SM vacuum stability

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Outline

- Past and present informations on the Higgs boson
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- Vacuum stability in the SM, the role of the top
- Minimal extensions of the SM that can stabilize the scalar potential
- Conclusions

The past: LEP



$$Q = \frac{\mathcal{L}(s+b)}{\mathcal{L}(b)}$$

The past: LEP+ Tevatron

Combining direct and indirect information: D'Agostini, G.D.1999





courtesy of S. Di Vita

The consistency of the (minimal) SM at the quantum level predicts a Higgs boson with mass between 110 and 160 GeV

The present: LHC 4th of July 2012 news



Clear evidence of a new particle with properties compatible with those of the SM Higgs boson

The present: LHC Studying the properties of the new particle



 $M_h = 125.5 \pm 0.2(stat) {}^{+0.5}_{-0.6}(syst)$ GeV

 $M_h = 125.7 \pm 0.3(stat) \pm 0.3(syst)$ GeV

SM is constrained

At the time of LEP we could envisage specific type of NP that could allow a heavy Higgs in the EW fit ("conspiracy").

$$\begin{split} \hat{\rho} &= \rho_0 + \delta\rho \left(\rho_0^{\mathrm{SM}} = 1, \delta\rho \leftrightarrow (\epsilon_1, T) \right) \\ \Delta \hat{r}_w &\leftrightarrow (\epsilon_3, S) \end{split}$$

$$\begin{split} \sin^2 \theta_{eff}^{lept} &\sim & \frac{1}{2} \left\{ 1 - \left[1 - \frac{4A^2}{M_Z^2 \,\hat{\rho} \left(1 - \Delta \hat{r}_w \right)} \right]^{1/2} \right\} \\ &\sim & \left(\sin^2 \theta_{eff}^{lept} \right)^\circ + c_1 \ln \left(\frac{M_H}{M_H^\circ} \right) + c_2 \left[\frac{(\Delta \alpha)_h}{(\Delta \alpha)_h^\circ} - 1 \right] - c_3 \left[\left(\frac{M_t}{M_t^\circ} \right)^2 - 1 \right] + \dots \\ c_i &> 0 \end{split}$$

$$\begin{split} \mathbf{C}_i &> 0 \\ \mathsf{To} \text{ increase the fitted } \mathsf{M}_{\mathsf{H}} \colon \left\{ \begin{array}{c} \hat{\rho} > 1 \rightarrow \\ \Delta \hat{r}_W < 0 \end{array} \right\} \left\{ \begin{array}{c} \rho_0 > 1 & \longleftarrow \\ \delta \rho > 0 & \longleftarrow \\ \mathsf{Light sleptons} \end{aligned} \right\} \begin{array}{c} \mathsf{Extra Z} \\ \mathsf{Isosplitt} (\mathsf{s}) \mathsf{fermions}, \\ \mathsf{Multi Higgs models}, \end{split}$$

NP (if there) seems to be of the decoupling type



Ciuchini, Franco, Mishima, Silvestrini (13)

Vacuum Stability bound

Quantum corrections to the classical Higgs potential can modify its shape



If B were constant at large values of Φ the potential would become negative and unbounded. But B runs Various possibilities:

B is negative at the weak scale but not large enough to make B negative at a large scale such that the potential can become negative. SM vacuum is stable

B is very negative at the weak scale and stays negative till the Planck scale SM vacuum is unstable N.P. should appear below the Planck scale

to rescue our lives

B is sufficient negative at the weak scale that the potential will become negative at a certain scale. However, increasing more the scale B turns positive. The potential develops a second deeper minimum at a large scale

SM is unstable, but

Other case: B ~ 0, M_H large

$$V_{eff}^{1l} \sim \lambda(M)\phi^4 + \frac{3\lambda^2}{4\pi^2}\phi^4 \ln \frac{\phi^2}{M^2} \Rightarrow V_{eff}^{RGE} = \frac{\lambda\phi^4}{1 - \frac{3\lambda}{4\pi^2} \ln \frac{\phi^2}{M^2}}$$





Which values of the Higgs mass ensure vacuum stability and perburbativity up to the Planck scale ?

Given the initial values for the couplings obtained from the experimental results we look for: Vacuum stability $\rightarrow V_{_{eff}} = 0$ ($\sim \lambda = 0$)

Perturbativity \rightarrow when λ becomes large



 $M_{_{H}} \sim 125-126 \text{ GeV: -Y}_t^4 \text{ wins: } \lambda(M_t) \sim 0.14 \text{ runs towards smaller values and can}$ eventually become negative. If so the potential is either unbounded from below or can develop a second (deeper) minimun at large field values

Illustrative



If your mexican hat turns out to be a dog bowl you have a problem...

from A. Strumia

The problem

There is a transition probability between the false and true vacua



It is really a problem?

It is a problem that must be cured via the appearance of New Physics at a scale below that where the potential become unstable ONLY if the transition probability is smaller than the life of the universe.

Metastability condition: if λ becomes negative provided it remains small in absolute magnitude the SM vacuum is unstable but sufficiently long-lived compared to the age of the Universe

If our vacuum is only a local minimum of the potential, quantum tunneling towards the true minimum can happen. Bubbles of true vacuum can form in the false vacuum and possibly expand throughout the universe converting false vacuum to true. These bubbles are nothing but the solution of the e.o.m. that interpolate between the two vacua (bounces)

Coleman 79

Transition probability
$$p \sim \tau_U^4 \Lambda_B^4 e^{-S(\Lambda_B)}$$
 $S(\Lambda_B) = \frac{8\pi^2}{3|\lambda(\Lambda_B)|}$

 $S(\Lambda_{B})$ action of the bounce of size R = Λ_{B}^{-1}

Tunneling is dominated by the bounce of size R such that $\lambda(\Lambda_{_B})$ is minimized, i.e. $\beta_{_{\lambda}}(\Lambda_{_B}) = 0$.

$$p \sim (e^{140} \frac{\Lambda_B}{M_P})^4 e^{-\frac{2600}{|\lambda/0.01|}}$$

Caveat: unknown Planckian dynamics could affect the tunneling rate.

Branchina, Messina (13)

Vacuum stability analyses

Long history, back to the middle seventies

Linde (76); Weinberg (76); Cabibbo, Maiani, Parisi, Petronzio (79); Hung (79); Lindner (86); Sher(89)



Fig. 1. Bounds on the mass of the Higgs boson $(m_{\rm H})$ as a function of the top quark mass $(M_{\rm f})$ in the case of three generations. We have taken $\sin^2 \theta_{\rm W} \approx 0.2$. The dashed line and the full line represent the upper and the lower bound, respectively. The dotted line is the prediction of the massless theory. The curves end in correspondence to the upper bound on $M_{\rm f}$, eq. (4.2).

Cabibbo, Maiani, Parisi, Petronzio (79);

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NNLO

- Two-loop effective potential (complete) Ford, Jack, Jones 92,97; Martin (02)
- Three-loop beta functions

gauge	Mihaila, Salomon, Steinhauser (12)	
g ₃ (NNLO)	v. Ritbergen, Vermaseren, Larin (97); Czakon (05)	
Yukawa, Higgs	Chetyrkin, Zoller (12, 13,); Bednyakov et al. (13)	

- Two-loop threshold corrections at the weak scale
 - y_t: g₃ (NNLO) Chetyrkin, Steinhauser (00); Melnikov, v. Ritbergen (00)
 gauge x QCD Bezrukov, Kalmykov, Kniehl, Shaposhnikov (12)
 α_w² Buttazzo,Giardino, Giudice, Sala, Salvio, Strumia, G.D. (13) (new)
 λ: Yuk x QCD, Bezrukov et al. (12), Di Vita et al. (12)
 SM gaugeless Di Vita, Elias-Miro, Espinosa, Giudice, Isisodri, Strumia, G.D. (12)
 Full SM Buttazzo,Giardino, Giudice, Sala, Salvio, Strumia, G.D. (13) (new)

Dominant theory uncertainty on the Higgs mass value that ensures vacuum stability comes from the threshold corrections at the weak scale



NNLO Calculation





 λ never becomes too negative at M_{p_1} . Both λ and β_{λ} are very close to zero around M_{p_1}

 $\lambda(M_{Pl.}) = -0.0128 + 0.0010 \left(\frac{M_h - 125.66 \,\mathrm{GeV}}{0.34 \,\mathrm{GeV}}\right) - 0.0043 \left(\frac{M_t - 173.35 \,\mathrm{GeV}}{0.65 \,\mathrm{GeV}}\right) + 0.0018 \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right)$

All analyses agree with this result, no one is claiming stability for $M_t = 173.4 \text{ GeV} \rightarrow M_t^{\overline{MS}}(M_t) = 163.3 \text{ GeV} \quad \lambda < 0 \text{ at } \Lambda \sim 10^{10} \text{--} 10^{11} \text{ GeV}$ Stability requires $Y_t(M_t) = 0.927$, present value (including 3 loop QCD) $Y_t(M_t) = 0.937$ difference as large as the full (QCD + EW) two-loop contribution







Top pole vs. MC mass

Is the Tevatron -LHC number really the "pole" (what is?) mass? Monte Carlo are used to reconstruct the top pole mass form its decays products. Modeling of the event that contain jets, missing energy and initial state radiation is required

 $M_t = M_t^{MC} + \Delta, \qquad \delta M_t^{MC} = \pm 0.7 \text{ GeV}, \quad \Delta = ?$

 M_t^{MC} is interpreted as M_t within the intrinsic ambiguity in the definition of M_t^{MC} $\Delta \sim O(\Lambda_{_{QCD}}) \sim 250-500 \text{ MeV}$

Extraction of the top mass in hadron collisions with a precision below $O(\Gamma_{top}) \sim 1$ GeV is extremely challenging

Alternative:

 $M_t^{\overline{MS}}$ is free of renormalon ambiguity $M_t^{\overline{MS}}$ can be extracted from total production cross section $M_t^{\overline{MS}}(M_t) = 163.3 \pm 2.7 \,\text{GeV} \rightarrow M_t = 173.3 \pm 2.8 \,\text{GeV}$

Consistent with the standard value albeit with a larger error.

Alekhin, Djouadi, Moch, (12)

Caution

Fermion masses are parameters of the QCD Lagrangian, not of the EW one. The Yukawa (and gauge) couplings are the parameters of the EW Lagrangian. The vacuum is not a parameter of the EW Lagrangian.

 $\overline{\text{MS}}$ masses are gauge invariant objects in QCD, not in EW, Yukawas are A $\overline{\text{MS}}$ mass in the EW theory has not a unique definition (RGE is not unique). It depends upon the definition of the vacuum:

- Minimum of the tree-level potential
 - $\rightarrow M_t^{\overline{MS}}$ g.i. but large EW corrections in the relation pole- $\overline{\text{MS}}$ mass (~ M_t^4) But direct extraction of $M_t^{\overline{MS}}$ requires EW correction

- Minimum of the radiatively corrected potential
- → $M_t^{\overline{MS}}$ not g.i. (problem? \overline{MS} mass is not a physical quantity) no large EW corrections in the relation pole- \overline{MS} mass

RGEs are written in terms of \overline{MS} gauge, Yukawa and λ couplings not in terms of masses.

N.B. The top pole mass is the same object that enters in the EW fit

Jegerlehner, Kalmykov, Kniehl, (12)

Is M₁ ~ 171 GeV compatibile?

Indirect determination of M₊

Indirect determination of M_h



SM phase diagram



Type of error	Estimate of the error	Impact on M_h	
M_t	experimental uncertainty in M_t	$\pm 1.4 \mathrm{GeV}$	-
$lpha_{f s}$	experimental uncertainty in $\alpha_{ m s}$	$\pm 0.5~{ m GeV}$	
Experiment	Total combined in quadrature	$\pm 1.5 {\rm GeV}$	
λ	scale variation in λ	0.7 GeV	-
h_t	${\cal O}(\Lambda_{ m QCD})$ correction to M_t	$\pm 0.6~{ m GeV}$	
h_t	QCD threshold at 4 loops	$\pm 0.3~{ m GeV}$	
RGE	EW at $3 \text{ loops} + \text{QCD}$ at 4 loops	$\pm 0.2 \text{ GeV}$	
Theory	Total combined in quadrature	$\pm 1.0 \text{ GeV}$	±0.7 GeV



Alekhin, Djouadi, Moch, 12



 $\lambda(M_{_{Pl}})$ and $y_t(M_{_{Pl}})$ almost at the minimum of the funnel An accident or deep meaning?

The mass term in the Higgs potential



 m^2 renormalizes multiplicative It stays basically flat till M_{pl}

No jumps because no new particle thresholds

 $\begin{aligned} & \text{Veltman's condition (Poles at d = 4-2/L)} \\ & M_h^2 = M_0^2 + \frac{3\Lambda^2}{16\pi^2 v^2} (\underbrace{2M_W^2 + M_Z^2 + M_h^2 - 4M_t^2}_{=0}) + \dots \end{aligned} \\ & \underbrace{ \begin{pmatrix} 9/4g^2(\mu) + 3/4g'^2(\mu) + 6\lambda(\mu) - 6Y_t^2(\mu) \end{pmatrix} \Lambda_{NP}^2 = 0 \\ \\ & 9/4g^2(\mu) + 3/4g'^2(\mu) + 6\lambda(\mu) - 6Y_t^2(\mu) = 0 \\ \\ & \mu < \mathsf{M}_{\mathsf{pl}} \to \mathsf{M}_\mathsf{t} < 170 \text{ GeV} \end{aligned}$

Stabilising the electroweak vacuum

Simplest model: SM + a complex singlet scalar

$$V_0(H,S) = m^2 |H|^2 + \lambda |H|^4 + \lambda_{SH} |H|^2 |S|^2 + m_S^2 |S|^2 + \lambda_S |S|^4$$

$$\swarrow portal$$

Effect of the portal is to increase the vacuum stability adding a positive contribution to β_{λ}

 $\beta_{\lambda} = \beta_{\lambda}^{SM} + 2\lambda_{SH}^2$

Many models that differ by: i) m_s obtained via a vev? ii) mass scale of S roughly the same as the H or much larger?





Elias-Miro', Espinosa, Giudice, Lee, Strumia 12



 $m_h = 125 \text{ GeV}, M_t = 173.2 \text{ GeV}$

 $m_s \sim v$

S mixes with H: rate $H \rightarrow$ diboson smaller than in the SM

- If $\lambda_{_{SH}}$ <0 the vev of S can generate the negative mass term needed for EWSB via the portal.
- Scale invariant model (m=0, m_s =0) can be constructed.

Ex:

 $V_0(H,S) = \lambda |H|^4 + \lambda_{SH} |H|^2 |S|^2 + \lambda_S |S|^4 + \lambda'_S (S^4 + (S^{\dagger})^4) + \lambda''_S |S|^2 (S^2 + (S^{\dagger})^2) + \lambda'_{SH} |H|^2 (S^2 + (S^{\dagger})^2)$

Ishiwata, (12); Frazinnia, He, Ren (13); Gabrielli et al (13)

The vev of S can be generate radiatively a la Coleman-Weinberg, which then causes EWSB. We obtain a relatively light CP-even boson, η (pseudo Goldstone boson of scale symmetry) that mixes with the 125 GeV Higgs and a heavier CP-odd boson, χ , that can be interpreted as a dark matter candidate. Via the vev of S a Majorana mass term for the neutrino can be constructed.

But

- Scalar couplings have the tendency to grow towards a Landau Pole
- η is light (300-500 GeV) in the LHC run 1 range
- Mixing of H with η is experimentally constrained (especially by ATLAS)

Vacuum stability and SUSY

MSSM variant: High-Scale Supersymmetry: All SUSY particle with mass m

Split SUSY:

Susy fermions at the weak scale Susy scalars with mass \widetilde{m}

(m: Supersymmetry breaking scale)

$\lambda(\mu)$ as a result of a matching with a high-scale theory

Predicted range for the Higgs mass

$$\lambda(\tilde{m}) = \frac{1}{8} \left[g^2(\tilde{m}) + g'^2(\tilde{m}) \right] \cos^2 2\beta$$



Supersymmetry broken at very large scale is disfavored

Conclusions

- SM is quite OK
- M_{h} -125/6 GeV is a very intriguing value.
- The SM potential is at the "border" of the stability region. The exact value of the top mass plays the central role between the full stability or metastability (preferred) options.
- All the analyses based on λ > 0 up to M_{pl} are assuming $M_t \sim 171$ GeV, a value not preferred by the EW fit
- Model-independent conclusion about the scale of NP cannot be derived.
 λ is small at high energy: NP (if exists) should have a *weakly interacting* Higgs particle
- λ and β_{λ} are very close to zero around the Planck mass: deep meaning or coincidence?
- Minimal extensions of the SM can stabilize the potential