Chiral effective field theory approach to electromagnetic transitions in light nuclei

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- Nuclear EM currents from χEFT
- EM observables in $A \leq 10$ systems
  - Magnetic moments and M1 transitions
- EM transitions in ^{8}Be
  - E2 transitions involving the first two excited states
  - M1 transitions involving isospin-mixed states
- Summary and outlook

The Basic Model

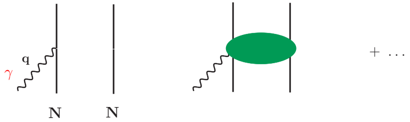
- The nucleus is a system made of A interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

where υ_{ij} and V_{ijk} are 2- and 3-nucleon interaction operators

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1−, 2−, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots \,, \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$



- Calculations with w.f.'s from "traditional (or conventional)" potentials and currents from χEFTs are called "hybrid calculations"

Currents and nuclear electroweak properties:

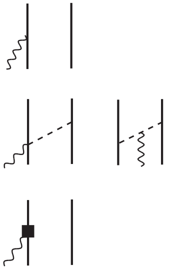
- Park, Rho *et al.* (1996–2009);
  hybrid studies in A=2–4 by Song *at al.* (2009-2011)
- Meissner *et al.* (2001), Kölling *et al.* (2009–2011);
  applications to *d* and ^{3}He photodisintegration by Rozpedzik *et al.* (2011);
  applications to *d* magnetic f.f. by Kölling, Epelbaum, Phillips (2012)
- Phillips (2003-2007);
  applications to deuteron static properties and f.f.'s

χEFT EM current up to $n = 1$ (or up to N3LO)

LO : $\mathbf{j}^{(-2)} \sim eQ^{-2}$

NLO : $\mathbf{j}^{(-1)} \sim eQ^{-1}$

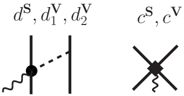
N^{2}LO : $\mathbf{j}^{(-0)} \sim eQ^{0}$



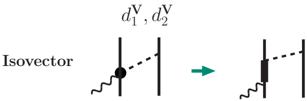
- $n = -2, -1, 0$, and 1-(loops only): depend on known LECs (g_A, F_π, and $\mu_{p/n}$)
- $n = 0$: $(Q/m_N)^2$ relativistic correction to $\mathbf{j}^{(-2)}$
- Strong contact LECs at $n = 1$ fixed from fits to *np* phases shifts—PRC**68**, 041001 (2003)
- Unknown EM LECs enter the $n = 1$ contact and tree-level currents
- No three-body EM currents at this order
- NLO and N3LO loop-contributions lead to purely isovector operators
- $\mathbf{j}^{(n\leq 1)}$ satisfies the CCR with χEFT two-nucleon potential $\upsilon^{(n\leq 2)}$

N^{3}LO: $\mathbf{j}^{(1)} \sim eQ$

unknown LEC′s →



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ-resonance saturation

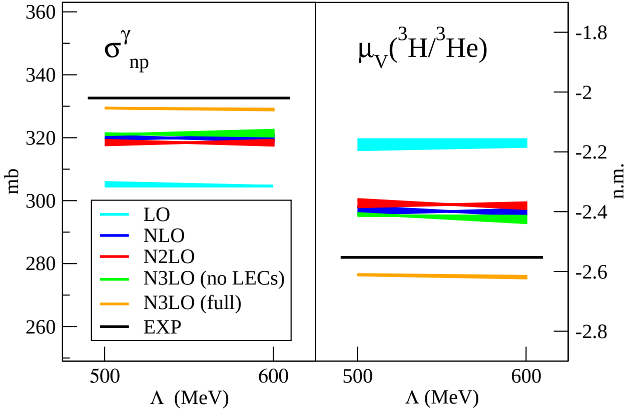
Left with 3 LECs: Fixed in the $A = 2-3$ nucleons' sector

- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\text{H}/^3\text{He})$
- Isovector sector:
  - model I = c^V from EXPT $npd\gamma$ xsec.

    or
  - model II = c^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m.

Predictions with χEFT EM currents for $A = 2$–3 systems- Piarulli *et al.*

np capture xsec. (using model II) / μ_V of $A = 3$ nuclei (using model I)
bands represent nuclear model dependence (N3LO/N2LO – AV18/UIX)



- *npd*γ xsec. and $\mu_V(^3\text{H}/^3\text{He})$ m.m. are within 1% and 3% of EXPT
- EM χEFT currents provide good description of $A = 2$ and 3 f.f.'s Piarulli *et al.*

trinucleon w.f.'s from hyperspherical harmonics expansion of Kievsky *et al.*, FBS**22**, 1 (1997); Viviani *et al.*, FBS**39**, 59 (2006); Kievsky *et al.*, J. Phys. G: Nucl. Part. Phys. **35**, 063101 (2008)

Predictions with χEFT EM currents for $A = 6$–10 systems: Variational Monte Carlo (VMC)

Minimize expectation value of H

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[\mathscr{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

- single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
- central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
- pair correlation operators U_{ij} reflect influence of υ_{ij} (AV18)
- triple correlation operator U_{ijk} added when V_{ijk} (IL7) is present

Ψ_V are spin-isospin vectors in $3A$ dimensions with $\sim 2^A \binom{A}{Z}$ components

Lomnitz-Adler, Pandharipande, Smith, NP **A361**, 399 (1981) Wiringa, PRC **43**, 1585 (1991)

Predictions with χEFT EM currents for $A = 6$–10 systems: Green's function Monte Carlo (GFMC)

Given a decent trial function Ψ_V, we can further improve it by "filtering" out the remaining excited state contamination:

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

$$\Psi(\tau \to \infty) = a_0 \psi_0$$

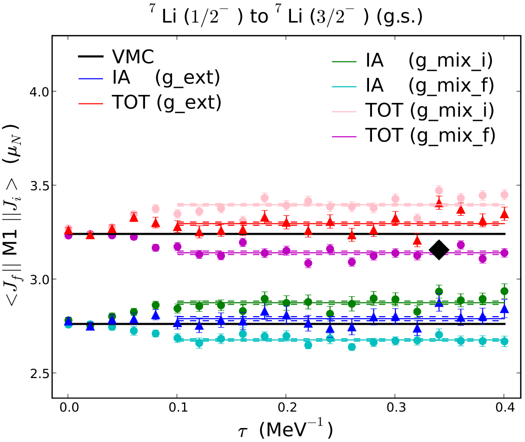
Evaluation of $\Psi(\tau)$ is done stochastically (Monte Carlo method) in small time steps $\Delta\tau$ using a Green's function formulation.
In practice, we evaluate a "mixed" estimates

$$\langle O(\tau) \rangle = \frac{{}_f\langle \Psi(\tau) | O | \Psi(\tau) \rangle_i}{\langle \Psi(\tau) | \Psi(\tau) \rangle} \approx \langle O(\tau) \rangle^i_{\text{Mixed}} + \langle O(\tau) \rangle^f_{\text{Mixed}} - \langle O \rangle_V$$

$$\langle O(\tau) \rangle^i_{\text{Mixed}} = \frac{{}_f\langle \Psi_V | O | \Psi(\tau) \rangle_i}{{}_f\langle \Psi_V | \Psi(\tau) \rangle_i} \; ; \; \langle O(\tau) \rangle^f_{\text{Mixed}} = \frac{{}_f\langle \Psi(\tau) | O | \Psi_V \rangle_i}{{}_f\langle \Psi(\tau) | \Psi_V \rangle_i}$$

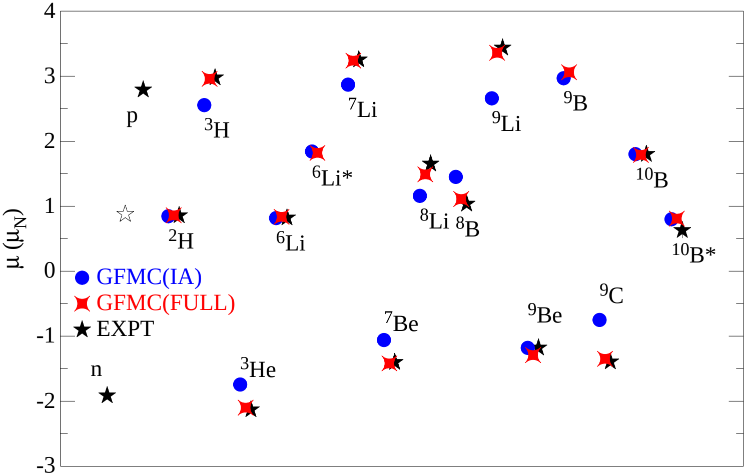
Pudliner, Pandharipande, Carlson, Pieper, & Wiringa, PRC **56**, 1720 (1997)
Wiringa, Pieper, Carlson, & Pandharipande, PRC **62**, 014001 (2000)
Pieper, Wiringa, & Carlson, PRC **70**, 054325 (2004)

Example of GFMC propagation: M1 Transition in $A = 7$



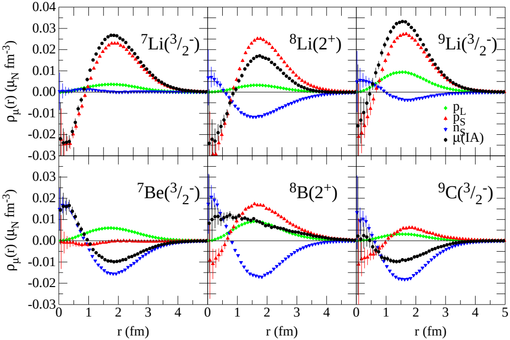
Magnetic moments in $A \leq 10$ nuclei

Predictions for $A > 3$ nuclei



$$\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

One-body magnetic densities



- IA magnetic moment operator

$$\mu(\mathrm{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

Magnetic radii from VMC

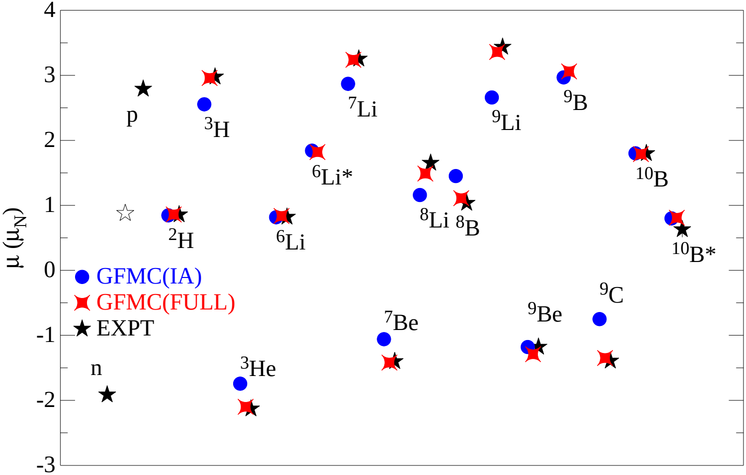
From the one-body magnetic densities $\rho_M(r)$ we evaluate magnetic radii $\langle r_M^2 \rangle$

$$\langle r_M^2 \rangle \propto \int d^3r \, \left(r^2 + \langle r_n^2 \rangle \right) \rho_M(r)$$

Nucleus	$\langle r_M^2 \rangle^{1/2}$ (fm)	EXPT
p		0.777(16)
n		0.862(9)
^{2}H	2.14	1.90(14)
^{3}H	1.92	1.84(18)
^{3}He	2.01	1.965(153)
^{6}Li	3.42	
^{7}Li	2.88	2.98(5)
^{9}Be	3.06	3.2(3)
^{10}B	2.77	

Magnetic moments in $A \leq 10$ nuclei - bis

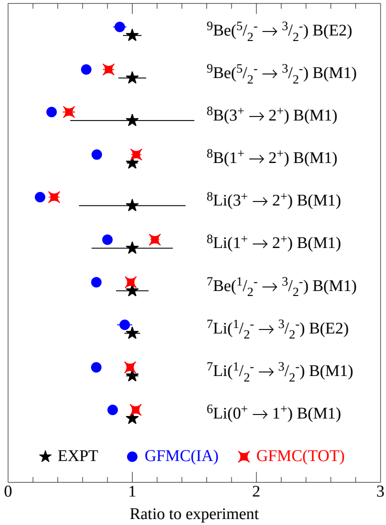
Predictions for $A > 3$ nuclei



- ^{9}C (^{9}Li) dominant spatial symmetry [s.s.] = [432] = [α,^{3}He(^{3}H),pp(nn)] $\rightarrow$ Large MEC
- ^{9}Be (^{9}B) dominant spatial symmetry [s.s.] = [441] = [$\alpha, \alpha, n(p)$]

EM transitions in $A \leq 9$ nuclei

- Two-body EM currents bring the theory in a better agreement with the EXP
- Significant correction in $A = 9$, $T = 3/2$ systems. Up to $\sim 40\%$ correction found in ^{9}C m.m.
- Major correction ($\sim 60 - 70\%$ of total MEC) is due to the one-pion-exchange currents at NLO – purely isovector



One M1 prediction: ^{9}Li$(1/2 \to 3/2)$*

$\Gamma(\mathrm{IA}) = 0.59(2)$ eV
$\Gamma(\mathrm{TOT}) = 0.79(3)$ eV

+ a number of B(E2)s in IA

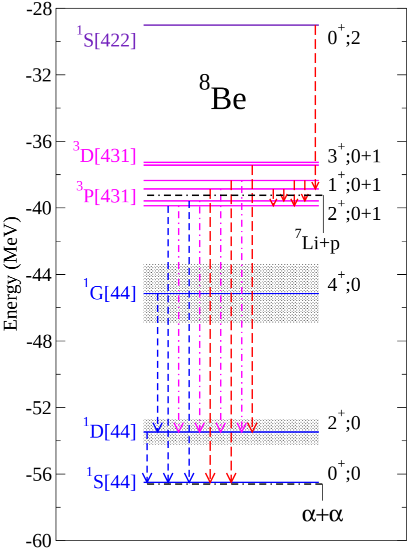
*Ricard-McCutchan *et al.* TRIUMF proposal 2014

EM transitions in low-lying states of ^{8}Be

^{8}Be energy spectrum

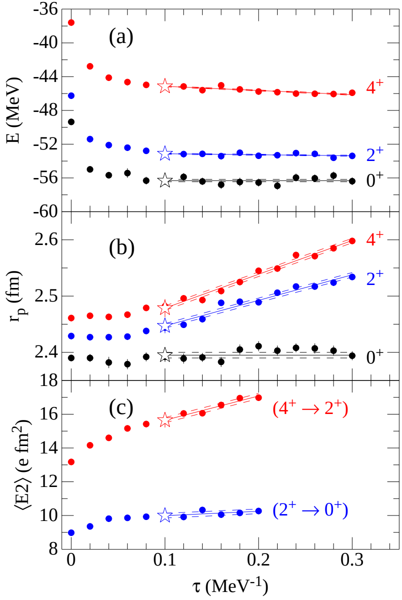
- 2^+ and 4^+ broad states at ~ 3 MeV and ~ 11 MeV
- isospin-mixed states at ~ 16 MeV, ~ 17 MeV, ~ 19 MeV
- M1 transitions
- E2 transitions
- E2 + M1 transitions

$J^\pi;T$	GFMC	Iso-mixed	Experiment
0^+	-56.3(1)		-56.50
2^+	+ 3.2(2)		+ 3.03(1)
4^+	+11.2(3)		+11.35(15)
$2^+;0$	+16.8(2)	+16.746(3)	+16.626(3)
$2^+;1$	+16.8(2)	+16.802(3)	+16.922(3)
$1^+;1$	+17.5(2)	+17.67	+17.640(1)
$1^+;0$	+18.0(2)	+18.12	+18.150(4)
$3^+;1$	+19.4(2)	+19.10	+19.07(3)
$3^+;0$	+19.9(2)	+19.21	+19.235(10)



E2 transitions in ^{8}Be

- 2^+ and 4^+ broad rotational states at ~ 3 MeV and ~ 11 MeV
- $4^+ \to 2^+$ transition recently measured at BARC*, Mumbai
- Calculational challenge: 2^+ and 4^+ states tend to break up into two α as τ increases
- Results obtained by linear fitting the GFMC points and extrapolating at $\tau = 0.1$ MeV where stability is observed in the g.s. energy propagation



$J^\pi;T$	E [MeV]	B(E2) [e^2 fm^4]
0^+	-56.3(1)	
2^+	+ 3.2(2)	20.0 (8)– [$2^+ \to 0^+$]
4^+	+11.2(3)	27.2(15)– [$4^+ \to 2^+$]*

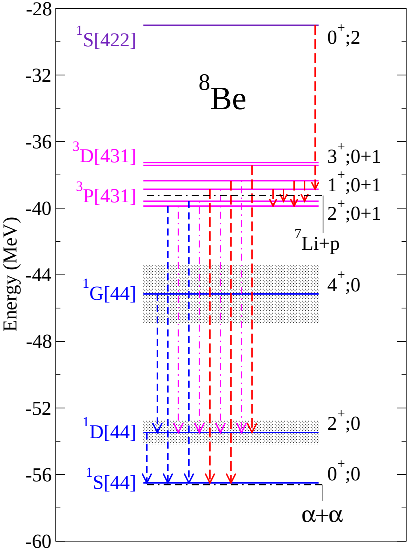
*Bhabha Atomic Research Centre

*EXPT B(E2) = 21 $\pm$ 2.3 e^2 fm^4

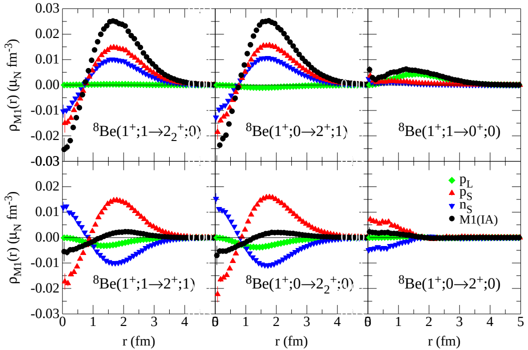
^{8}Be energy spectrum - bis

- isospin-mixed states at ~ 16 MeV, ~ 17 MeV, ~ 19 MeV
- M1 transitions: 4 classes from largest to smallest
  - conserve w.f. [s.s.]* and $\Delta T = 1$
  - conserve w.f. [s.s.] and $\Delta T = 0$
  - change w.f. [s.s.] and $\Delta T = 1$
  - change w.f. [s.s.] and $\Delta T = 0$
- E2 transitions
- E2 + M1 transitions

*[s.s.] = dominant spatial symmetry



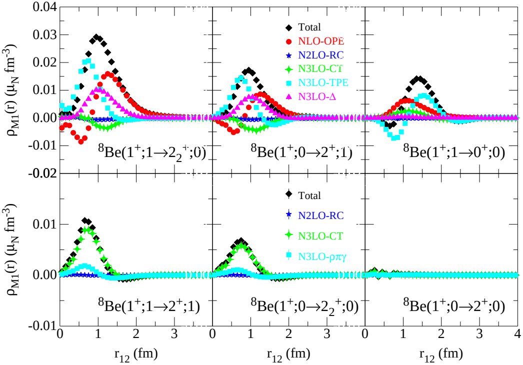
One-body M1 transitions densities



- [s.s.]-conserving transitions are enhanced due to overlap between large components of the initial and final w.f.'s
- Isospin-conserving transitions are suppressed w.r.t. isospin-changing transitions due to a cancellation between proton and neutron spin magnetization terms

$$\mathrm{M1(IA)} = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

Two-body M1 transitions densities



$(J_i, T_i) \rightarrow (J_f, T_f)$	IA	NLO-OPE	N2LO-RC	N3LO-TPE	N3LO-CT	N3LO-Δ	MEC
$(1^+; 1) \rightarrow (2_2^+; 0)$	2.461 (13)	0.457 (3)	-0.058 (1)	0.095 (2)	-0.035 (3)	0.161 (21)	0.620 (5)

M1 transitions in ^{8}Be isospin-mixed states

- 2^+, 1^+, and 3^+ states are isospin mixed, with mixing coefficients $\alpha_J^2 + \beta_J^2 = 1$

$$\psi^a = \alpha_J\, \psi_{T=0} + \beta_J\, \psi_{T=1}$$
$$\psi^b = \beta_J\, \psi_{T=0} - \alpha_J\, \psi_{T=1}$$

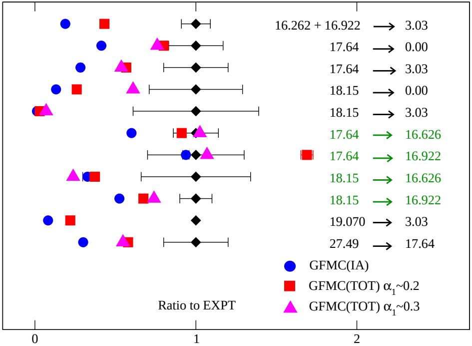
- Mixing angles α, β are from experimental decay widths as $\Gamma^a/\Gamma^b = \alpha_J^2/\beta_J^2$
  - ($\alpha_2 \sim 0.77$, β_2) well known through EXP α-decay widths, which is the only channel energetically allowed and available via $T = 0$
  - ($\alpha_1 \sim 0.21$, β_1) and ($\alpha_3 \sim 0.41$, β_3) involve multiple decay channels $\rightarrow$ hard to extract them with great accuracy (Barker NUCL. PHYS. **83**, 418 (1966))
- GFMC energies and m.e.'s are calculated for pure $T = 0$ and $T = 1$ states
- Results are 'mixed' using the 'empirical' mixing angles and then compared with EXP

Example: $1^+; T = 0+1 \rightarrow 2^+; T = 0+1$, $[431] \rightarrow [431]$

$< JT\|M1\|JT >$	IA	TOT	$E_i \rightarrow E_f$ [MeV]	B(M1)$_{\rm IA}$	B(M1)$_{\rm TOT}$	EXP [μ_N^2]
$< 10\|M1\|20 >$	0.17(0)	0.19(0)	18.15→16.626	0.56(1)	0.62(1)	1.88(46)
$< 10\|M1\|21 >$	2.60(1)	2.89(1)	18.15→16.922	1.56(2)	2.01(2)	2.89(33)
$< 11\|M1\|20 >$	2.29(1)	2.91(1)	17.64→16.626	1.65(2)	2.54(3)	2.65(25)
$< 11\|M1\|21 >$	0.14(0)	0.18(1)	17.64→16.922	0.25(1)	0.46(1)	0.30(7)

MEC contribute $\sim$ 20–30% of the total m.e.'s

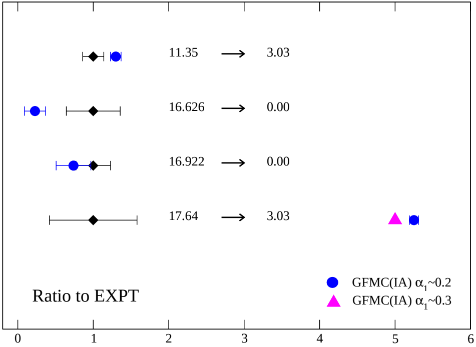
- Predictions for [s.s.]-conserving transitions are in fair agreement with EXPT
- For M1 transitions that connect two different [s.s.], GFMC calculations underpredict the EXPT even when MEC are accounted for



* We minimize the χ^2 w.r.t. EXPT for the four [s.s.]-conserving transitions

* We find that the best fit to EXPT favors larger mixing $\alpha_1(\text{NEW}) \sim 0.3$ as opposed to $\alpha_1(\text{OLD}) \sim 0.21$

- We attempt to evaluate a number of E2 transitions (predictions not shown in the figure)
- Complications are due to large cancellations among large m.e.'s $\rightarrow$ E2s very sensitive to small components
- One more complication: make sure that the first and second $(J^\pi, T) = (2^+, 0)$ states are orthogonal



  * We orthogonalize the second $(J^\pi, T) = (2^+, 0)$ via

$$|\Psi^{2_2^+}(\text{ortho})\rangle_G = |\Psi^{2_2^+}\rangle_G - {}_G\langle\Psi^{2_2^+}|\Psi^{2^+}\rangle_V|\Psi^{2^+}\rangle_G$$

Summary

- N3LO χEFT EM currents tested in the $A \leq 10$ nuclei
- MEC or two-body EM current corrections are important to bring theory in agreement with EXPT
- Large χEFT two-body corrections in ^{9}C's m.m.
- A number of M1 and E2 transitions in low-lying states of ^{8}Be have been calculated
- M1 transitions that preserve the dominant spatial symmetry of the w.f.'s are in good agreement with EXPT provided that MEC are included
- A best fit to EXPT favors more mixing in the iso-mixed $1^+, T = 0 + 1$ states at ~ 17 MeV

Outlook

* EM structure of light nuclei
  - Extend hybrid calculations to different combinations of 2N and 3N potentials to study charge radii, charge and magnetic form factors of $A \leq 10$ systems (on going project)

* Weak structure of light nuclei
  - Extend hybrid calculations to weak properties of light nuclei