Proton–deuteron scattering lengths in pionless effective field theory

Sebastian König

in collaboration with H.-W. Hammer

Bound States and Resonances in Effective Field Theories and Lattice QCD calculations

Centro de Ciencias de Benasque Pedro Pascual. Benasque, Spain

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The Ohio State University

NUCLEI
Nuclear Computational Low-Energy Initiative

Current status of p–d scattering lengths

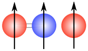
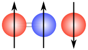
Proton

- spin $1/2$
- isospin $1/2$

Deuteron

- spin 1
- isospin 0

$\hookrightarrow$ **two S-wave channels:**

$\mathbf{1} \otimes \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{3}}{\mathbf{2}} \Big(\sim$  $\Big) \oplus \frac{\mathbf{1}}{\mathbf{2}} \Big(\sim$  $+ \cdots \Big)$

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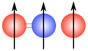
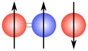
$$\mathbf{1} \otimes \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{3}}{\mathbf{2}} \left(\sim\right.$$  $$\left.\right) \oplus \frac{\mathbf{1}}{\mathbf{2}} \left(\sim\right.$$  $$\left. + \cdots \right)$$

Quartet channel

Ref.	${}^4a_{p\text{–}d}$ (fm)
van Oers, Brockman (1967)	$11.4^{+1.8}_{-1.2}$
Arvieux (1973)	11.88 ± 0.4
Huttel *et al.* (1983)	≈ 11.1
Chen *et al.* (1989)	13.8
Kievsky *et al.* (1994)	13.76
Black *et al.* (1999)	14.7 ± 2.3

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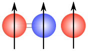
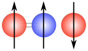
$$\mathbf{1} \otimes \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{3}}{\mathbf{2}} \Big(\sim$$  $$\Big) \oplus \frac{\mathbf{1}}{\mathbf{2}} \Big(\sim$$  $$+ \cdots \Big)$$

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Doublet channel

Ref.	$^2a_{p-d}$ (fm)
van Oers, Brockman (1967)	1.2 ± 0.2
Arvieux (1973)	2.73 ± 0.10
Huttel *et al.* (1983)	≈ 4.0
Black *et al.* (1999)	-0.13 ± 0.04
Orlov, Orevkov (2006)	≈ 0.024

Motivation

Goal

Precise and controlled extraction from EFT calculation!

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Precise and controlled extraction from EFT calculation!

Scope of method

- **Nuclear astrophysics**
  - Scattering parameters $\leftrightarrow$ shallow bound states

    SK, Lee, Hammer 2013

    Sparenberg, Capel, Baye 2010
  - Low-energy nuclear reactions in Halo-EFT
  - $\rightarrow$ one-neutron halo states in $^{11}\mathrm{Be}$
  - $\rightarrow$ one-proton halo state in $^{8}\mathrm{B}$?
- **Cold-atom systems**
  - EFT with van-der-Waals tails?

Outline

1. **Pionless effective field theory**
2. **Coulomb-modified effective range expansion**
3. **Quartet-channel scattering length**
4. **Doublet-channel scattering length**
5. **Summary and outlook**

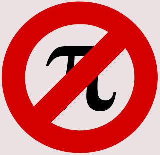
SK, H.-W. Hammer, arXiv:1312.2573

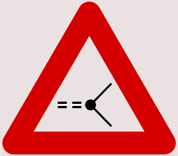
SK, Ph.D. thesis (Bonn U, 2013)

SK, H.-W. Hammer, PRC **83** (2011) 064001

Foundation and basic features

Pionless EFT

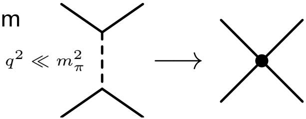




Foundation and basic features

Pionless EFT

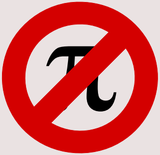
- at very low energies even pions can be integrated out
  ↪ only nucleons left as effective degrees of freedom
- non-relativistic framework
- large scattering lengths in N-N scattering
  ↪ additional low-energy scale

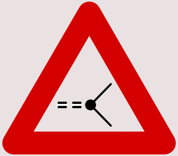


$$\gamma_d = \frac{1}{a_d}\left(1 + \mathcal{O}(a_0/r_d)\right)$$

Kaplan, Savage, Wise 1998; van Kolck 1997/98

Pionless EFT





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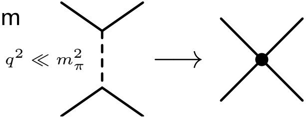

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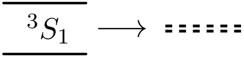


- convenient description of three-body sector with dibaryon fields

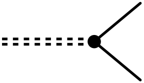
Bedaque, Hammer, van Kolck 1998

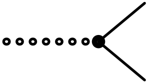
Effective Lagrangian

$$\mathcal{L} = N^\dagger \left(\mathrm{i}D_0 + \frac{\vec{D}^2}{2M_N} \right) N + \mathcal{L}_{\text{photon}} + \mathcal{L}_3$$

$$- d^{i\dagger} \left[\sigma_d + \ldots \right] d^i \quad - t^{A\dagger} \left[\sigma_t + \ldots \right] t^A$$

$$- y_d \left[d^{i\dagger} \left(N^T P_d^i N \right) + \text{h.c.} \right] - y_t \left[t^{A\dagger} \left(N^T P_t^A N \right) + \text{h.c.} \right]$$





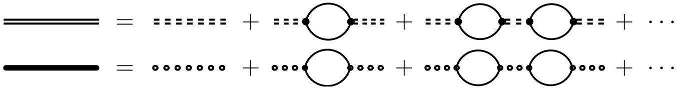
- **nucleon** field N, doublet in spin and isospin space
- auxiliary **dibaryon** fields d^i $(^3S_1,\ I = 0)$ and t^A $(^1S_0,\ I = 1)$
  $\leftrightarrow$ channels in N–N scattering
- **coupling constants** $y_{d,t}$ and $\sigma_{d,t}$
- dibaryon propagators are just constants at leading order

Dibaryon propagators

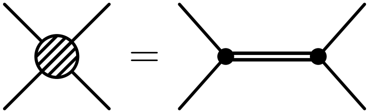
Bubble chains

$^3S_1$: $\Delta_d =$

$^1S_0$: $\Delta_t =$



Fix parameters from N-N scattering!



$$\mathrm{i}\mathcal{A}_{d,t}(k) = -y_{d,t}^2 \Delta_{d,t}\left(p_0 = \frac{\mathbf{k}^2}{2M_N}, \mathbf{p} = 0\right) = \frac{4\pi}{M_N} \frac{\mathrm{i}}{k \cot \delta_{d,t} - \mathrm{i}k}$$

- $k \cot \delta_d(k) = -\gamma_d + \frac{\rho_d}{2}(k^2 + \gamma_d^2) + \ldots \longrightarrow y_d, \sigma_d$
- $k \cot \delta_t(k) = -\gamma_t + \frac{r_{0t}}{2} k^2 + \ldots$ with $\gamma_t \equiv \frac{1}{a_t} \longrightarrow y_t, \sigma_t$

Range corrections

Dibaryon kinetic-energy terms

 $\sim\ \mathrm{i}\Delta_d^{\mathrm{LO}}(p)\times(-\mathrm{i})\left(p_0-\frac{\mathbf{p}^2}{4M_N}\right)\times\mathrm{i}\Delta_d^{\mathrm{LO}}(p)$

$\hookrightarrow$ effective-range corrections

$$\Delta_d(p)\sim\frac{1}{-\gamma_d+\sqrt{\frac{\mathbf{p}^2}{4}-M_N p_0-\mathrm{i}\varepsilon}-\frac{\rho_d}{2}\left(\frac{\mathbf{p}^2}{4}-M_N p_0-\gamma_d^2\right)}$$

$\mathcal{O}(Q/\Lambda)\sim\mathcal{O}(\gamma_d\rho_d)$

expand in ρ_d, r_{0t} $\rightarrow$ NLO, N^{2}LO, ...

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$\hookrightarrow$ effective-range corrections

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$\mathcal{O}(Q/\Lambda) \sim \mathcal{O}(\gamma_d \rho_d)$

expand in ρ_d, r_{0t} $\rightarrow$ NLO, N^{2}LO, ...

$$D_d(E;q) = D_d^{(0)}(E;q) + D_d^{(1)}(E;q) + \cdots$$

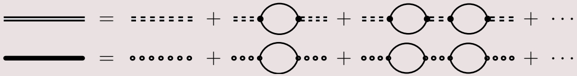
$$= -\frac{4\pi}{M_N y_d^2} \frac{1}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}} \times \left[1 + \frac{\rho_d}{2} \frac{\left(3q^2/4 - M_N E - \gamma_d^2\right)}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}} + \cdots\right]$$

Power counting $\hookrightarrow$ resum certain classes of diagrams!

Full dibaryon propagators

$^3S_1: \quad \Delta_d =$

$^1S_0: \quad \Delta_t =$



Resummations

Power counting $\hookrightarrow$ resum certain classes of diagrams!

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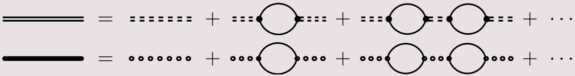
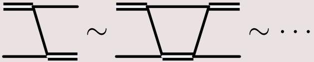
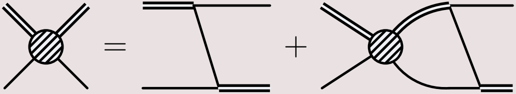


Scattering amplitude



all of same order $\rightarrow$ **Integral equation!**



Lippmann–Schwinger equation $\rightsquigarrow$ **solve numerically!**

What about Coulomb effects?

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2-body sector

- p–p scattering — Kong, Ravndal 1999, 2000
- ... at higher order — Ando, Shin, Hyun, Hong 2007

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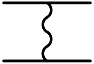
3-body sector

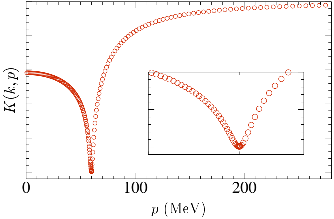
- p–d quartet-channel scattering — Rupak, Kong 2003
- ^{3}He binding energy (LO only) — Ando, Birse 2010
- p–d scattering (quartet + doublet) and ^{3}He — SK, Hammer, 2011; Vanasse, Egolf, Kerin, SK, Springer, 2014; SK, Grießhammer, Hammer, 2014

Coulomb contributions

Coulomb photons:  $\sim\ (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2}\,(\mathrm{ie})\ \longrightarrow\ (\mathrm{ie})\,\frac{\mathrm{i}}{\mathbf{q}^2+\lambda^2}\,(\mathrm{ie})$

$\mathcal{O}(\alpha)$ diagrams

Coulomb peak



$\rightarrow$ re-shuffle mesh points!

SK, Hammer, 2011

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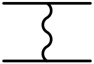
Coulomb photons:  $\sim \; (\mathrm{ie})\,\dfrac{\mathrm{i}}{\mathbf{q}^2}\,(\mathrm{ie}) \;\longrightarrow\; (\mathrm{ie})\,\dfrac{\mathrm{i}}{\mathbf{q}^2+\lambda^2}\,(\mathrm{ie})$

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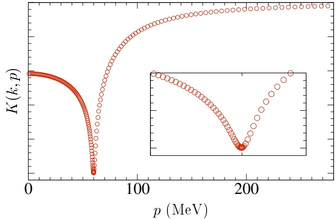

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generated by dibaryon kinetic term!

$$\mathcal{L} \supset d^{i\dagger}\left[\sigma_d + \left(\mathrm{i}D_0 + \frac{\boldsymbol{D}^2}{4M_N}\right)\right] d^i$$

$\hookrightarrow$ **range correction!**

Coulomb-subtracted phase shifts

Coulomb force

- long (infinite) range $\rightarrow$ very strong at small momentum transfer
- pure Coulomb scattering can be solved analytically

$\hookrightarrow$ **subtract the known pure Coulomb contribution!**

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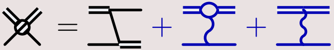
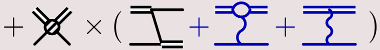
Bottom line

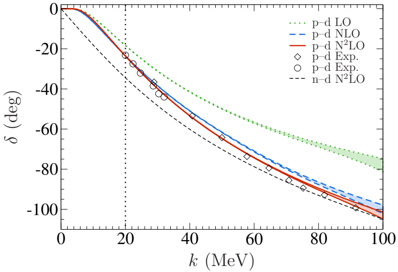




→ full amplitude $\mathcal{T}_{\text{full}}$

→ Coulomb amplitude $\mathcal{T}_{\text{c}}$

$\tilde{\delta}(k) \approx \delta_{\text{diff}}(k) \equiv \delta_{\text{full}}(k) - \delta_{\text{c}}(k)$



Rupak, Kong (2001); SK, Hammer (2011)

Modified effective range expansion

Ordinary effective range expansion

$$k\cot\delta(k) = -\frac{1}{a_0} + \frac{r_0}{2}k^2 + \cdots$$

a = scattering length
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Modified effective range expansion

$$C_{\eta,0}^2\, k\cot\delta_{\text{diff}}(k) + \alpha\mu\, h_0(\eta) = -\frac{1}{a_0^C} + \cdots$$

Gamow factor

$$C_{\eta,0}^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$$

$$\eta = \alpha\mu/k$$

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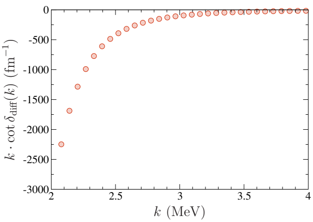
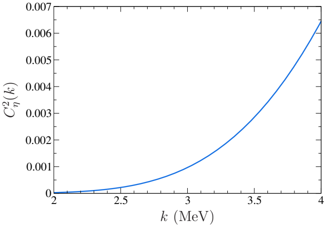


$\times$



= **finite value**

The Gamow factor

$$C_{\eta,0}^2 = \frac{2\pi\eta}{\mathrm{e}^{2\pi\eta} - 1}$$

But we have a screened Coulomb potential!

$$\frac{1}{q^2 + \lambda^2} \leftrightarrow \frac{\mathrm{e}^{-\lambda r}}{r}$$

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- furthermore: $\psi_{\mathbf{k},0}^{(+)}(p) = \frac{2\pi^2}{k^2}\delta(k - p) - \frac{2\mu\, Z_0 \mathcal{T}(E; p, k)}{k^2 - p^2 + i\varepsilon}$, $E = E(k)$

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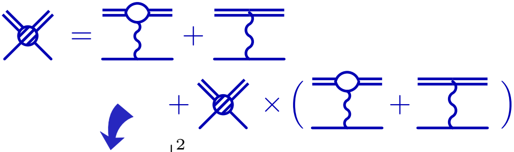
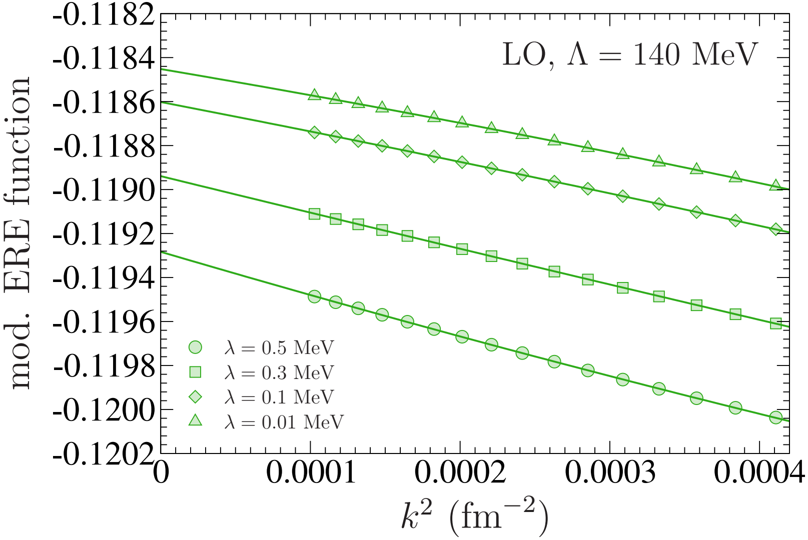


Solution

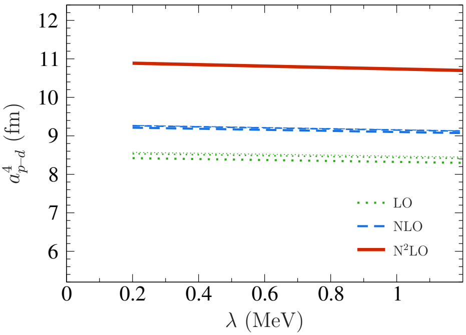
$$\rightsquigarrow C_{\eta,\lambda}^2 = \left|1 + \frac{2M_N}{3\pi^2}\int_0^\Lambda \frac{\mathrm{d}p\, p^2}{p^2-k^2-\mathrm{i}\varepsilon} Z_0 \mathcal{T}_{\mathrm{c}}(E;p,k)\right|^2$$

$\hookrightarrow$ **consistent extraction from numerical calculation!**

Quartet-channel scattering length

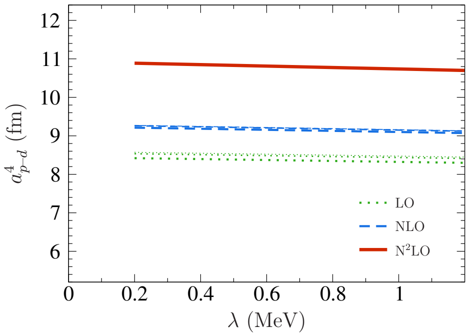


Convergence pattern



- right order of magnitude ✓
- nice (weak) photon-mass dependence ✓

Convergence pattern



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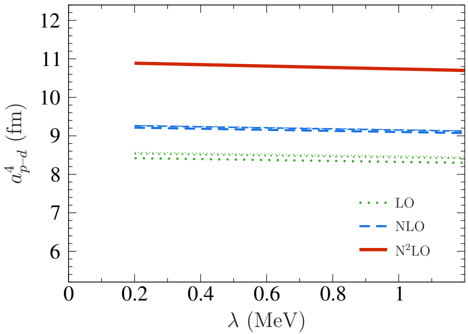
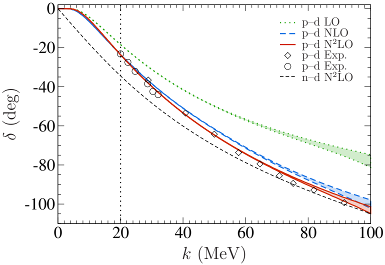
Convergence pattern



$\overset{?}{\Leftrightarrow}$



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Fully perturbative calculation (I)

So far…

Partial-resummation approach Bedaque, Grießhammer, Hammer, Rupak 2003

- $\mathcal{T}_{\mathsf{LO}} = K_{\mathsf{LO}} + \mathcal{T}_{\mathsf{LO}} \otimes (D_{\mathsf{LO}} K_{\mathsf{LO}})$
- $\mathcal{T}_{\mathsf{NLO}} = K_{\mathsf{NLO}} + \mathcal{T}_{\mathsf{NLO}} \otimes (D_{\mathsf{NLO}} K_{\mathsf{NLO}})$
- etc. $\hookrightarrow$ resums certain higher-order corrections!

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Better (cleaner) approach

Fully perturbative calculation see, *e.g.*, Ji, Phillips 2012

- $\mathcal{T}_{\text{NLO}} = \mathcal{T}_{\text{LO}} + \Delta\mathcal{T}_{\text{NLO}}$
- $\Delta\mathcal{T}_{\text{NLO}} = \mathcal{T}_{\text{LO}} \otimes (D^{(1)} K_{\text{LO}}) \otimes \mathcal{T}_{\text{LO}} + \cdots$
- $\delta(k) = \delta^{(0)} + \delta^{(1)} + \cdots$
- complicated at N^2LO!

Fully perturbative calculation (I)

So far...

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- $\delta(k) = \delta^{(0)} + \delta^{(1)} + \cdots$
- complicated at N^2LO!
- $\Delta\mathcal{T}_{\mathrm{NLO}} = \mathcal{T}_{\mathrm{LO}} \otimes (D^{(1)} K_{\mathrm{LO}}) \otimes \mathcal{T}_{\mathrm{LO}} + \cdots$

Much more efficient calculation with re-shuffling of terms!

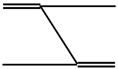
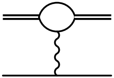
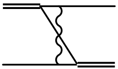
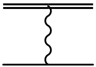
Vanasse 2013

Fully perturbative calculation (II)

$$\mathcal{T}_{\rm full}^{(0)} = K^{(0)} + \mathcal{T}_{\rm full}^{(0)} \otimes D_d^{(0)}\,K^{(0)}$$

$$\mathcal{T}_{\rm full}^{(1)} = K^{(1)} + \mathcal{T}_{\rm full}^{(0)} \otimes \left[D_d^{(0)}\,K^{(1)} + D_d^{(1)}\,K^{(0)}\right] + \mathcal{T}_{\rm full}^{(1)} \otimes D_d^{(0)}\,K^{(0)}$$

$$\mathcal{T}_{\rm full}^{(2)} = \mathcal{T}_{\rm full}^{(0)} \otimes \left[D_d^{(1)}\,K^{(1)} + D_d^{(2)}\,K^{(0)}\right] + \mathcal{T}_{\rm full}^{(1)} \otimes \left[D_d^{(0)}\,K^{(1)} + D_d^{(1)}\,K^{(0)}\right] + \mathcal{T}_{\rm full}^{(2)} \otimes D_d^{(0)}\,K^{(0)}$$

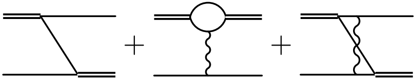
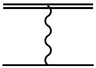
$K^{(0)} =$  $+$  $+$  , $K^{(1)} =$ 

Fully perturbative calculation (II)

$$\mathcal{T}^{(0)}_{\text{full}} = K^{(0)} + \mathcal{T}^{(0)}_{\text{full}} \otimes D^{(0)}_d K^{(0)}$$

$$\mathcal{T}^{(1)}_{\text{full}} = K^{(1)} + \mathcal{T}^{(0)}_{\text{full}} \otimes \left[D^{(0)}_d K^{(1)} + D^{(1)}_d K^{(0)}\right] + \mathcal{T}^{(1)}_{\text{full}} \otimes D^{(0)}_d K^{(0)}$$

$$\mathcal{T}^{(2)}_{\text{full}} = \mathcal{T}^{(0)}_{\text{full}} \otimes \left[D^{(1)}_d K^{(1)} + D^{(2)}_d K^{(0)}\right] + \mathcal{T}^{(1)}_{\text{full}} \otimes \left[D^{(0)}_d K^{(1)} + D^{(1)}_d K^{(0)}\right] + \mathcal{T}^{(2)}_{\text{full}} \otimes D^{(0)}_d K^{(0)}$$

$K^{(0)} =$  , $K^{(1)} =$ 

$$[k\cot\delta_{\text{diff}}]^{(0)} = \frac{2\pi}{\mu}\frac{\mathrm{e}^{2\mathrm{i}\delta_{\text{c}}^{(0)}}}{T^{(0)}_{\text{diff}}} + \mathrm{i}k$$

$$[k\cot\delta_{\text{diff}}]^{(1)} = \frac{2\pi}{\mu}\mathrm{e}^{2\mathrm{i}\delta_{\text{c}}^{(0)}} \times \left[\frac{2\mathrm{i}\delta_{\text{c}}^{(1)}}{T^{(0)}_{\text{diff}}} - \frac{T^{(1)}_{\text{diff}}}{(T^{(0)}_{\text{diff}})^2}\right]$$

$$[k\cot\delta_{\text{diff}}]^{(2)} = -\frac{2\pi}{\mu}\mathrm{e}^{2\mathrm{i}\delta_{\text{c}}^{(0)}} \times \left[\frac{2(\delta_{\text{c}}^{(1)})^2 - 2\mathrm{i}\delta_{\text{c}}^{(2)}}{T^{(0)}_{\text{diff}}} + \frac{2\mathrm{i}\delta_{\text{c}}^{(1)}T^{(1)}_{\text{diff}} + T^{(2)}_{\text{diff}}}{(T^{(0)}_{\text{diff}})^2} - \frac{(T^{(1)}_{\text{diff}})^2}{(T^{(0)}_{\text{diff}})^3}\right]$$

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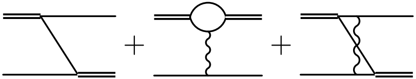
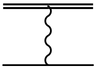
$K^{(0)} =$  , $K^{(1)} =$ 

$$[k\cot\delta_{\rm diff}]^{(0)} = \frac{2\pi}{\mu}\frac{e^{2i\delta_c^{(0)}}}{T_{\rm diff}^{(0)}} + ik$$

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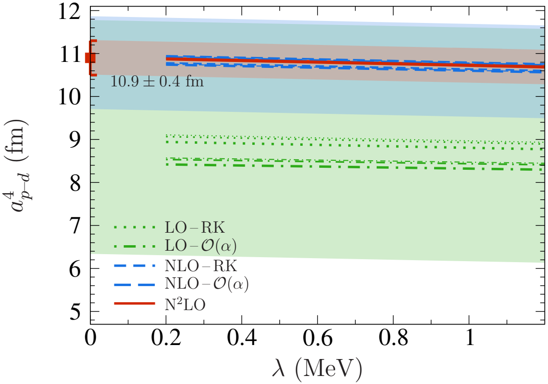
$$[k\cot\delta_{\rm diff}]^{(2)} = -\frac{2\pi}{\mu} e^{2i\delta_c^{(0)}} \times \left[\frac{2(\delta_c^{(1)})^2 - 2i\delta_c^{(2)}}{T_{\rm diff}^{(0)}} + \frac{2i\delta_c^{(1)} T_{\rm diff}^{(1)} + T_{\rm diff}^{(2)}}{(T_{\rm diff}^{(0)})^2} - \frac{(T_{\rm diff}^{(1)})^2}{(T_{\rm diff}^{(0)})^3}\right]$$

Scattering length

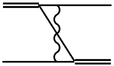
$$C_{\eta,\lambda}^2\,[k\cot\delta_{\rm diff}(k)] + \gamma\, h(\eta) = -\frac{1}{a_{p-d}} + \mathcal{O}(k^2)$$

Combine with $C_{\eta,\lambda}^2 = [C_{\eta,\lambda}^2]^{(0)} + [C_{\eta,\lambda}^2]^{(1)} + \cdots$

Fully perturbative result

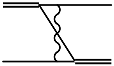


"RK"



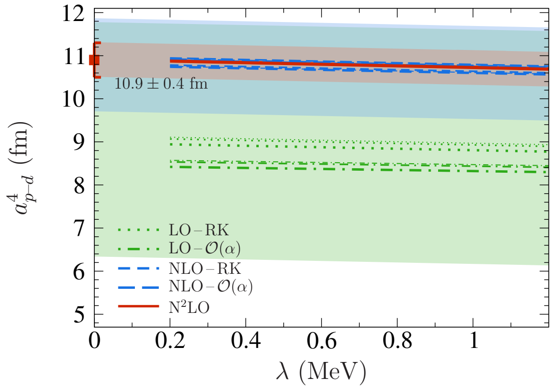
as Q/Λ correction

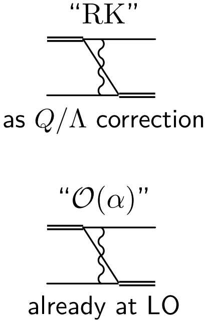
"$\mathcal{O}(\alpha)$"



already at LO

Fully perturbative result





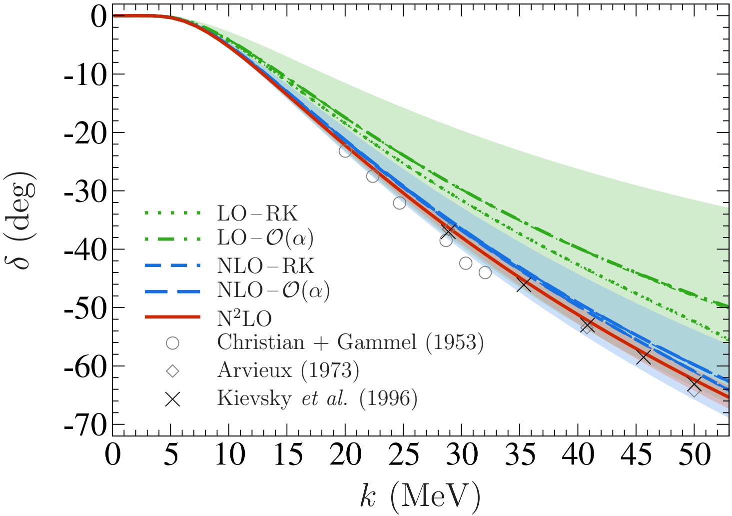
Older experimental determinations

- $a^4_{p\text{–}d} = 11.88 \pm 0.4$ fm Arvieux (1973)
- $a^4_{p\text{–}d} = 11.1$ fm Huttel *et al.* (1983)

More recent results

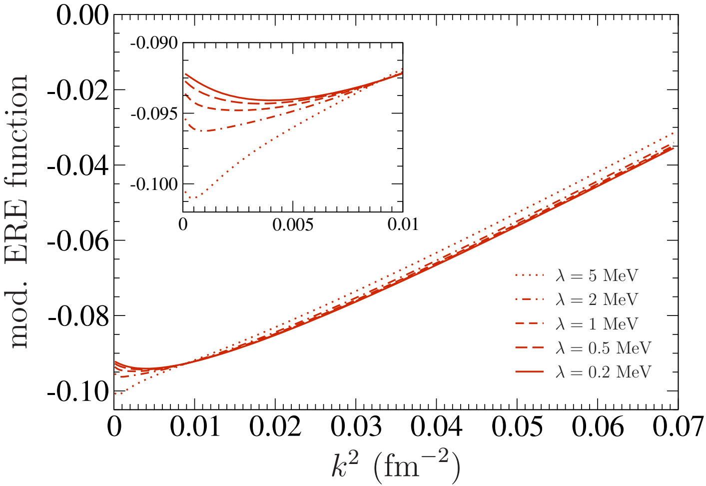
- $a^4_{p\text{–}d} \approx 13.8$ fm Chen *et al.*(1989) Kievsky *et al.*(1994)
- $a^4_{p\text{–}d} = 14.7 \pm 2.3$ fm Black *et al.* (1999)

Phase shifts



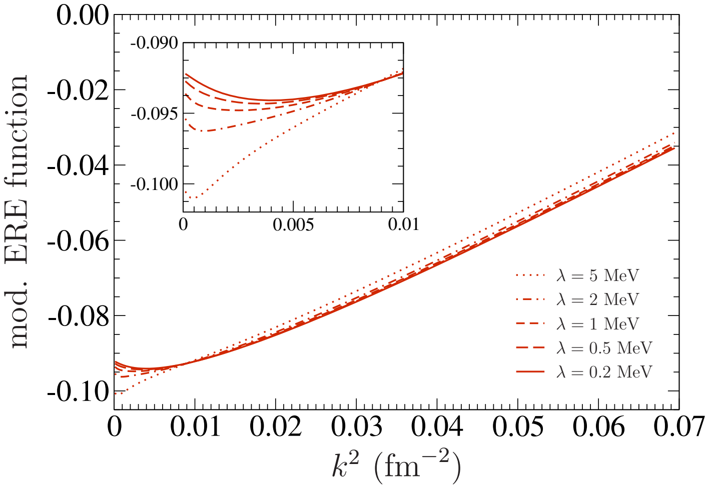
No relevant discrepancy here!

Step back: effective range function



- convergent in the limit $\lambda \to 0$ ✓
- curvature $\leftrightarrow$ missing screening corrections in $h(\eta)$?

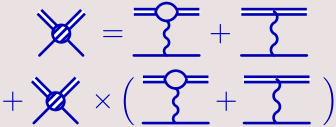
Step back: effective range function



- convergent in the limit $\lambda \to 0$ ✓ $\quad C^2_{\eta,\lambda}\left[k\cot\delta_{\text{diff}}(k)\right] + \gamma\, h(\eta) = -\frac{1}{a_{p\text{–}d}} + \mathcal{O}(k^2)$
- curvature $\leftrightarrow$ missing screening corrections in $h(\eta)$?

Coulomb subtraction

EFT calculation (momentum space)



- $k\cot\delta_{\mathrm{diff}} = \frac{2\pi}{\mu}\frac{\mathrm{e}^{2\mathrm{i}\delta_c}}{T_{\mathrm{full}}-T_c} + \mathrm{i}k$
- $C^2_{\eta,\lambda} = \left|1 + \frac{2M_N}{3\pi^2}\int_0^\Lambda \frac{\mathrm{d}p\,p^2}{p^2-k^2-\mathrm{i}\varepsilon} T_c(E;p,k)\right|^2$

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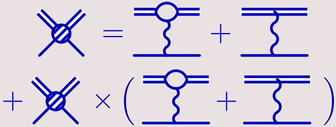

- $k \cot \delta_{\text{diff}} = \frac{2\pi}{\mu} \frac{e^{2i\delta_c}}{T_{\text{full}} - T_c} + ik$
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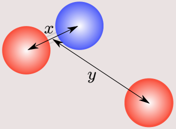
Potential-model calculation (configuration space)

$$\psi(x,y) \xrightarrow{y\to\infty} \left[F(\eta, ky) \cot \tilde{\delta}(k) + G(\eta, ky)\right] u(x)$$

cf.. Chen, Payne, Friar, Gibson 1989



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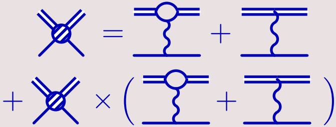

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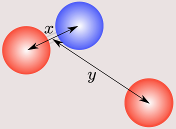

Equivalent? → **probably not. . .**

try using a simple two-body Yukawa potential for the pure Coulomb sector

Subtraction with simple Yukawa potential

Just get T_c from a two-body Lippmann–Schwinger equation

$\rightarrow$ **no perturbative expansion!**

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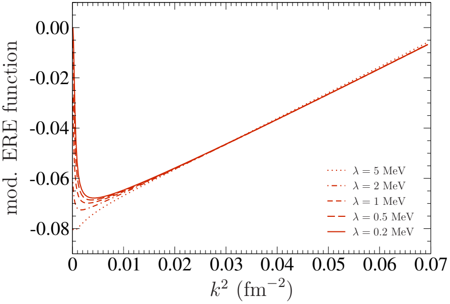

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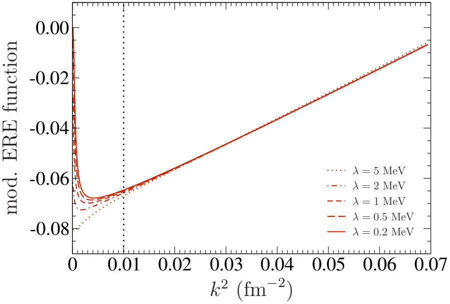

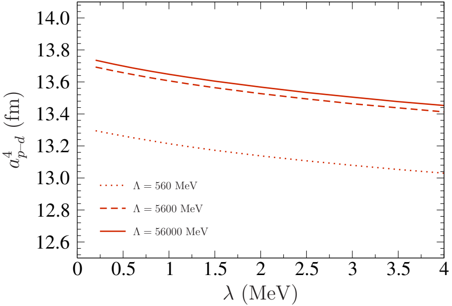

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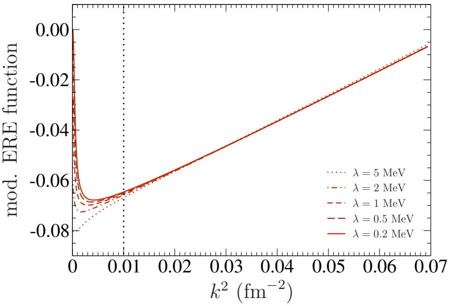

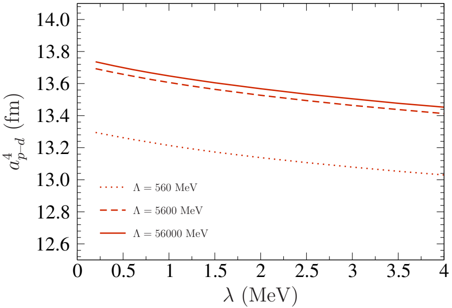

- seems to give a value consistent with potential-model calculations
- artifacts at small k and stronger cutoff dependence
- no longer a pure EFT calculation (?)

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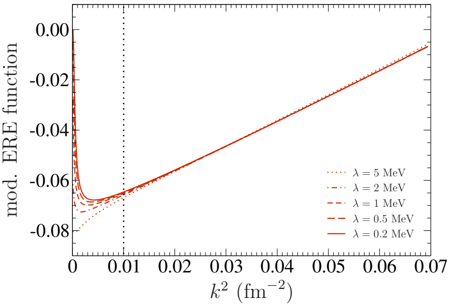
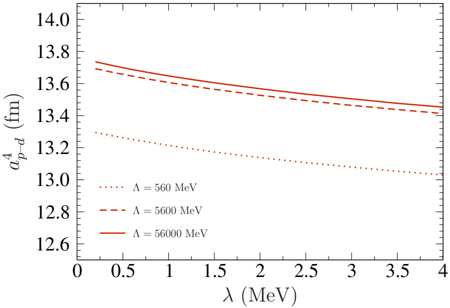
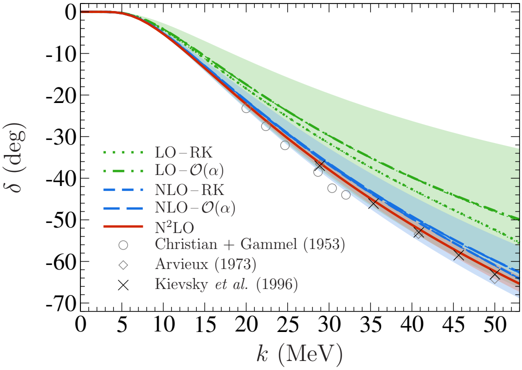


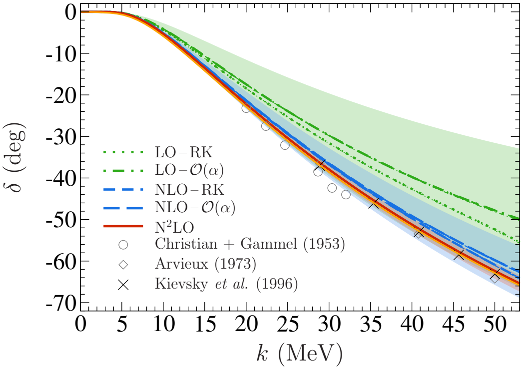


- seems to give a value consistent with potential-model calculations
- artifacts at small k and stronger cutoff dependence
- no longer a pure EFT calculation (?)
- shouldn't one calculate subtracted phase shift the same way?

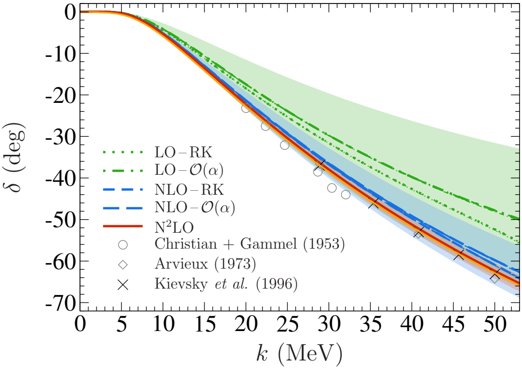
Phase shifts revisited

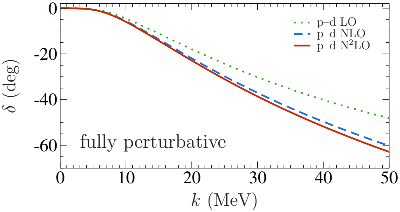


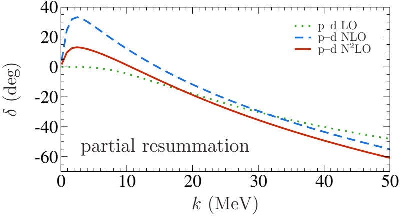
Phase shifts revisited



Phase shifts revisited







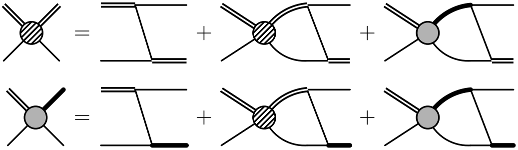
Part IV

Doublet channel

- **Coupled channels**
- **Three-nucleon forces**
- **Results (preliminary)**

Complications

1. coupled channels!



Complications

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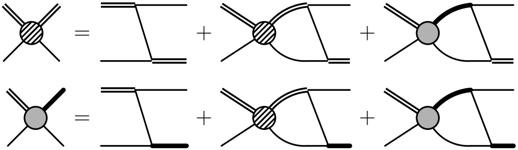
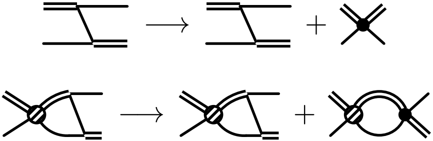


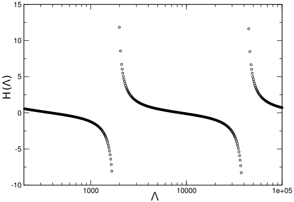
2. strong cutoff dependence!

$\hookrightarrow$ renormalize with leading order 3N-force force ($SU(4)$-symmetric)

Bedaque, Hammer, van Kolck 1999

$$\mathcal{L}_3 = -M_N \frac{H(\Lambda)}{\Lambda^2}\left(y_d^2 N^\dagger (\vec{d}\cdot\vec{\sigma})^\dagger (\vec{d}\cdot\vec{\sigma}) N + \ldots\right)$$

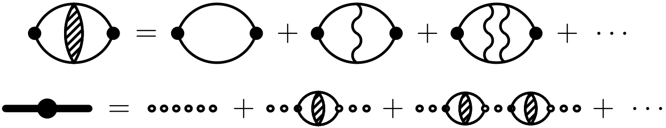




... fix $H(\Lambda)$ with three-body input $\to$ triton binding energy, ${}^2a_{n-d}$

Coulomb effects in the proton–proton channel

In doublet channel, the singlet dibaryon can be in a pure $p-p$ state



$$\Delta_{t,pp}(p) \sim \frac{1}{-1/a_C - 2\kappa H(\kappa/p')} \ , \ \kappa = \frac{\alpha M_N}{2} \ , \ p' = \mathrm{i}\sqrt{\mathbf{p}^2/4 - M_N p_0 - \mathrm{i}\varepsilon}$$

Kong, Ravndal 1999

→ Coulomb-modified effective range expansion

Bethe 1949

cf. Ando, Birse 2010

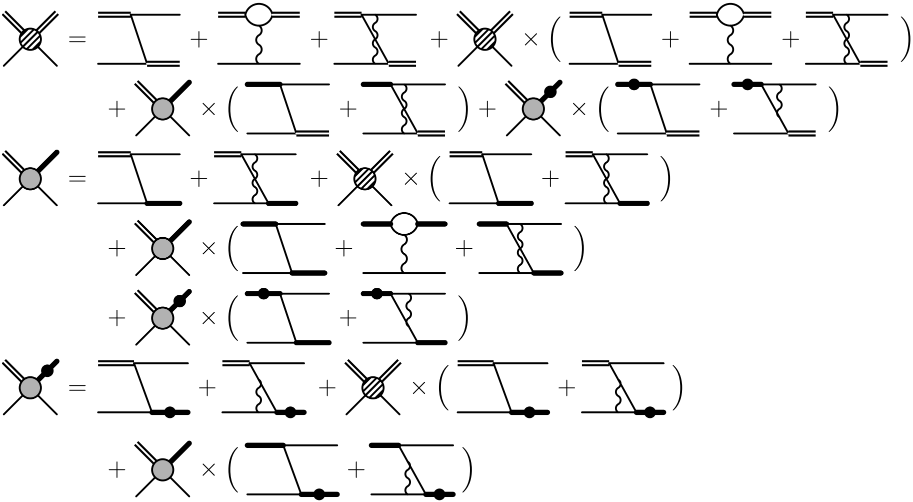
The third nucleon neccessarily has to be a neutron!

→ no additional Coulomb-photon exchange!

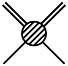
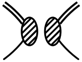
↪ 3-channel integral equation

Full doublet-channel integral equation

Include all $\mathcal{O}(\alpha)$ Coulomb diagrams. . .

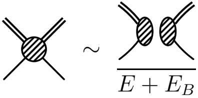


bound-sate regime:

 $\sim$  / $E + E_B$ $+$ regular terms

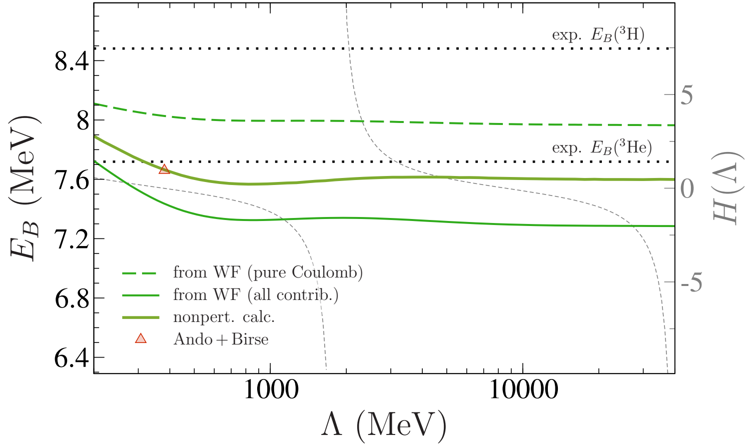
He-3 binding energy (LO)

bound-sate regime:



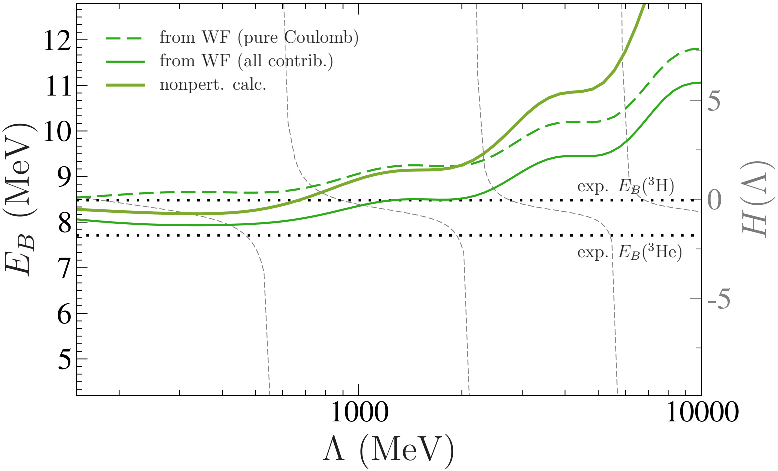
$+$ regular terms

$\hookrightarrow$ calculate ^{3}He binding energy!



He-3 binding energy (NLO)

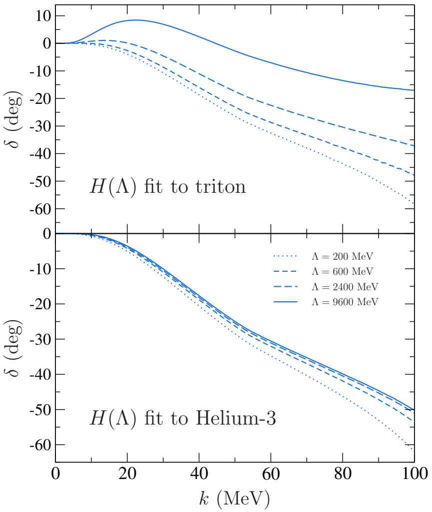
At NLO, things don't work so well...



↪ **incomplete renormalization!**

New "Coulomb" counterterm

Re-fit $H(\Lambda)$ to ^{3}He energy at NLO



SK, Grießhammer, Hammer 2014

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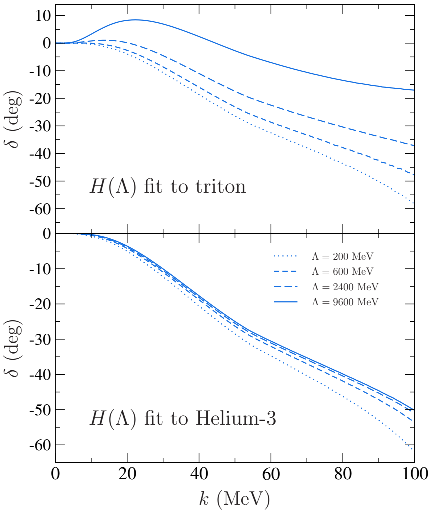
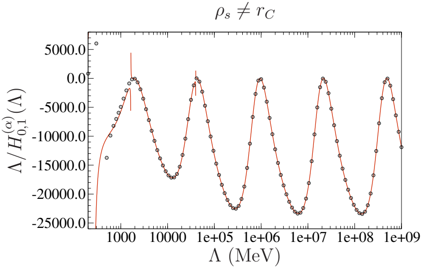


SK, Grießhammer, Hammer 2014

Can be shown analytically!

Vanasse, Egolf, Kerin, SK, Springer 2014

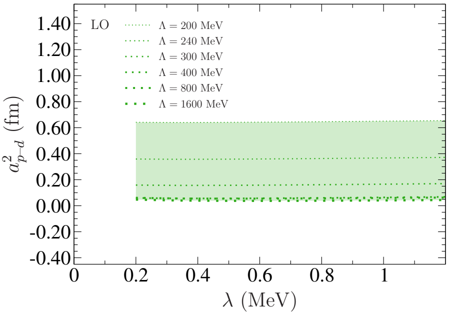
$$H(\Lambda) = H_{0,0}(\Lambda) + H_{0,1}(\Lambda) + H_{0,1}^{(\alpha)}(\Lambda)$$



Doublet-channel scattering length

Back to the fully perturbative approach. . .

- fit $H_1^{(\alpha)}(\Lambda)$ to ^{3}He binding energy
- predict doublet-channel p–d scattering length



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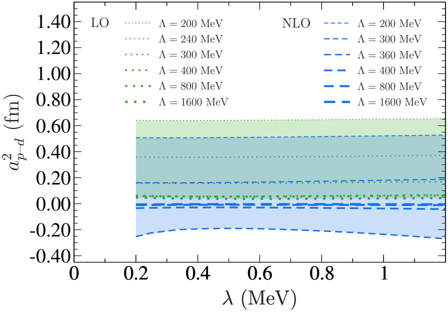

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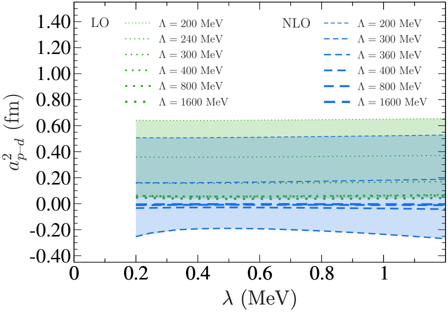

Other determinations

Ref.	$^2a_{p\text{–}d}$ (fm)
van Oers, Brockman (1967)	1.2 ± 0.2
Arvieux (1973)	2.73 ± 0.10
Huttel *et al.* (1983)	≈ 4.0
Black *et al.* (1999)	-0.13 ± 0.04
Orlov, Orevkov (2006)	≈ 0.024

Summary and outlook

- Non-perturbative Coulomb effects are hard to include consistently
- Screened Gamow factor can be calculated numerically
- Clean perturbative expansion very important at low energies
- Quartet-channel scattering length agrees well with older experimental determinations. . .
- . . . but it may be a scheme-dependent quantity
- Discrepancy with potential-model calculations may be resolved this way
- Need to go to higher orders to nail down doublet-channel scattering length

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Thanks for your attention!