Baryon-baryon interactions in chiral EFT

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Outline

The YN (YY) interaction

- study the role of strangeness in low and medium energy nuclear physics
- test $SU(3)_{flavor}$ symmetry
- H dibaryon
  - Jaffe (1977) → deeply bound 6-quark state with $I = 0$, $J = 0$, $S = -2$
  - many experimental searches but no convincing signal
  - Lattice QCD (2010) → evidence for a bound H dibaryon
- prerequisite for studies of (Λ, Σ) hypernuclei
- quest for $\Lambda\Lambda$ hypernuclei and Ξ hypernuclei
  → J-PARC, FAIR
- implications for astrophysics
  → hyperon stars
  stability/size of neutron stars

Experimental status

YN data [$S = -1$]

- about 35 data points, all from the 1960s
- 10 new data points, from the KEK-PS E251 collaboration (from $\approx$ 2000)

YY data [$S = -2$]

- a few rough estimations of ΞN cross sections from the 1970s
- "more precise" cross sections (for $\Xi^- p$ and $\Xi^- p \to \Lambda\Lambda$) published in 2006
- constraints on the $\Lambda\Lambda$ scattering length from $^{12}C(K^-, K^+\Lambda\Lambda X)$

$S = -3, -4$: uncharted territory

- ? information from heavy ion collisions ?
  ? S=-3 physics at J-PARC ?

YN, YY, ΞY, $\Xi\Xi$ in chiral effective field theory

We* follow the scheme of S. Weinberg (1990)
in complete analogy to the study of NN in χEFT by
E. Epelbaum, W. Glöckle, U.-G. Meißner (NPA 671 (2000) 295)

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

Obstacle: YN data base is rather poor

- only about 40 data points
- no polarization data $\Rightarrow$ no phase shift analysis
- need to fit directly to YN data
- constraints from hypernuclei ($^3_\Lambda$H binding energy)

$\rightarrow$ impose $SU(3)_f$ constraints

(*J.H, N. Kaiser, U.-G. Meißner, A. Nogga, S. Petschauer, W. Weise)

Power counting

$$V_{\rm eff} \equiv V_{\rm eff}(Q, g, \mu) = \sum_{\nu} (Q/\Lambda)^{\nu} \, \mathcal{V}_{\nu}(Q/\mu, g)$$

- Q ... soft scale (baryon three-momentum, Goldstone boson four-momentum, Goldstone boson mass)
- Λ ... hard scale
- g ... pertinent low–energy constants
- μ ... regularization scale
- $\mathcal{V}_{\nu}$ ... function of order one
- $\nu \geq 0$ ... chiral power

Leading order (LO): $\nu = 0$

a) non-derivative four-baryon contact terms

b) one-meson (Goldstone boson) exchange diagrams

Next-to-leading order (NLO): $\nu = 2$

a) four-baryon contact terms with two derivatives

b) two-meson (Goldstone boson) exchange diagrams e.g., LO contact terms for BB:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right) \quad \Rightarrow \quad \begin{aligned} \mathcal{L}^1 &= \tilde{C}_i^1 \left\langle \bar{B}_a \bar{B}_b \left(\Gamma_i B\right)_b \left(\Gamma_i B\right)_a \right\rangle , \\ \mathcal{L}^2 &= \tilde{C}_i^2 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle , \\ \mathcal{L}^3 &= \tilde{C}_i^3 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \right\rangle \left\langle \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle \end{aligned}$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

a, b ... Dirac indices of the particles

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

C_i, $\tilde{C}_i$ ... low-energy constants (LECs) spin-momentum structure of the contact term potential:

BB contact terms without derivatives (LO):

$$V^{(0)}_{BB\to BB} = C_{S,BB\to BB} + C_{T,BB\to BB}\,\vec{\sigma}_1\cdot\vec{\sigma}_2$$

BB contact terms with two derivatives (NLO):

$$\begin{aligned} V^{(2)}_{BB\to BB} &= C_1\vec{q}^{\,2} + C_2\vec{k}^{\,2} + (C_3\vec{q}^{\,2} + C_4\vec{k}^{\,2})(\vec{\sigma}_1\cdot\vec{\sigma}_2) \\ &+ \frac{i}{2}C_5(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}\times\vec{k}) + C_6(\vec{q}\cdot\vec{\sigma}_1)(\vec{q}\cdot\vec{\sigma}_2) \\ &+ C_7(\vec{k}\cdot\vec{\sigma}_1)(\vec{k}\cdot\vec{\sigma}_2) + \frac{i}{2}C_8(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{q}\times\vec{k}) \end{aligned}$$

note: $C_i \longrightarrow C_{i,BB\to BB}$

$\vec{q} = \vec{p}\,' - \vec{p}$; $\vec{k} = (\vec{p}\,' + \vec{p})/2$

$SU(3)$ symmetry

10 independent spin-isospin channels in NN and YN (for L=0)
(NN (I=0), NN (I=1), ΛN, ΣN (I=1/2), ΣN (I=3/2), $\Lambda N \leftrightarrow \Sigma N$)

$\Rightarrow$ in principle (at LO), 10 low-energy constants

$SU(3)$ symmetry $\Rightarrow$ only 5 independent low-energy constants

$SU(3)$ structure for scattering of two octet baryons:
direct product:

$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

$C_{S,i}$, $C_{T,i}$, $C_{1,i}$, etc., can be expressed by LECs corresponding to the $SU(3)_f$ irreducible representations:
C^1, C^{8_a}, C^{8_s}, C^{10^*}, C^{10}, C^{27}

$SU(3)$ structure of contact terms for BB

	Channel	I	V_{1S0}, $V_{3P0,3P1,3P2}$	V_{3S1}, $V_{3S1-3D1}$, V_{1P1}	$V_{1P1-3P1}$
$S = 0$	$NN \to NN$	0	–	C^{10^*}	–
	$NN \to NN$	1	C^{27}	–	–
$S = -1$	$\Lambda N \to \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Lambda N \to \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27} + C^{8_s}\right)$	$\frac{1}{2}\left(-C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
					$\frac{-1}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \to \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27} + 9C^{8_s}\right)$	$\frac{1}{2}\left(C^{8_a} + C^{10^*}\right)$	$\frac{3}{\sqrt{20}} C^{8_s 8_a}$
	$\Sigma N \to \Sigma N$	$\frac{3}{2}$	C^{27}	C^{10}	–

Number of contact terms:

NN: 2 (LO) 7 (NLO)
YN: +3 (LO) +11 (NLO)
YY: +1 (LO) + 4 (NLO)
$\Rightarrow C^1$ contributes only to $I = 0$, $S = -2$ channels!!

$SU(3)$ structure of contact terms for BB

Examples:

$$
\begin{aligned}
V_{NN\to NN}(^1S_0) &= \tilde{C}^{27}_{^1S_0} + C^{27}_{^1S_0}(p^2+p'^2) \\
V_{\Lambda N\to \Lambda N}(^1S_0) &= \frac{1}{10}\left[(9\tilde{C}^{27}_{^1S_0} + \tilde{C}^{8_s}_{^1S_0}) + (9C^{27}_{^1S_0} + C^{8_s}_{^1S_0})(p^2+p'^2)\right] \\
V^{I=1/2}_{\Sigma N\to \Sigma N}(^1S_0) &= \frac{1}{10}\left[(\tilde{C}^{27}_{^1S_0} + 9\tilde{C}^{8_s}_{^1S_0}) + (C^{27}_{^1S_0} + 9C^{8_s}_{^1S_0})(p^2+p'^2)\right] \\
V^{I=3/2}_{\Sigma N\to \Sigma N}(^1S_0) &= \tilde{C}^{27}_{^1S_0} + C^{27}_{^1S_0}(p^2+p'^2) \\
V_{NN\to NN}(^1P_1) &= C^{10^*}_{^1P_1}(p\,p') \\
V_{\Lambda N\to \Lambda N}(^1P_1) &= \frac{1}{2}(C^{8_a}_{^1P_1} + C^{10^*}_{^1P_1})(p\,p') \\
&\dots
\end{aligned}
$$

Pseudoscalar-meson exchange

$\mathrm{SU(3)}_f$-invariant pseudoscalar-meson-baryon interaction Lagrangian:

$$\mathcal{L} \quad = \quad -\left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\{\partial_\mu P, B\} + \frac{g_A\alpha}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5[\partial_\mu P, B]\right\rangle$$

$$f = g_A/(2F_\pi); \quad g_A \simeq 1.26, \quad F_\pi \approx 93 \text{ MeV}$$

$$\alpha = F/(F+D) \text{ with } g_A = F + D$$

$$P \quad = \quad \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\begin{array}{llllll} f_{NN\pi} = & f & f_{NN\eta_8} = & \frac{1}{\sqrt{3}}(4\alpha-1)f & f_{\Lambda NK} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f \\ f_{\Xi\Xi\pi} = & -(1-2\alpha)f & f_{\Xi\Xi\eta_8} = & -\frac{1}{\sqrt{3}}(1+2\alpha)f & f_{\Xi\Lambda K} = & \frac{1}{\sqrt{3}}(4\alpha-1)f \\ f_{\Lambda\Sigma\pi} = & \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma\Sigma\eta_8} = & \frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Sigma NK} = & (1-2\alpha)f \\ f_{\Sigma\Sigma\pi} = & 2\alpha f & f_{\Lambda\Lambda\eta_8} = & -\frac{2}{\sqrt{3}}(1-\alpha)f & f_{\Xi\Sigma K} = & -f \end{array}$$

Pseudoscalar-meson (boson) exchange

One-pseudoscalar-meson exchange (V^{OBE}) [LO]

$$V^{OBE}_{B_1 B_2 \to B_1' B_2'} = -f_{B_1 B_1' P} f_{B_2 B_2' P} \frac{(\vec{\sigma}_1 \cdot \vec{q})\,(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^{\,2} + m_P^2}$$

$f_{B_1 B_1' P}$ ... coupling constants
m_P ... mass of the exchanged pseudoscalar meson

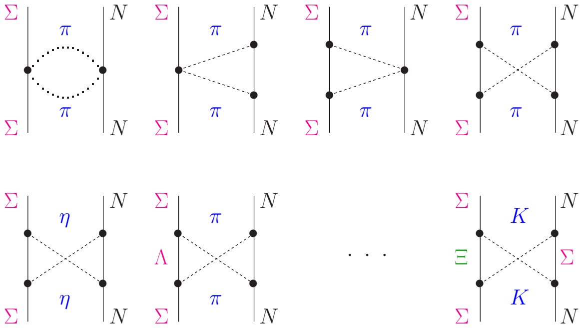
- dynamical breaking of $SU(3)$ symmetry due to the mass splitting of the ps mesons
  (m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV)
  taken into account already at LO!

Details:

(H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244; PLB 653 (2007) 29)

Two-pseudoscalar-meson exchange diagrams

Two-pseudoscalar-meson exchange diagrams (V^{TBE}) [NLO]



⇒ J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24

Coupled channels Lippmann-Schwinger Equation

$$T^{\nu'\nu,J}_{\rho'\rho}(p',p) = V^{\nu'\nu,J}_{\rho'\rho}(p',p) + \sum_{\rho'',\nu''}\int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V^{\nu'\nu'',J}_{\rho'\rho''}(p',p'')\frac{2\mu_{\rho''}}{p^2-p''^2+i\eta}\, T^{\nu''\nu,J}_{\rho''\rho}(p'',p)$$

ρ', ρ = ΛN, ΣN
$\Lambda\Lambda$, ΞN, $\Lambda\Sigma$, $\Sigma\Sigma$

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

The potential in the LS equation is cut off with the regulator function:

$$V^{\nu'\nu,J}_{\rho'\rho}(p',p) \to f^\Lambda(p')\,V^{\nu'\nu,J}_{\rho'\rho}(p',p)\,f^\Lambda(p); \quad f^\Lambda(p) = e^{-(p/\Lambda)^4}$$

consider values Λ = 450 - 700 MeV [500 - 650 MeV]

NLO Results for YN

Pseudoscalar-meson exchange

- All one- and two-pseudoscalar-meson exchange diagrams are included
- $SU(3)$ symmetry is broken by using the physical masses of the pion, kaon, and eta
- $SU(3)$ breaking in the coupling constants is ignored
  $F_\pi = F_K = F_\eta = F_0 = 93$ MeV; $g_A = 1.26$
- assume that $\eta \equiv \eta_8$ (i.e. $\theta_P = 0^o$ and $f_{BB\eta_1} \equiv 0$)
- assume that $\alpha = F/(F+D)$ = 2/5
  (semi-leptonic decays $\Rightarrow \alpha \approx 0.364$)
- Correction to V^{OBE} due to baryon mass differences are ignored

- (A fit with two-pion-meson exchange diagrams is possible!)
- (A fit with physical values for F_π, F_K, F_η is possible!)

Contact terms

- $SU(3)$ symmetry is assumed (for ΛN, ΣN)
- (at NLO $SU(3)$ breaking corrections to the LO contact terms arise!)
- 10 contact terms in S-waves
  no SU(3) constraints from the NN sector are imposed!
- 12 contact terms in P-waves and in ${}^3S_1$–${}^3D_1$
  SU(3) constraints from the NN sector are imposed!
- 1 contact term in ${}^1P_1$–${}^3P_1$ (singlet-triplet mixing) is set to zero

- contact terms in S-waves: can be fairly well fixed from data
- some correlations between NLO and LO LECs

Contact terms in P-waves

- contact terms in P-waves are much less constrained
- use SU(3) and fix some (5) LECs from NN
- the others (7) are fixed from "bulk" properties:

  (1) $\sigma_{\Lambda p} \approx 10$ mb at $p_{lab} \approx 700 - 900$ MeV/c

  (2) $d\sigma/d\Omega_{\Sigma^- p \to \Lambda n}$ at $p_{lab} \approx 135 - 160$ MeV/c

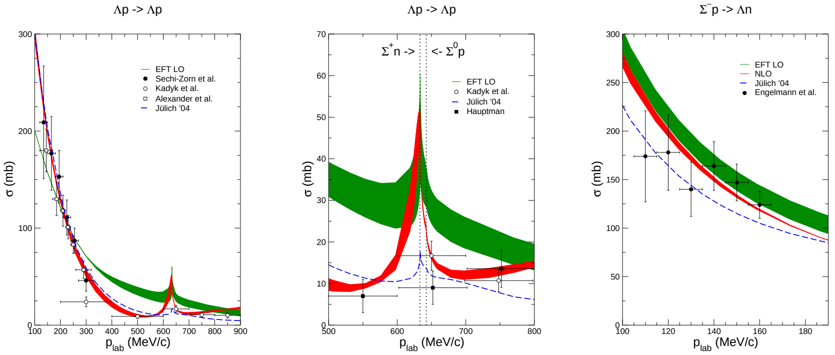
Other (future) options:

- consider matter properties:

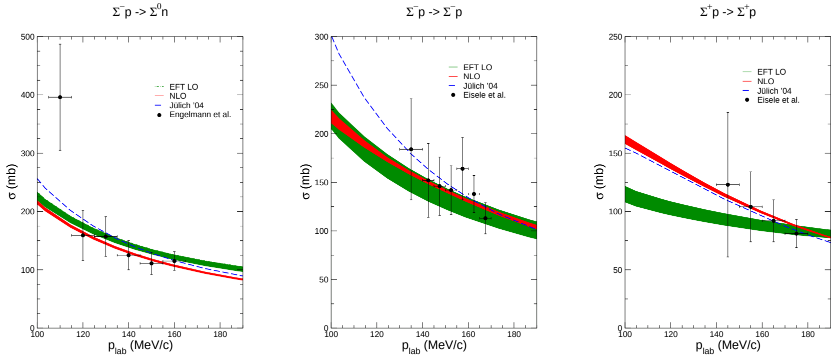
  use spin-orbit splitting of the Λ single particle levels in nuclei

  Consider the Scheerbaum factor S_Λ calculated in nuclear matter to relate the strength of the Λ-nucleus spin-orbit potential to the two body ΛN interaction
  (R.R. Scheerbaum, NPA 257 (1976) 77)

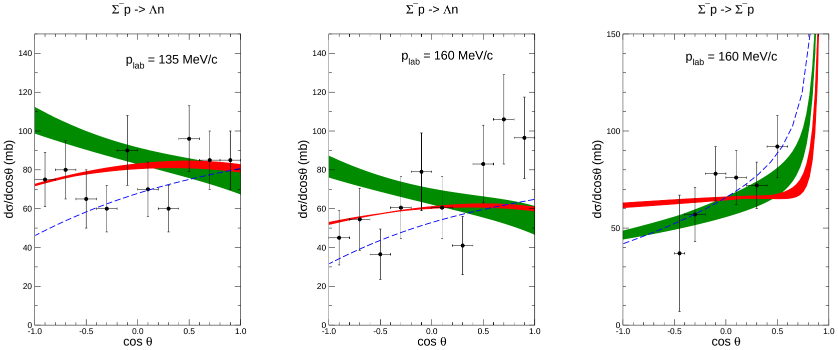
YN integrated cross sections



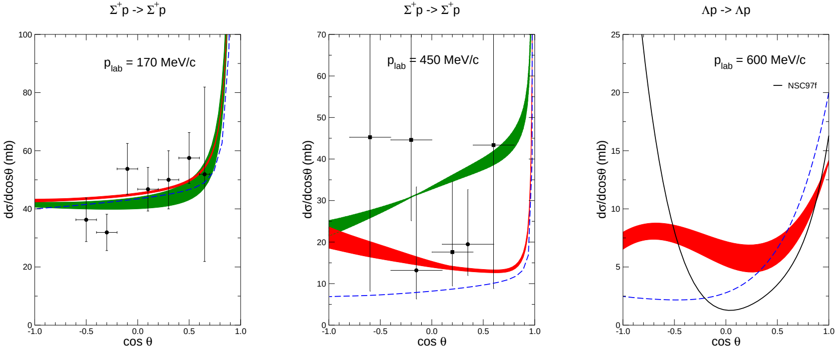
YN integrated cross sections



YN differential cross sections



YN differential cross sections



YN scattering lengths [fm]

	EFT LO	EFT NLO	Jülich '04	NSC97f	experiment*
Λ [MeV]	$550 \cdots 700$	$500 \cdots 650$			
$a_s^{\Lambda p}$	$-1.90 \cdots -1.91$	$-2.90 \cdots -2.91$	-2.56	-2.51	$-1.8^{+2.3}_{-4.2}$
$a_t^{\Lambda p}$	$-1.22 \cdots -1.23$	$-1.51 \cdots -1.61$	-1.66	-1.75	$-1.6^{+1.1}_{-0.8}$
$a_s^{\Sigma^+ p}$	$-2.24 \cdots -2.36$	$-3.46 \cdots -3.60$	-4.71	-4.35	
$a_t^{\Sigma^+ p}$	$0.60 \cdots 0.70$	$0.48 \cdots 0.49$	0.29	-0.25	
χ^2	≈ 30	$15.7 \cdots 16.8$	≈ 25	16.7	
$({}^3_\Lambda\mathrm{H})\ E_B$	$-2.34 \cdots -2.36$	$-2.30 \cdots -2.33$	-2.27	-2.30	-2.354(50)

*A. Gasparyan et al., PRC 69 (2004) 034006
$\Rightarrow$ extract ΛN scattering lengths from final-state interaction:

$pp \to K^+ \Lambda p$ (COSY-Jülich: M. Röder et al., EPJA 49 (2013) 157)

$\gamma d \to K^+ \Lambda n$ (SPring-8, CLAS)

ΣN (I=3/2)

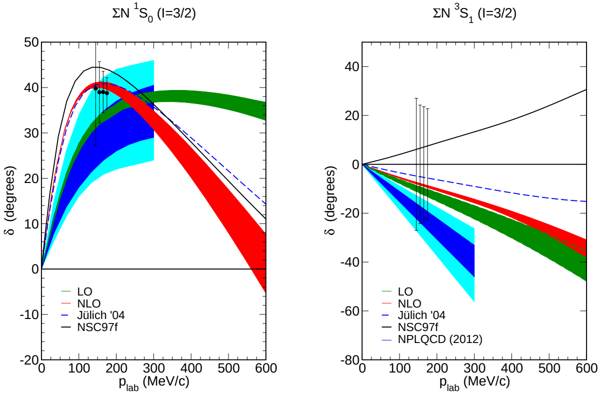
$^1S_0$: test for $SU(3)$ symmetry $\rightarrow V_{NN} \equiv V_{\Sigma N}$

- LEC's that are fitted to the pp $^1S_0$ phase shift produce a bound state in $\Sigma^+ p$
  $\rightarrow \sigma_{^1S_0} \approx 4 \times \sigma_{\Sigma^+ p}$
- simultaneous fit is possible if we assume that there is $SU(3)$ breaking in the LO contact term only.

$^3S_1$–$^3D_1$: decisive for Σ properties in nuclear matter

- A description of YN data is possible with an attractive as well as a repulsive $^3S_1$–$^3D_1$ interaction

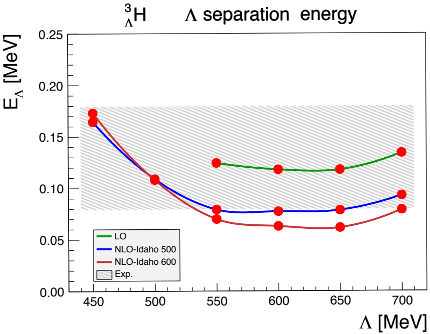
ΣN (I=3/2) phase shifts



partial cross section: $\sigma_{\Sigma^+p;\,J} = \frac{(2J+1)\pi}{p_{cm}^2}\ \sin^2\delta_J$

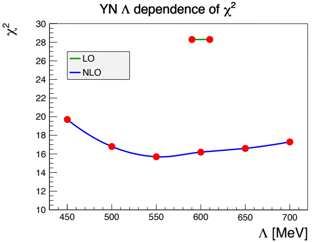
NPLQCD: S. Beane et al., PRL 109 (2012) 172001

Hypertriton

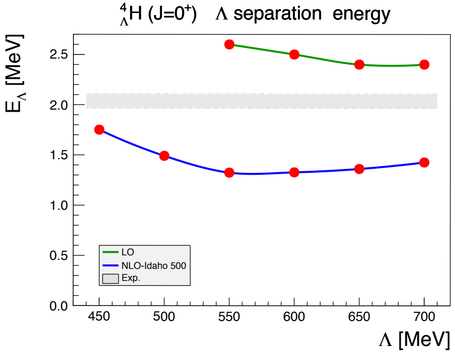


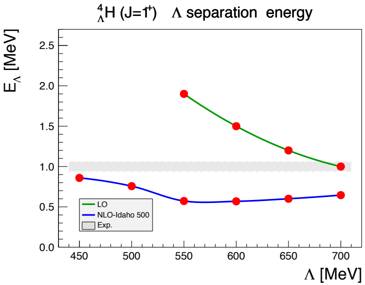
separation energies:

$$E_\Lambda = E(core) - E(hypernucleus)$$



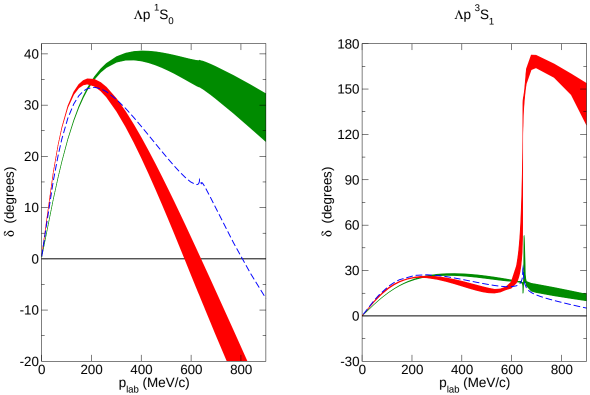
- singlet scattering length for one cutoff chosen so that hypertriton binding energy is OK
- cutoff variation
  - is **lower bound** for magnitude of higher order contributions
  - correlation with χ^2 of YN interaction ?
- long range 3BFs need to be explicitly estimated





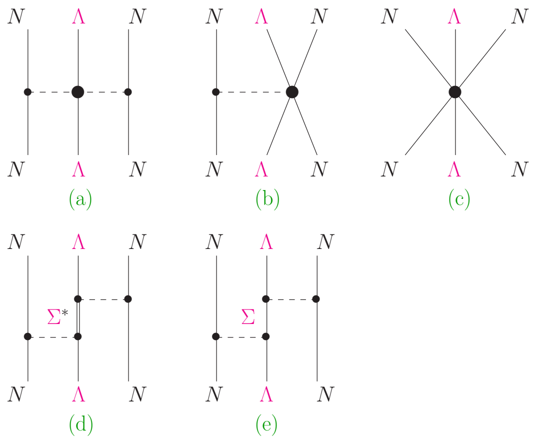
- LO/NLO results: LO uncertainty in 0^+ is underestimated by cutoff variation
- NLO results in line with model results, implies underbinding
- long range 3BFs need to be explicitly estimated

Λp S-wave phase shifts



⇒ less repulsion in ${}^1S_0$ at short distances – and/or 3BFs ?

Three-body forces



(a) - (c) appear at N2LO

(d) appears at NLO – in EFT that includes decuplet baryons

(e) is already included by solving coupled-channel Faddeev equations

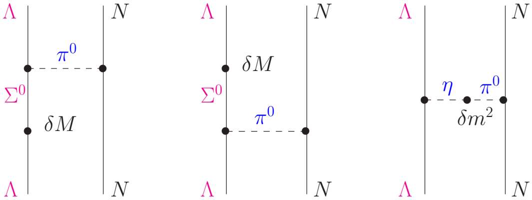
Charge symmetry breaking

Contributions to the difference of ${}^4_\Lambda\mathrm{H}\,(0^+) - {}^4_\Lambda\mathrm{He}\,(0^+)$ separation energies

Λ [MeV]	450	500	550	600	650	700	Jülich 04	Nijm SC97	Nijm SC89	Expt.
ΔT [keV]	44	50	52	51	46	40	0	47	132	-
ΔV_{NN} [keV]	-3	-2	5	5	3	0	-31	-9	-9	-
ΔV_{YN} [keV]	-11	-11	-11	-10	-8	-7	2	37	228	-
tot [keV]	30	37	46	46	41	33	-29	75	351	350
$P_{\Sigma-}$	1.0%	1.1%	1.2%	1.2%	1.1%	0.9%	0.3%	1.0%	2.7%	-
$P_{\Sigma 0}$	0.6%	0.6%	0.7%	0.7%	0.6%	0.5%	0.3%	0.5%	1.4%	-
$P_{\Sigma+}$	0.1%	0.1%	0.2%	0.2%	0.2%	0.1%	0.3%	0.0%	0.1%	-

- **kinetic energy contribution is driven by Σ component**
- NN force contribution due to small deviation of Coulomb
- YN force contribution:
  - *SC89 CSB is strong*
  - *NLO CSB is zero, only Coulomb acts (Σ component)*

$\Lambda - \Sigma^0$ mixing



Electromagnetic mass matrix:

$\langle \Sigma^0 | \delta M | \Lambda \rangle = [M_{\Sigma^0} - M_{\Sigma^+} + M_p - M_n]/\sqrt{3}$

$\langle \pi^0 | \delta m^2 | \eta \rangle = [m^2_{\pi^0} - m^2_{\pi^+} + m^2_{K^+} - m^2_{K^0}]/\sqrt{3}$

(R.H. Dalitz & F. von Hippel, PL 10 (1964) 153)

$\Lambda - \Sigma^0$ mixing

$$f_{\Lambda\Lambda\pi} = [-2\frac{\langle\Sigma^0|\delta M|\Lambda\rangle}{M_{\Sigma^0}-M_\Lambda} + \frac{\langle\pi^0|\delta m^2|\eta\rangle}{m^2_\eta - m^2_{\pi^0}}]\, f_{\Lambda\Sigma\pi}$$

latest PDG mass values $\Rightarrow$

$$f_{\Lambda\Lambda\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi} \approx -0.0403\, f_{\Lambda\Sigma\pi}$$

Scattering lengths (in fm)

		Isospin basis	particle basis		+CSB	
		ΛN	Λp	Λn	Λp	Λn
EFT NLO (600)	$^1S_0$	-2.902	-2.906	-2.907	-2.866	-2.948
NSC97f		-2.60			-2.51	-2.68
EFT NLO (600)	$^3S_1$	-1.520	-1.541	-1.517	-1.547	-1.512
NSC97f		-1.72			-1.75	-1.66

Summary

YN interaction based on chiral *EFT*

- approach is based on a modified Weinberg power counting, analogous to the NN case
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints
- Good description of the empirical YN data was achieved already at LO (only 5 free parameters!)
- Excellent results at next-to-leading order (NLO)
- YN data are reproduced with a quality comparable to phenomenological models
- $SU(3)$ symmetry for the LEC's can be maintained in the YN system (ΛN, ΣN) but not between YN and NN

Outlook

systematic investigation of YNN and $YNNN$ systems

- binding energies are influenced by:

  (1) relative strength of the ΛN $^1S_0$ and $^3S_1$ interactions

  (2) strength of the ΛN–ΣN coupling ($^3S_1$–$^3D_1$)

  (3) possible three-body forces (beyond intermediate Σ)

  (appear formally at N2LO!)
  $\Rightarrow$ S. Petschauer, PhD thesis

extension to N2LO

- same number of LECs that need to be determined