The large-scale structure of the Higgs vacuum

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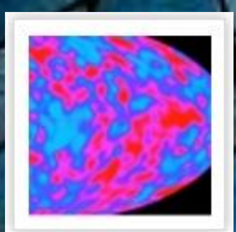


Modern Cosmology: Early Universe, CMB and LSS
Benasque, August 3-16, 2014

A.L.M and F. Prada to appear

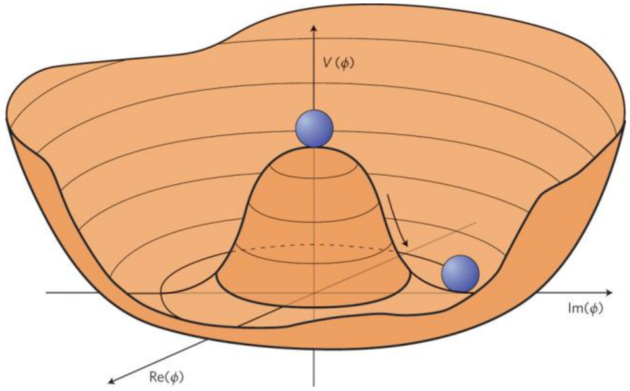
Albareti, Cembranos, A.L.M. arXiv:1404.5946 and 1405.3900

Introduction

- The discovery at LHC of a scalar resonance with m_H= 126 GeV compatible with the SM Higgs gives support to the simplest EW Symmetry Breaking Sector (SBS) with just one scalar doublet

- Future measurements at ATLAS and CMS will constrain the Higgs mass and branching ratios with better precision

- However the Higgs self-couplings, i.e. the **shape of the potential,** difficult to constrain with LHC luminosity design value



- We need alternative probes of the SBS. Cosmology?

Introduction

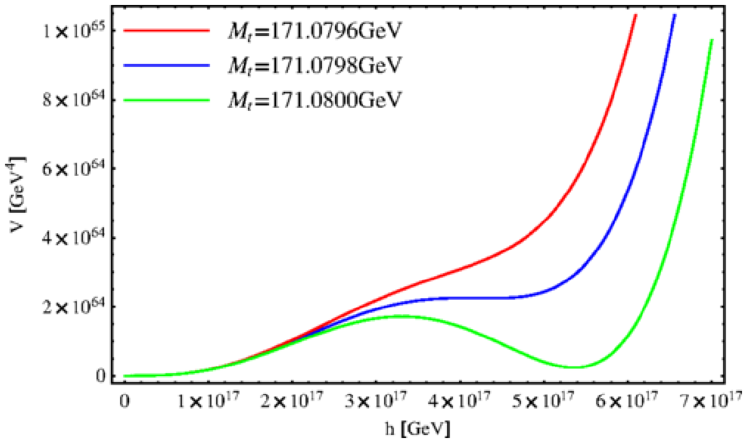
- EW phase transition difficult to explore, but still some effects: EW inflation, baryogenesis, magnetogensis, … Vachaspati, 91; Knox, Turner, 93; Kuseneko, 99

- Non-minimal couplings and Higgs inflation (higher energies)

  Bezrukov, Shaposhnikov, 08

- Do metric perturbations modify the Higgs potential (VEV)?.

- NO at classical level (scalar field)
- But **quantum fluctuations** feel the geometry



Phys. Rev. Lett. 112, 241301, Y. Hamada, et al.

- Calculate the one-loop effective potential with metric perturbations

OUTLINE

- Higgs effective potential in Minkowski space-time
  I. *Refreshing basic ideas*

- Effective potential in curved space-time
  I. *Locally inertial vs. comoving observers*

- Perturbed Robertson-Walker backgrounds
  I. *Perturbative solutions*
  II. *Homogeneous and inhomogeneous contributions*

- Higgs VEV, varying fundamental constants and phenomenology

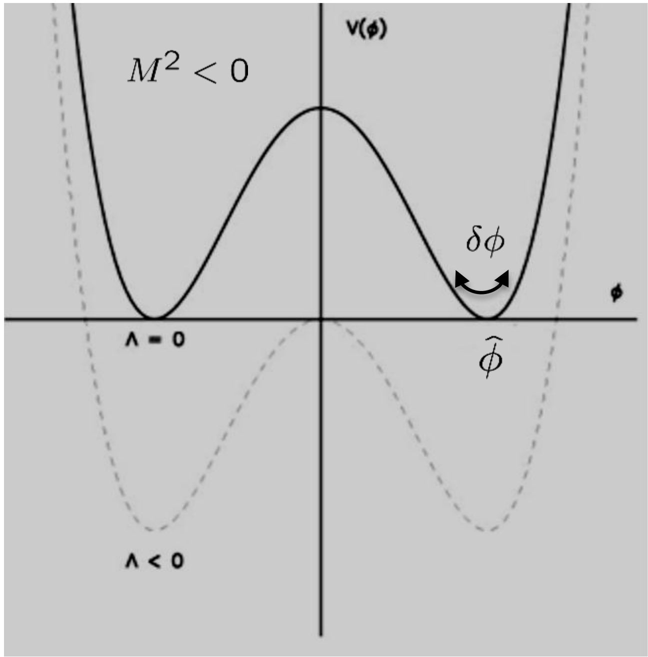
Effective potential in Minkowski space-time

$$\mathcal{L}_{SBS} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - M^2(\Phi^\dagger \Phi) - \lambda(\Phi^\dagger \Phi)^2 + \mathcal{L}_{YK}$$

Higgs doublet $\quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \theta_2 + i\theta_1 \\ \phi + i\theta_3 \end{pmatrix}$

$$\phi(t, \mathbf{x}) = \widehat{\phi} + \delta\phi(t, \mathbf{x})$$

$$\Box \phi + V'(\phi) = 0$$



$$V(\phi) = V_0 + \frac{1}{2} M^2 \phi^2 + \frac{\lambda}{4} \phi^4$$

Tree-level potential

Effective potential in Minkowski space-time

$$\Box\,\phi + V'(\phi) = 0$$

$\langle \delta\phi \rangle = 0$

VEV one-loop equation of motion

$$\Box\,\hat{\phi} + \underbrace{V'(\hat{\phi}) + \frac{1}{2}V'''(\hat{\phi})\langle \delta\phi^2 \rangle}_{V'_{eff}(\hat{\phi})} = 0\,.$$

Fluctuations equation

$$\Box\,\delta\phi + \underbrace{V''(\hat{\phi})}_{m^2(\hat{\phi})}\delta\phi = 0$$

Effective potential

$$V_{eff}(\hat{\phi}) = V(\hat{\phi}) + \underbrace{\frac{1}{2}\int dm^2 \langle 0|\delta\phi^2(\eta, \mathbf{x})|0\rangle}_{\text{one-loop contribution}}$$

Effective potential in Minkowski space-time

$$\langle 0|\delta\phi^2|0\rangle$$

$$\omega^2 = k^2 + m^2(\hat{\phi})$$

Canonical quantization

$$\delta\phi(t,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}\sqrt{2\omega}} \left(a_{\mathbf{k}}\, e^{i\,(\mathbf{k}\,\mathbf{x}-\omega t)} + a^\dagger_{\mathbf{k}} e^{-(i\,\mathbf{k}\,\mathbf{x}-\omega t)}\right).$$

$$[a_{\mathbf{p}}, a^\dagger_{\mathbf{q}}] = \delta^{(3)}(\mathbf{p}-\mathbf{q}) \qquad a_{\mathbf{p}}|0\rangle = 0, \quad \forall \mathbf{p}$$

$$\langle 0|\delta\phi^2|0\rangle = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{k^2+m^2(\hat{\phi})}}$$

$$V_{eff}(\hat{\phi}) = V(\hat{\phi}) + \underbrace{\frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^2(\hat{\phi})}}_{V_1(\hat{\phi}) = \rho_{vac}(\hat{\phi})}$$

Regularization

Cutoff regularization

$$V_1(\hat{\phi}) = \frac{1}{4\pi^2}\int_0^{\Lambda} dk\, k^2\sqrt{k^2 + m^2(\hat{\phi})}$$

$$V_1(\hat{\phi}) \;=\; \frac{1}{16\pi^2}\left(\Lambda^4 + m^2(\hat{\phi})\Lambda^2 - \frac{m^4(\hat{\phi})}{4}\ln\left(\frac{\Lambda^2}{\mu^2}\right)\right) \;+\; \frac{m^4(\hat{\phi})}{64\pi^2}\left[\ln\left(\frac{m^2(\hat{\phi})}{\mu^2}\right) + \left(\frac{1}{2} - 2\ln(2)\right)\right] \;+\; \mathcal{O}(\Lambda^{-2})$$

$\Lambda \to \infty$

$$m^2(\hat{\phi}) = M^2 + 3\lambda\hat{\phi}^2$$

Dimensional regularization

$$V_1(\hat{\phi}) = \frac{\mu^{\epsilon}}{2}\int \frac{d^{3-\epsilon}k}{(2\pi)^{3-\epsilon}}\sqrt{k^2 + m^2(\hat{\phi})}$$

$\epsilon \to 0$

$$V_1(\hat{\phi}) \;=\; \frac{m^4(\hat{\phi})}{64\pi^2}\left(-\frac{2}{\epsilon} - \ln(4\pi) + \gamma\right) + \frac{m^4(\hat{\phi})}{64\pi^2}\left[\ln\left(\frac{m^2(\hat{\phi})}{\mu^2}\right) - \frac{3}{2}\right] + \mathcal{O}(\epsilon)$$

Renormalization

Countertems $$V_{eff}(\hat{\phi}) = V(\hat{\phi}) + V_1(\hat{\phi}) + \Delta_{cc} + \frac{1}{2}\Delta_M \hat{\phi}^2 + \frac{1}{4}\Delta_\lambda \hat{\phi}^4$$

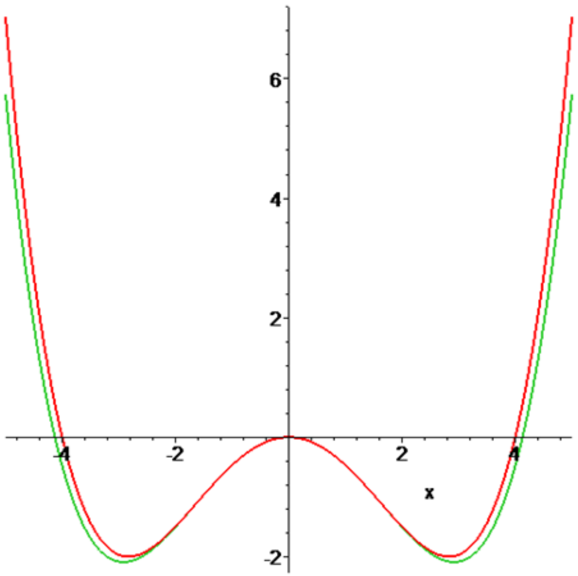
$$V_{eff}(\hat{\phi}) \;=\; V_0 + \frac{1}{2}M^2\hat{\phi}^2 + \frac{\lambda}{4}\hat{\phi}^4 + \;\frac{m^4(\hat{\phi})}{64\pi^2}\left(\ln\left(\frac{m^2(\hat{\phi})}{\mu^2}\right) + C\right)$$

Running coupling and mass

$$\frac{dV_{eff}}{d\ln\mu} = 0$$

$$\beta(\lambda) \equiv \frac{d\lambda}{d(\ln\mu)} = \frac{18\lambda^2}{(4\pi)^2}$$

$$\gamma_M(\lambda) \equiv \frac{d\ln M^2}{d(\ln\mu)} = \frac{6\lambda}{(4\pi)^2}$$



Effective potential in curved space-time

- In Minkowski space-time there is a privileged class of observers (inertial observers) which share the same vacuum state (Poincaré invariance).

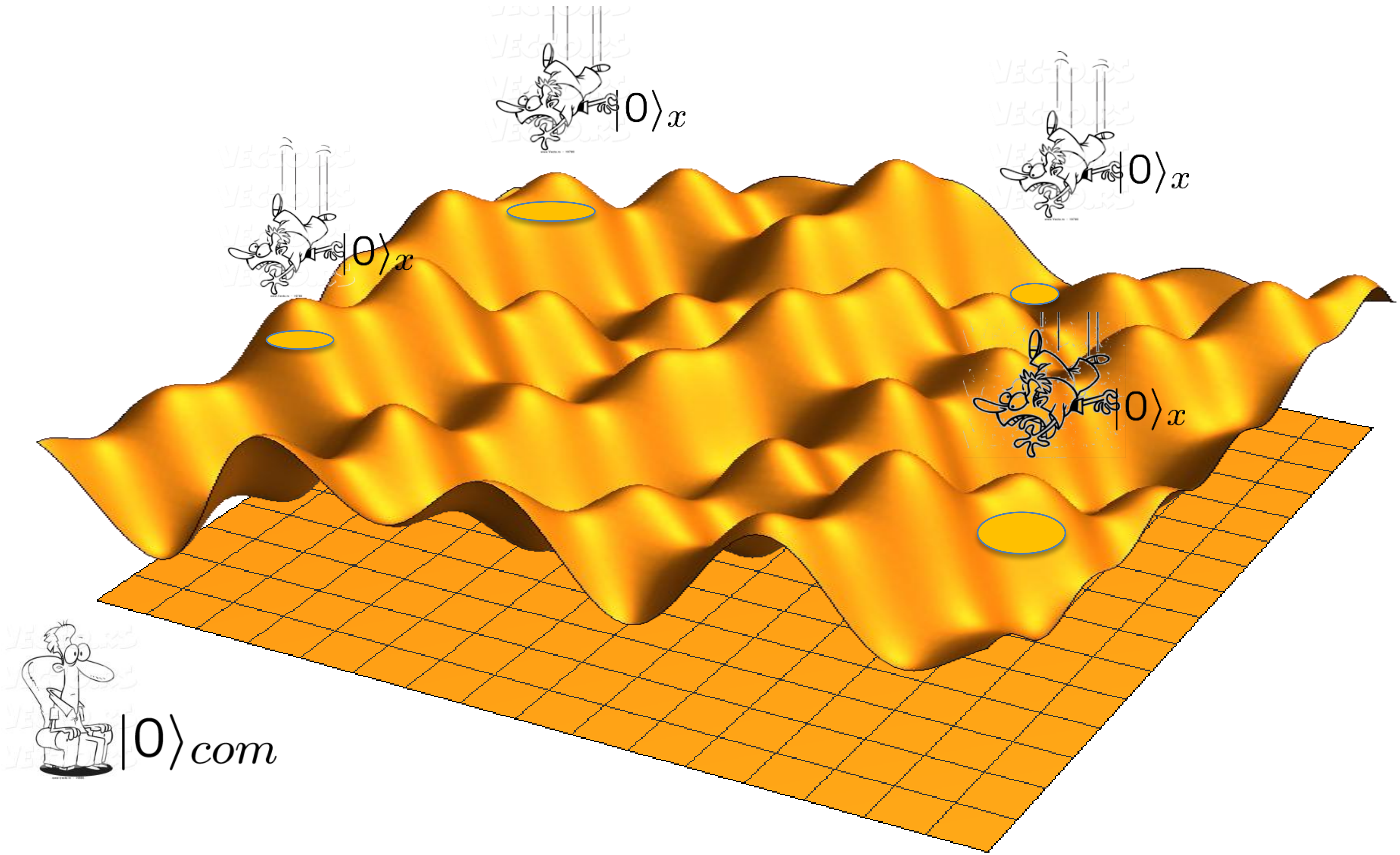
- In curved space-time there is no natural definition of vacuum

- If we are interested in calculating a local expression for the effective potential, we can use the coordinates (and vacuum) associated to a free-falling observer (Schwinger- de Witt **local** effective action) valid in a local neighborhood of the observer.

- However we are interested in comparing the effective potential at space-time points with large separations (beyond the curvature radius) and the local expansion might not be appropriate. Adiabatic vacuum for comoving observers.

Locally inertial vs. comoving adiabatic vacua

$$\delta\phi(\eta,\mathbf{x}) = \int \mathrm{d}^3\mathbf{k}\left(a_{\mathbf{k}}\delta\phi_k(\eta,\mathbf{x}) + a^\dagger_{\mathbf{k}}\delta\phi^*_k(\eta,\mathbf{x})\right)$$



Locally inertial vs. comoving adiabatic vacua

$|0\rangle_x$

Locally inertial observer: Schwinger-de Witt representation explicitly covariant but **local**

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{3}R_{\mu\alpha\nu\beta}x^\alpha x^\beta - \frac{1}{6}R_{\mu\alpha\nu\beta;\gamma}x^\alpha x^\beta x^\gamma + \left(-\frac{1}{20}R_{\mu\alpha\nu\beta;\gamma\delta} + \frac{2}{45}R_{\alpha\mu\beta\lambda}R^\lambda{}_{\gamma\nu\delta}\right)x^\alpha x^\beta x^\gamma x^\delta + \cdots$$

$|0\rangle_{com}$

Comoving observer: **global** but not explicitly covariant

$$ds^2 = a^2(\eta)\left\{[1+2\Phi(\eta,\mathrm{x})]\,d\eta^2 - [1-2\Psi(\eta,\mathrm{x})]\,d\mathrm{x}^2\right\}$$

Effective potential in curved space-time

Locally inertial observers

Effective action W

$$e^{iW[g_{\mu\nu}]} = \int [d\phi] e^{iS[g_{\mu\nu},\phi]} = \int [d\phi] e^{-\frac{i}{2}\int dx\sqrt{g}\phi\overbrace{(\Box + m^2 + \xi R - i\epsilon)}^{O}\phi} = (\det O)^{-1/2}$$

O

For constant fields

$$V_{eff}(\hat{\phi}) = V(\hat{\phi}) + \frac{1}{32\pi^2}\left(\frac{1}{2}m^4(\hat{\phi}) + \left(\frac{1}{6} - \xi\right) m^2(\hat{\phi})R(x)\right)\ln\left(\frac{m^2(\hat{\phi})}{\mu^2}\right) + \mathcal{O}(R^2)$$

Valid in a normal neighborhood

Effective potential in curved space-time

Comoving observers

$$ds^2 = a^2(\eta)\left\{[1 + 2\Phi(\eta, \mathbf{x})]\, d\eta^2 - [1 - 2\Psi(\eta, \mathbf{x})]\, d\mathbf{x}^2\right\}$$

- Since the background metric and fields are now space-time dependent, the effective potential is unsuitable to determine the Higgs VEV dynamics (we need to know the full effective action).

- However, for slowly varying background fields compared to the frequency of quantum fluctuations. i.e $\omega^2 \gg \mathcal{H}^2$ and $\omega^2 \gg \{\nabla^2\Phi,\ \nabla^2\Psi\}$ we can neglect the gradient terms and work with a quasi-potential. Sinha and Hu, 88

- In this case we can use the adiabatic (WKB) approximation for the quantum fluctuations modes

Equations in perturbed Robertson-Walker background

Spatially flat RW longitudinal gauge

$$ds^2 = a^2(\eta)\left\{[1+2\Phi(\eta,\mathbf{x})]\,d\eta^2 - [1-2\Psi(\eta,\mathbf{x})]\,d\mathbf{x}^2\right\}$$

$$\Box\,\phi + V'(\phi) = 0\,.$$

First-order equations

$$\phi'' + (2\mathcal{H} - \Phi' - 3\Psi')\phi' - (1 + 2(\Phi+\Psi))\nabla^2\phi$$
$$- \vec{\nabla}\phi\cdot\vec{\nabla}(\Phi-\Psi) + a^2(1+2\Phi)V'(\phi) = 0\,.$$

$$\phi(\eta,\mathbf{x}) = \hat{\phi}(\eta,\mathbf{x}) + \delta\phi(\eta,\mathbf{x})$$

Equations in perturbed Robertson-Walker background

VEV one-loop equation of motion

$$\hat{\phi}'' + (2\mathcal{H} - \Phi' - 3\Psi')\hat{\phi}' - (1 + 2(\Phi + \Psi))\nabla^2\hat{\phi}$$
$$- \vec{\nabla}\hat{\phi} \cdot \vec{\nabla}(\Phi - \Psi)$$
$$+ a^2(1 + 2\Phi)\underbrace{\left(V'(\hat{\phi}) + \frac{1}{2}V'''(\hat{\phi})\langle 0|\delta\phi^2|0\rangle\right)}_{V'_{eff}(\hat{\phi})} = 0\,.$$

$V'_{eff}(\hat{\phi}) = 0$

slowly varying back.

Quantum fluctuations equation

$$\delta\phi'' + (2\mathcal{H} - \Phi' - 3\Psi')\delta\phi' - (1 + 2(\Phi + \Psi))\nabla^2\delta\phi$$
$$- \vec{\nabla}\delta\phi \cdot \vec{\nabla}(\Phi - \Psi) + a^2(1 + 2\Phi)V''(\hat{\phi})\delta\phi = 0\,.$$

Quantization

Canonical quantization

$$\delta\phi(\eta,\mathbf{x}) = \int \mathrm{d}^3\mathbf{k}\left(a_{\mathbf{k}}\delta\phi_k(\eta,\mathbf{x}) + a^\dagger_{\mathbf{k}}\delta\phi^*_k(\eta,\mathbf{x})\right)$$

Scalar product

$$(\delta\phi_p,\delta\phi_q) = -i\int_\Sigma \left[\delta\phi_p(x)\partial_\mu\delta\phi^*_q(x) - (\partial_\mu\delta\phi_p(x))\delta\phi^*_q(x)\right]\sqrt{g_\Sigma}\,\mathrm{d}\Sigma^\mu$$

$$\mathrm{d}\Sigma^\mu = \mathrm{d}^3\mathbf{x}\left(\frac{1-\Phi}{a},0,0,0\right)$$

$$(\delta\phi_p,\delta\phi_q) = \delta^{(3)}(\mathbf{p}-\mathbf{q}) \quad \Rightarrow \quad [a_{\mathbf{p}},a^\dagger_{\mathbf{q}}] = \delta^{(3)}(\mathbf{p}-\mathbf{q})$$

$$a_{\mathbf{p}}|0\rangle = 0, \quad \forall\mathbf{p}$$

Adiabatic + perturbative expansion

Adiabatic expansion

$$\delta\phi_k(\eta, \mathbf{x}) = f_k(\eta, \mathbf{x})\, e^{i\theta_k(\eta, \mathbf{x})}$$

slowly varying amplitude. (f_k)

fast varying phase (θ_k)

- Leading adiabatic order $\mathcal{O}(\omega^2)$

$$-\theta_k'^2 + (\vec{\nabla}\theta_k)^2(1+2(\Phi+\Psi)) + m^2a^2(1+2\Phi) = 0\,.$$

- Next to leading adiabatic order $\mathcal{O}(\omega)$

$$2f_k'\theta_k' + f_k\theta_k'' + f_k\theta_k'(2\mathcal{H} - \Phi' - 3\Psi') - 2\vec{\nabla} f_k \cdot \vec{\nabla}\theta_k - f_k\nabla^2\theta_k = 0$$

Adiabatic + perturbative expansion

Perturbative expansion

$$\delta\phi_k(\eta,\mathbf{x}) = f_k(\eta,\mathbf{x})\, e^{i\theta_k(\eta,\mathbf{x})}$$

$$f_k(\eta,\mathbf{x}) = F_k(\eta) + \delta f_k(\eta,\mathbf{x})$$

$$\theta_k(\eta,\mathbf{x}) = -\int^{\eta} \omega(\eta')\,\mathrm{d}\eta' + \mathbf{k}\cdot\mathbf{x} + \delta\theta_k(\eta,\mathbf{x})$$

Lowest-order solution

$$\delta\phi_k^{(0)}(\eta,\mathbf{x}) = F_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x} - i\int^{\eta}\omega(\eta')\mathrm{d}\eta'}$$

$$F_k(\eta) = \frac{1}{a(2\pi)^{3/2}\sqrt{2\omega}}$$

$$\omega^2 = k^2 + m^2 a^2$$

Perturbative solutions

Quantum fluctuations

Metric perturbations

Defining

$$\alpha_k(\eta, \mathbf{p}) = \frac{\mathbf{k}\cdot\mathbf{p}}{\omega}$$

$$\beta_k(\eta, \mathbf{p}) = \int \alpha_k(\eta, \mathbf{p}) d\eta$$

$$G_k(\eta, \mathbf{p}) = -\omega\Phi(\eta, \mathbf{p}) - \frac{k^2}{\omega}\Psi(\eta, \mathbf{p})$$

Phase perturbation

$$\delta\theta_k(\eta, \mathbf{p}) = e^{-i\beta_k(\eta,\mathbf{p})} \int_0^\eta e^{i\beta_k(\eta',\mathbf{p})} G_k(\eta', \mathbf{p}) d\eta'$$

Amplitude perturbation

$$\delta f_k(\eta, \mathbf{p}) = F_k(\eta) P_k(\eta, \mathbf{p})$$

$$P_k(\eta, \mathbf{p}) = e^{-i\beta_k(\eta,\mathbf{p})} \int e^{i\beta_k(\eta,\mathbf{p})} \frac{H_k(\eta, \mathbf{p})}{2\omega} d\eta$$

$$H_k(\eta, \mathbf{p}) = \omega \left(-i\frac{\alpha_k(\eta, \mathbf{p})}{\omega}\delta\theta_k(\eta, \mathbf{p}) + \Psi(\eta, \mathbf{p})\left(3 - \frac{k^2}{\omega^2}\right)\right)' + \quad p^2\delta\theta_k(\eta, \mathbf{p})$$

Effective potential

$$V_1 = \frac{1}{2}\int dm^2 \langle 0|\delta\phi^2(\eta,\mathbf{x})|0\rangle$$

$$V_1 = \frac{1}{2}\int dm^2 \int \mathrm{d}^3\mathbf{k}(F_k^2(\eta) + 2F_k(\eta)\left(\mathrm{R}e\,\delta f_k(\eta,\mathbf{x}) + \mathrm{I}m\,\delta\theta(\eta,\mathbf{x})\right)$$
$$= V_1^h(\eta) + V_1^i(\eta,\mathbf{x})$$

Homogeneous

Inhomogeneous

Homogeneous contribution: renormalized potential

Three-momentum cutoff vs. dimensional regularization

$$V^h_{eff}(\hat{\phi}) = V(\hat{\phi}) + \frac{m^4(\hat{\phi})}{64\pi^2}\left(\ln\left(\frac{m^2(\hat{\phi})}{\mu^2_{ph}}\right) + C\right)$$

Dimensional regularization	μ_{ph} constant renormalization scale	$\omega_V = -1$ constant equation of state
Cutoff regularization	$\mu_{ph} = \frac{\mu}{a}$ evolving renormalization scale	$\omega_V = -1 - \frac{D}{\ln a}$ evolving equation of state

Inhomogeneous contribution

Matter dominated universe

Φ and Ψ time independent

Cutoff regularized potential

$$V^i_{eff}(\hat{\phi}) = \frac{m^2(\hat{\phi})\Lambda^2}{16\pi^2 a^2}\mathcal{F}\left[\frac{\sin(p\eta)}{p\eta}(\Phi+3\Psi)-(\Phi+\Psi)\right] + \frac{m^4(\hat{\phi})}{64\pi^2}\ln\left(\frac{m^2(\hat{\phi})a^2}{\Lambda^2}\right)\mathcal{R}(\eta,\mathbf{x}) + \mathcal{O}(\Lambda^{-2})$$

$$\mathcal{R}(\eta,\mathbf{x}) = \mathcal{F}\left[(\Phi(\mathbf{p})-\Psi(\mathbf{p}))\left(1-\frac{1}{5}\left(\cos(p\eta)+4\frac{\sin(p\eta)}{p\eta}\right)\right)\right]$$

Renormalized potential

$$V_{eff}(\hat{\phi}) = V(\hat{\phi}) + \frac{m^4(\hat{\phi})}{64\pi^2}\ln\left(\frac{m^2(\hat{\phi})}{\mu^2_{ph}}\right)(1+\mathcal{R}(\eta,\mathbf{x}))$$

Super-Hubble modes: no contribution to leading order

$p\eta \ll 1$ $\qquad$ $\mathcal{R}(\eta,\mathbf{x}) \simeq \mathcal{O}(p^2\eta^2)$

Renormalization

Space-time dependent
beta function

$$\beta(\lambda) \equiv \frac{d\lambda}{d(\log\mu)} = \frac{18\lambda^2}{(4\pi)^2}(1+\mathcal{R}(\eta,\mathbf{x}))$$

$$\gamma_M(\lambda) \equiv \frac{d\log M^2}{d(\log\mu)} = \frac{6\lambda}{(4\pi)^2}(1+\mathcal{R}(\eta,\mathbf{x}))$$

Inhomogeneous renormalization: space-time dependence of the renormalized mass and coupling constant (renormalization conditions)

Dimensional regularization not feasible in this case.
Regularization dependence?

Zero-point energy

Vacuum energy per field mode
homogeneous space-time
(leading adiabatic order)

$$E_{vac}(k) = \frac{1}{2}\hbar\omega$$

Vacuum energy per mode
perturbed space-time

$$E_{vac}(k) = \hbar\omega\left(\frac{1}{2} + \frac{3}{2}\Psi + \frac{1}{2}\nabla^2\int\left(\frac{1}{a}\int a\Phi\,\mathrm{d}\eta\right)\mathrm{d}\eta\right)$$

$k \ll m$

Albareti, Cembranos, A.L.M. arXiv:1404.5946

Higgs vacuum expectation value

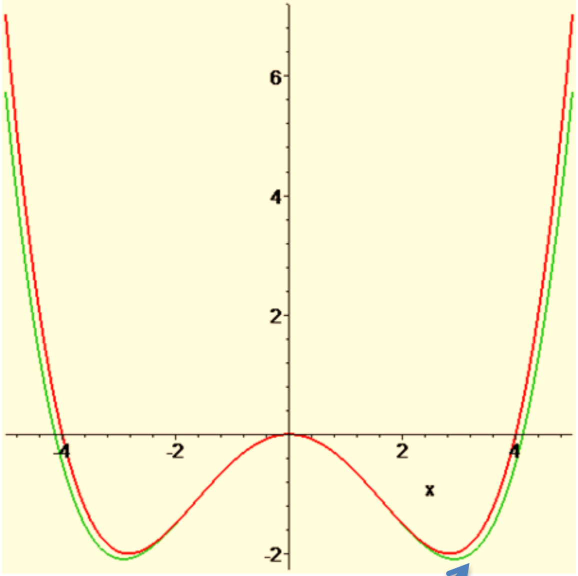
$$V_{eff}{}'(\hat{\phi}_{vac}) = 0$$

$$\hat{\phi}_{vac} = \hat{\phi}_0 + \Delta\hat{\phi}$$

Homogeneous VEV

$$V^h_{eff}{}'(\hat{\phi}_0) = 0$$

Perturbations contribution



Space-time dependent VEV

Assuming constant tree-level parameters and

$$\mu_{ph} = m_H$$

$$\frac{\Delta\hat{\phi}}{\hat{\phi}_0} = -\frac{3\lambda}{32\pi^2}\mathcal{R} = -\frac{3}{256\pi^2}\mathcal{R}$$

Higgs vacuum expectation value

Gravitational slip
(Eddington parameter)

$$\varpi = 1 - \eta = \frac{\Phi - \Psi}{\Phi}$$

Higgs VEV variation

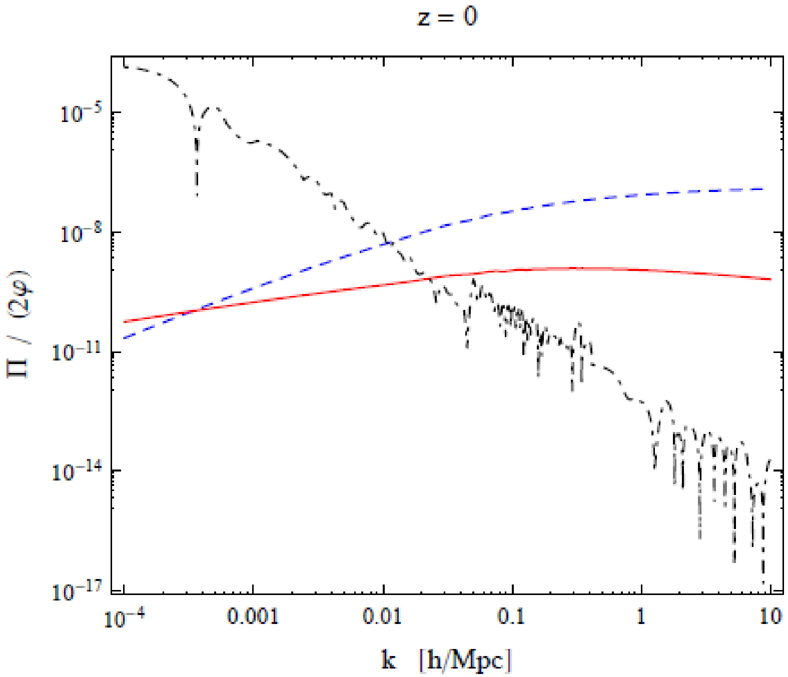
$$\frac{\Delta\phi}{\phi} \simeq 10^{-3}\Phi\varpi$$

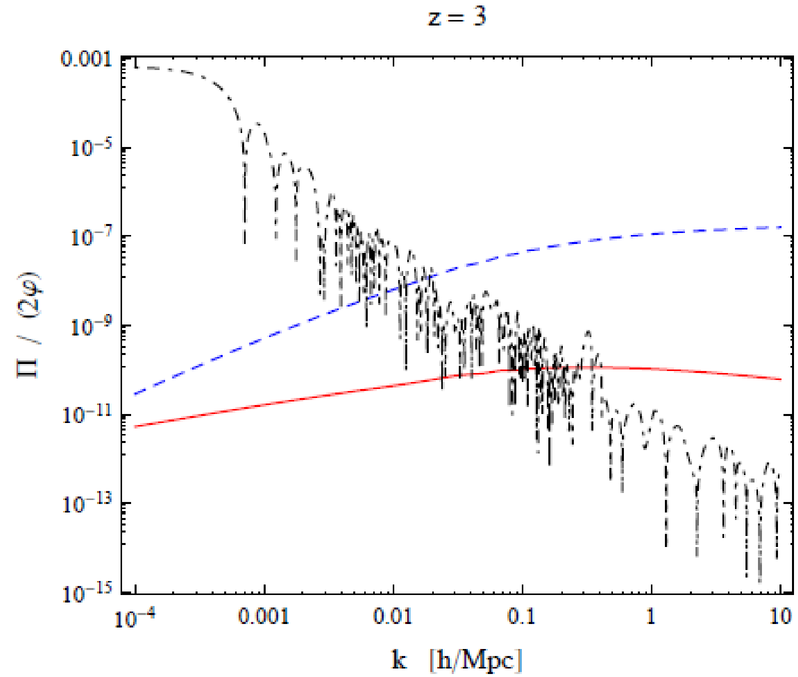
Gravitational slip is small in standard cosmology with contributions from neutrino diffusion and second order perturbations

Saltas, et al. arXiv:1406.7139

$$|\varpi| \lesssim 10^{-3}$$

Gravitational slip



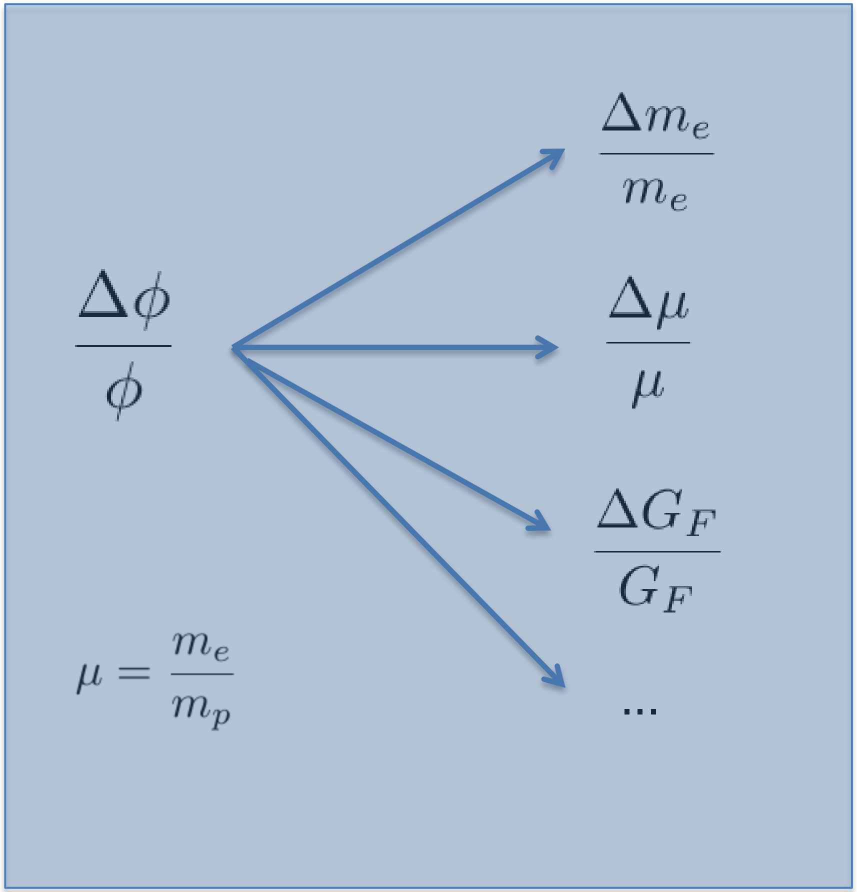


Ballesteros, et al. arXiv:1112.4837

$$\frac{\Delta\phi}{\phi} \simeq 10^{-11}$$

Cosmological scales

Phenomenological consequences



Astrophysical and cosmological constraints

- Quasar absorption spectra $\lesssim 10^{-6}$
- CMB $\lesssim 10^{-2}$
- BBN $\lesssim 10^{-3}$

Local constraints

- Atomic clocks on Earth $\lesssim 10^{-16}$
- Milky Way molecular clouds $\lesssim 10^{-8}$

Uzan, 2010

Conclusions

- We have calculated the Higgs one-loop effective potential in a perturbed Robertson-Walker background using the adiabatic vacuum of comoving observers

- We have obtained an inhomogeneous contribution to the effective potential and also to the running of the renormalized mass and coupling constant

- Quantum field theory does not predict the value of any physical parameter (they must be measured). However, the results suggest a space-time dependence of the Higgs mass and VEV.

- Variations in the Higgs VEV imply variations in the particle masses, which could reach $10^{-3} \varpi \Phi$. Potential signals or limits on gravitational slip.

- Improved atomic spectra data required. Other tests?