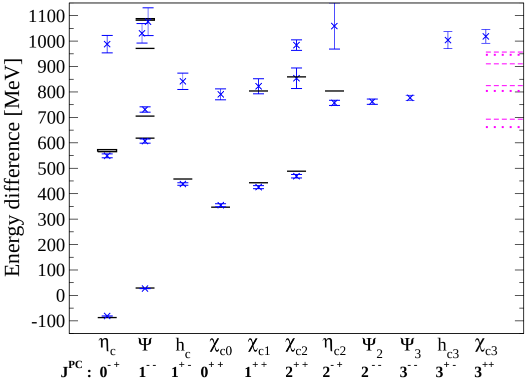
$$M = \Delta M + M_{sa,phys}$$

Testing our tuning ...

	Ensemble (1)	Ensemble (2)	Experiment
m_π	266(3)(3)	156(7)(2)	139.5702(4)
m_K	552(1)(6)	504(1)(7)	493.677(16)
m_ϕ	1015.8(1.8)(10.7)	1018.4(2.8)(14.6)	1019.455(20)
m_{η_s}	732.3(0.9)(7.7)	692.9(0.5)(9.9)	688.5(2.2)*
$m_{J/\Psi} - m_{\eta_c}$	107.9(0.3)(1.1)	107.1(0.2)(1.5)	113.2(0.7)
$m_{D_s^*} - m_{D_s}$	120.4(0.6)(1.3)	142.1(0.7)(2.0)	143.8(0.4)
$m_{D^*} - m_D$	129.4(1.8)(1.4)	148.4(5.2)(2.1)	140.66(10)
$2m_{\overline{D}} - m_{\overline{cc}}$	890.9(3.3)(9.3)	882.0(6.5)(12.6)	882.4(0.3)
$2M_{\overline{D_s}} - m_{\overline{cc}}$	1065.5(1.4)(11.2)	1060.7(1.1)(15.2)	1084.8(0.6)
$m_{D_s} - m_D$	96.6(0.9)(1.0)	94.0(4.6)(1.3)	98.87(29)

- A single ensemble: Discrepancies due to discretization and unphysical light-quark masses expected

Low-lying charmonium spectrum on Lattice (1)



D. M., S. Prelovsek, R. M. Woloshyn, PRD 87 034501 (2013);

- Serves as further confirmation of our heavy-quark approach
- Data from 1 ensemble; Errors statistical + scale setting

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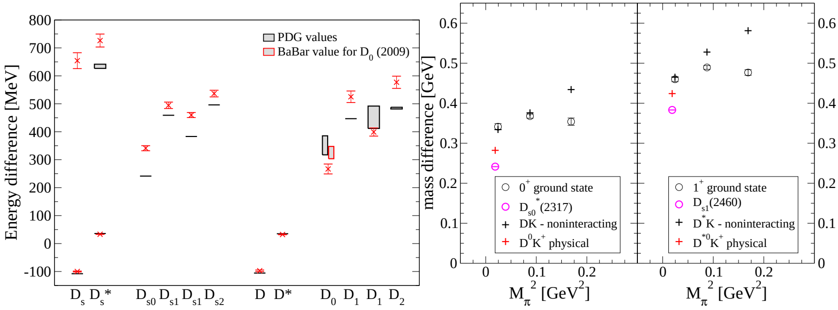
Motivation: Experimental D_s spectrum

- Established states:
  - D_s ($J^P = 0^-$) and D_s^* (1^-)
  - $D_{s0}^*(2317)$ (0^+), $D_{s1}(2460)$ (1^+), $D_{s1}(2536)$ (1^+), $D_{s2}^*(2573)$ (2^+)
- More recent discoveries:
  - $D_{s1}^*(2710)$ seen by BaBar, Belle, LHCb (1^-)
  - $D_{sJ}^*(2860)$ seen by BaBar, LHCb (3^-?,0^+?)
  - $D_{sJ}^*(3040)$ seen by BaBar (1^+?,2^-?)
  - $D_{sJ}^*(2632)$ seen by SELEX (1^-?)
- $j = \frac{1}{2}$ doublet almost mass-degenerate with non-strange states
- Some models suggest a tetraquark/molecular interpretations for controversial states
- (Most) lattice studies using single hadron ($c\bar{s}$) interpolators get large or badly determined masses
- Large m_π: $D_{s0}^*(2317)$ below DK threshold;
  Small m_π: $D_{s0}^*(2317) \approx DK$ threshold

Our previous attempt on ensemble (2)...

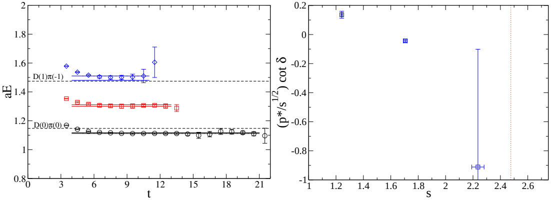


Mohler and Woloshyn, PRD 84 054503, 2011

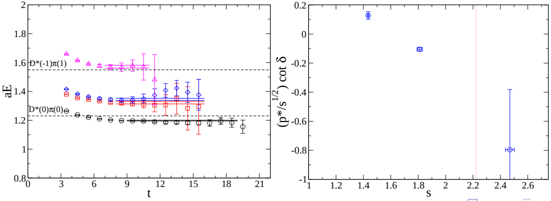
- DK threshold turned out to be unphysical
- Even with light sea-quark masses the lowest states with $J^P = 0^+, 1^+$ remained unphysical
- Including the DK threshold explicitly might be vital

DM, Prelovsek, Woloshyn, PRD 87 034501 (2013)

- In the $J^P = 0^+$ $D_0^\star$ channel we extract three levels



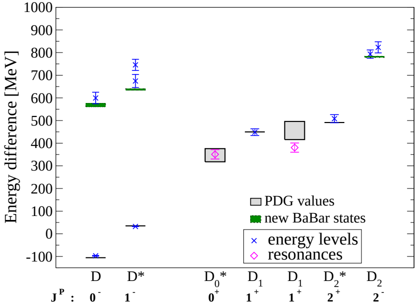
- For the $J^P = 1^+$ channel there are two resonances $D_1(2420)$ and $D_1(2430)$



$D\pi$ and $D^\star\pi$ scattering

`DM, Prelovsek, Woloshyn, PRD 87 034501 (2013)`

- Motivated by the heavy quark limit, We assume one state is given by the naive energy level and fit the remaining data to obtain

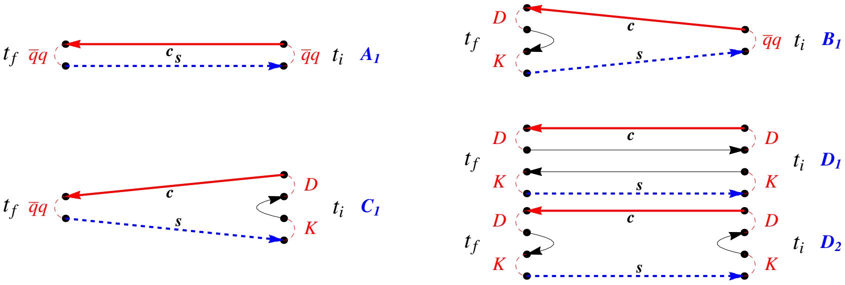


	$D_0^\star(2400)$	$D_1(2430)$
g^{lat} [GeV]	2.55 ± 0.21	2.01 ± 0.15
g^{exp} [GeV]	$\leq 1.92 \pm 0.14$	$\leq 2.50 \pm 0.40$

Revisiting the $D_{s0}^*(2317)$ and $D_{s1}(2460)$

- Use almost physical light quarks
- Work with a partially quenched strange quark
  - Use ϕ meson and η_s to set strange quark mass
  - We obtain $\kappa_s = 0.13666$
- Improve charm quark tuning used for Fermilab charm
  - Use Landau link for $c_{sw,c} = \frac{1}{u_0^3}$
  - Empirically this reduces discretization effects
- Explicitly include DK interpolators into the basis

Contractions

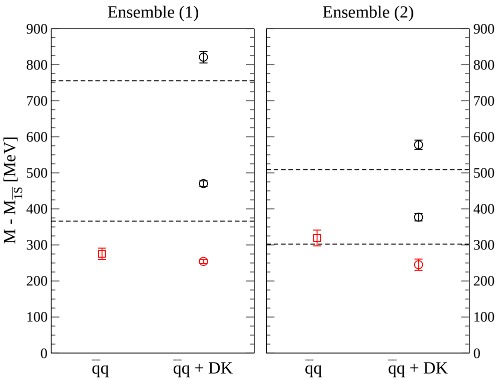


- Handled efficiently within the distillation method

`Peardon et al. PRD 80, 054506 (2009)`
`Morningstar et al. PRD 83, 114505 (2011)`

Energy levels for D_s with $J^P = 0^+$

DM, Lang, Leskovec, Prelovsek, Woloshyn, PRL 111 222001 (2013)



- With the combined basis we obtain a much better quality of the ground state plateau
- The variational method yields two low-lying levels and fits are unambiguous

Possible interpretations

(1) A sub-threshold state stable under the strong interaction

- We call this “bound state scenario”
- This is irrespective of the nature of the state
- One expects a negative scattering length in this case

See `Sasaki and Yamazaki, PRD 74 114507 (2006)` for details.

(2) A resonance in a channel with attractive interaction

- The lowest state corresponds to the scattering level shifted below threshold in finite volume
- The additional level would indicate a QCD resonance
- One expects a positive scattering length in this case

This is the situation for the $D_0^*(2400)$

`DM, Prelovsek, Woloshyn, PRD 87 034501 (2013).`

Using Lüscher's formula

- We can test the plausibility of these scenarios using Lüscher's formula and an effective range approximation

  M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

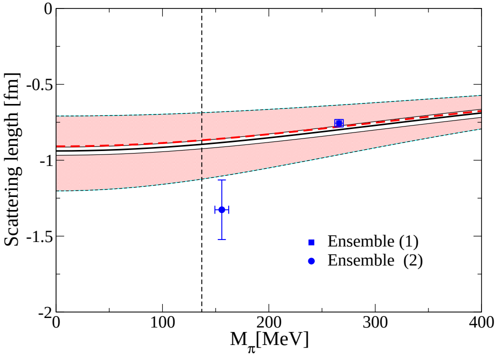
$$
\begin{aligned}
K^{-1} = p\cot\delta(p) &= \frac{2}{\sqrt{\pi}L} Z_{00}(1;q^2)\ , \\
&\approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2\ ,
\end{aligned}
$$

- Results for ensembles (1) and (2)

$$
\begin{aligned}
a_0 &= -0.756 \pm 0.025\mathrm{fm} & r_0 &= 0.056 \pm 0.031\mathrm{fm} & (1)\\
a_0 &= -1.33 \pm 0.20\mathrm{fm} & r_0 &= 0.27 \pm 0.17\mathrm{fm} & (2)
\end{aligned}
$$

Results for the scattering length a_0

DM, Lang, Leskovec, Prelovsek, Woloshyn, PRL 111 222001 (2013)

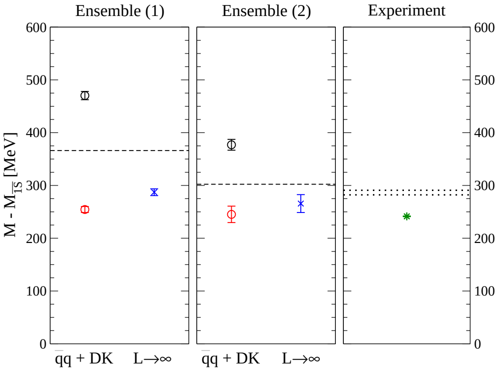


- We compare to the predictions from an indirect calculation

  Liu *et al.* PRD 87 014508 (2013).
- Our determination robustly leads to negative values.

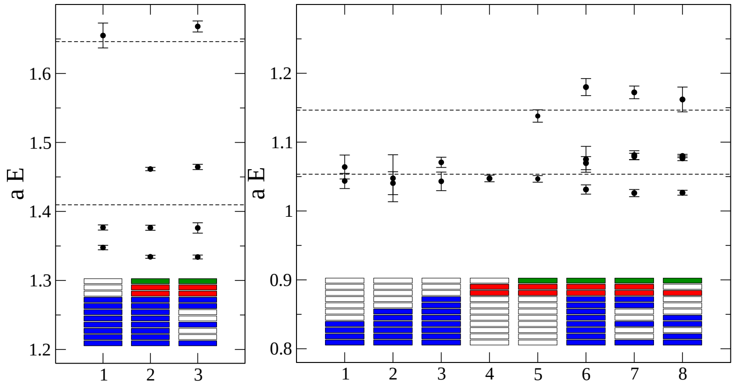
Infinite volume bound states *vs.* experiment

- (Infinite volume)bound state: T-matrix pole for $\cot\delta(i|p_b|) = i$
- Using our a_0 and r_0 we can determine the binding momentum and calculate the corresponding Energy level

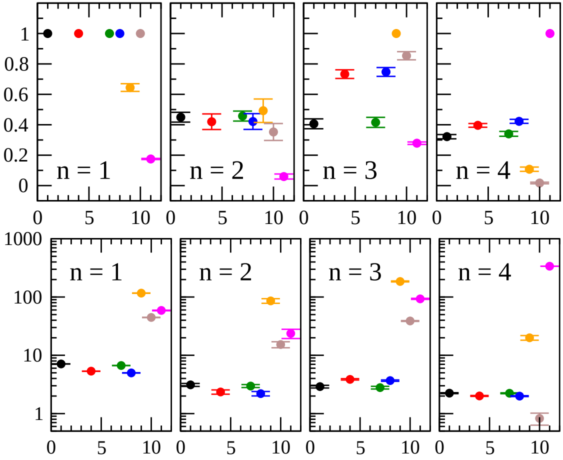


Extending our calculation to the $D_{s1}(2460)$

- Assume the heavy quark limit is a good approximation
  $\rightarrow$ $D_{s1}(2536)$ decays only in D-wave and we extract just a naive energy level



Composition of eigenstates



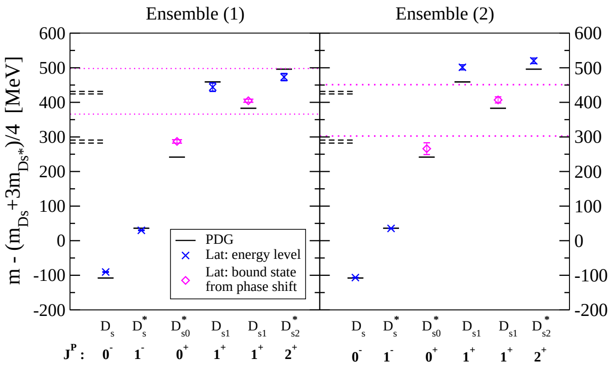
- Beware: Ambiguity in the normalization (eliminated by ratios)

Results in T_1^+

set	$a_0^{D^*K}$ [fm]	$r_0^{D^*K}$ [fm]	$(ap_B)^2$	am_B	$m_K + m_{D^*} - m_B$ [MeV]	$m_B - \frac{1}{4}(m_{D_s} + 3m_{D_s^*})$ [MeV]
Ensemble (1)						
	-0.665(25)	-0.106(37)	-0.0301(15)	1.3511(35)	93.2(4.7)(1.0)	404.6(4.5)(4.2)
Ensemble (2)						
set 1	-1.15(19)	0.13(22)	-0.0071(22)	1.0336(60)	43.2(13.8)(0.6)	408(13)(5.8)
set 2	-1.11(11)	0.10(10)	-0.0073(16)	1.0331(41)	44.2(9.9)(0.6)	407.0(8.8)(5.8)
Experiment						
					44.7	383

set	$m_{D_{s1}(2536)} - \frac{1}{4}(m_{D_s} + 3m_{D_s^*})$ [MeV]	$m_{D_{s1}(2536)} - m_K - m_{D^*}$ [MeV]
Ensemble (1)		
	444(12)	-53(12)
Ensemble (2)		
set 1	507(10)	56(11)
set 2	501(8)	50(8)
Experiment		
	459	31

Resulting D_s P-wave spectrum



- Remaining discrepancies of the size of discretization uncertainties
- Possible improvement down the road: S-wave - D-wave mixing for 1^+ and 2^+ states.

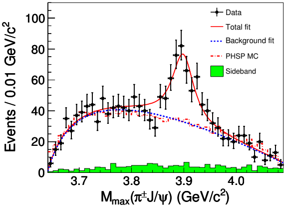
Outline

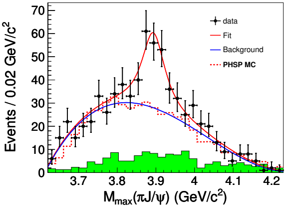
Experimental evidence

[from Brambilla *et al.*, arXiv:1404.3723]

particle	C	J^P	decay	collab.
$Z^+(4430)$	-1	1^+	$\psi(2S)\pi^+$	Belle[08], LHCb[14]
$Z_c^+(3900)$	-1	$?^?$	$J/\psi\pi^+$	BESIII[13], Belle[13], CLEO-c[13]
$Z_c^+(3885)$	-1	1^+	$(DD^*)^+$	BESIII[13]
$Z_c^+(4020)$	-1	$?^?$	$h_c(1P)\pi^+$	BESIII[13]
$Z_c^+(4025)$	-1	$?^?$	$(D^*D^*)^+$	BESIII[13]
$Z^+(4200)$	-1	1^+	$J/\psi\pi^+$	Belle[14] @ Moriond14

- States contain a charm-anticharm pair and carry charge
  $\rightarrow$ at least four quarks





Previous studies

- First search for $Z_c^+(3900)$

  `Sasa Prelovsek, Luka Leskovec PLB 727 172 (2013)`

  - $J/\Psi\pi$ and DD^* interpolators
  - only levels close to two-meson states found

- Search for resonance in DD^* scattering

  `Chen et al., PRD 89 094506 (2014)`

  - no $J/\Psi\pi$ interpolators,
  - twisted mass ensembles with $m\_\pi = 300, 420, 485$ MeV
  - Phase shift analysis assuming only DD^* is relevant
  - S-wave - P-wave mixing taken into account

Our approach

- Search for a Z_c^+ in the $I^G J^{PC} = 1^+ 1^{+-}$ channel
- Aim at simulating all meson-meson states below ≈ 4.1GeV
- Include tetraquark interpolators of type $3_c \times \bar{3}_c$
- Count energy levels and identify them according to their overlaps
- Hope: See an extra level, as would be expected for a (narrow) resonance

More rigorous approach (a la Lüscher) quite challenging

- Coupled channel system with many channels
- Small shifts in finite volume and (largish) discretization effects
- Thresholds should be close to physical
- Suitable ensembles are (probably) not available at the moment.

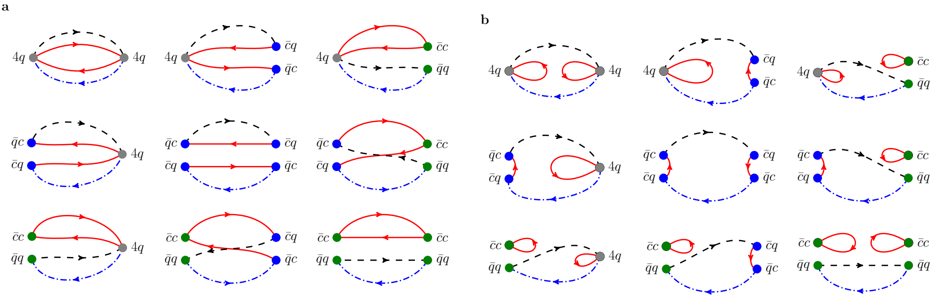
Analysis details

- We use ensemble (1)

$$m_\pi = 266\mathrm{MeV}$$
$$a = 0.1239(13)\mathrm{fm}$$
$$L \approx 1.98\mathrm{fm}$$

- 18×18 basis with $J/\Psi\pi$, $\eta_c\rho$, $D^{(*)}D^{(*)}$ and Tetraquark interpolators
- Neglects 3-particle states
- Determine energy levels and calculate overlaps Z_i^n of the i-th operator to the n-th state

Wick contractions



- We neglect contractions with a backtracking c-quark
- OZI suppressed and difficult to handle (tower of light states)

A look at the spectrum of scattering states

- Expect level close to non-interacting scattering states

$J/\Psi\pi$

$\eta_c\rho$

$J\Psi(1)\pi(-1)$

DD^*

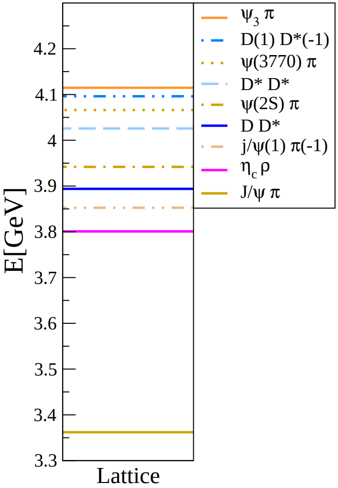
$\Psi_{2S}\pi$

D^*D^*

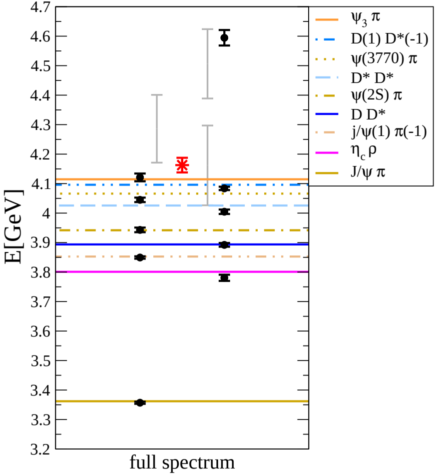
$\Psi_{3770}\pi$

$D(1)D^*(-1)$

$\Psi_3\pi$

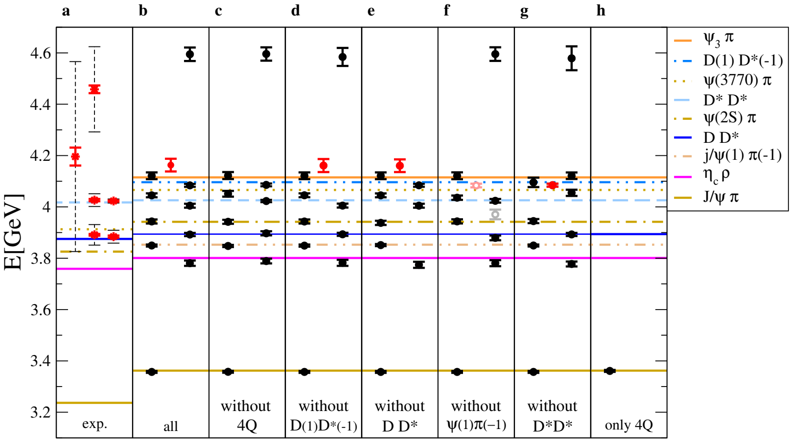


An additional state!



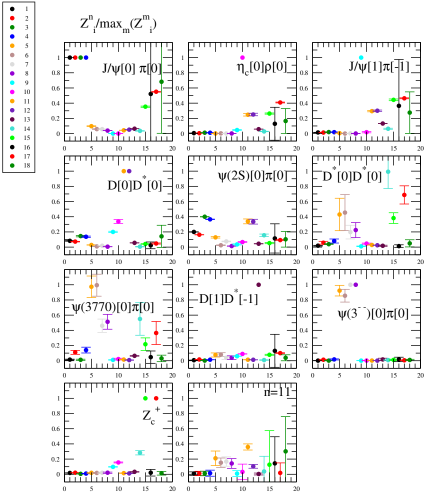
- We see energy levels close to scattering states we expected to see
- We see an additional state with a good signal
- Would ideally want to have higher scattering levels like $J/\Psi(2)\pi(-2)$
- What is the additional state?

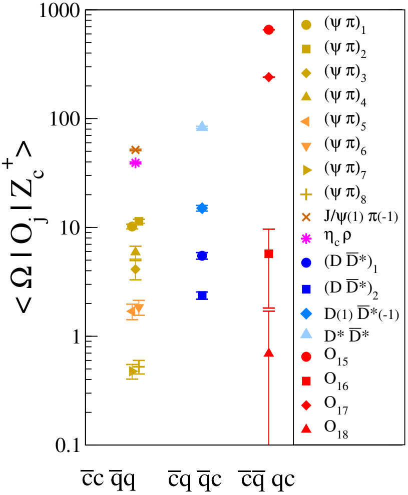
Playing with the interpolator basis



- The level disappears when tetraquark-like interpolators are dropped
- For our pion mass, some scattering channels seem to decouple

Overlap ratios and composition of our extra-state





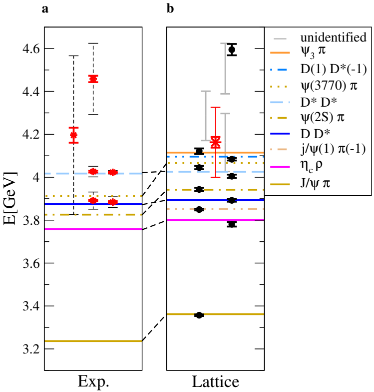
Result and uncertainty estimate

Additional state at

$$m_{Z_c^+} = 4.16 \pm 0.16 \pm \mathcal{O}(\Gamma_{Z_c^+})\mathrm{GeV}$$

Uncertainty includes

- A crude estimate of the pion-mass dependence
- An estimate of heavy quark discretization effects
- A flat percent uncertainty from the lattice scale determination



- Result not compatible with the $Z_c(3900)$ but additional level (exotic state!) observed

Outline

Conclusions

- Determining the the resonance spectrum from QCD has just begun
- Meson and baryon states close to threshold(s) can be attacked
- It will be a lot of incremental progress working our way up the spectrum (Coupled channel results encouraging $\rightarrow$ Christopher)
- Precision results for charm will require full control of systematics (physical quark masses, continuum extrapolation, multiple volumes)
- Extracting resonance parameters from lattice scattering phase shifts will need (some degree) of modeling, just like in experiment.
- Brute force calculations may not be tractable
- Lattice QCD calculations $\leftrightarrow$ Phenomenology/ EFT's

Backup slides

• • •

Angular momentum (mesons)

- *Reminder:* No unique spin assignment on the lattice. Five irreducible representations:

Irrep of O	J	Spinors in irrep
A_1	0,4,...	$1, \gamma_t, \gamma_5, \gamma_t\gamma_5$
A_2	3,6,...	
E	2,4,5,...	
T_1	1,3,4,5,...	$\gamma_i, \gamma_t\gamma_i, \gamma_5\gamma_i, \gamma_t\gamma_5\gamma_i$
T_2	2,3,4,5,...	

- Classification of interpolator basis by representations
- Unique identification of spin nontrivial

  `Dudek et al., PRL 103 262001 (2009)`

Z_c Interpolator basis I

$$\left.\begin{aligned}
\mathcal{O}_1 &= \bar{c}\gamma_i c[0]\ \bar{d}\gamma_5 u[0] \\
\mathcal{O}_2 &= \bar{c}\gamma_i\gamma_t c[0]\ \bar{d}\gamma_5 u[0] \\
\mathcal{O}_3 &= \bar{c}\overleftarrow{\nabla}_j\gamma_i\overrightarrow{\nabla}_j c[0]\ \bar{d}\gamma_5 u[0] \\
\mathcal{O}_4 &= \bar{c}\overleftarrow{\nabla}_j\gamma_i\gamma_t\overrightarrow{\nabla}_j c[0]\ \bar{d}\gamma_5 u[0] \\
\mathcal{O}_5 &= |\epsilon_{ijk}||\epsilon_{klm}|\ \bar{c}\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c[0]\ \bar{d}\gamma_5 u[0] \\
\mathcal{O}_6 &= |\epsilon_{ijk}||\epsilon_{klm}|\ \bar{c}\gamma_t\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c[0]\ \bar{d}\gamma_5 u[0] \\
\mathcal{O}_7 &= R_{ijk}Q_{klm}\ \bar{c}\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c[0]\ \bar{d}\gamma_5 u[0] \\
\mathcal{O}_8 &= R_{ijk}Q_{klm}\ \bar{c}\gamma_t\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c[0]\ \bar{d}\gamma_5 u[0] \\
\mathcal{O}_9 &= \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c[e_k]\ \bar{d}\gamma_5 u[-e_k]
\end{aligned}\right\} [\bar{c}\Gamma_1 c]_{1_c}\ [\bar{d}\Gamma_2 u]_{1_c}\quad J/\psi\pi\text{-like}$$

$$\mathcal{O}_{10} = \bar{c}\gamma_5 c[0]\ \bar{d}\gamma_i u[0] \quad \} \ [\bar{c}\Gamma_1 c]_{1_c}\ [\bar{d}\Gamma_2 u]_{1_c}\quad \eta_c\rho$$

$$\left.\begin{aligned}
\mathcal{O}_{11} &= \bar{c}\gamma_5 u[0]\ \bar{d}\gamma_i c[0] \\
\mathcal{O}_{12} &= \bar{c}\gamma_5\gamma_t u[0]\ \bar{d}\gamma_i\gamma_t c[0] \\
\mathcal{O}_{13} &= \bar{c}\gamma_5 u[1]\ \bar{d}\gamma_i c[-1]
\end{aligned}\right\} [\bar{c}\Gamma_1 u]_{1_c}\ [\bar{d}\Gamma_2 c]_{1_c}\quad DD^*$$

$$\mathcal{O}_{14} = \epsilon_{ijk}\bar{c}\gamma_j u[0]\ \bar{d}\gamma_k c[0] \quad \} \ [\bar{c}\Gamma_1 u]_{1_c}\ [\bar{d}\Gamma_2 c]_{1_c}\quad D^* D^*$$

$$\mathcal{O}_{15} = N_L^3\ \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5\bar{d}_c\ c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i\bar{d}_c\ c_{b'}\gamma_5 C u_{c'}) \quad \} \ [\bar{c}\Gamma_1\bar{d}]_{3_c}\ [c\Gamma_2 u]_{\bar{3}_c}$$

$$\mathcal{O}_{16} = N_L^3\ \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\bar{d}_c\ c_{b'}\gamma_i\gamma_5 C u_{c'} - \bar{c}_b C\gamma_i\gamma_5\bar{d}_c\ c_{b'} C u_{c'}) \quad \} \ [\bar{c}\Gamma_1\bar{d}]_{3_c}\ [c\Gamma_2 u]_{\bar{3}_c}$$

$$\mathcal{O}_{17} = \mathcal{O}_{15} \qquad \mathcal{O}_{18} = \mathcal{O}_{16} \quad \} \ Nv = 32$$

Technicalities: The "Distillation" method

Peardon et al. PRD 80, 054506 (2009)
Morningstar et al. PRD 83, 114505 (2011)

- Idea: Construct separable quark smearing operator using low modes of the 3D lattice Laplacian
  Spectral decomposition for an $N \times N$ matrix:

$$f(A) = \sum_{k=1}^{N} f(\lambda^{(k)})\, v^{(k)} v^{(k)\dagger}.$$

  With $f(\nabla^2) = \Theta(\sigma_s^2 + \nabla^2)$ (Laplacian-Heaviside (LapH) smearing):

$$q_s \equiv \sum_{k=1}^{N} \Theta(\sigma_s^2 + \lambda^{(k)}) v^{(k)} v^{(k)\dagger}\, q \;=\; \sum_{k=1}^{N_v} v^{(k)} v^{(k)\dagger}\, q\,.$$

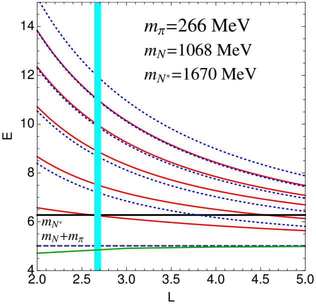
- Advantages: momentum projection at source; large interpolator freedom, small storage
- Disadvantages: expensive; unfavorable volume scaling
- Stochastic approach (mostly) eliminates bad volume scaling

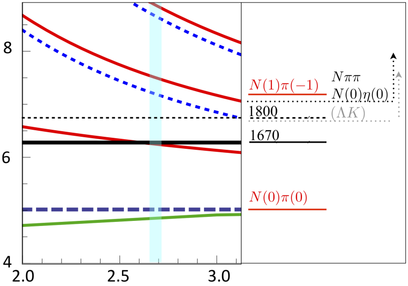
Pion Nucleon scattering in $J^P = \frac{1}{2}^-$

`Lang and Verducci, PRD 87 054502 (2013)`

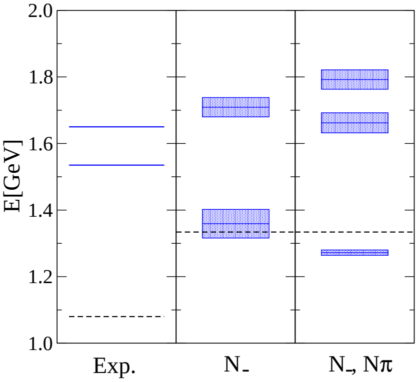
- Puzzle: Do we extract an energy level related to the $N^\star(1535)$ or do we see the $N\pi$ threshold?
- To address this: Combined basis of 3-quark and $N\pi$ interpolators
- Simulation at $M_\pi = 266$ MeV, $m_N = 1068$ MeV on a 2-flavor sea

Expected energy levels for a single resonance with $\Gamma_r = 150\mathrm{MeV}$:





Pion Nucleon scattering in $J^P = \frac{1}{2}^-$ II



- Spectrum using just 3-quark interpolators can be misleading
- Pattern using combined basis is very similar to experiment
- At physical masses problem is inelastic $\rightarrow$ much harder