(a) Power Counting Comparison

prepared by Grießhammer 7.3.2013, Orsay Workshop, based on priv comms.

<table>
<tr><th>wave</th><th>order</th><th>Yang/Long</th><th>Pavon Valderrama</th><th>Birse</th></tr>
<tr><td>$^1S_0$</td><td>LO</td><td colspan="3">-1</td></tr>
<tr><td></td><td>NLO</td><td colspan="3">0</td></tr>
<tr><td></td><td>N^{2}LO</td><td>1</td><td colspan="2">2</td></tr>
<tr><td>$^3S_1$</td><td>LO</td><td colspan="3">-1</td></tr>
<tr><td></td><td>NLO</td><td>1</td><td>2</td><td>$\frac{1}{2}$</td></tr>
<tr><td></td><td>N^{2}LO</td><td></td><td></td><td>$\frac{5}{2}$</td></tr>
<tr><td>$^3SD_1$</td><td>LO</td><td>1</td><td>$-\frac{1}{2}$</td><td>-1</td></tr>
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<tr><td>$^3P_0$</td><td>LO</td><td colspan="2">-1</td><td>$-\frac{1}{2}$</td></tr>
<tr><td></td><td>NLO</td><td>1</td><td>2</td><td>$\frac{3}{2}$</td></tr>
<tr><td>OPE</td><td>LO</td><td colspan="3">-1</td></tr>
<tr><td></td><td>NLO</td><td></td><td></td><td>2</td></tr>
<tr><td>TPE</td><td>LO</td><td>1</td><td colspan="2">2</td></tr>
<tr><td></td><td>NLO</td><td>2</td><td colspan="2">3</td></tr>
</table>

Grießhammer, Benasque 24.7.2014
based on by Saclay 4.3.2013
and nucl-th/0404073

Notes: Testing a Power Counting

PC provides quantitative, falsifiable predictions

$\Rightarrow$ chance to convince people who can/do not reproduce your derivation!

Usually, cutoff Λ bad – here: use it to your advantage!

$$\text{observable}\quad \mathcal{O}(\Lambda) = \underbrace{\sum_i^n \left(\frac{p_{typ.}}{\underbrace{\bar{\Lambda}}_{\text{breakdown scale}}}\right)^i \mathcal{O}_i}_{\substack{\text{renormalised, cutoff-indep.}\\ \text{result at order } n.}} + \underbrace{\mathcal{C}(\Lambda)\left(\frac{p_{typ.}}{\bar{\Lambda}}\right)^{n+1}}_{\substack{\text{residual cutoff}\\ \text{dependence: order } (n+1)}}$$

[can be generalised to include $\ln\bar{\Lambda}$, $\ln k$ etc]

$$\Rightarrow \frac{\mathcal{O}(\Lambda_1)-\mathcal{O}(\Lambda_2)}{\mathcal{O}(\Lambda_1)} = \underbrace{\left(\frac{p_{typ.}}{\bar{\Lambda}}\right)^{n+1}}_{\substack{\text{predict slope in}\\ \ln-\ln \text{ plot}}} \underbrace{\frac{\mathcal{C}(\Lambda_1)-\mathcal{C}(\Lambda_2)}{\mathcal{C}(\Lambda_1)}}_{\substack{\text{cf derivative w.r.t. to}\\ \Lambda:\ \text{RG evolution plot!}}}$$

for $\underset{\text{ext. momentum}}{k} \gg$ any other low scale (eg m_π, γ, ...) and $k \ll \bar{\Lambda}$,

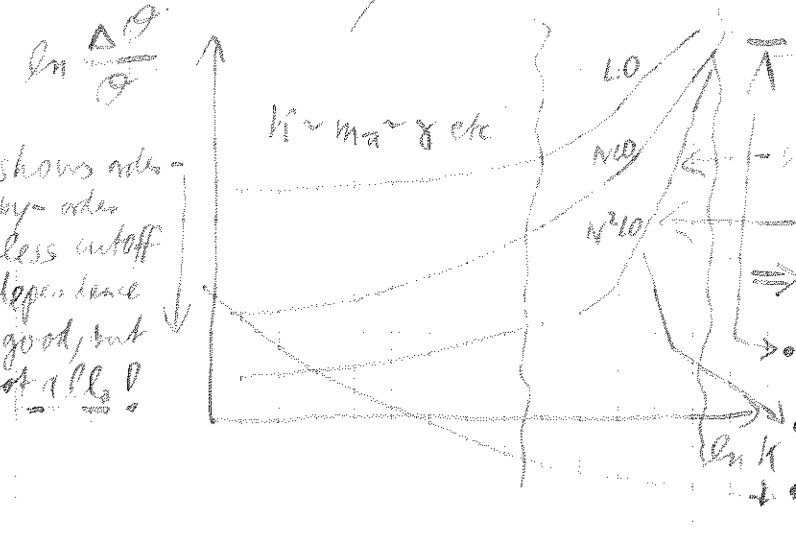
can determine slope $(n+1)$: predicted by PC!

and $\bar{\Lambda}$: breakdown scale

any two cutoffs Λ_1, Λ_2 should work, but in practise, want a large leverage arm for good numerics.

NB: Not what Lepage did in nucl-th/9706029 cf. Steele nucl-th/9802069, Epelbaum et al nucl-th/9910064: He compared theory to (fake) data:

We only want to consider data after showing consistency.



shows order-by-order less cutoff dependence – good, but not all!

$\bar{\Lambda}$ breakdown scale

– slope (n_0+1) !!

– region $k \gg$ any other scale

$\Rightarrow$ Read off 3 essentials:

- breakdown scale $\bar{\Lambda}$
- PC slope $(n+1)$
- low-k expansion param. $\left(\frac{p_{typ}}{\bar{\Lambda}}\right)$

What observable $\mathcal{O}$ to choose?

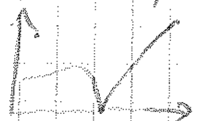
: should be as unconstrained as possible

<u>not</u> phase shift since $\delta(k\to 0)\to 0$: point for free.

$\Rightarrow$ eg $k^{2l+1}\cot\delta$ for l-th partial wave.

: nice if $\mathcal{O}$ is positive or negative definite for all k

$\to$ otherwise get



↳ artificial zero

: If LECs need to be fitted, do that in small-k region ($k\to 0$, $k \ll p_{typ}$ all other scales):

Then slope can still emerge nicely as $k\to\bar{\Lambda}$

$\Rightarrow$ utilise maximally

k-dependence of prediction!

↳ fit point

$\mathcal{O}(\Lambda)$ fixed!

: after fit, data at higher k "irrelevant"

to determine internal consistency of PC!

$\Rightarrow$ <u>do not even consider convergence to experiment!</u>

<u>Illustration:</u> Lesson in 3N EFT($\not\pi$): H_2 is N^{2}LO from slope

$\longrightarrow$ nucl-th/040473.

<u>Limitations:</u> • Cannot see LECs which do <u>not</u> serve as CTs

• Does not solve <u>all</u> problems, can be numerically indecisive

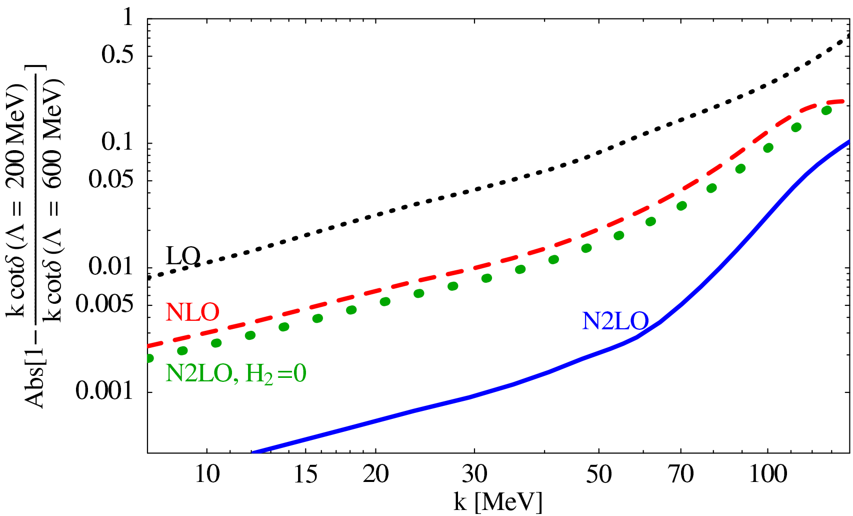
– But one should <u>try</u> it at least!

<u>Challenge:</u> Produce <u>and publish</u> such plots using your PC,

<u>in particular</u> if you do <u>not</u> like the answer!

(h) Doublet-S Wave nd Phase Shift

Bedaque/hg/Hammer/Rupak 2002, hg 2004



Wilson's Renormalisability Criterion quantified by "non-Lepage plot"

$$\left|1 - \frac{k\cot\delta(\Lambda = 200\ \text{MeV})}{k\cot\delta(\Lambda = \infty)}\right| \sim \underbrace{\left(\frac{p_{\text{typ.}}}{\Lambda_{\not\pi}}\right)^{n}}_{Q^{n}}$$

	LO	NLO	N^{2}LO	N^{2}LO without H_2
n fitted	~ 1.9	2.9	4.8	3.1
n expected	2	3	4	$4!?!$

$\Longrightarrow$ Fit to $k \in [70;100\ldots130]$ MeV