

# Hadron-hadron molecules in the heavy hadron spectrum



*Bound states and resonances in effective field theory and lattice QCD calculations*



*Benasque 2014*

**D.R. Entem**

Thanks to my collaborators:

**P.G. Ortega, J. Segovia, F. Fernández**



**University of Salamanca**

# Outline

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- **Introduction**
- **hadron-hadron states**
  - **The two-baryon sector:**
    - **$NN$  sector**
    - **$\Delta\Delta$  states**
  - **The two-meson sector:**
    - **The  $X(3872)$**
    - **The  $0^{++}$  and  $1^{--}$  sectors**
  - **The baryon-meson sector:**
    - **The  $\Lambda_c(2940)^+$**
    - **The  $X_c(3250)$**
- **Summary**



# The Model

J. Vijande *et al.*, J. Phys. G 31

■ Spontaneous Chiral Symmetry Breaking →

- Goldstone bosons
- Goldstone bosons exchange
- Scalar boson exchanges

■ Gluon coupling

$$\mathcal{L}_{gqq} = i\sqrt{4\pi\alpha_s} \bar{\Psi} \gamma_\mu G_c^\mu \lambda^c \Psi$$

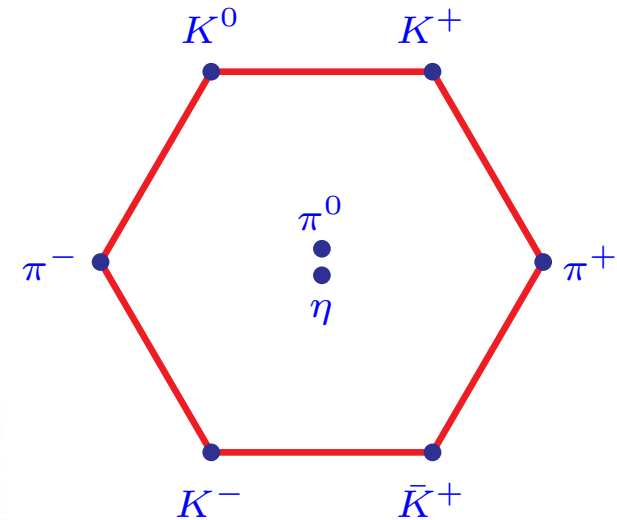
- One gluon exchange

■ Confinement

Color screened confinement

■ Interactions:

$$V_{q_i q_j} = \begin{cases} q_i q_j = nn \Rightarrow V_{CON} + V_{OGE} + V_{GBE} + V_{SBE} \\ q_i q_j = nQ \Rightarrow V_{CON} + V_{OGE} \\ q_i q_j = QQ \Rightarrow V_{CON} + V_{OGE} \end{cases}$$



# Model Results for $1^{--}$ sector.

(nL)	States	QM	Exp.
(1S)	$J/\psi$	3096	$3096,916 \pm 0,011$
(2S)	$\psi(2S)$	3703	$3686,09 \pm 0,04$
(1D)	$\psi(3770)$	3796	$3772 \pm 1,1$
(3S)	$\psi(4040)$	4097	$4039 \pm 1$
(2D)	$\psi(4160)$	4153	$4153 \pm 3$
(4S)		4389	
(3D)	$\psi(4415)$	4426	$4421 \pm 4$
(5S)		4614	
(4D)		4641	

Masses in  $MeV$  of  $J^{PC} = 1^{--} c\bar{c}$  mesons ( $nL$ ) refers to the dominant partial wave and QM denotes the results of the model.

# XYZ Mesons

N.Brambilla et al. Eur. Phys. J. C 71, 1534 (2011)

State	$m$ (MeV)	$\Gamma$ (MeV)	$J^{PC}$	Process (mode)	Experiment ( $\# \sigma$ )	Year	Status
X(3872)	$3871.52 \pm 0.20$	$1.3 \pm 0.6$ ( $< 2.2$ )	$1^{++}/2^{-+}$	$B \rightarrow K(\pi^+\pi^-J/\psi)$ $p\bar{p} \rightarrow (\pi^+\pi^-J/\psi) + \dots$ $B \rightarrow K(\omega J/\psi)$ $B \rightarrow K(D^{*0}\bar{D}^0)$ $B \rightarrow K(\gamma J/\psi)$ $B \rightarrow K(\gamma\psi(2S))$	Belle [85, 86] (12.8), BABAR [87] (8.6) CDF [88–90] (np), DØ [91] (5.2) Belle [92] (4.3), BABAR [93] (4.0) Belle [94, 95] (6.4), BABAR [96] (4.9) Belle [92] (4.0), BABAR [97, 98] (3.6) BABAR [98] (3.5), Belle [99] (0.4)	2003	OK
X(3915)	$3915.6 \pm 3.1$	$28 \pm 10$	$0/2^{2+}$	$B \rightarrow K(\omega J/\psi)$ $e^+e^- \rightarrow e^+e^-(\omega J/\psi)$	Belle [100] (8.1), BABAR [101] (19) Belle [102] (7.7)	2004	OK
X(3940)	$3942_{-8}^{+9}$	$37_{-17}^{+27}$	$?^{2+}$	$e^+e^- \rightarrow J/\psi(D\bar{D}^*)$ $e^+e^- \rightarrow J/\psi(\dots)$	Belle [103] (6.0) Belle [54] (5.0)	2007	NC!
G(3900)	$3943 \pm 21$	$52 \pm 11$	$1^{--}$	$e^+e^- \rightarrow \gamma(D\bar{D})$	BABAR [27] (np), Belle [21] (np)	2007	OK
Y(4008)	$4008_{-49}^{+121}$	$226 \pm 97$	$1^{--}$	$e^+e^- \rightarrow \gamma(\pi^+\pi^-J/\psi)$	Belle [104] (7.4)	2007	NC!
Z <sub>1</sub> (4050) <sup>+</sup>	$4051_{-43}^{+24}$	$82_{-55}^{+51}$	$?$	$B \rightarrow K(\pi^+\chi_{c1}(1P))$	Belle [105] (5.0)	2008	NC!
Y(4140)	$4143.4 \pm 3.0$	$15_{-7}^{+11}$	$?^{2+}$	$B \rightarrow K(\phi J/\psi)$	CDF [106, 107] (5.0)	2009	NC!
X(4160)	$4156_{-25}^{+29}$	$139_{-65}^{+113}$	$?^{2+}$	$e^+e^- \rightarrow J/\psi(D\bar{D}^*)$	Belle [103] (5.5)	2007	NC!
Z <sub>2</sub> (4250) <sup>+</sup>	$4248_{-45}^{+185}$	$177_{-72}^{+321}$	$?$	$B \rightarrow K(\pi^+\chi_{c1}(1P))$	Belle [105] (5.0)	2008	NC!

# XYZ Mesons

N.Brambilla et al. Eur. Phys. J. C 71, 1534 (2011)

State	$m$ (MeV)	$\Gamma$ (MeV)	$J^{PC}$	Process (mode)	Experiment ( $\# \sigma$ )	Year	Status
$Y(4260)$	$4263 \pm 5$	$108 \pm 14$	$1^{--}$	$e^+e^- \rightarrow \gamma(\pi^+\pi^- J/\psi)$	<i>BABAR</i> [108, 109] (8.0) CLEO [110] (5.4) Belle [104] (15)	2005	OK
				$e^+e^- \rightarrow (\pi^+\pi^- J/\psi)$	CLEO [111] (11)		
				$e^+e^- \rightarrow (\pi^0\pi^0 J/\psi)$	CLEO [111] (5.1)		
$Y(4274)$	$4274.4^{+8.4}_{-6.7}$	$32^{+22}_{-15}$	$?^{?+}$	$B \rightarrow K(\phi J/\psi)$	CDF [107] (3.1)	2010	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0,2^{++}$	$e^+e^- \rightarrow e^+e^-(\phi J/\psi)$	Belle [112] (3.2)	2009	NC!
$Y(4360)$	$4353 \pm 11$	$96 \pm 42$	$1^{--}$	$e^+e^- \rightarrow \gamma(\pi^+\pi^- \psi(2S))$	<i>BABAR</i> [113] (np), Belle [114] (8.0)	2007	OK
$Z(4430)^+$	$4443^{+24}_{-18}$	$107^{+113}_{-71}$	?	$B \rightarrow K(\pi^+\psi(2S))$	Belle [115, 116] (6.4)	2007	NC!
$X(4630)$	$4634^{+9}_{-11}$	$92^{+41}_{-32}$	$1^{--}$	$e^+e^- \rightarrow \gamma(\Lambda_c^+\Lambda_c^-)$	Belle [25] (8.2)	2007	NC!
$Y(4660)$	$4664 \pm 12$	$48 \pm 15$	$1^{--}$	$e^+e^- \rightarrow \gamma(\pi^+\pi^- \psi(2S))$	Belle [114] (5.8)	2007	NC!
$Y_b(10888)$	$10888.4 \pm 3.0$	$30.7^{+8.9}_{-7.7}$	$1^{--}$	$e^+e^- \rightarrow (\pi^+\pi^- \Upsilon(nS))$	Belle [37, 117] (3.2)	2010	NC!

# XYZ Mesons

Meson	Mass (Exp)	Candidate?	$J^{PC}$	Mass (Th)
$Y(4360)$	$4353 \pm 11$	$\psi(4S)$	$1^{--}$	4389
$X(4630)$	$4634_{-11}^{+9}$	$\psi(5S)$	$1^{--}$	4614
$Y(4660)$	$4664 \pm 12$	$\psi(4D)$	$1^{--}$	4641
$X(4160)$	$4156 \pm 15$	$\eta_{c2}$	$2^{-+}$	4166

Candidates for some XYZ mesons in our CQM  $c\bar{c}$  spectrum.

No  $c\bar{c}$  candidates for  $X(3872)$ ,  $X(3915)$ ,  $X(3940)$ ,  $G(3900)$ ,  $Y(4008)$ ,  $Y(4140)$ ,  $Y(4260)$ ,  $Y(4274)$ ,  $X(4350)$ ,



# $^3P_0$ model

## ■ Pair creation Hamiltonian:

$$\mathcal{H} = g \int d^3x \bar{\psi}(x) \psi(x)$$

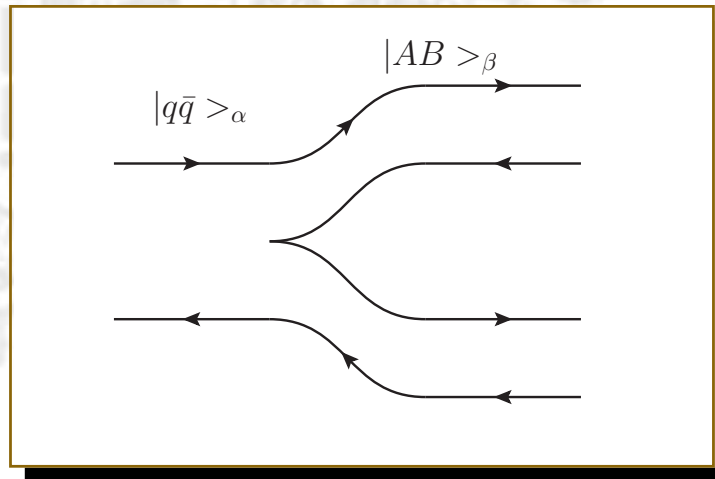
## ■ Non relativistic reduction:

$$T = -3\sqrt{2}\gamma' \sum_{\mu} \int d^3p d^3p' \delta^{(3)}(p + p') \left[ \mathcal{Y}_1 \left( \frac{p - p'}{2} \right) b_{\mu}^{\dagger}(p) d_{\nu}^{\dagger}(p') \right]^{C=1, I=0, S=1, J=0}$$

with  $\gamma' = 2^{5/2} \pi^{1/2} \gamma$ ,  $\gamma = \frac{g}{2m}$  (in the light quark sector)

## ■ Transition potential:

$$\langle \phi_{M_1} \phi_{M_2} \beta | T | \psi_{\alpha} \rangle = P h_{\beta\alpha}(P) \delta^{(3)}(\vec{P}_{cm})$$





# $^3P_0$ results for $c\bar{c}$ strong decays

$\gamma$  parameter fitted to  $\psi(3770) \rightarrow DD$

Phys. Rev. D 78, 114033 (2008).

Meson	Dominant Mode	$\Gamma_{QM}$ (MeV)	$\Gamma_{exp}$ (MeV)
$\psi(3770)$	$DD$	22,2	$22,4 \pm 2,5$
	$D^+D^-$	9,5	$9,5 \pm 1,4$
	$D^0\bar{D}^0$	12,7	$12,8 \pm 1,8$
$\psi(4040)$	$D^*D^*$	92,9	$80 \pm 10$
$\psi(4160)$	$D^*D^*$	96,8	$103 \pm 8$
$\psi(4360)$	$DD_1$	89,8	$103 \pm 11$
$\psi(4415)$	$DD_1$	113,1	$119 \pm 16(*)$
$\psi(4660)$	$D^*D^*$	107,9	$42 \pm 6$

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# The baryon-baryon sector



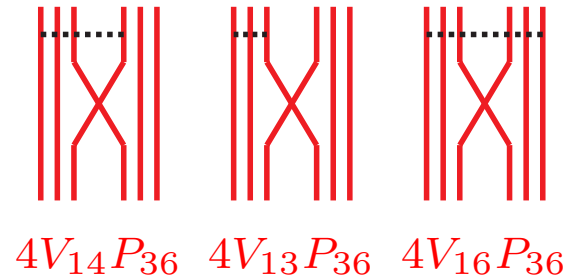
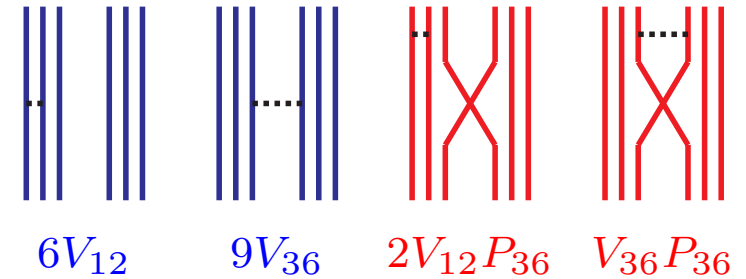
# The $NN$ interaction

$$\psi_B = \phi_B(\vec{p}_{\xi_1}, \vec{p}_{\xi_2}) \chi_{B\xi_c}[1^3]$$

$$\phi_B(\vec{p}_{\xi_1}, \vec{p}_{\xi_2}) = \left[ \frac{2b^2}{\pi} \right]^{\frac{3}{4}} e^{-b^2 p_{\xi_1}^2} \left[ \frac{3b^2}{2\pi} \right]^{\frac{3}{4}} e^{-\frac{3b^2}{4} p_{\xi_2}^2}$$

$$\psi_{B_1 B_2} = \mathcal{A} \left[ \chi(\vec{P}) \psi_{B_1 B_2}^{ST} \right]$$

$$= \mathcal{A} \left[ \phi_{B_1}(\vec{p}_{\xi_{B_1}}) \phi_{B_2}(\vec{p}_{\xi_{B_2}}) \chi(\vec{P}) \chi_{B_1 B_2}^{ST} \xi_c[2^3] \right]$$



## Rayleigh-Ritz variational principle (Resonating Group Method)

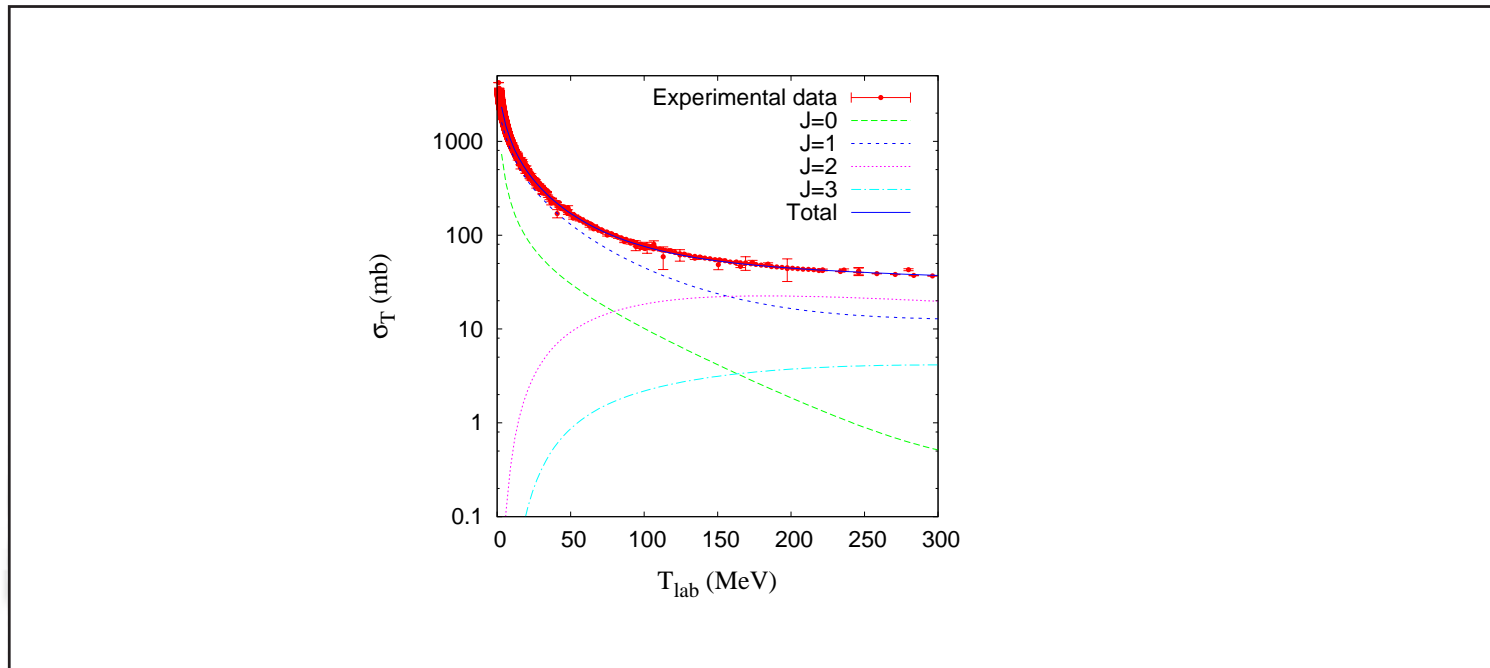
$$(\mathcal{H} - E_T) |\psi\rangle = 0 \quad \Rightarrow \quad \langle \delta\psi | (\mathcal{H} - E_T) |\psi\rangle = 0$$

$$\left( \frac{\vec{P}'^2}{2\mu} - E \right) \chi(\vec{P}') + \int \left( {}^{\text{RGM}}V_D(\vec{P}', \vec{P}_i) + {}^{\text{RGM}}K(\vec{P}', \vec{P}_i) \right) \chi(\vec{P}_i) d\vec{P}_i = 0$$

$$T_\alpha^{\alpha'}(z; p', p) = V_\alpha^{\alpha'}(p', p) + \sum_{\alpha''} \int dp'' p''^2 V_{\alpha''}^{\alpha'}(p', p'') \frac{1}{z - E_{\alpha''}(p'')} T_\alpha^{\alpha''}(z; p'', p)$$

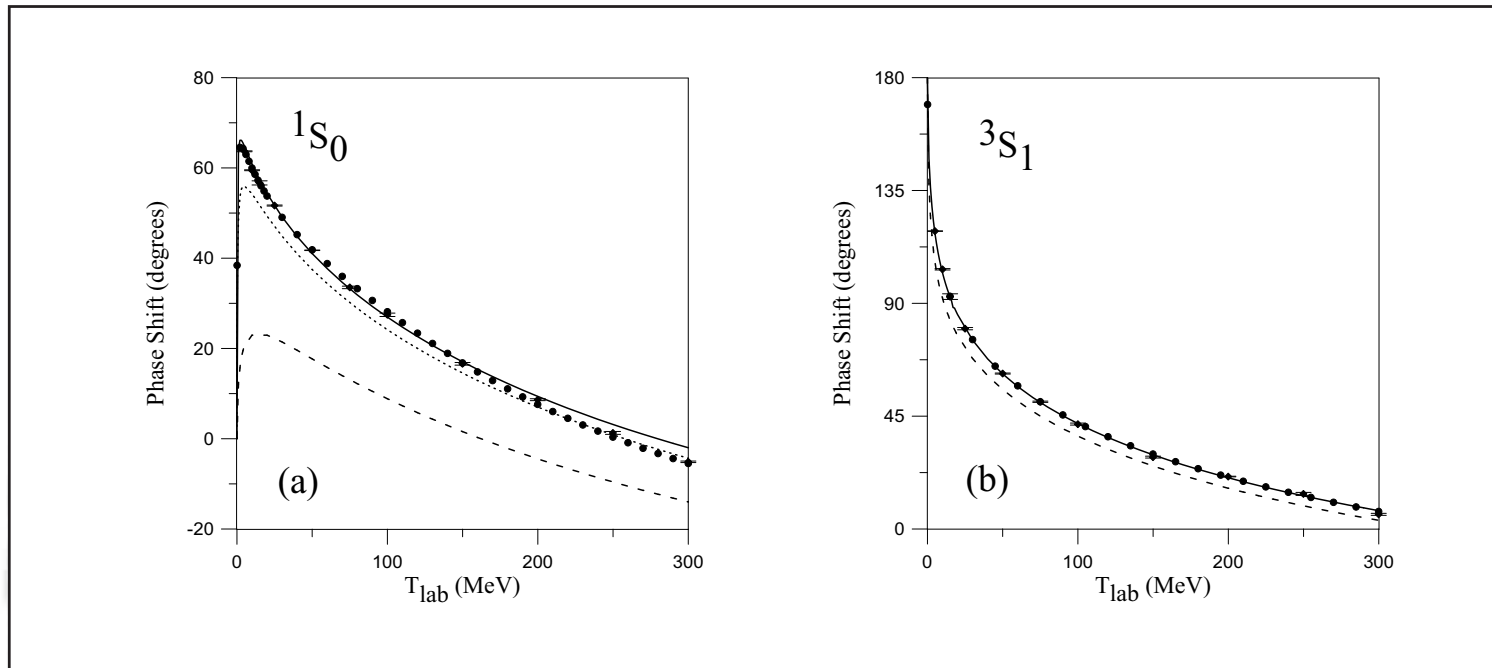
## Lippmann-Schwinger Equation

# NN System



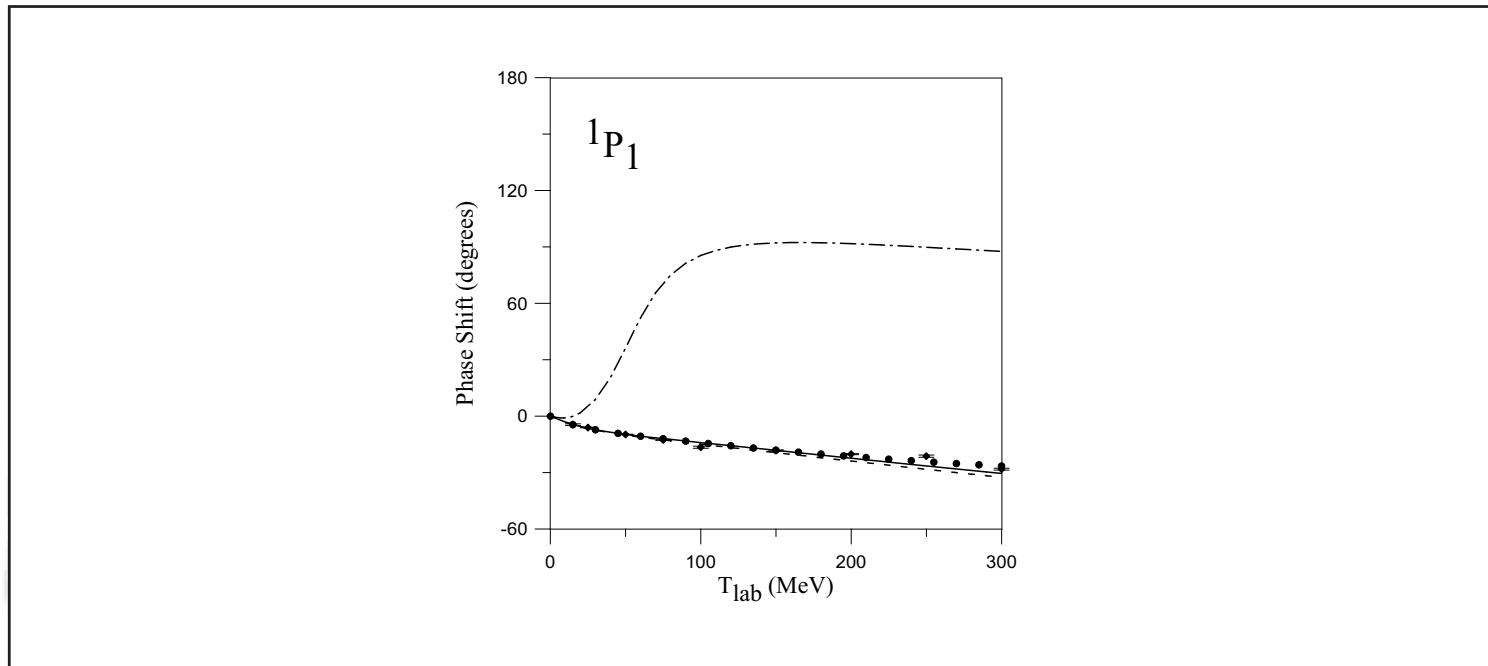
- Constituent quark model, *Phys. Rev C* 62, 034002 (2000)

# $NN$ System



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# $NN$ System



- Constituent quark model, *Phys. Rev C 62, 034002 (2000)*
- Antisymmetry gives repulsion

# NN System

	Quark	$\chi\text{N}^3\text{LO}$	CD-Bonn	Exp.
$E_D$ (MeV)	2.2246	2.224575	2.224575	2.224575(9)
$r_m$ (fm)	1.985	1.978	1.970	1.97535(85)
$A_S$ (fm $^{-1/2}$ )	0.8941	0.8843	0.8846	0.8846(9)
$\eta$	0.0250	0.0256	0.0256	0.0256(4)

- Constituent quark model, *Phys. Rev C* 62, 034002 (2000)
- Antisymmetry gives repulsion
- One bound state in  $NN$ , what happens in channels where antisymmetry is not present?



# $\Delta\Delta$ states

INSTITUTE OF PHYSICS PUBLISHING

JOURNAL OF PHYSICS G: NUCLEAR AND PARTICLE PHYSICS

J. Phys. G: Nucl. Part. Phys. **27** (2001) L1–L7

www.iop.org/Journals/jg PII: S0954-3899(01)14892-9

## LETTER TO THE EDITOR

### $\Delta\Delta$ and $\Delta\Delta\Delta$ bound states

A Valcarce<sup>1</sup>, H Garcilazo<sup>2</sup>, R D Mota<sup>2</sup> and F Fernández<sup>1</sup>

**Table 2.** Binding energies  $B_2$  (in MeV) of the  $\Delta\Delta$  states with total angular momentum  $j$  and isospin  $i$  obtained in the chiral quark cluster model using only the direct term or the direct plus exchange terms of the interaction and in the meson-exchange model.

$(j, i)$	$B_2$		
	Quark direct	Quark direct + exchange	Meson exchange
(0, 1)	188.8	108.4	2035.3
(0, 3)	6.0	0.4	Unbound
(1, 0)	193.9	138.5	2651.7
(1, 2)	70.0	5.7	Unbound
(2, 1)	76.4	30.5	43.0
(2, 3)	35.6	Unbound	Unbound
(3, 0)	17.4	29.9	8.2
(3, 2)	30.7	Unbound	Unbound

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Also in the QDCSM the  $3^+ I = 0$  state appears

J.L. Ping *et al.*, Phys. Rev. C **78** (2009)

# $\Delta\Delta$ states

## The ABC effect (WASA/CELSIUS Collaboration)

PRL 106, 242302 (2011)

PHYSICAL REVIEW LETTERS

week ending  
17 JUNE 2011

### Abashian-Booth-Crowe Effect in Basic Double-Pionic Fusion: A New Resonance?

P. Adlarson,<sup>1</sup> C. Adolph,<sup>2</sup> W. Augustyniak,<sup>3</sup> V. Baru,<sup>4,5</sup> M. Bashkanov,<sup>6</sup> T. Bednarski,<sup>7</sup> F. S. Bergmann,<sup>8</sup>

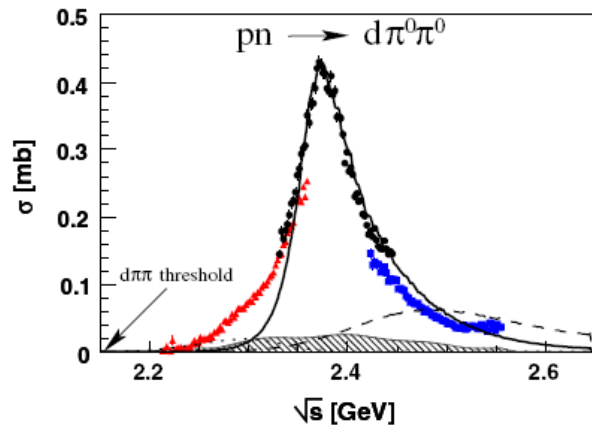


FIG. 2 (color online). Total cross sections obtained from this experiment on  $pd \rightarrow d\pi^0\pi^0 + p_{\text{spectator}}$  for the beam energies  $T_p = 1.0$  GeV (triangles), 1.2 GeV (dots), and 1.4 GeV (squares) normalized independently. Shown are the total cross section data after acceptance, efficiency and Fermi motion corrections. The hatched area indicates systematic uncertainties. The drawn lines represent the expected cross sections for the Roper excitation process (dotted) and the  $t$ -channel  $\Delta\Delta$  contribution (dashed) as well as a calculation for a  $s$ -channel resonance with  $m = 2.37$  GeV and  $\Gamma = 68$  MeV (solid).

- The observables are consistent with a  $I(J^P) = 0(3^+)$  resonance
- Inconsistency with the  $NN$  inelastic cross section?
- G. Faldt and C. Wilkin, Phys, Lett. B 701 for  $\pi^0\pi^0$
- M. Albaladejo and E. Oset arXiv:1304.7698 for  $\pi^+\pi^-$
- A possible candidate for a  $\Delta\Delta$  state A. Pricking, M. Bashkanov and H. Clement arXiv:1310:5532
- A new proposal as a  $NN^*$  state D. Bugg Eur. Phys. J A50



# The meson-meson sector



# Measured Properties of $X(3872)$

- Quantum Numbers  $J^{PC} = 1^{++}$  (confirmed by LHCb)
- Width :  $\Gamma < 1,2 \text{ MeV}$
- Mass :  $M_X = 3871,68 \pm 0,17 \text{ MeV}/c^2 \rightarrow$  below  $D^0 D^{*0}$  mass threshold of  $3871,80 \pm 0,35 \text{ MeV}/c^2$
- $\frac{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 0,8 \pm 0,3$
- $\frac{\mathcal{B}(X \rightarrow J/\psi \gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 0,33 \pm 0,12$
- $\frac{\mathcal{B}(X \rightarrow \psi(2S) \gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 1,1 \pm 0,4$

$$\frac{\mathcal{B}(X \rightarrow \psi(2S) \gamma)}{\mathcal{B}(X \rightarrow J/\psi \gamma)} < 2,1$$



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- $\frac{\mathcal{B}(X \rightarrow \psi(2S) \gamma)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 1,1 \pm 0,4$

We perform a couple channel calculation with  $DD^*$  and  $P$ -wave  $c\bar{c}$  states.

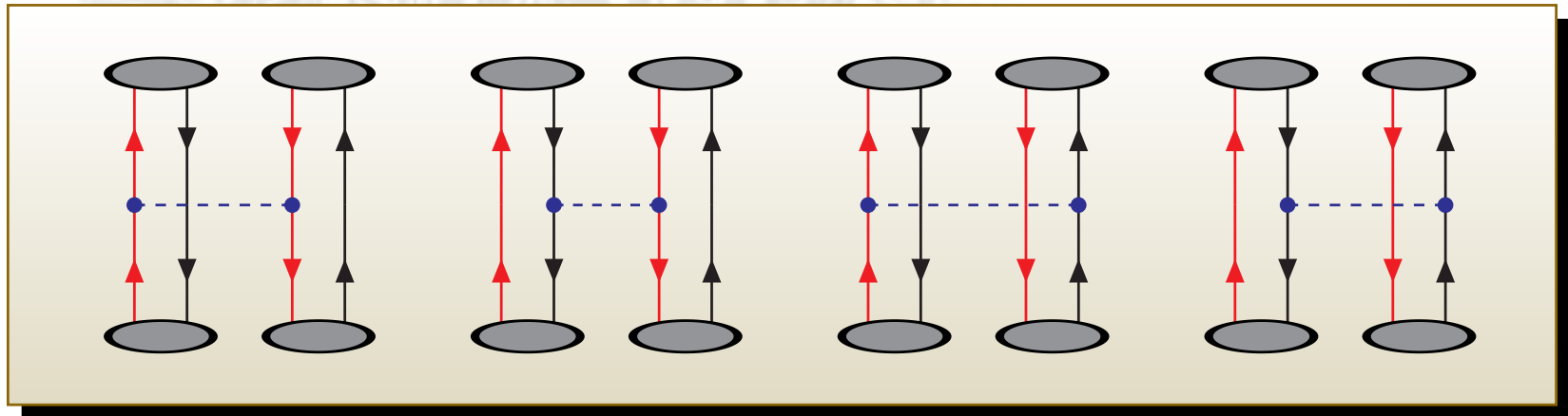
# The $M_1 M_2$ system

- **Quark interactions** → **Cluster interaction.**

- For the  $DD^*$  system only **direct RGM Potential:**

$${}^{RGM}V_D(\vec{P}', \vec{P}_i) = \sum_{i \in A, j \in B} \int d\vec{p}_{\xi'_A} d\vec{p}_{\xi'_B} d\vec{p}_{\xi_A} d\vec{p}_{\xi_B} \phi_A^*(\vec{p}_{\xi'_A}) \phi_B^*(\vec{p}_{\xi'_B}) V_{ij}(\vec{P}', \vec{P}_i) \phi_A(\vec{p}_{\xi_A}) \phi_B(\vec{p}_{\xi_B})$$

- $\phi_C(\vec{p}_C)$  is the wave function for cluster  $C$  **solution of Schrödinger's equation using Gaussian Expansion Method.**





# The $M_1 M_2$ system

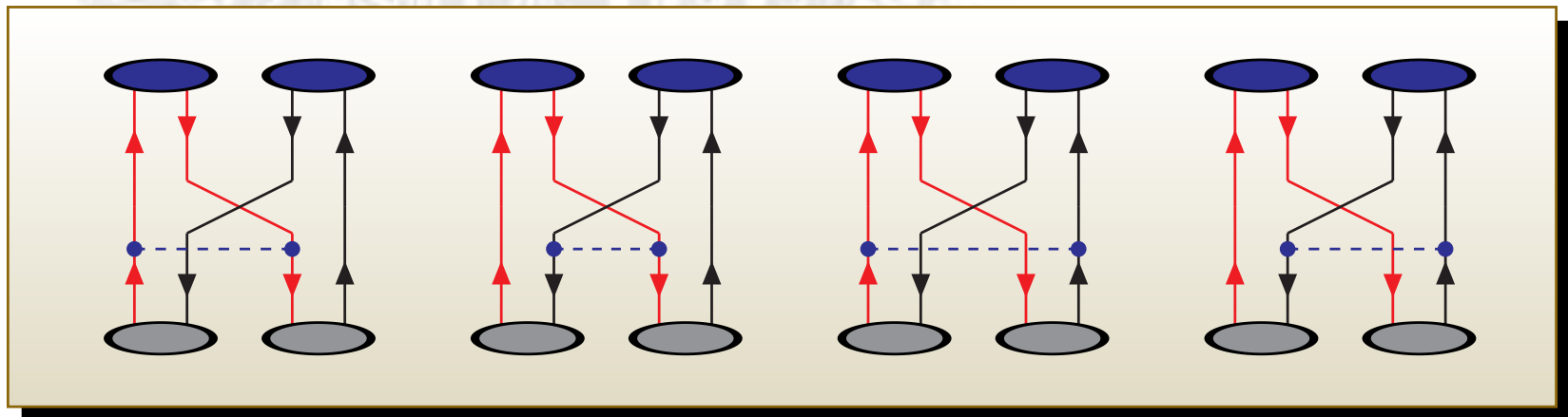
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- $\phi_C(\vec{p}_C)$  is the wave function for cluster  $C$  **solution of Schrödinger's equation using Gaussian Expansion Method.**

Rearrangement processes (like  $DD^* \rightarrow J/\psi\omega$ )



# Coupling $q\bar{q}$ and $q\bar{q}q\bar{q}$ sectors

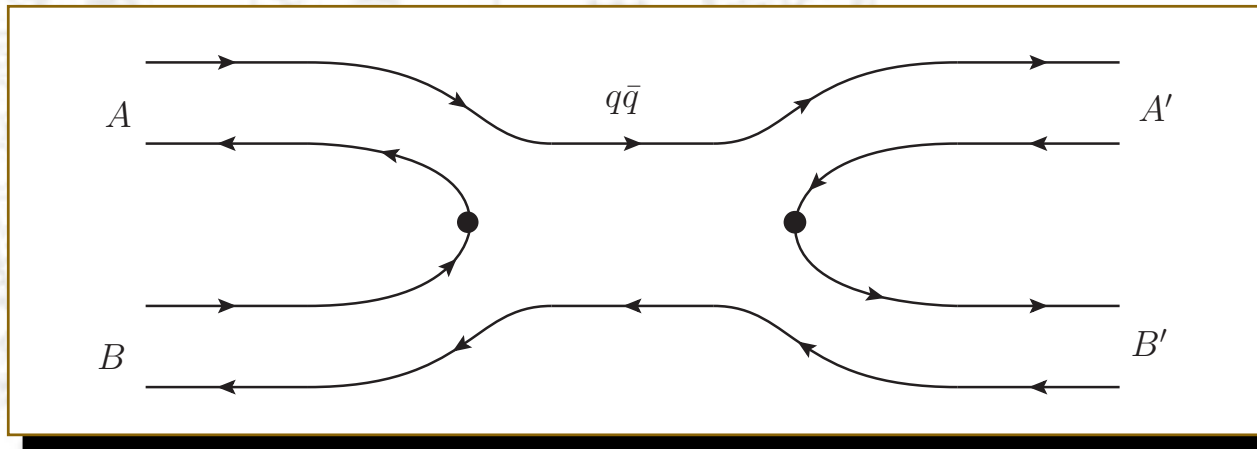
■ Hadronic state:  $|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\psi\rangle + \sum_{\beta} \chi_{\beta}(P) |\phi_{M_1}\phi_{M_2}\beta\rangle$

■ Solving the coupling with  $c\bar{c}$  states  $\rightarrow$  **Schrödinger type equation:**

$$\sum_{\beta} \int \left( H_{\beta'\beta}^{M_1 M_2}(P', P) + V_{\beta'\beta}^{eff}(P', P) \right) \chi_{\beta}(P) P^2 dP = E \chi_{\beta'}(P')$$

with

$$V_{\beta'\beta}^{eff}(P', P) = \sum_{\alpha} \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_{\alpha}}$$



■ The  $c\bar{c}$  amplitudes are given by,

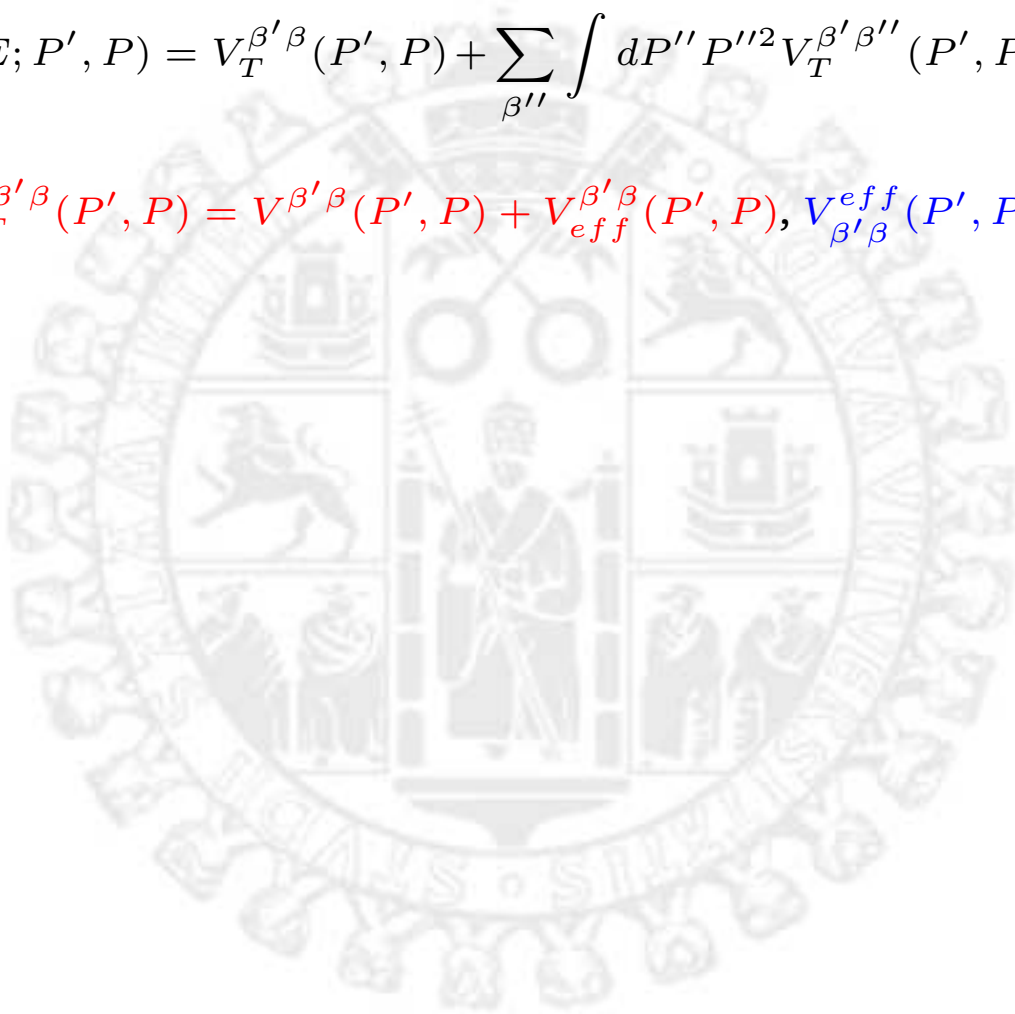
$$c_{\alpha} = \frac{1}{E - M_{\alpha}} \sum_{\beta} \int h_{\alpha\beta}(P) \chi_{\beta}(P) P^2 dP$$

# Resonance states

## Lippman-Schwinger equation

$$T^{\beta'\beta}(E; P', P) = V_T^{\beta'\beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V_T^{\beta'\beta''}(P', P'') \frac{1}{E - E_{\beta''}(P'')} T^{\beta''\beta}(E; P'', P)$$

with  $V_T^{\beta'\beta}(P', P) = V^{\beta'\beta}(P', P) + V_{eff}^{\beta'\beta}(P', P)$ ,  $V_{eff}^{\beta'\beta}(P', P) = \sum_{\alpha} \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_{\alpha}}$



# Resonance states

## Lippman-Schwinger equation

$$T^{\beta'\beta}(E; P', P) = V_T^{\beta'\beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V_T^{\beta'\beta''}(P', P'') \frac{1}{E - E_{\beta''}(P'')} T^{\beta''\beta}(E; P'', P)$$

with  $V_T^{\beta'\beta}(P', P) = V^{\beta'\beta}(P', P) + V_{eff}^{\beta'\beta}(P', P)$ ,  $V_{eff}^{\beta'\beta}(P', P) = \sum_{\alpha} \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_{\alpha}}$

**Solution** (Baru et al. Eur. Phys. Jour. A 44, 93 (2010))

$$T^{\beta'\beta}(E; P', P) = T_V^{\beta'\beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(E; P') \Delta_{\alpha'\alpha}^{-1}(E) \bar{\phi}^{\alpha\beta}(E; P)$$

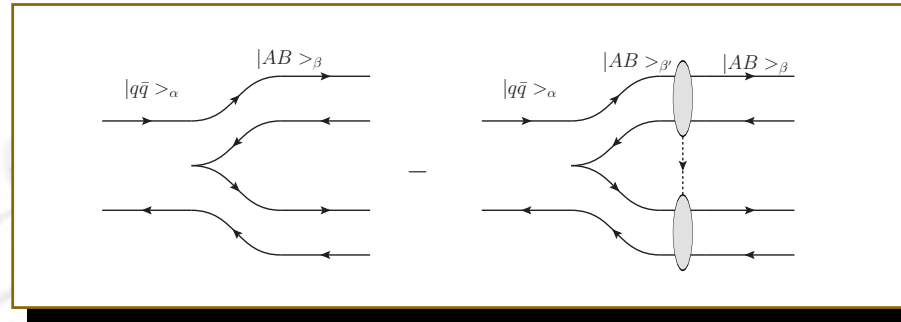
■ **Non resonant contribution**

■ **Resonant contribution**

with

$$T_V^{\beta'\beta}(E; P', P) = V^{\beta'\beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V^{\beta'\beta''}(P', P'') \frac{1}{z - E_{\beta''}(P'')} T_V^{\beta''\beta}(E; P'', P)$$

# Resonance states



**Solution** (Baru et al. Eur. Phys. Jour. A 44, 93 (2010))

$$T^{\beta'\beta}(E; P', P) = T_V^{\beta'\beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(E; P') \Delta_{\alpha'\alpha}^{-1}(E) \bar{\phi}^{\alpha\beta}(E; P)$$

■ **Non resonant contribution**

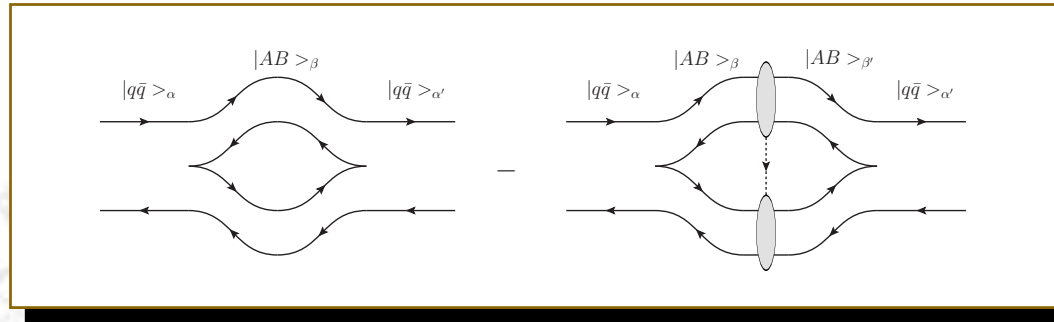
■ **Resonant contribution**

with

$$\phi^{\alpha\beta'}(E; P) = h_{\alpha\beta'}(P) - \sum_{\beta} \int \frac{T_V^{\beta'\beta}(E; P, q) h_{\alpha\beta}(q)}{q^2/2\mu - E} q^2 dq,$$

$$\bar{\phi}^{\alpha\beta}(E; P) = h_{\alpha\beta}(P) - \sum_{\beta'} \int \frac{h_{\alpha\beta'}(q) T_V^{\beta'\beta}(E; q, P)}{q^2/2\mu - E} q^2 dq$$

# Resonance states



**Solution** (Baru et al. Eur. Phys. Jour. A 44, 93 (2010))

$$T^{\beta'\beta}(E; P', P) = T_V^{\beta'\beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(E; P') \Delta_{\alpha'\alpha}^{-1}(E) \bar{\phi}^{\alpha\beta}(E; P)$$

■ **Non resonant contribution**

■ **Resonant contribution**

with

$$\Delta^{\alpha'\alpha}(E) = \left\{ (E - M_{\alpha}) \delta^{\alpha'\alpha} + \mathcal{G}^{\alpha'\alpha}(E) \right\}$$

$$\mathcal{G}^{\alpha'\alpha}(E) = \sum_{\beta} \int dq q^2 \frac{\phi^{\alpha\beta}(q, E) h_{\beta\alpha'}(q)}{q^2/2\mu - E}$$

# Resonance states

## ■ Resonance mass (pole position)

$$\left| \Delta^{\alpha'\alpha}(\bar{E}) \right| = \left| (\bar{E} - M_\alpha) \delta^{\alpha'\alpha} + \mathcal{G}^{\alpha'\alpha}(\bar{E}) \right| = 0$$

## ■ Bare $c\bar{c}$ probabilities

$$\left\{ M_\alpha \delta^{\alpha\alpha'} - \mathcal{G}^{\alpha'\alpha}(\bar{E}) \right\} c_{\alpha'}(\bar{E}) = \bar{E} c_\alpha(\bar{E})$$

## ■ Molecular wave function

$$\chi_{\beta'}(P') = -2\mu_{\beta'} \sum_\alpha \frac{\phi_{\beta'\alpha}(E; P') c_\alpha}{P'^2 - k_{\beta'}^2}$$

## ■ Normalization

$$\sum_\alpha |c_\alpha|^2 + \sum_\beta \langle \chi_\beta | \chi_\beta \rangle = 1$$



# Isospin symmetric calculation

- $^3S_1$  and  $^3D_1$   $DD^*$  partial waves included.
- Coupling to  $1^{++}$  ground and first excited  $c\bar{c}$  states with bare masses within the model:

$$c\bar{c}(1^3P_1) \rightarrow M = 3503,9 \text{ MeV}$$

$$c\bar{c}(2^3P_1) \rightarrow M = 3947,4 \text{ MeV}.$$

First results:

$M \text{ (MeV)}$	$c\bar{c}(1^3P_1)$	$c\bar{c}(2^3P_1)$	$D^0 D^{*0}$	$D^\pm D^{*\mp}$	Assignment
3936	0 %	79 %	10,5 %	10,5 %	$\rightarrow X(3940)$
3865	1 %	32 %	33,5 %	33,5 %	$\rightarrow X(3872)$
3467	95 %	0 %	2,5 %	2,5 %	$\rightarrow \chi_{c1}(3510)$

Parameter free calculation.

# Isospin breaking

Charge basis → Isospin breaking:

$$|D^\pm D^{*\mp}\rangle = \frac{1}{\sqrt{2}} (|DD^* I = 0\rangle - |DD^* I = 1\rangle)$$

$$|D^0 D^{*0}\rangle = \frac{1}{\sqrt{2}} (|DD^* I = 0\rangle + |DD^* I = 1\rangle)$$

$M$ (MeV)	$c\bar{c}(1^3P_1)$	$c\bar{c}(2^3P_1)$	$D^0 D^{*0}$	$D^\pm D^{*\mp}$	Assignment
3937	0 %	79 %	7 %	14 %	→ $X(3940)$
3863	1 %	30 %	46 %	23 %	→ $X(3872)$
3467	95 %	0 %	2,5 %	2,5 %	→ $\chi_{c1}(3510)$

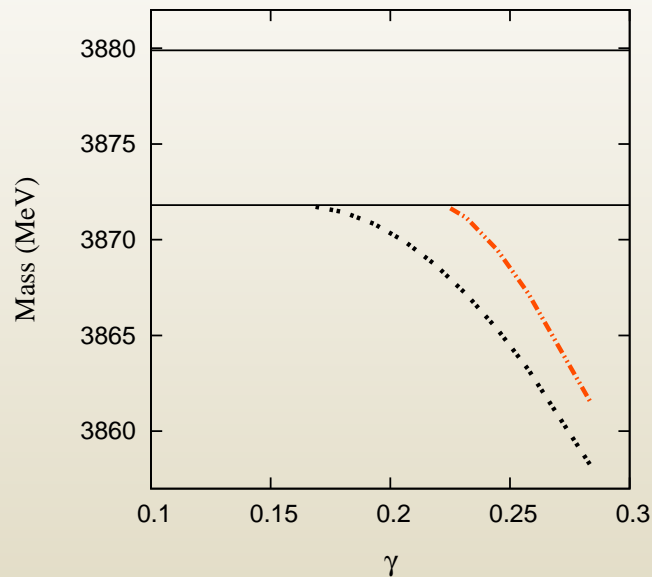
Isospin probabilities:

- $I = 0 \rightarrow \mathcal{P} = 66 \%$ ,
- $I = 1 \rightarrow \mathcal{P} = 3 \%$ .

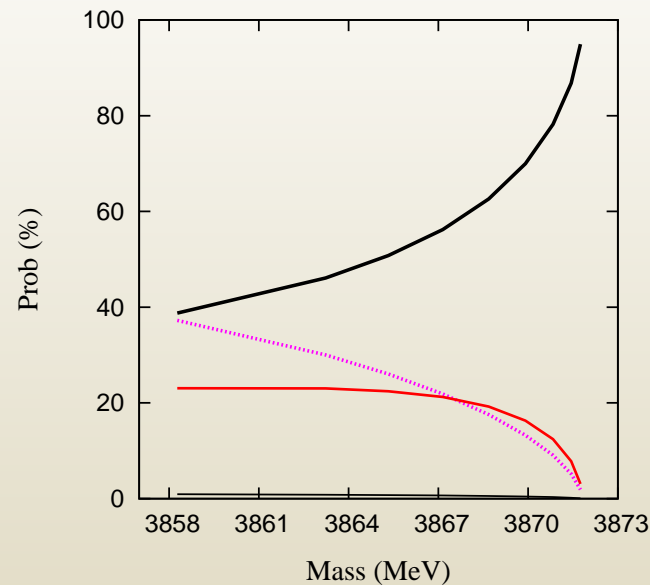
# Dependence on $\gamma$

Binding energy very sensitive  $\rightarrow$  Variation of  $\gamma$ .

$X(3872)$  Mass vs.  $\gamma$



Probabilities for different channels vs.  $X(3872)$  Mass



**No  $DD^*$  interaction included.**  
 $DD^*$  interaction included.

$D^0 \bar{D}^{*0}$  component  
 $D^+ D^{*-}$  component  
 $c\bar{c}(2P)$  component  
 $c\bar{c}(1P)$  component

# Final results

$^3P_0$   $\gamma$  strength parametre 25 % smaller  $\rightarrow E_{bind} = -0,6 \text{ MeV}$ .

$M \text{ (MeV)}$	$c\bar{c}(1^3P_1)$	$c\bar{c}(2^3P_1)$	$D^0 D^{*0}$	$D^\pm D^{*\mp}$	Assignment
3942	0 %	88 %	4 %	8 %	$\rightarrow X(3940)$
3871	0 %	7 %	83 %	10 %	$\rightarrow X(3872)$
3484	97 %	0 %	1,5 %	1,5 %	$\rightarrow \chi_{c1}(3510)$

## Isospin probabilities:

- $I = 0 \rightarrow \mathcal{P} = 70 \%$ ,
- $I = 1 \rightarrow \mathcal{P} = 23 \%$ .

P.G. Ortega, J. Segovia, DRE, F. Fernández, *Phys. Rev. D* 81 (2010)

P.G. Ortega, DRE, F. Fernández, *J. Phys. G* 40 (2013)

M. Takizawa, S. Takeuchi, *PTEP* 9 (2013) at hadron level

# Comparison with data

## Flatte parametrization

Following V. Baru *et al.* Phys. Lett. B 586, 53 (2004)

$$F_{DD^*}^\beta(P, P; E) = -\pi\mu \sum_\alpha \frac{h_{\beta\alpha}^2(P)}{E - M_\alpha + g_{DD^*}^\alpha(E)}$$

$$g_{DD^*}^\alpha(E) = \sum_\beta \int \frac{h_{\beta\alpha}^2(P)}{\frac{P^2}{2\mu} - E - i0^+} P^2 dP \sim \bar{E}_{DD^*}^\alpha + \frac{i}{2}\Gamma_{DD^*}^\alpha + \mathcal{O}(4\mu^2\epsilon/\Lambda^2)$$

for small binding energies

$$\frac{dBr(B \rightarrow KD^0 D^{*0})}{dE} = \mathcal{B} \frac{1}{2\pi} \frac{\Gamma_{D^0 D^{*0}}(E)}{|D(E)|^2}$$

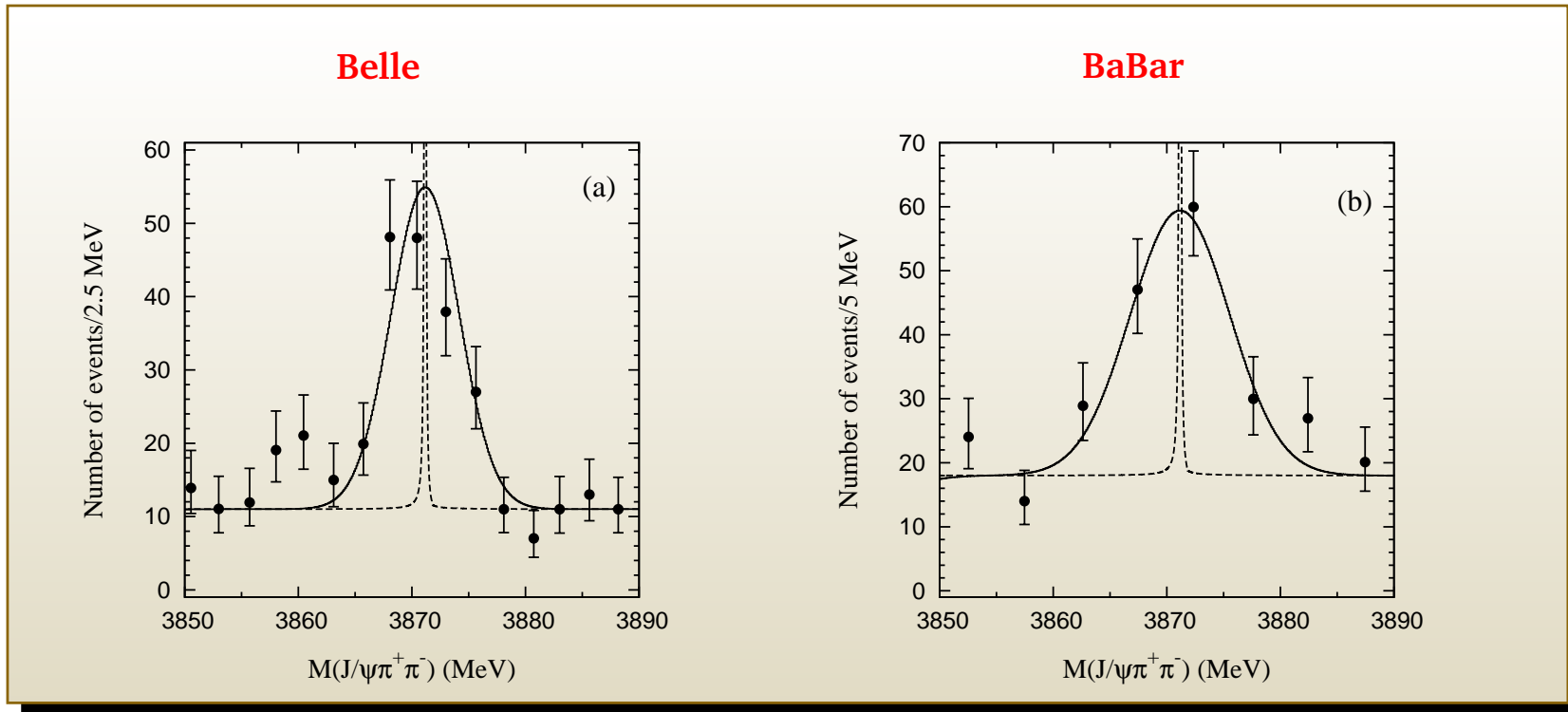
$$\frac{dBr(B \rightarrow K\pi^+ \pi^- J/\Psi)}{dE} = \mathcal{B} \frac{1}{2\pi} \frac{\Gamma_{\pi^+ \pi^- J/\Psi}(E)}{|D(E)|^2}.$$

$$D(E) = E - E_f + \frac{i}{2}(\Gamma_{D^0 D^{*0}} + \Gamma_{D^+ D^{*-}} + \Gamma(E)) + \mathcal{O}(4\mu^2\epsilon/\Lambda^2)$$

We calculate

$$\Gamma_{\pi^+ \pi^- J/\Psi} = \sum_{JL} \int_0^{k_{max}} dk \frac{\Gamma_\rho}{(M_X - E_\rho - E_{J/\Psi})^2 + \frac{\Gamma_\rho^2}{4}} \left| \mathcal{M}_{X \rightarrow \rho J/\Psi}^{JL}(k) \right|^2.$$

# Belle and BaBar data



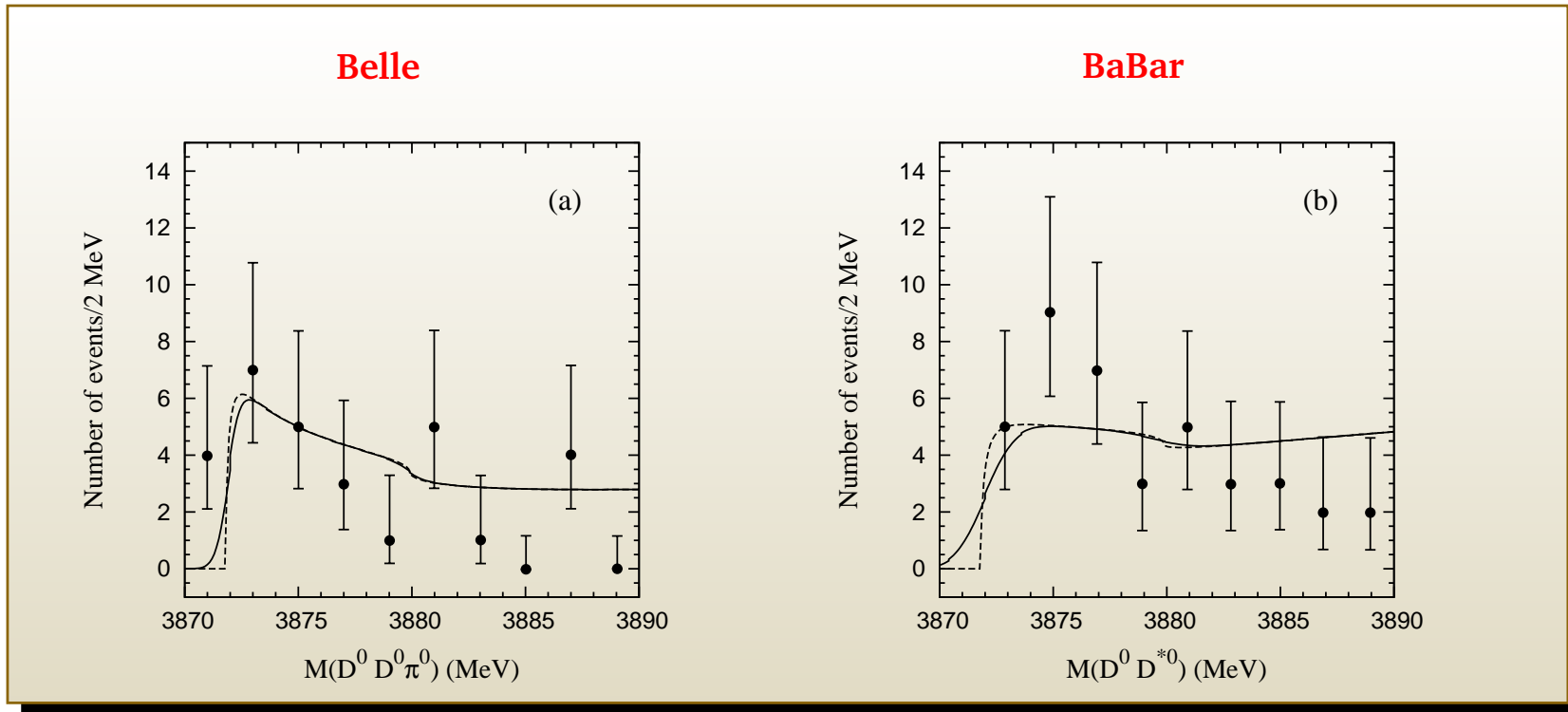
$B \rightarrow K\pi^+\pi^- J/\Psi$  data

solid (dashed) line with (without) resolution function

$$N_{Belle}^{\pi\pi J/\Psi}(E) = 2,5[\text{MeV}] \left( \frac{131}{8,3 \cdot 10^{-6}} \right) \frac{dBr(B \rightarrow K\pi^+\pi^- J/\Psi)}{dE}$$

$$N_{BaBar}^{\pi\pi J/\Psi}(E) = 5[\text{MeV}] \left( \frac{93,4}{8,4 \cdot 10^{-6}} \right) \frac{dBr(B \rightarrow K\pi^+\pi^- J/\Psi)}{dE}$$

# Belle and BaBar data



$B \rightarrow K D^0 \bar{D}^0 \pi^0$  data

solid (dashed) line with (without) resolution function

$$N_{Belle}^{D^0 \bar{D}^0 \pi^0}(E) = 2,0[\text{MeV}] \left( \frac{48,3}{0,73 \cdot 10^{-4}} \right) \frac{dBr(B \rightarrow K D^0 \bar{D}^0 \pi^0)}{dE}$$

$$N_{BaBar}^{D^0 D^{*0}}(E) = 2,0[\text{MeV}] \left( \frac{33,1}{1,67 \cdot 10^{-4}} \right) \frac{dBr(B \rightarrow K D^0 \bar{D}^{*0})}{dE}$$



# The $X(3872)$ branching ratios

$\gamma$	$E_{bind}$	$c\bar{c}(2^3P_1)$	$D^0 D^{*0}$	$D^\pm D^{*\mp}$	$J/\psi\rho$	$J/\psi\omega$
0,231	-0,60	12,40	79,24	7,46	0,49	0,40
0,226	-0,25	8,00	86,61	4,58	0,53	0,29



# The $X(3872)$ branching ratios

$E_{bind}(\text{MeV})$	$\Gamma_{\pi^+\pi^-J/\psi}$	$\Gamma_{\pi^+\pi^-\pi^0J/\psi}$	$R_1$
-0,60	27,61	14,40	0,52
-0,25	24,18	10,64	0,44

$$R_1 = \frac{X(3872) \rightarrow \pi^+\pi^-\pi^0J/\psi}{X(3872) \rightarrow \pi^+\pi^-J/\psi} = 0,8 \pm 0,3$$



# The $X(3872)$ branching ratios

$E_{bind}(\text{MeV})$	$\Gamma_{J/\psi\gamma}^{VMD}$	$\Gamma_{J/\psi\gamma}^{ANN}$	$R_2^M$	$\Gamma_{J/\psi\gamma}^{c\bar{c}}$	$R_2^{c\bar{c}}$	$R_2$
-0,60	0,014	0,056	0,0025	8,15	0,29	0,30
-0,25	0,011	0,045	0,0023	5,25	0,22	0,22

$$R_1 = \frac{\Gamma(X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi)}{\Gamma(X(3872) \rightarrow \pi^+\pi^- J/\psi)} = 0,8 \pm 0,3$$

$$R_2 = \frac{\Gamma(X(3872) \rightarrow \gamma J/\psi)}{\Gamma(X(3872) \rightarrow \pi^+\pi^- J/\psi)} = 0,14 \pm 0,05 \quad 0,33 \pm 0,12$$

# The $X(3872)$ branching ratios

$E_{bind}(\text{MeV})$	$\Gamma_{\Psi(2S)\gamma}^{ANN}$	$R_3^M$	$\Gamma_{\Psi(2S)\gamma}^{c\bar{c}}$	$R_3^{c\bar{c}}$	$R_3$
-0,60	0,134	0,0048	9,80	0,35	0,34
-0,25	0,101	0,0042	6,31	0,26	0,26

$$R_1 = \frac{\Gamma(X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi)}{\Gamma(X(3872) \rightarrow \pi^+\pi^- J/\psi)} = 0,8 \pm 0,3$$

$$R_2 = \frac{\Gamma(X(3872) \rightarrow \gamma J/\psi)}{\Gamma(X(3872) \rightarrow \pi^+\pi^- J/\psi)} = 0,14 \pm 0,05 \quad 0,33 \pm 0,12$$

$$R_3 = \frac{\Gamma(X(3872) \rightarrow \gamma \psi(2S))}{\Gamma(X(3872) \rightarrow \pi^+\pi^- J/\psi)} = 1,1 \pm 0,4$$

# The $X(3872)$ branching ratios

$E_{bind}(\text{MeV})$	$R_3/R_2$
-0,60	1,13
-0,25	1,18

$$R_1 = \frac{\Gamma(X(3872) \rightarrow \pi^+ \pi^- \pi^0 J/\psi)}{\Gamma(X(3872) \rightarrow \pi^+ \pi^- J/\psi)} = 0,8 \pm 0,3$$

$$R_2 = \frac{\Gamma(X(3872) \rightarrow \gamma J/\psi)}{\Gamma(X(3872) \rightarrow \pi^+ \pi^- J/\psi)} = 0,14 \pm 0,05 \quad 0,33 \pm 0,12$$

$$R_3 = \frac{\Gamma(X(3872) \rightarrow \gamma \psi(2S))}{\Gamma(X(3872) \rightarrow \pi^+ \pi^- J/\psi)} = 1,1 \pm 0,4$$

$$\frac{R_3}{R_2} = \frac{\mathcal{B}(X \rightarrow \psi(2S)\gamma)}{\mathcal{B}(X \rightarrow J/\psi\gamma)} < 2,1$$

# The $0^{++}$ sector

Bare  $c\bar{c} 2^3P_0$  (3909)

Meson channels:  $DD + J/\psi\omega + D_sD_s + J/\psi\phi$

Mass(MeV)	$2^3P_0$	$DD$	$J/\psi\omega$	$D_sD_s$	$J/\psi\phi$	$\Gamma_{DD}$	$\Gamma_{J/\psi\omega}$	$\Gamma_{D_sD_s}$
3896,05 – $i2,10$	34,22	46,67	9,41	9,67	0,03	3,37	0,83	–
3970,07 – $i94,67$	57,27	35,32	0,15	5,72	1,54	38,69	2,89	147,76
<b>E.J. Eichten et al. Phys. Rev. D 73 014014 (2005) (<math>C^3</math>)</b>								
3881,4 – $i30,75$	49	34,22	–	4,41	–			

$X(3945)$  and  $Y(3940) \rightarrow X(3915)$

Uehara et al. PRL 104, 092001  $M = 3915 \pm 3 \pm 2$   $\Gamma = 17 \pm 10 \pm 3$   $e^+e^- \rightarrow e^+e^-\omega J/\psi$

Choi et al. PRL 94, 182002  $M = 3943 \pm 11 \pm 13$   $\Gamma = 87 \pm 22 \pm 26$   $B \rightarrow \omega J/\psi K$

Assuming  $\Gamma_{\gamma\gamma}(X(3915)) \sim 1$  KeV and  $\Gamma_{\gamma\gamma}(X(3915)) \times \mathcal{B}(X(3915) \rightarrow J/\psi\omega) = 52 \pm 10 \pm 3$  eV implies  $\Gamma_{J\psi\omega} \sim 1$  MeV too big for an OZI suppress decay

F.-K. Guo, Ulf-G. Meissner, PRD86, 091501(R)

For us  $\Gamma_{\gamma\gamma}(X(3915)) \times \mathcal{B}(X(3915) \rightarrow J/\psi\omega) = 125$  eV

# The $1^{--}$ sector

Bare  $c\bar{c}$   $3^3S_1$  (4097) and  $2^3D_1$  (4153)

Meson channels:  $DD + DD^* + D^*D^* + D_sD_s + D_sD_s^* + D_s^*D_s^*$

$M$ (MeV)	$3^3S_1$	$2^3D_1$	$DD$	$DD^*$	$D^*D^*$	$D_sD_s$	$D_sD_s^*$	$D_s^*D_s^*$
3994,6 – $i$ 11,60	31,56	3,00	2,49	36,44	17,75	7,53	0,52	0,71
4048,4 – $i$ 7,54	0,92	36,15	2,99	23,49	25,81	8,86	0,92	0,85
4123,9 – $i$ 71,11	59,01	0,98	2,13	6,84	19,19	0,75	3,37	7,73
<b>E.J. Eichten et al. Phys. Rev. D 73 014014 (2005) (<math>C^3</math>)</b>								
4038 – $i$ 37	44,89	0,16	2,87	20,36	23,10	0,98	1,58	1,08
(4160) – $i$ 24,6	0,09	47,61	8,37	4,24	8,87	0,55	0,96	1,31

$M$	$\Gamma$	$\Gamma(DD)$	$\Gamma(DD^*)$	$\Gamma(D^*D^*)$	$\Gamma(D_sD_s)$	$\Gamma(D_sD_s^*)$
3994,6	23,37	0,12	19,09	–	4,16	–
4048,4	15,09	0,51	7,24	4,42	2,92	–
4123,9	142,23	4,73	7,51	100,03	3,82	26,15

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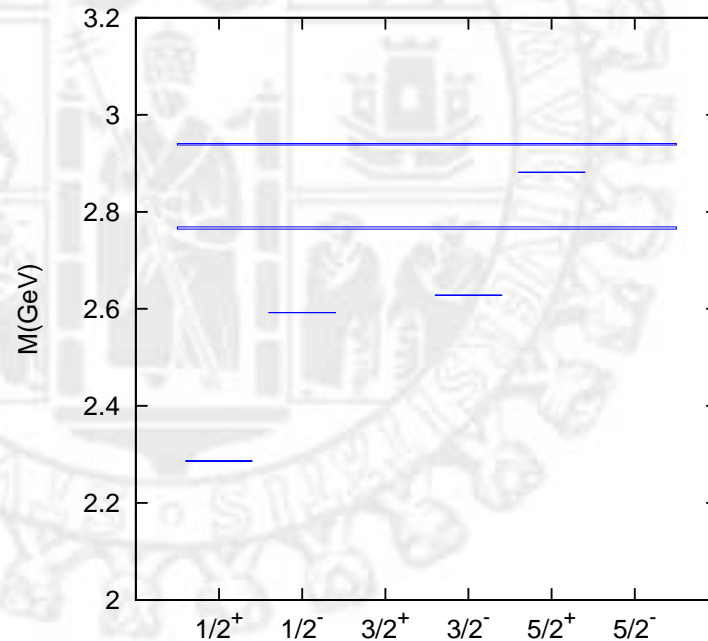
# The meson-baryon sector





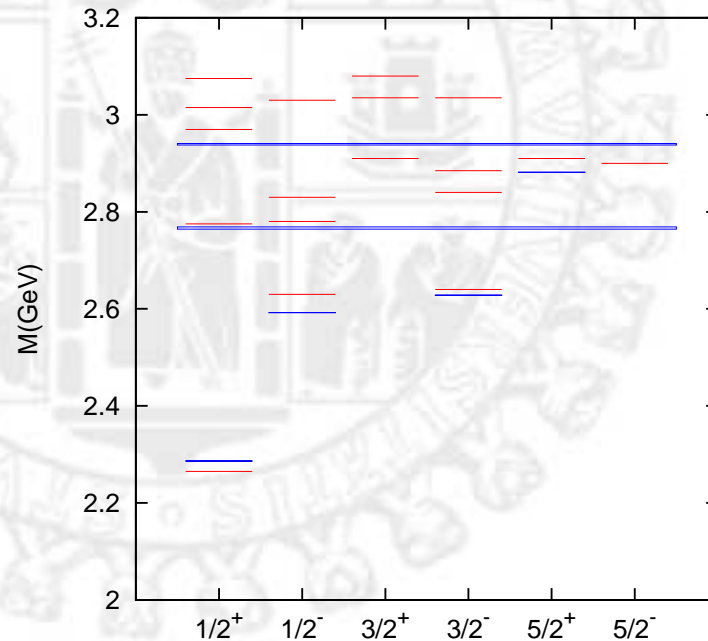
# The $\Lambda_c$ spectrum

- $\Lambda_c^+ I(J^P) = 0(\frac{1}{2}^+)$   $M = 2286,46 \pm 0,14$  MeV
- $\Lambda_c^+(2595) I(J^P) = 0(\frac{1}{2}^-)$   $M = 2592,25 \pm 0,28$  MeV  $\Gamma = 2,59 \pm 0,30 \pm 0,47$  MeV
- $\Lambda_c^+(2625) I(J^P) = 0(\frac{3}{2}^-)$   $M = 2628,11 \pm 0,19$  MeV  $\Gamma < 0,97$  MeV
- $\Lambda_c^+(2765)$  or  $\Sigma_c(2765) I(J^P) = ?(??)$   $M = 2766,6 \pm 2,4$  MeV  $\Gamma = 50$  MeV
- $\Lambda_c^+(2880) I(J^P) = 0(\frac{5}{2}^+)$   $M = 2881,53 \pm 0,35$  MeV  $\Gamma = 5,8 \pm 1,1$  MeV
- $\Lambda_c^+(2940) I(J^P) = 0(??)$   $M = 2939,3_{-1,5}^{+1,4}$  MeV  $\Gamma = 17_{-6}^{+8}$  MeV



# The $\Lambda_c$ spectrum

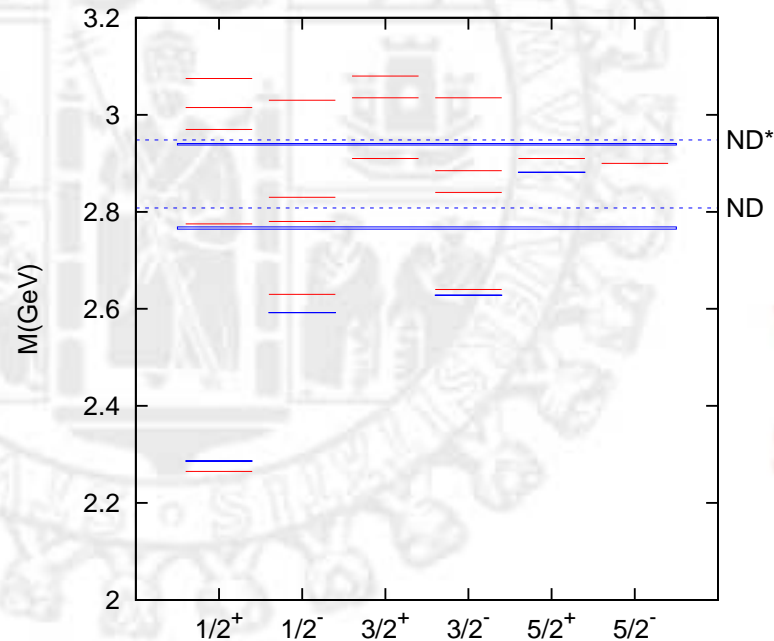
- $\Lambda_c^+ I(J^P) = 0(\frac{1}{2}^+)$   $M = 2286,46 \pm 0,14$  MeV
- $\Lambda_c^+(2595) I(J^P) = 0(\frac{1}{2}^-)$   $M = 2592,25 \pm 0,28$  MeV  $\Gamma = 2,59 \pm 0,30 \pm 0,47$  MeV
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S. Capstick and N. Isgur, Phys. Rev. D 34

# The $\Lambda_c$ spectrum

- $\Lambda_c^+ I(J^P) = 0(\frac{1}{2}^+)$   $M = 2286,46 \pm 0,14$  MeV
- $\Lambda_c^+(2595) I(J^P) = 0(\frac{1}{2}^-)$   $M = 2592,25 \pm 0,28$  MeV  $\Gamma = 2,59 \pm 0,30 \pm 0,47$  MeV
- $\Lambda_c^+(2625) I(J^P) = 0(\frac{3}{2}^-)$   $M = 2628,11 \pm 0,19$  MeV  $\Gamma < 0,97$  MeV
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- $\Lambda_c^+(2880) I(J^P) = 0(\frac{5}{2}^+)$   $M = 2881,53 \pm 0,35$  MeV  $\Gamma = 5,8 \pm 1,1$  MeV
- $\Lambda_c^+(2940) I(J^P) = 0(??)$   $M = 2939,3_{-1,5}^{+1,4}$  MeV  $\Gamma = 17_{-6}^{+8}$  MeV



S. Capstick and N. Isgur, Phys. Rev. D 34

# The $\Lambda_c(2940)^+$

B. Aubert *et al.* (BABAR Collaboration), Phys. Rev. Lett. 98, 012001 (2007)

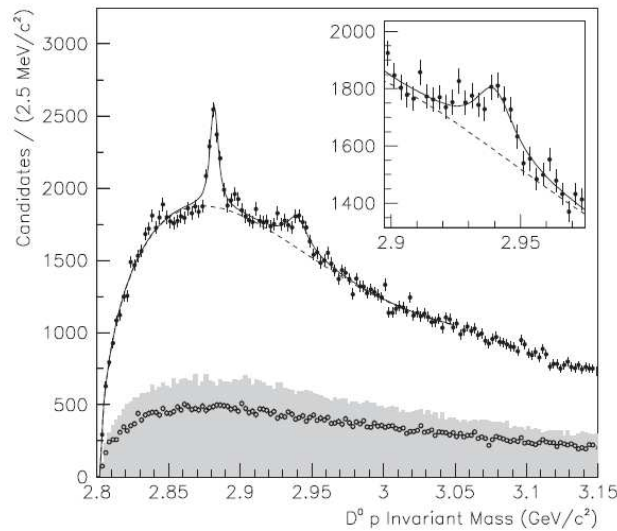


FIG. 1. The solid points are the  $D^0 p$  invariant mass distribution of the final sample. Also shown are (gray) the contribution from false  $D^0$  candidates estimated from  $D^0$  mass sidebands and (open points) the mass distribution from wrong-sign  $\bar{D}^0 p$  candidates. The solid curve is the fit described in the text. The dashed curve is the portion of that fit attributed to combinatorial background.

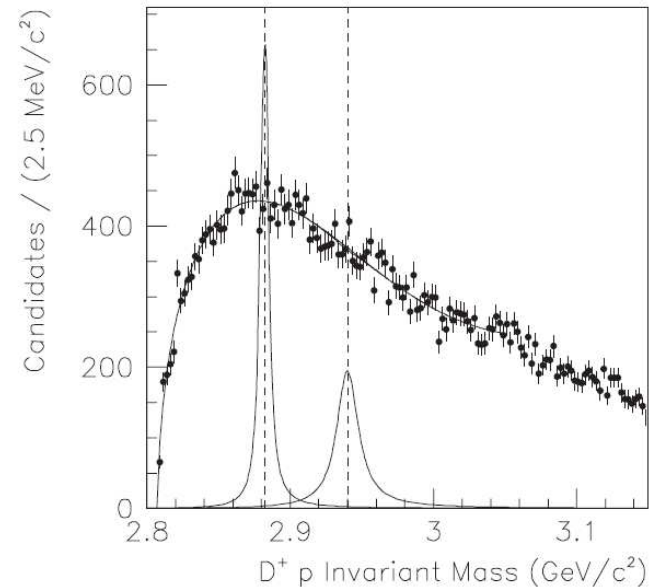


FIG. 3. The invariant mass distribution of selected  $D^+ p$  candidates. The curve is the result of the fit described in the text. The curves below are the line shapes of the  $\Lambda_c(2880)^+$  and  $\Lambda_c(2940)^+$  baryons obtained from the  $D^0 p$  data, drawn approximately to scale after correcting for selection efficiency and  $D^0$  and  $D^+$  branching fractions.

- $e^+ e^-$  annihilation near  $\sqrt{s} = 10,58$  GeV
- $D^0 p$  signal found
- No  $D^+ p$  signal found
- No  $\bar{D}^0 p$  signal found

# The $\Lambda_c(2940)^+$

R. Mizuk *et al.* (Belle Collaboration), Phys. Rev. Lett. 98, 262001 (2007)

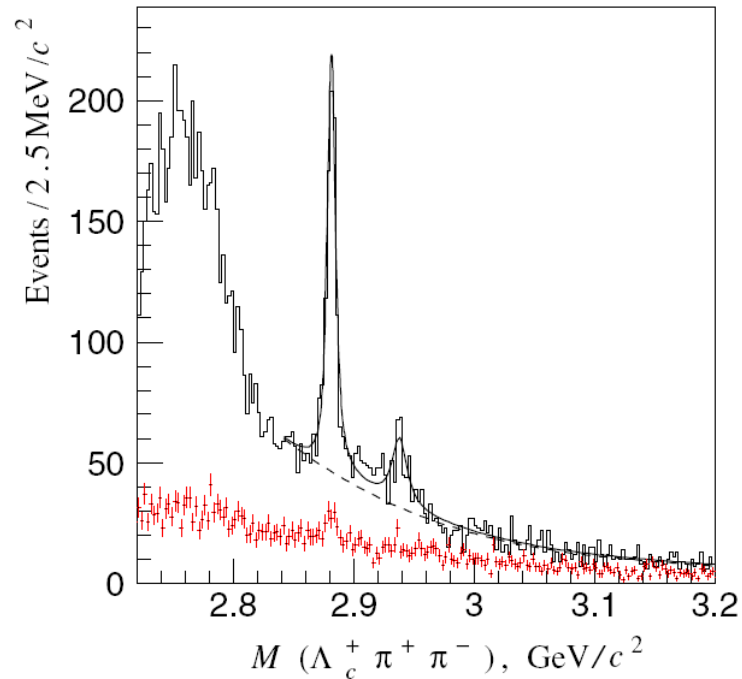


FIG. 1 (color online). The invariant mass of the  $\Lambda_c^+ \pi^+ \pi^-$  combinations for the  $\Sigma_c(2455)$  signal region (histogram) and scaled sidebands (dots with error bars). The fit result (solid curve) and its combinatorial component (dashed curve) are also presented.



# The $\Lambda_c(2940)^+$

## ■ *cnn* candidates

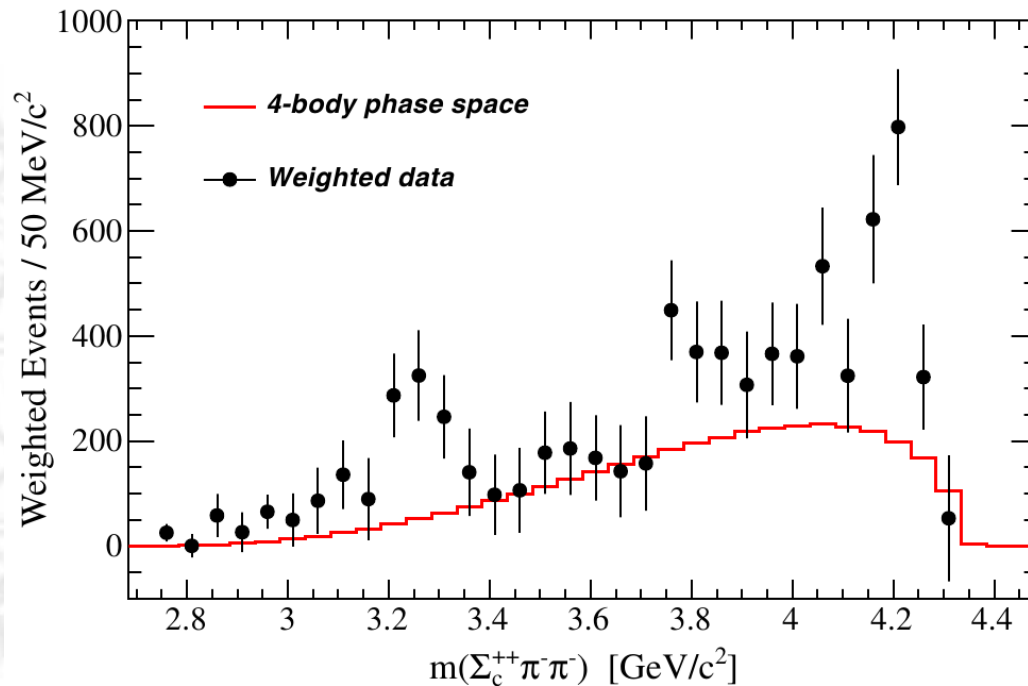
- S. Capstick and N. Isgur, Phys. Rev. D 34,  $3/2^+$  or  $5/2^-$
- C. Chen, Phys. Rev. 75
  - Strong decays using the  $^3P_0$  model
  - Possible *D*-wave states in  $1/2^+$  or  $3/2^+$
  - First radial excitation of the  $\Lambda_c(2286)^+$  fully excluded
- X.-H. Zhong, Phys. Rev. 77
  - Chiral quark model
  - Possible *D*-wave in  $5/2^+$  (studying strong decays)
  - Strong decays too small

## ■ Possible *ND*\* molecule

- X.-G. He *et al.*, Eur. Phys. J. C 51
  - Effective Lagrangian at hadron level
  - Possible  $1/2^-$  also a possible  $3/2^-$
- Y. Dong *et al.*, Phys. Rev. D 81
  - Effective Lagrangian at hadron level
  - Rule out  $1/2^-$  due to very large widths
  - Possible  $1/2^+$  with a small width with dominant  $\Sigma_c \pi$  decay channels
- C. Garci-Recio *et al.*, Phys. Rev. D 79
  - Unitarized couple channel calculation
  - A possible candidate in  $3/2^-$  with a small width (two meson states only in *S*-waves)
- J. He *et al.*, Phys. Rev. D 82
  - OBE model
  - Possible  $1/2^\pm$  or  $3/2^\pm$

# The $X_c(3250)$

J.P.Lees *et al.* (BaBar Collaboration), Phys. Rev. D 86, 091102 (2012)



■  $B^- \rightarrow \Sigma_c^{++} \bar{p} \pi^- \pi^-$

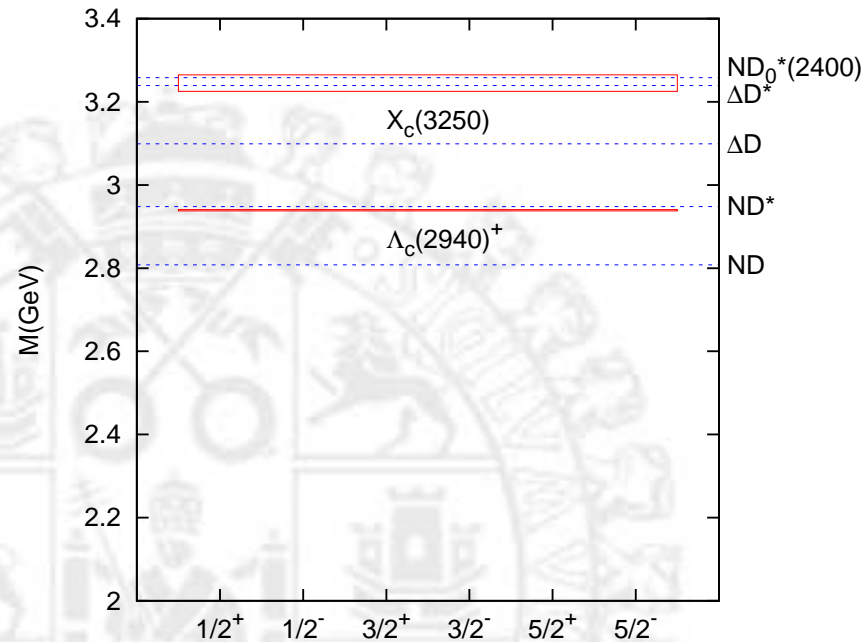
■  $\Sigma_c^{++} \pi^- \pi^-$  invariant mass distribution with peaks at 3.25 GeV/c<sup>2</sup>, 3.8 GeV/c<sup>2</sup> and 4.2 GeV/c<sup>2</sup>

■ Preliminary Breit-Wigner fit

$$M = 3245 \pm 20 \text{ MeV}/c^2 \quad \Gamma = 108 \pm 6 \text{ MeV}$$



# The $X_c(3250)$





# The $X_c(3250)$

$ND_0^*(2400)$  molecule?

■ **J. He *et al.*, Eur. Phys. J C 72 (2012)**

- Effective Lagrangian with  $\sigma$ ,  $\rho$  and  $\omega$ .
- $I = 1$  and  $J^P = 1/2^+$  with  $\Lambda \sim 1,2$  GeV.
- $I = 0$  with  $\Lambda \sim 4,2$  GeV.

■ **J-R. Zhang, Phys. Rev. D 87 (2013)**

- QCD sum rules.
- Released the OPE convergence criterion.
- $I = 1$   $M = 3,18 \pm 0,51$  GeV.
- Only weak conclusions can be obtained for the  $ND_0^*$  hypothesis.

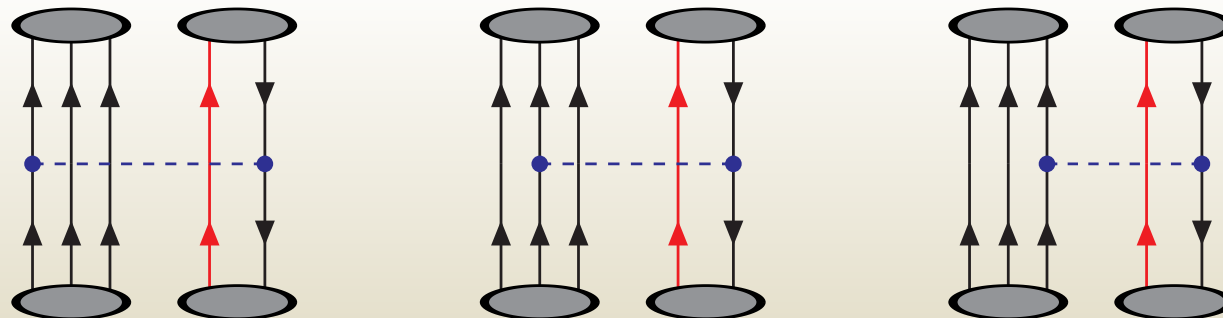
# The $BM$ system

- **Quark interactions** → **Cluster interaction.**

- For the  $ND^*$  system only **direct RGM Potential:**

$${}^{RGM}V_D(\vec{P}', \vec{P}_i) = \sum_{i \in N, j \in D^*} \int d\vec{p}_{\xi'_N} d\vec{p}_{\xi'_{D^*}} d\vec{p}_{\xi_N} d\vec{p}_{\xi_{D^*}} \phi_N^*(\vec{p}_{\xi'_N}) \phi_{D^*}^*(\vec{p}_{\xi'_{D^*}}) V_{ij}(\vec{P}', \vec{P}_i) \phi_N(\vec{p}_{\xi_N}) \phi_{D^*}(\vec{p}_{\xi_{D^*}})$$

- $\phi_C(\vec{p}_C)$  is the wave function for cluster  $C$  **solution of Schrödinger's equation using Gaussian Expansion Method.**



# The $BM$ system

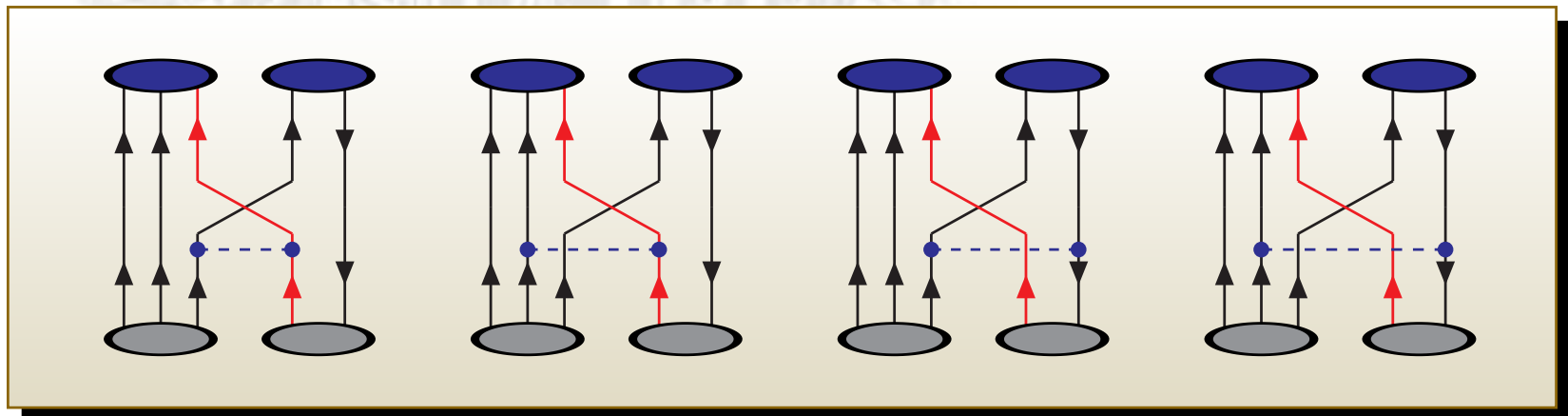
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- $\phi_C(\vec{p}_C)$  is the wave function for cluster  $C$  **solution of Schrödinger's equation using Gaussian Expansion Method.**

Rearrangement processes (like  $ND^* \rightarrow \Sigma_C \pi$ )



# Charm sector $J^P = 3/2^-$

## BaBar

$$M = 2939,8 \pm 1,3 \pm 1,0 \text{ MeV}/c^2$$

$$\Gamma = 17,5 \pm 5,2 \pm 5,9 \text{ MeV}/c^2$$

## Belle

$$M = 2938,0 \pm 1,3^{+2,0}_{-4,0} \text{ MeV}/c^2$$

$$\Gamma = 13^{+8}_{-5} \text{ }^{+27}_{-7} \text{ MeV}/c^2$$

$M \text{ (MeV)}$	$\mathcal{P}_{4S_{3/2}}$	$\mathcal{P}_{2D_{3/2}}$	$\mathcal{P}_{4D_{3/2}}$	$\mathcal{P}_{D^*0p}$	$\mathcal{P}_{D^*+n}$	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$
<b>2938.8</b>	<b>96.22</b>	<b>0.86</b>	<b>2.92</b>	<b>63.93</b>	<b>36.07</b>	<b>97.52</b>	<b>2.48</b>

Decay channel	Width (MeV)	decay channel	Width (keV)
$\Lambda_c^+ \rightarrow D^0 p$	<b>9.42</b>	$\Lambda_c^+ \rightarrow \Sigma_c^{++} \pi^-$	<b>29.7</b>
$\Lambda_c^+ \rightarrow D^+ n$	<b>10.74</b>	$\Lambda_c^+ \rightarrow \Sigma_c^+ \pi^0$	<b>25.2</b>
		$\Lambda_c^+ \rightarrow \Sigma_c^0 \pi^+$	<b>21.1</b>
$\Gamma(\text{total})$	<b>20.2</b>		

# Bottom sector $J^P = 3/2^-$

$M$ (MeV)	$\mathcal{P}_{4S_{3/2}}$	$\mathcal{P}_{2D_{3/2}}$	$\mathcal{P}_{4D_{3/2}}$	$\mathcal{P}_{B^{*-}p}$	$\mathcal{P}_{\bar{B}^{*0}n}$	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$
<b>6248.3</b>	<b>95.15</b>	<b>1.08</b>	<b>3.77</b>	<b>52.56</b>	<b>47.44</b>	<b>99.91</b>	<b>0.09</b>

Decay channel	Width (MeV)	Decay channel	Width (keV)
$\Lambda_b \rightarrow B^- p$	<b>3.69</b>	$\Lambda_b \rightarrow \Sigma_b^+ \pi^-$	<b>40.9</b>
$\Lambda_b \rightarrow \bar{B}^0 n$	<b>3.75</b>	$\Lambda_b \rightarrow \Sigma_b^0 \pi^0$	<b>39.5</b>
		$\Lambda_b \rightarrow \Sigma_b^- \pi^+$	<b>38.1</b>
$\Gamma(\text{total})$	<b>7.56</b>		

# $ND^{(*)}$ and $N\bar{B}^{(*)}$ states

$J^P$	<i>Isospin</i>	<i>Molecule</i>	<i>Mass(MeV)</i>	$E_b(\text{MeV})$	$P_{max}(\text{Channel})$
$\frac{1}{2}^-$	0	$DN$	2805,24	-1,70	$98,08(^2S_{1/2})$
$\frac{1}{2}^-$	1	$D^*N$	2947,61	-0,48	$99,93(^2S_{1/2})$
$\frac{3}{2}^-$	0	$D^*N$	2940,06	-8,02	$96,05(^4S_{3/2})$
$\frac{1}{2}^-$	0	$\bar{B}N$	6206,11	-12,09	$87,61(^2S_{1/2})$
$\frac{1}{2}^-$	1	$\bar{B}N$	6217,83	-0,36	$99,05(^2S_{1/2})$
$\frac{1}{2}^-$	1	$\bar{B}^*N$	6260,58	-3,43	$99,86(^2S_{1/2})$
$\frac{3}{2}^-$	0	$\bar{B}^*N$	6248,87	-15,15	$95,07(^4S_{3/2})$



# $\Delta D^{(*)}$ and $\Delta \bar{B}^{(*)}$ states

$J^P$	<i>Isospin</i>	<i>Molecule</i>	<i>Mass(MeV)</i>	$E_b$ (MeV)	$P_{max}$ (Channel)
$\frac{1}{2}^-$	2	$D^* \Delta$	3232,70	-6,47	99,71( ${}^2S_{1/2}$ )
$\frac{3}{2}^-$	1	$D\Delta$	3097,14	-0,88	99,13( ${}^4S_{3/2}$ )
$\frac{3}{2}^-$	2	$D^* \Delta$	3238,19	-0,98	99,69( ${}^2S_{1/2}$ )
$\frac{5}{2}^-$	1	$D^* \Delta$	3226,05	-13,12	97,25( ${}^6S_{5/2}$ )
$\frac{1}{2}^-$	2	$\bar{B}^* \Delta$	6540,88	-14,21	99,69( ${}^2S_{1/2}$ )
$\frac{3}{2}^-$	1	$\bar{B}\Delta$	6498,56	-10,72	88,14( ${}^4S_{3/2}$ )
$\frac{3}{2}^-$	2	$\bar{B}\Delta$	6505,61	-3,67	94,72( ${}^4S_{3/2}$ )
$\frac{3}{2}^-$	1	$\bar{B}^* \Delta$	6554,71	-0,39	97,10( ${}^4S_{3/2}$ )
$\frac{3}{2}^-$	2	$\bar{B}^* \Delta$	6550,25	-4,85	99,48( ${}^4S_{3/2}$ )
$\frac{5}{2}^-$	1	$\bar{B}^* \Delta$	6531,94	-23,16	96,76( ${}^6S_{5/2}$ )

# Decays of $\Delta D^{(*)}$ and $\Delta \bar{B}^{(*)}$ states

$J^P$	$I$		$\Gamma_{D\Delta}$	$\Gamma_{\Sigma_c\rho}$	$\Gamma_{\Sigma_c\pi\pi}$	$\Gamma_{D^*N}$	$\Gamma_{DN}$	$\Gamma_{D\pi\Delta}$	$\Gamma_{D^*N\pi}$	$\Gamma_{DN\pi}$
$\frac{1}{2}^-$	2	$D^*\Delta$	0,005	0,018	2,60	0	0	0	111	0
$\frac{3}{2}^-$	1	$D\Delta$	0	0	0	1,31	0,001	0	0,049	113
$\frac{3}{2}^-$	2	$D^*\Delta$	6,18	0,007	0	0	0	0,038	114	0
$\frac{5}{2}^-$	1	$D^*\Delta$	0,003	0	0	1,23	0,64	0	108	0

$$X_c(3250) \rightarrow (D^*N)\pi \rightarrow (\Sigma_c\pi)\pi$$

$J^P$	$I$		$\Gamma_{\bar{B}\Delta}$	$\Gamma_{\Sigma_b\eta}$	$\Gamma_{\bar{B}^*N}$	$\Gamma_{\bar{B}N}$	$\Gamma_{\bar{B}\pi\Delta}$	$\Gamma_{\bar{B}^*N\pi}$	$\Gamma_{\bar{B}N\pi}$
$\frac{1}{2}^-$	2	$\bar{B}^*\Delta$	0,002	0	0	0	0	111	0
$\frac{3}{2}^-$	1	$\bar{B}\Delta$	0	0,02	3,91	0,02	0	10	98
$\frac{3}{2}^-$	2	$\bar{B}\Delta$	0	0	0	0	0	5	108
$\frac{3}{2}^-$	1	$\bar{B}^*\Delta$	12,5	0,12	0,224	0,019	0,076	115	0
$\frac{3}{2}^-$	2	$\bar{B}^*\Delta$	19,9	0	0	0	0	114	0
$\frac{5}{2}^-$	1	$\bar{B}^*\Delta$	0,001	0,18	0	0,90	0	108	0



# $X_c \rightarrow \pi N D^*$ decay

## Decay width

$$\begin{aligned} \Gamma_{X_c^{\Delta D^*} \rightarrow (\pi N) D^*} &= 2\pi \int q^2 dq |\langle \pi N D^* | \Gamma_{\pi N \Delta} | X_c^{\Delta D^*} \rangle|^2 \delta(E_f - E_i) \\ &= \int_0^{k_{max}} dk k^2 |\chi_{\Delta D^*}^\alpha(k)|^2 \Gamma_\Delta(q) \end{aligned}$$

with  $\Gamma_\Delta(q) \sim \Gamma_\Delta$  or  $\Gamma_\Delta(q) \sim \Gamma_\Delta \left(\frac{q}{q_\Delta}\right)^3$ .

$J^P$	$I$	Mass (MeV/c <sup>2</sup> )	$\Gamma_{D^* N \pi}$ (MeV)	$\Gamma'_{D^* N \pi}$ (MeV)	$P_{max}$ (Channel)
$\frac{1}{2}^-$	2	3232.7	111	78	99.71( <sup>2</sup> $S_{1/2}$ )
$\frac{3}{2}^-$	2	3238.2	114	102	99.69( <sup>4</sup> $S_{3/2}$ )
$\frac{5}{2}^-$	1	3226.1	108	63	97.25( <sup>6</sup> $S_{5/2}$ )

$J^P$	$I$	Mass (MeV/c <sup>2</sup> )	$\Gamma_{\bar{B}^* N \pi}$ (MeV)	$\Gamma'_{\bar{B}^* N \pi}$ (MeV)	$P_{max}$ (Channel)
$\frac{1}{2}^-$	2	6540.9	111	63	99.69( <sup>2</sup> $S_{1/2}$ )
$\frac{3}{2}^-$	1	6554.7	115	109	97.10( <sup>4</sup> $S_{3/2}$ )
$\frac{3}{2}^-$	2	6550.2	114	87	99.48( <sup>4</sup> $S_{3/2}$ )
$\frac{5}{2}^-$	1	6531.9	108	49	96.76( <sup>6</sup> $S_{5/2}$ )

# Summary

- We have used a **chiral constituent quark model to study possible hadron-hadron molecules.**
- In the  $BB$  sector the model describes the **deuteron as a  $NN$  bound state** and **there is candidate for the resonance found by WASA as a  $\Delta\Delta$  state.**
- The model describes the  $X(3872)$  as a  $DD^*$  resonance state coupled to  $c\bar{c}$  states.
- In the charmonium  $0^{++}$  sector we find two resonances.
- In the  $ND^*$  sector we found **a bound state with  $J^P$   $3/2^-$  which can be identified with the  $\Lambda_c(2940)^+$  state.** There is an analog  $N\bar{B}^*$  state with the same quantum numbers which could be **a possible  $\Lambda_b(6248)$ .**
- We find several bound states in the  $ND^{(*)}$  ( $N\bar{B}^{(*)}$ ) and  $\Delta D^{(*)}$  ( $\Delta\bar{B}^{(*)}$ ) sectors.
- No  $ND_0^*(2400)$  molecule found in  $1/2^+$ ,  $1/2^-$  and  $3/2^-$
- Candidates for the  $X_c(3250)$  as  $\Delta D^*$  molecules
  - **Negative parity** against **positive parity for  $ND_0^*$  hypothesis**
  - **$I=2$  candidate** against  **$I = 1$  for  $ND_0^*$  hypothesis**
  - Bottom partner with  $M \sim 6,5 \text{ GeV}/c^2$  against  $M \sim 6,6 \text{ GeV}/c^2$
  - Main decay channel  $X_c \rightarrow D^* N\pi$  against  $X_c \rightarrow DN\pi$