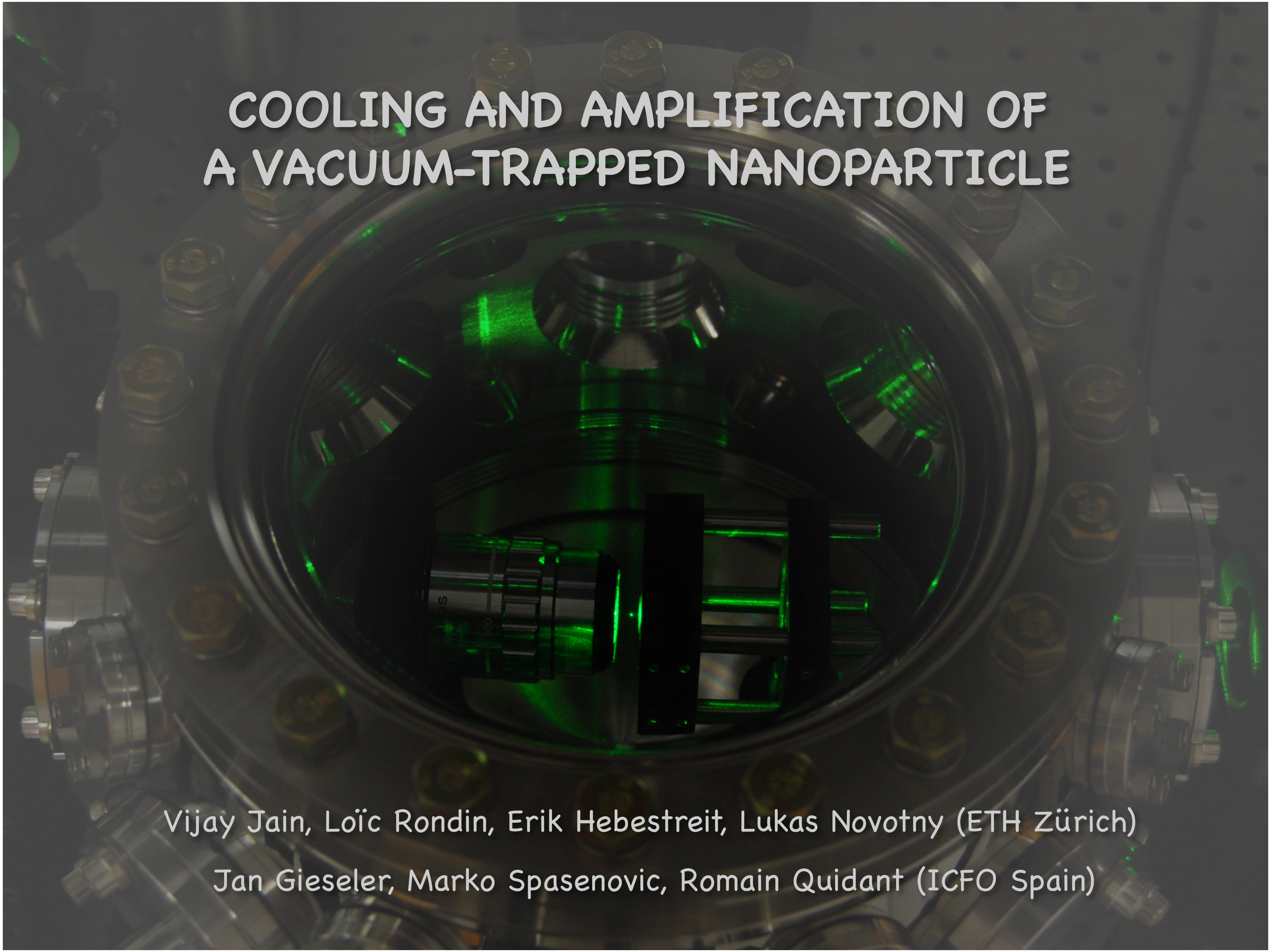
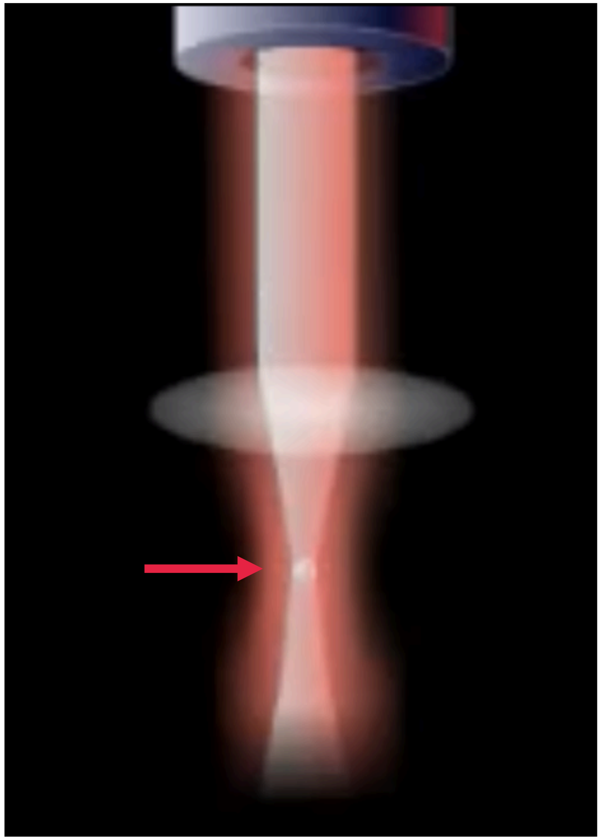


COOLING AND AMPLIFICATION OF A VACUUM-TRAPPED NANOPARTICLE

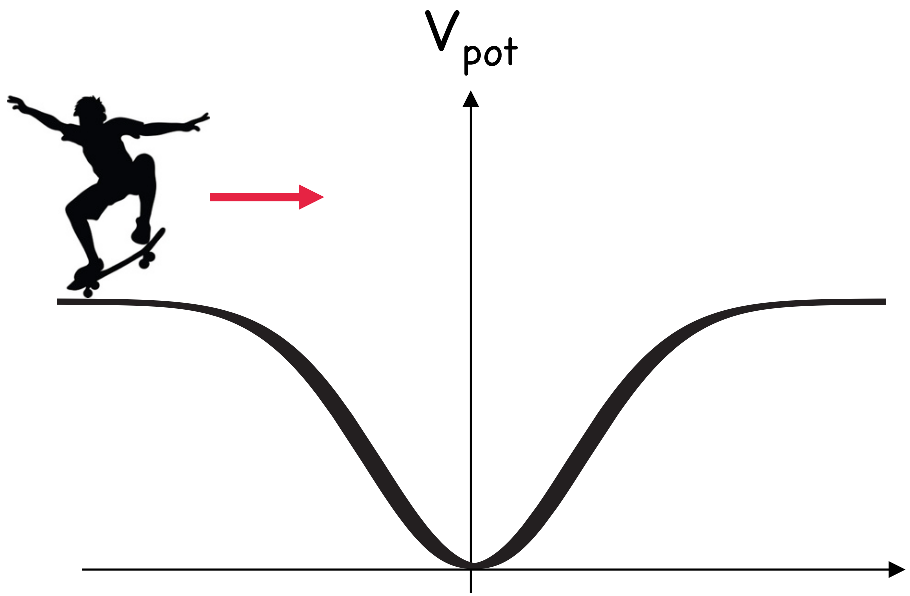
Vijay Jain, Loïc Rondin, Erik Hebestreit, Lukas Novotny (ETH Zürich)

Jan Gieseler, Marko Spasenovic, Romain Quidant (ICFO Spain)

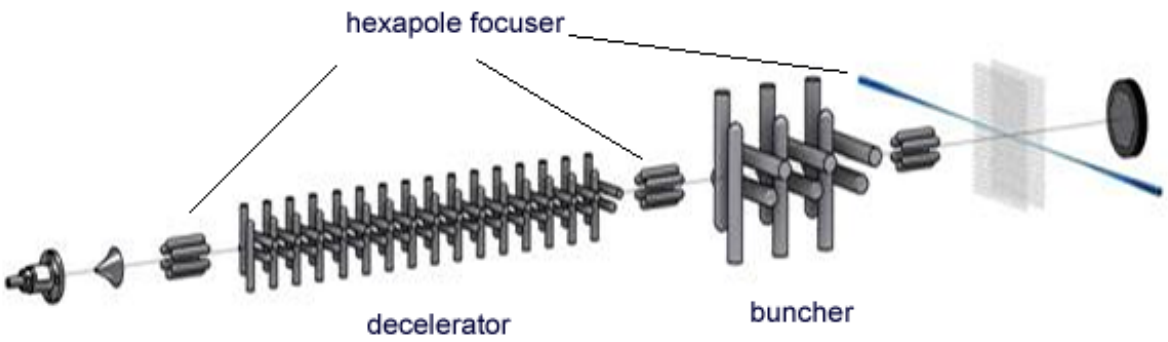
TRAPPING



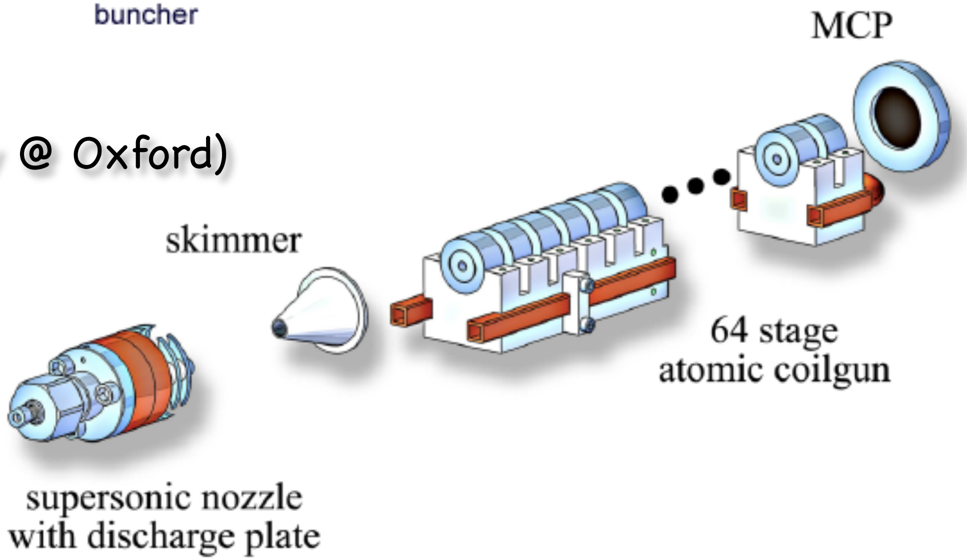
$$\langle \mathbf{F} \rangle = -(\alpha'/2)\, \nabla \langle |\mathbf{E}|^2 \rangle$$



DECELERATION



Stark decelerator (Tim Softley @ Oxford)

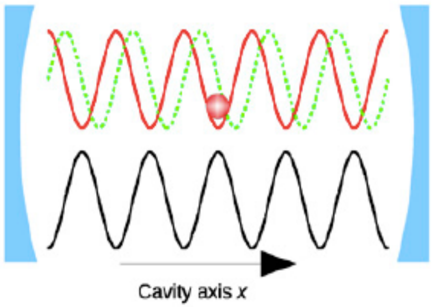


Atomic coilgun (Mark Raizen @ UT Austin,
Frédéric Merkt @ ETH)

Cavity opto-mechanics using an optically levitated nanosphere

D. E. Chang[a], C. A. Regal[b], S. B. Papp[b], D. J. Wilson[b], J. Ye[b,c], O. Painter[d], H. J. Kimble[b,1], and P. Zoller[b,e]

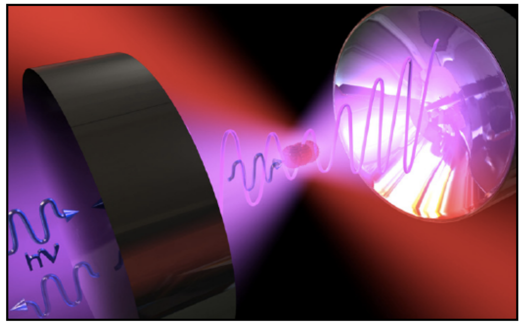
PNAS | January 19, 2010 | vol. 107 | no. 3 | 1005–1010



Toward quantum superposition of living organisms

Oriol Romero-Isart[1,4], Mathieu L Juan[2], Romain Quidant[2,3] and J Ignacio Cirac[1]

New Journal of Physics **12** (2010) 033015



PRL **105,** 173003 (2010) PHYSICAL REVIEW LETTERS week ending 22 OCTOBER 2010

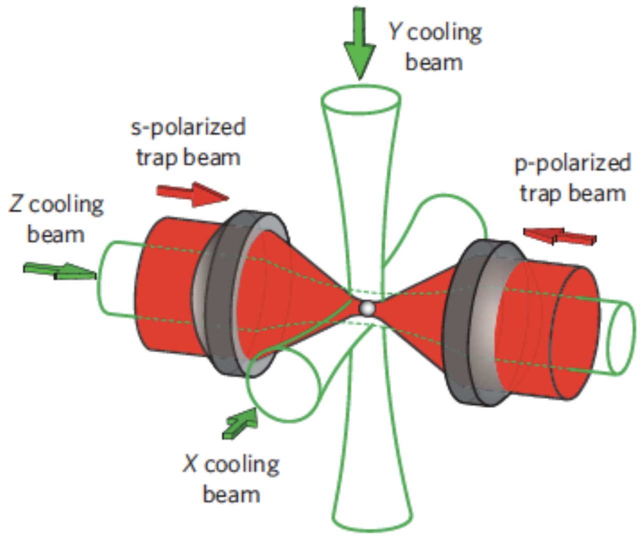
Feedback Cooling of a Single Neutral Atom

Markus Koch,[1,*] Christian Sames,[1] Alexander Kubanek,[1] Matthias Apel,[1] Maximilian Balbach,[1] Alexei Ourjoumtsev,[1,†] Pepijn W. H. Pinkse,[2] and Gerhard Rempe[1]

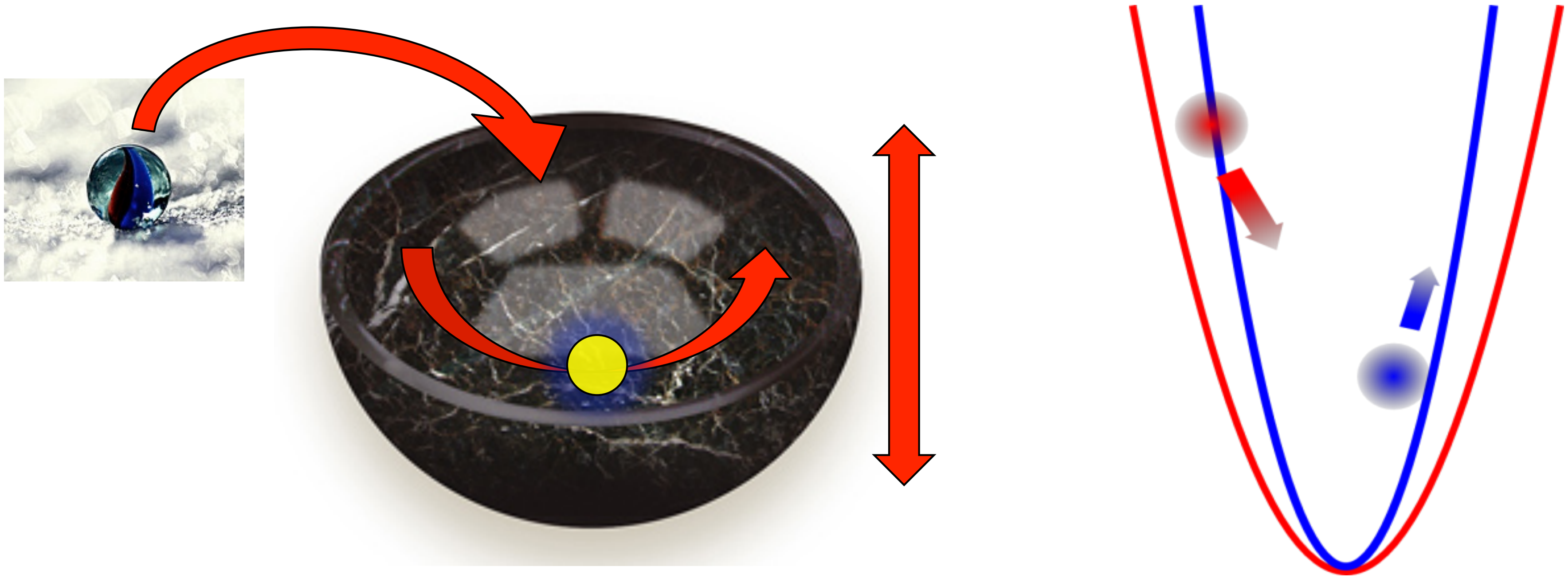
nature physics LETTERS
PUBLISHED ONLINE: 20 MARCH 2011 | DOI: 10.1038/NPHYS1952

Millikelvin cooling of an optically trapped microsphere in vacuum

Tongcang Li, Simon Kheifets and Mark G. Raizen*



PARAMETRIC COOLING



www.nano-optics.org

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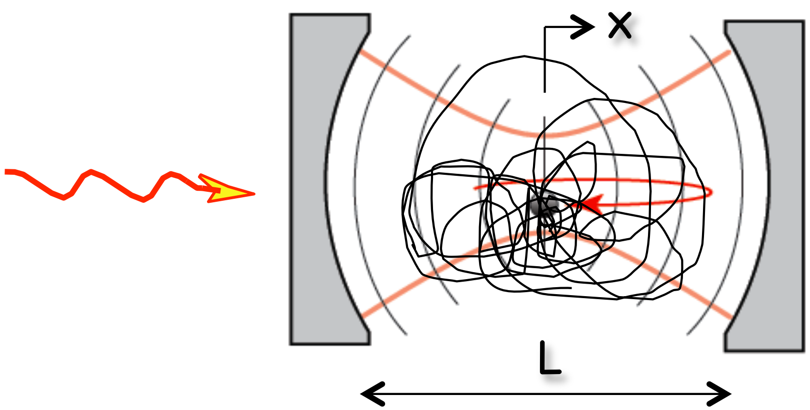
CAVITY OPTOMECHANICS



particle : Ω_0

cavity : ω_0

$$H_{\rm tot} = H_{\rm cavity} + H_{\rm mech} + H_{\rm int}$$

$$= \hbar\omega_0 [a^\dagger a + 1/2] + (1/2)\,[p^2/m + m\,\Omega_0^2 q^2] + \hbar g_0 q [a^\dagger a + 1/2]$$

$$= \hbar\omega_0 [a^\dagger a + 1/2] + \hbar\Omega_0 [b^\dagger b + 1/2] + \hbar g_0\, x_0 [b^\dagger + b][a^\dagger a + 1/2]$$

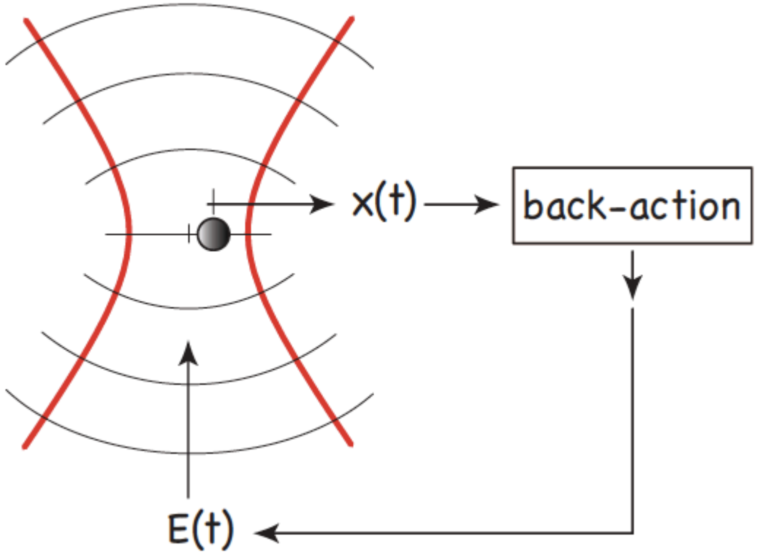
x_0: $\sqrt{\hbar / (2m\Omega_0)}$ zero-point oscillation amplitude

$[a^\dagger a + 1/2]$ → sidebands

$g_0 x_0 \longrightarrow$ single-photon coupling strength

Principles of Nano-Optics, 2nd ed. (Cambridge University Press 2012)

PROBLEM STATEMENT



$$\langle \mathbf{F}_{\mathrm{fluct}}(t)\,\mathbf{F}_{\mathrm{fluct}}(t')\rangle = 2m\Gamma_0\,k_B T\,\overleftrightarrow{\mathrm{I}}\,\delta(t-t')$$

$$\ddot{\mathbf{r}}(t) + \Gamma_0\,\dot{\mathbf{r}}(t) - \frac{\alpha}{2}\nabla E^2(\mathbf{r}) = \frac{1}{m}\left[\mathbf{F}_{\mathrm{fluct}}(t) + \mathbf{F}_{\mathrm{opt}}(t)\right]$$

$$\Gamma_0 = 4\pi R^2\, P_{\mathrm{gas}} \,/\, (m\, v_{\mathrm{gas}})$$

focal field

$$\oint F_{\mathrm{opt}}(x)\,dx \neq 0$$

$$\alpha = 4\pi\varepsilon_0 R^3[\varepsilon - 1]/[\varepsilon + 2]$$

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SMALL OSCILLATION AMPLITUDES

$$\mathbf{F}_{\text{trap}} = -\frac{\alpha}{2}\nabla E^2(\mathbf{r}) \approx \begin{bmatrix} k_{\text{trap}}^{(x)}\, x \\ k_{\text{trap}}^{(y)}\, y \\ k_{\text{trap}}^{(z)}\, z \end{bmatrix}$$

trap stiffness

$$\ddot{x}(t) + \Gamma_0\,\dot{x}(t) + \Omega_0^2\, x(t) = (1/m)\left[F_{\text{fluct}}(t) + F_{\text{opt}}(t)\right]$$

$$\Omega_0 = \sqrt{k_{\text{trap}}/m}.$$

$$k_{\text{trap}} = 4\pi^3\,\frac{\alpha P}{c\varepsilon_0}\,\frac{(\text{NA})^4}{\lambda^4}$$

EXPERIMENTAL PARAMETERS

$P = 50$ mW

$NA = 0.8$

$\lambda = 1064$ nm

$B = 300$ kHz

$R = 73$ nm

$n = 1.45 + i\, 8\; 10^{-9}$

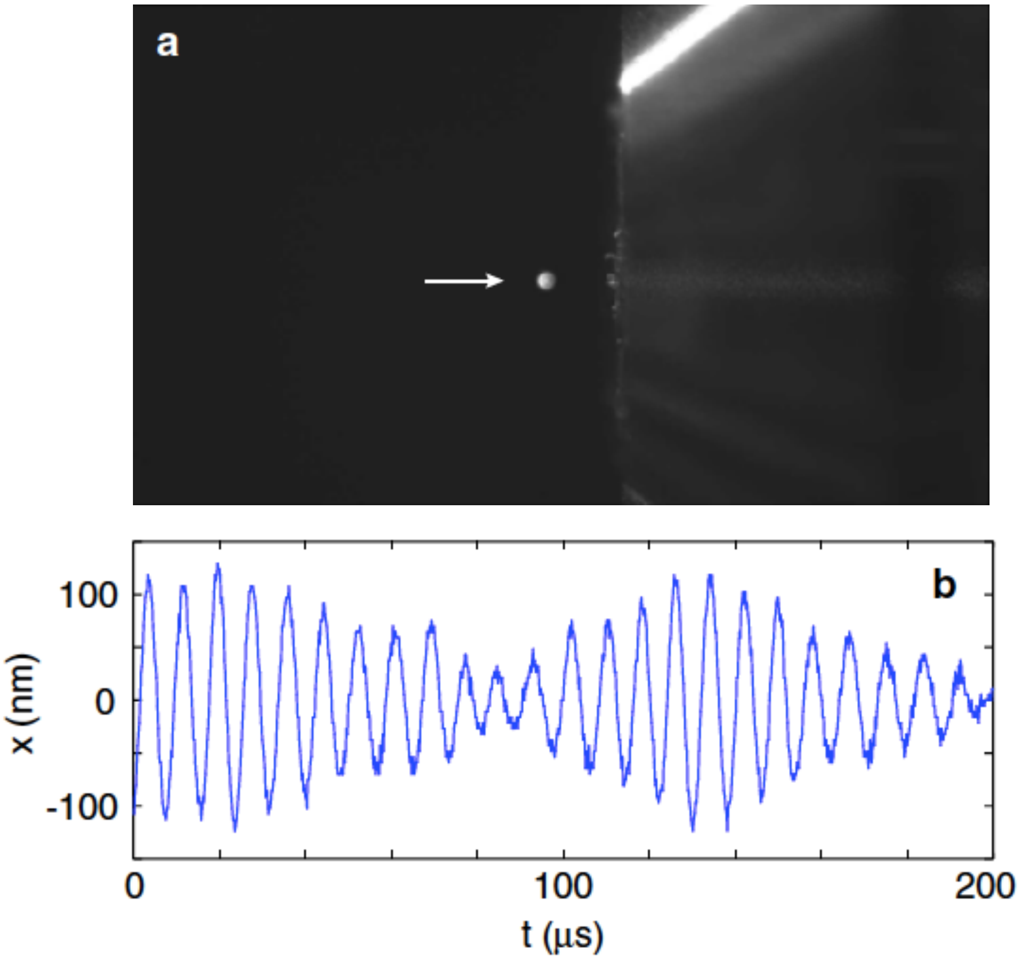
$m = 4\; 10^{-18}$ kg

$\sigma_{scatt} = (10\,\text{nm})^2$

$k_{trap} = 10\ \mu\text{N/m}$

$\Omega_0 = 2\pi\ 125$ kHz

MEASURING THE TRAP STIFFNESS



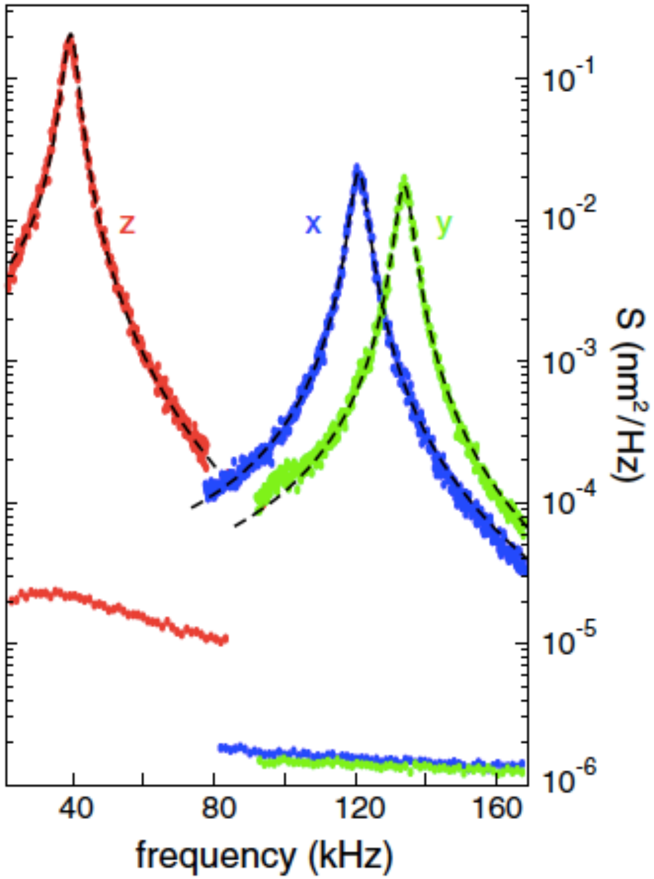
$$\Omega_0 = \sqrt{k_{\text{trap}}/m}.$$

POWER SPECTRAL DENSITY

$$\ddot{x}(t) + \Gamma_0 \dot{x}(t) + \Omega_0^2 x(t) = (1/m)\,[F_{\mathrm{fluct}}(t) + \underbrace{F_{\mathrm{opt}}(t)}_{=0}]$$

$$S_x(\Omega) = \int_{-\infty}^{\infty} \langle x(t)x(t-t')\rangle e^{-i\Omega t'} dt' = \frac{k_B T}{\pi m} \frac{\Gamma_0}{(\Omega_0^2 - \Omega^2)^2 + \Omega^2 \Gamma_0^2}$$

$\sim 1.2\ \mathrm{pm}/\sqrt{\mathrm{Hz}}$



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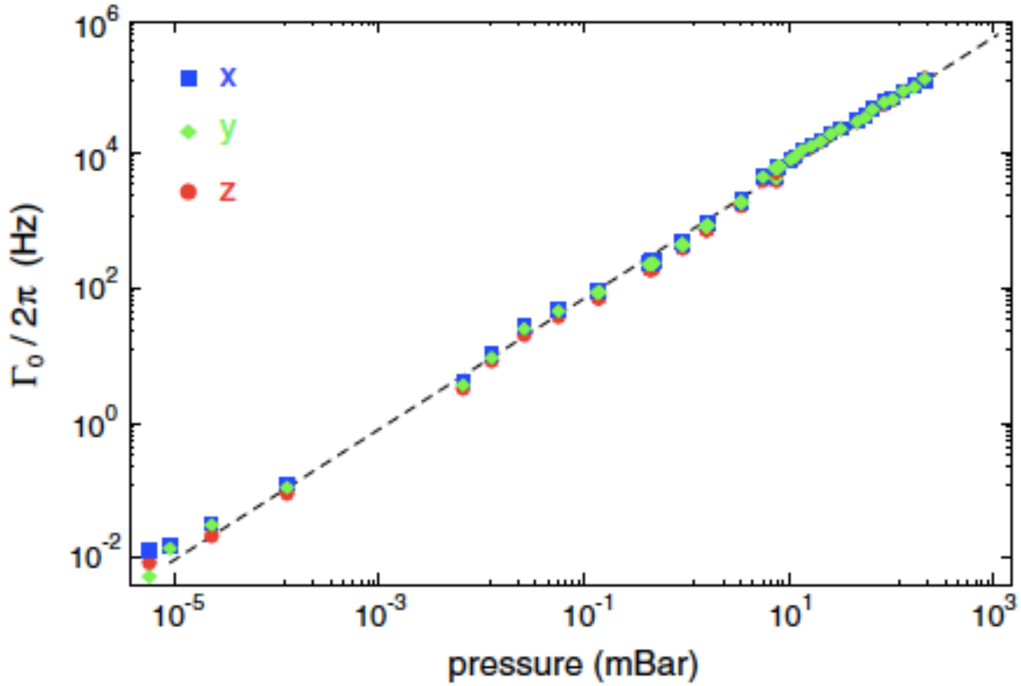
QUALITY FACTOR



$$P_{\text{gas}} = 10^{-5}\,\text{mbar} \longrightarrow \Gamma_0/2\pi = 10\ \text{mHz} \longrightarrow Q = 10^7$$

$$P_{\text{gas}} = 10^{-9}\,\text{mbar} \longrightarrow \boxed{Q \sim 10^{11}}$$

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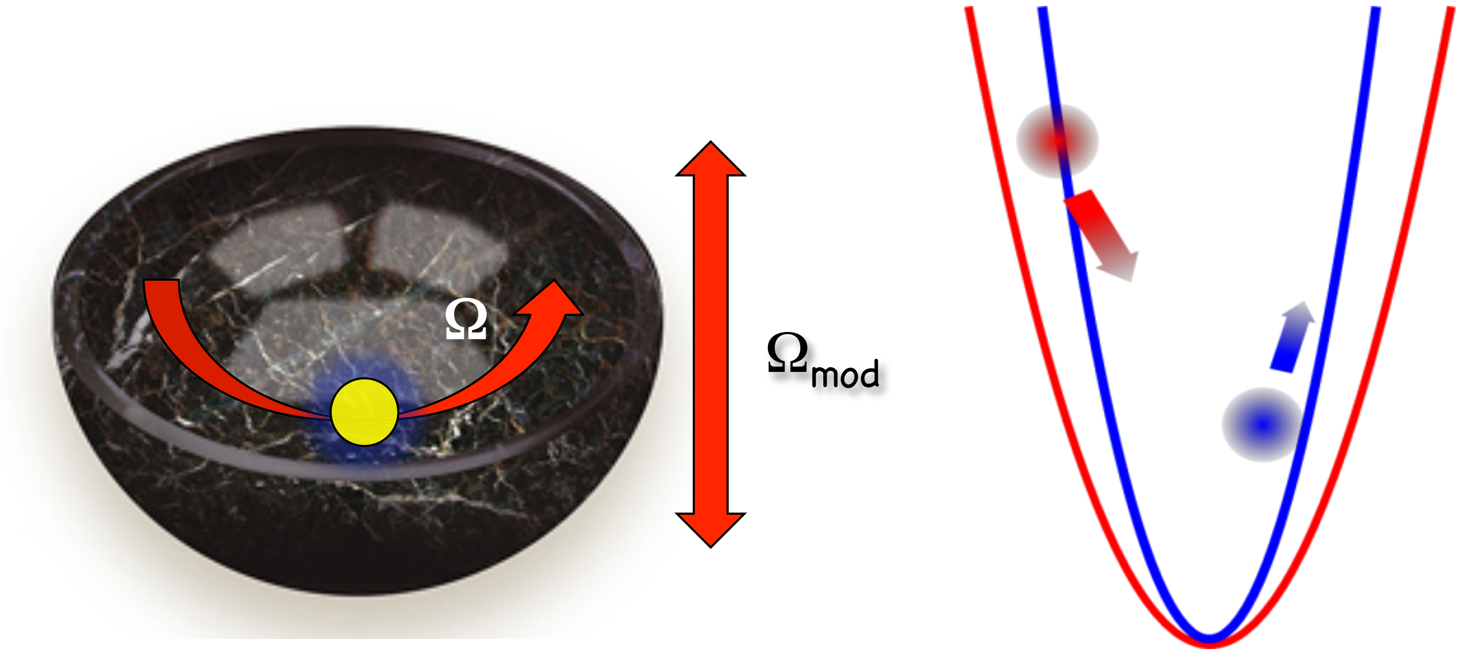
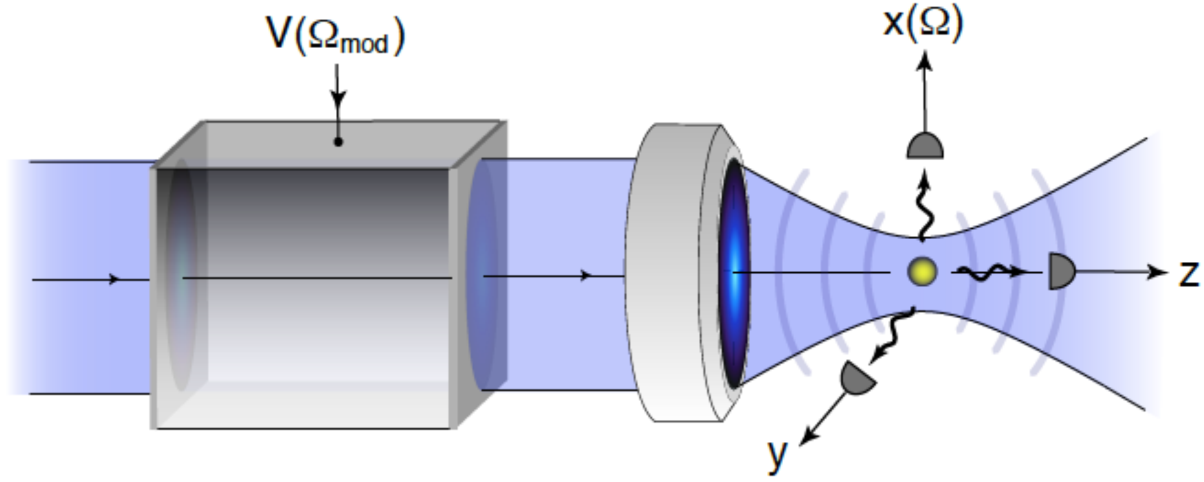
AMPLIFICATION



AMPLIFICATION

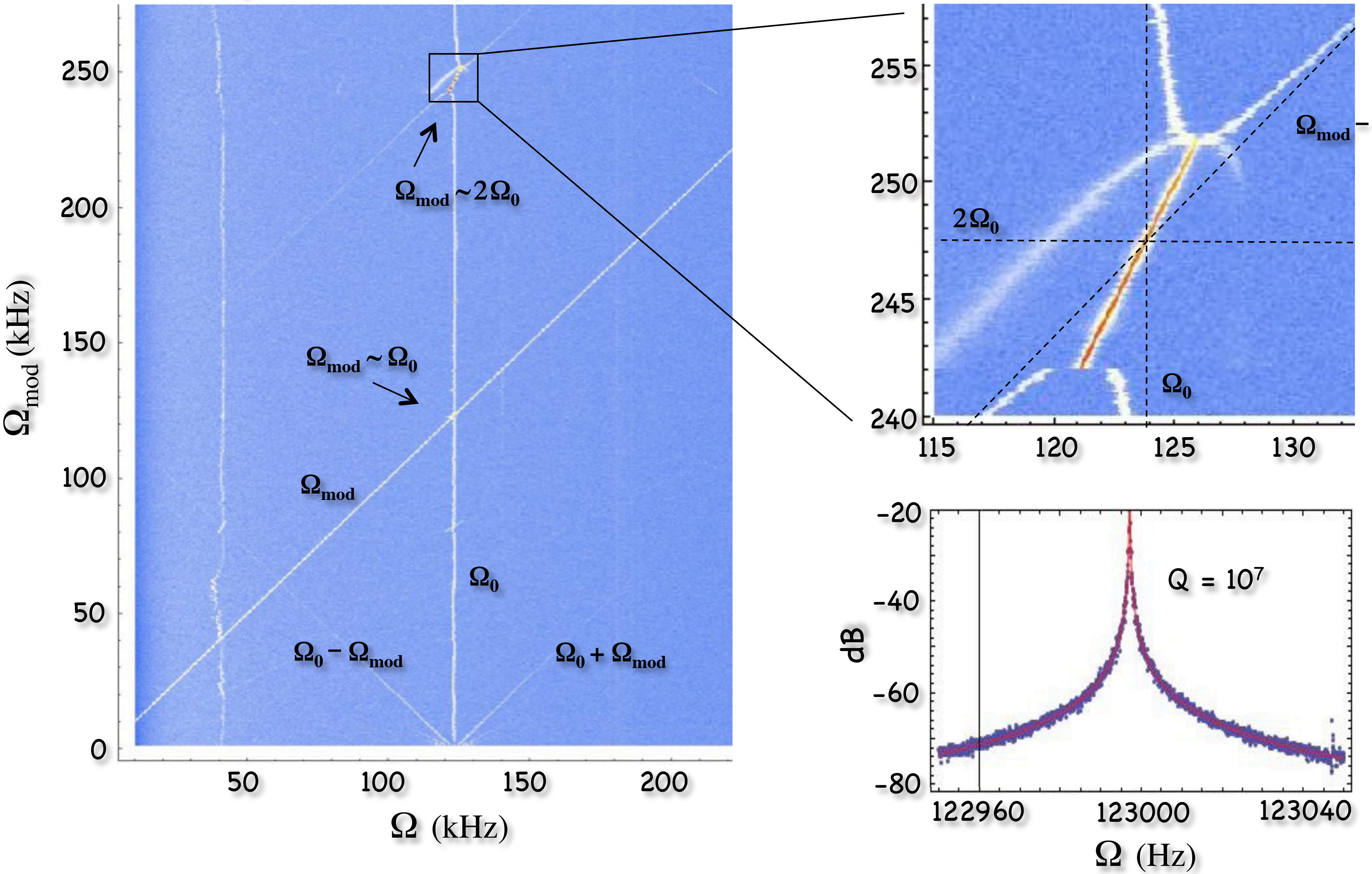


$$\ddot{x}(t) + \Gamma_0\,\dot{x}(t) + \Omega_0^2\,x(t) = (1/m)\,[F_{\text{fluct}}(t) + F_{\text{opt}}(t)]$$

$$F_{\text{opt}}(t) = \overbrace{\Delta k_{\text{trap}}(t)\,x(t)}^{\textbf{nonlinear!}}$$

$$k_0 \sin(\Omega_{\text{mod}} t) \qquad x_0 \sin(\Omega_0 t)$$

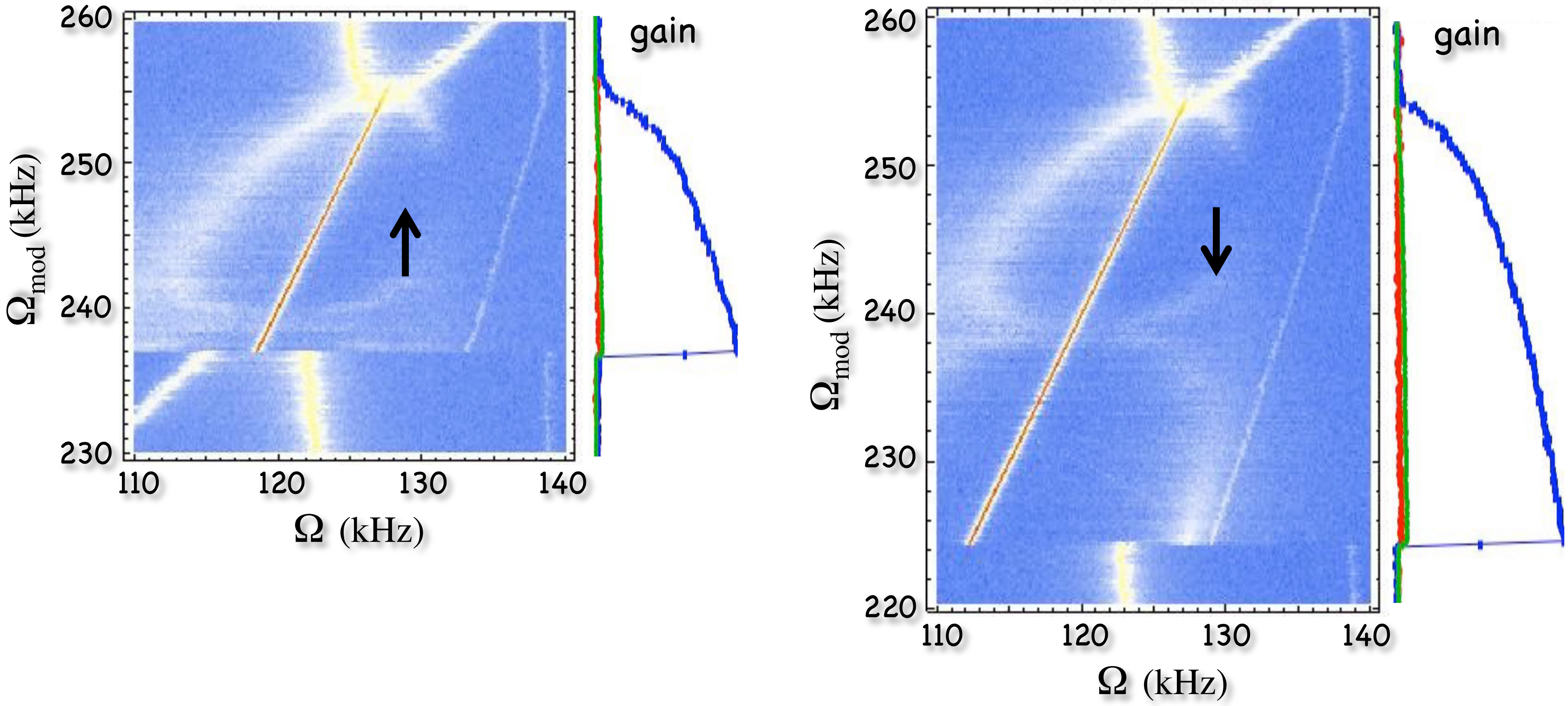
Power spectral density (x) :



Modulation: 0.1V / Pressure: 3 10^{-3} mBar

PRL, in print (2014)

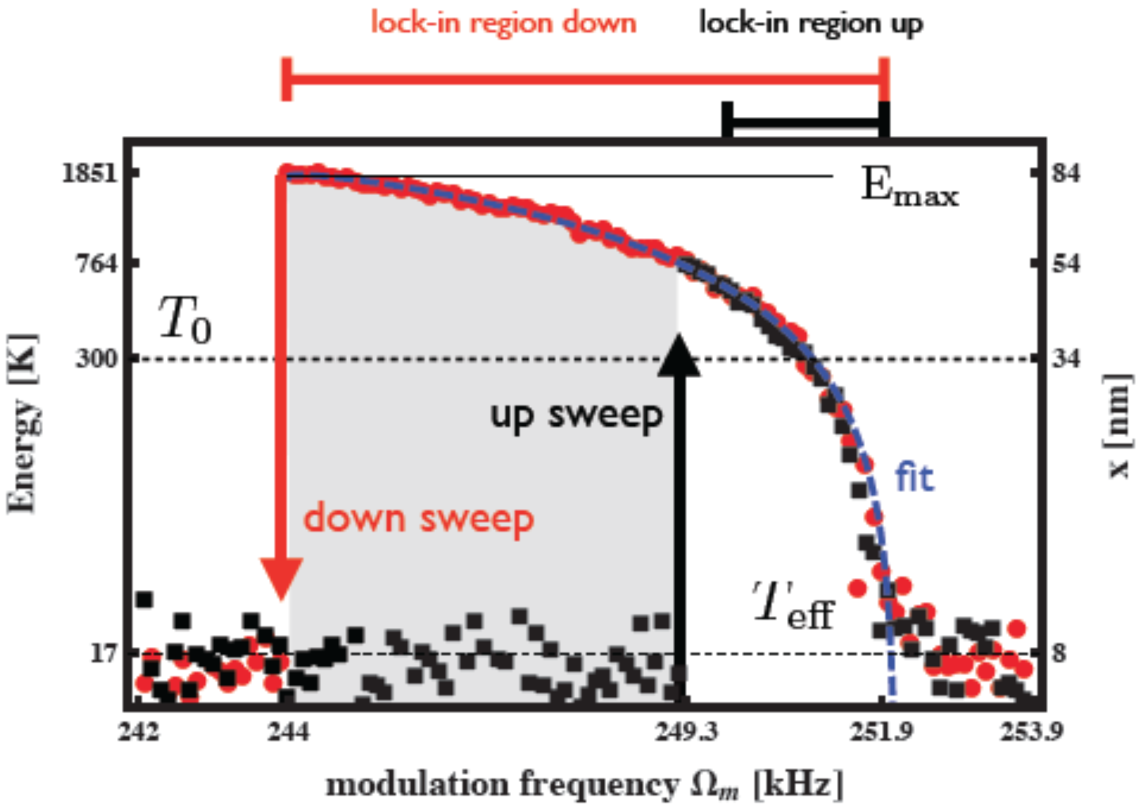
BISTABILITY / HYSTERESIS



Modulation: 0.1V / Pressure: 3 10^{-3} mBar

PRL, in print (2014)

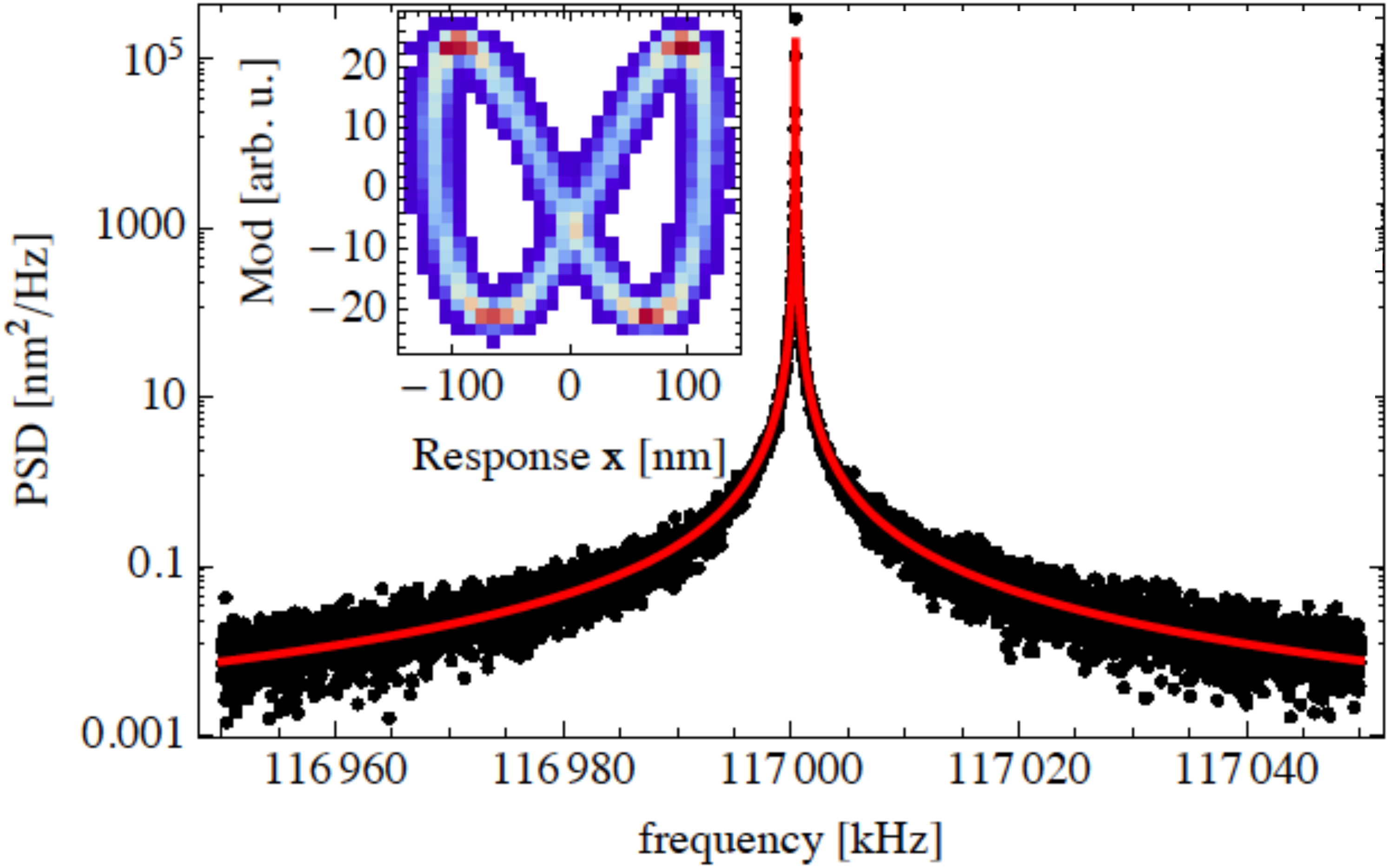
BISTABILITY / HYSTERESIS



Modulation: 0.1V / Pressure: 3 10^{-3} mBar

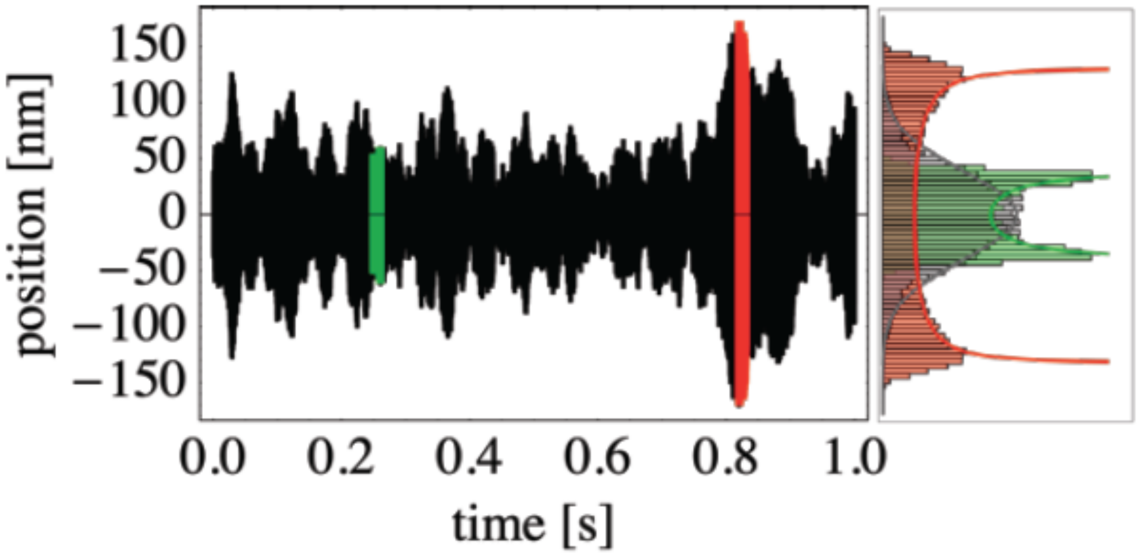
PRL, in print (2014)

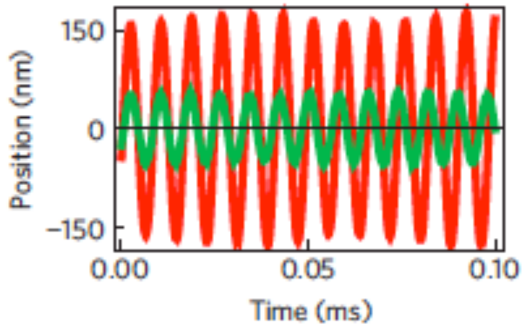
OSCILLATION IS PHASE-LOCKED

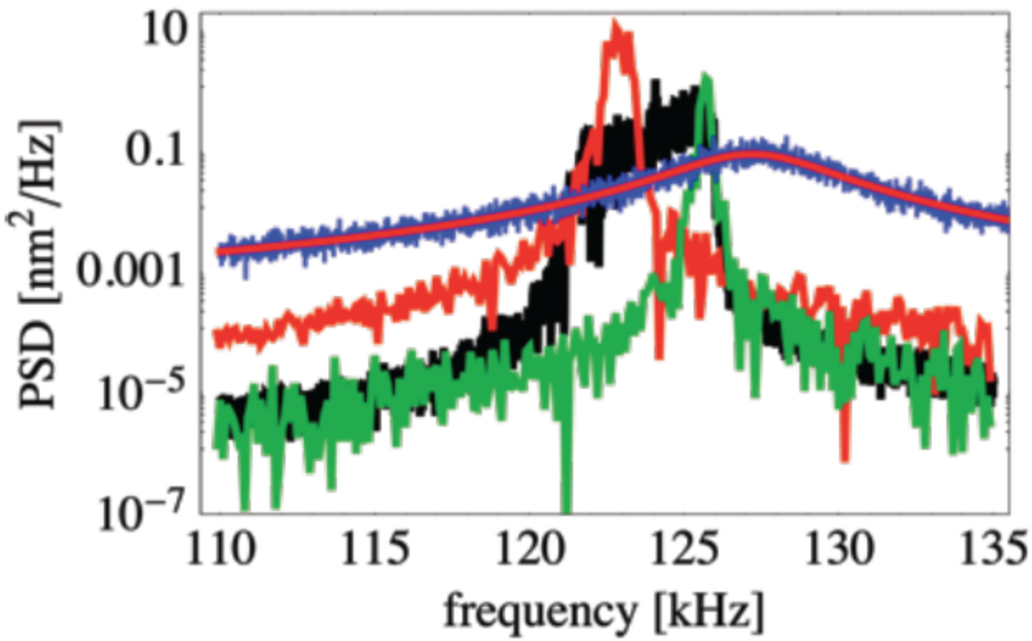


PRL, in print (2014)

NONLINEAR FREQUENCY FLUCTUATIONS



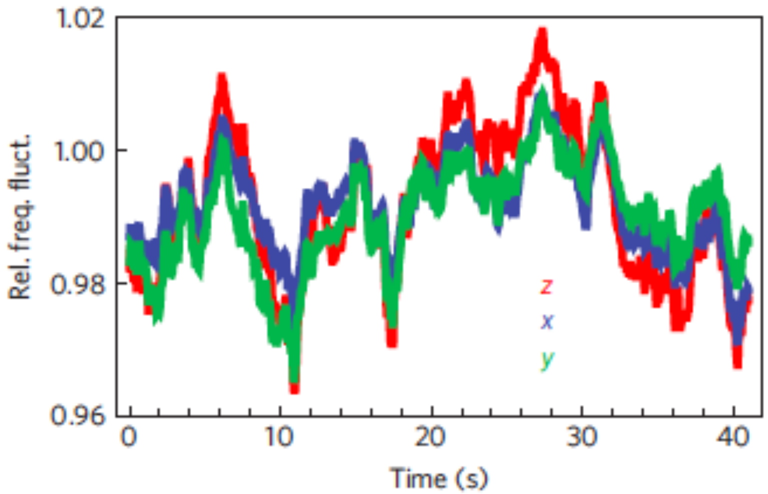




Nonlinear frequency fluctuations add frequency noise, but do not affect damping !

Nature Phys. **9**, 806 (2013)

NONLINEAR FREQUENCY FLUCTUATIONS



$$\ddot{x}_i + \Omega_i Q_i^{-1} \dot{x}_i + \Omega_i^2 \left(1 + \sum_{j=x,y,z} \xi_j x_j^2\right) x_i = \mathcal{F}_{\text{fluct}}/m$$

$$\Delta\Omega_{\text{L}} = \Omega_0 Q^{-1}$$

$$\Delta\Omega_{\text{NL}} = \frac{3}{8}\xi\,\Omega_0 r_{\text{th}}^2 \quad \longrightarrow \quad r_{\text{th}} = \sqrt{2k_{\text{B}}T/m\Omega_0^2}$$

$$\mathcal{R} = \frac{\Delta\Omega_{\text{NL}}}{\Delta\Omega_{\text{L}}} = \frac{3\xi Q k_{\text{B}} T}{4\Omega_0^2 m} \longrightarrow \text{QT}$$

Nature Phys. **9**, 806 (2013)

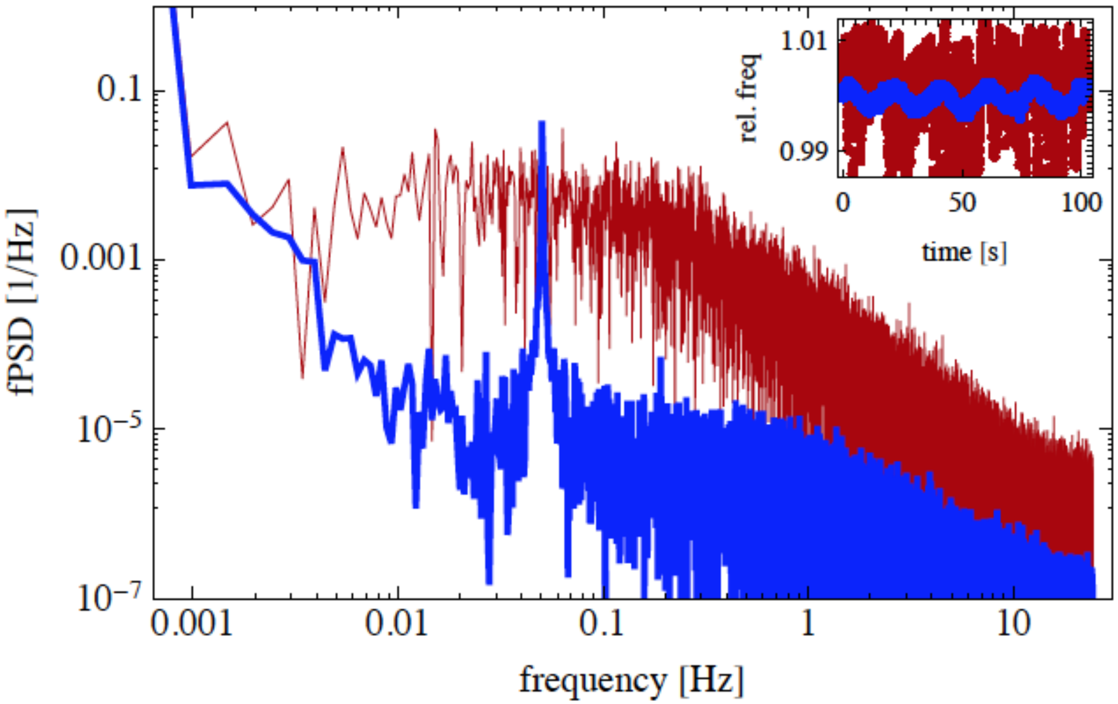
BOOSTING SENSITIVITY

Force noise spectral density : $S_F = 4k_B T m\Omega_0/Q$ $\longrightarrow$ Q / T

$$\longrightarrow F_{min} = S_F^{1/2} B^{1/2}$$

Reduce T and keep Q high:

$$S_F^{\text{eff}} = 4k_B m\Omega_0 \frac{T_{\text{eff}}}{Q_{\text{eff}}}$$
$$= S_F$$
$$\approx (20\text{zN})^2/\text{Hz}$$



Nature Phys. **9**, 806 (2013)

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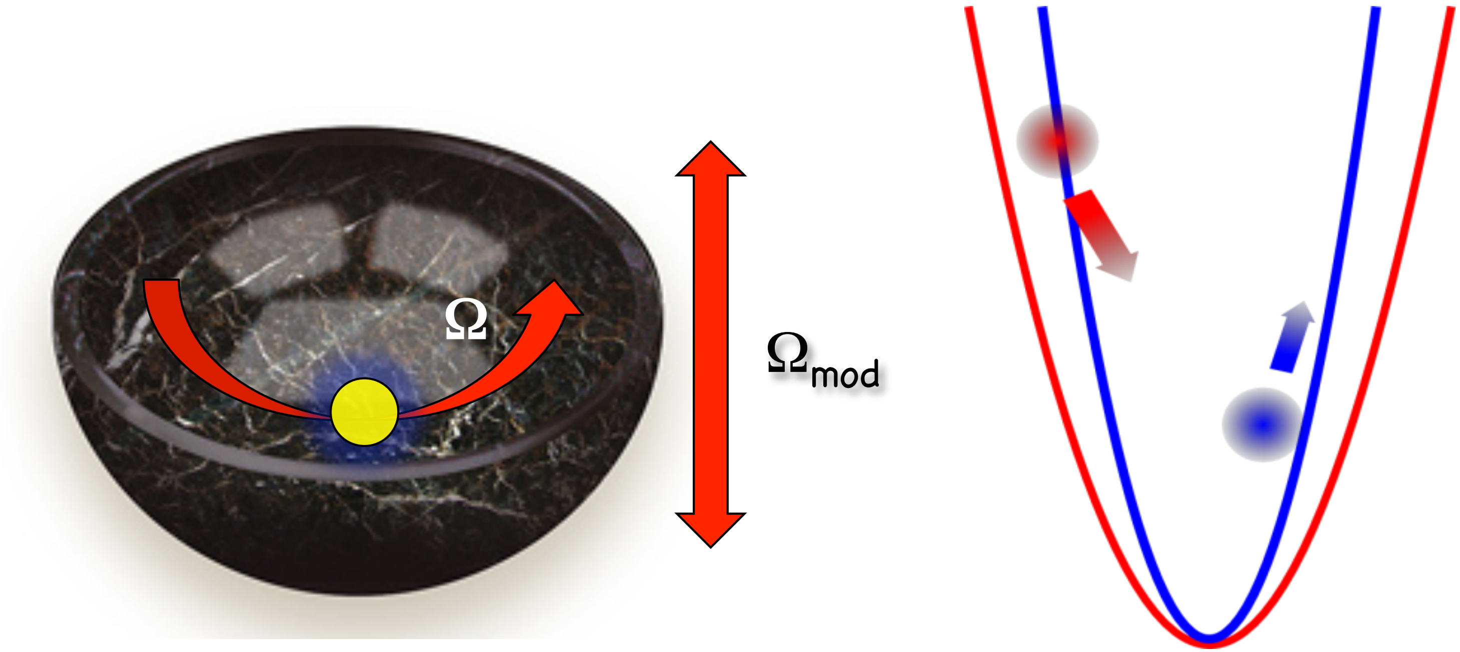
COOLING



-> GENERATION OF SIDEBANDS

PARAMETRIC FEEDBACK

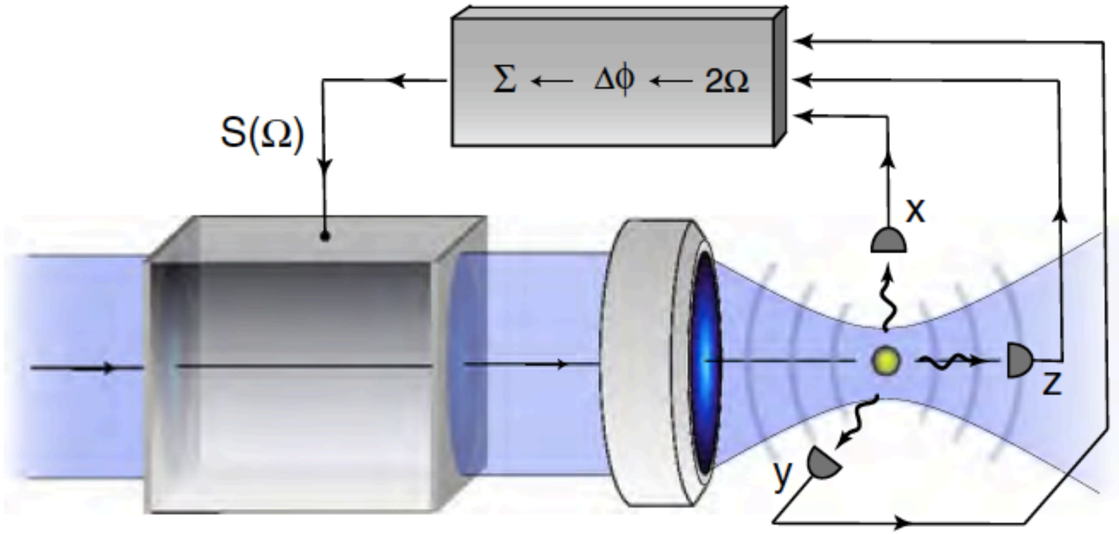
$$\ddot{x}(t) + \Gamma_0\,\dot{x}(t) + \Omega_0^2\,x(t) = (1/m)\,[F_{\text{fluct}}(t) + F_{\text{opt}}(t)]$$

$$\downarrow$$

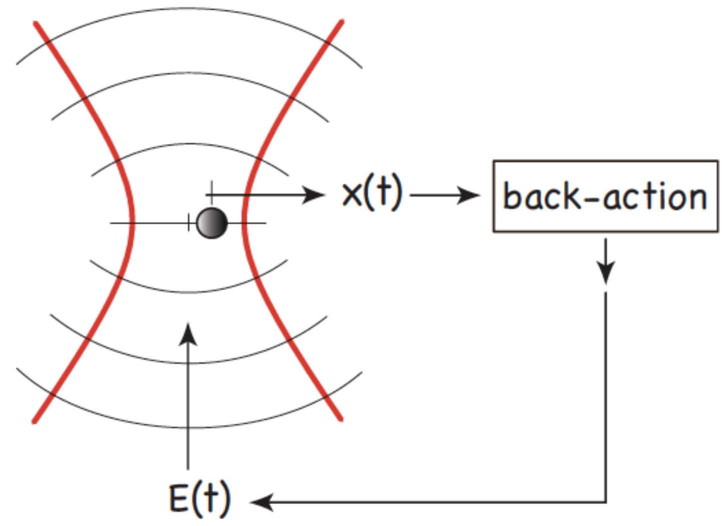
$$F_{\text{opt}}(t) = \Delta k_{\text{trap}}(t)\,x(t)$$

$$\downarrow$$

$$x(t)\dot{x}(t)$$



FEEDBACK LINEARIZED



$$\ddot{x}(t) + \Gamma_0\,\dot{x}(t) + \Omega_0^2\,x(t) = (1/m)\,[F_{\mathrm{fluct}}(t) + F_{\mathrm{opt}}(t)]$$

$$\ddot{x}(t) + [\Gamma_0 + \delta\Gamma]\,\dot{x}(t) + [\Omega_0 + \delta\Omega]^2\,x(t) = F_{\mathrm{fluct}}(t)\,/\,m$$

CENTER-OFF-MASS TEMPERATURE

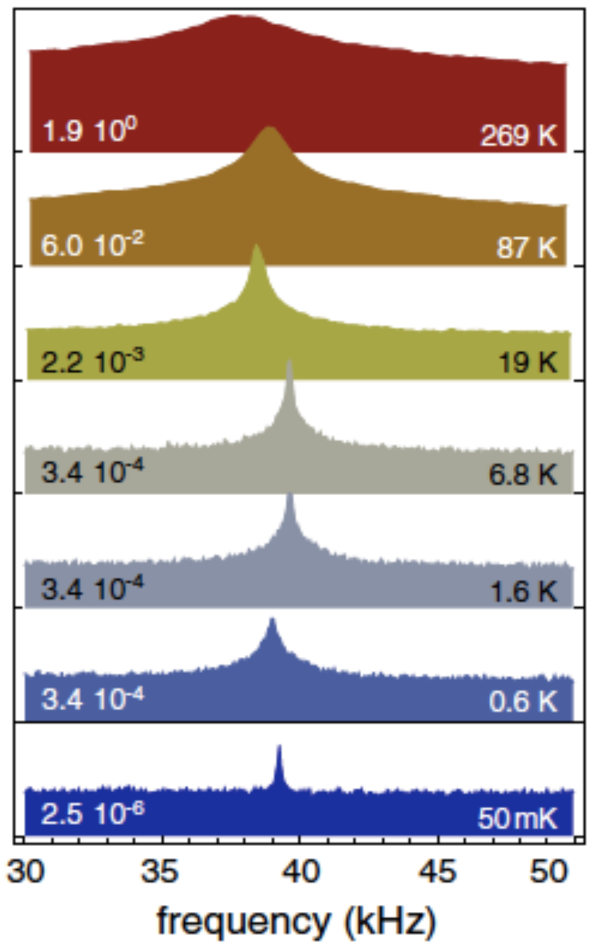
$$S_x(\Omega) = \int_{-\infty}^{\infty} \langle x(t)x(t-t')\rangle e^{-i\Omega t'} dt' \quad = \frac{\Gamma_0 k_B T/(\pi m)}{([\Omega_0 + \delta\Omega]^2 - \Omega^2)^2 + \Omega^2[\Gamma_0 + \delta\Gamma]^2}$$

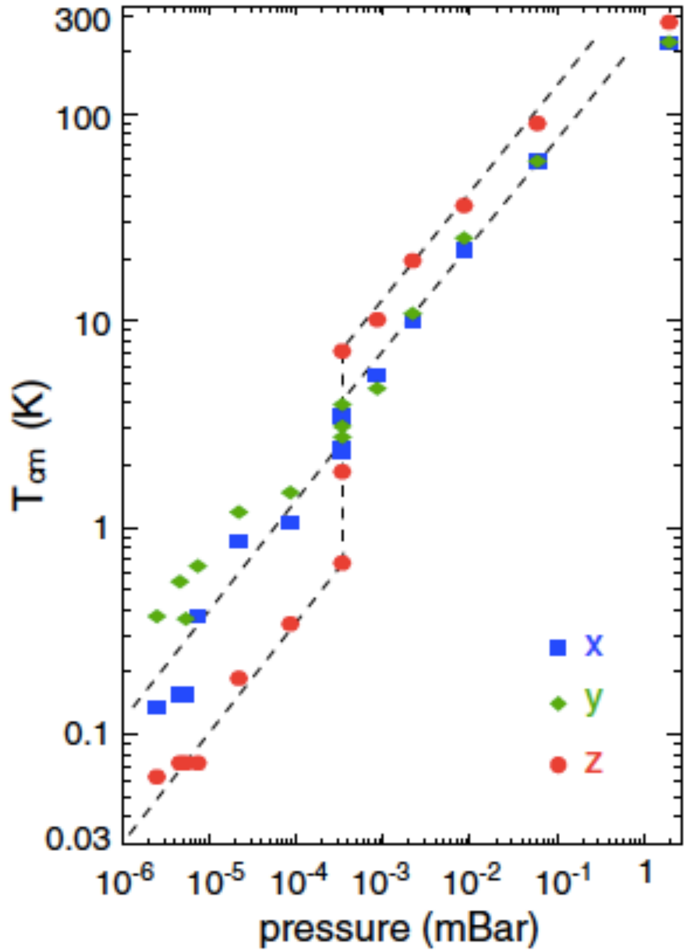
Integrating over Ω : $$\langle x^2\rangle = \langle x(0)x(0)\rangle = \frac{k_B T}{m(\Omega_0 + \delta\Omega)^2}\frac{\Gamma_0}{\Gamma_0 + \delta\Gamma}$$

Equipartition principle : $$k_B T_{c.m.} = m(\Omega_0 + \delta\Omega)^2\langle x^2\rangle$$

For $\delta\Omega \ll \Omega_0$: $$\boxed{T_{c.m.} = T\frac{\Gamma_0}{\Gamma_0 + \delta\Gamma}}$$

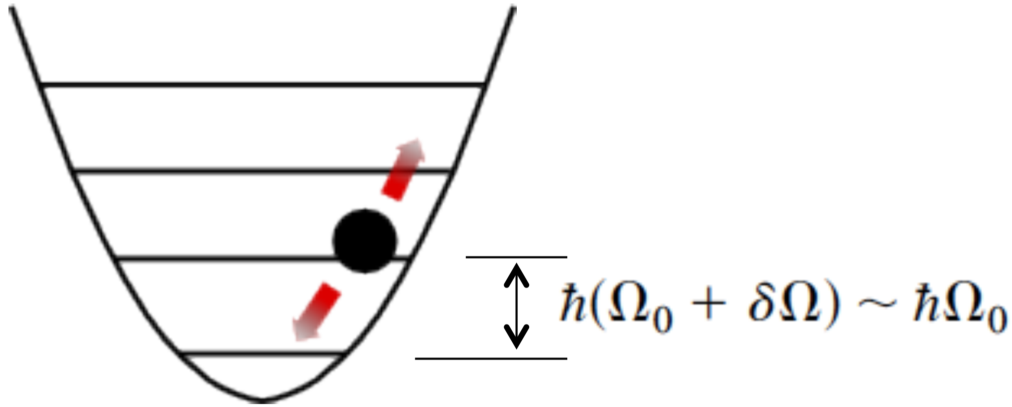
CENTER-OFF-MASS TEMPERATURE





PRL **109**, 103603 (2012)

QUANTUM GROUNDSTATE



Mean thermal occupancy : $\langle n \rangle = \dfrac{k_B T_{\mathrm{c.m.}}}{\hbar\Omega_0}$

Quantum groundstate : $\langle n \rangle < 1 \longrightarrow T_{\mathrm{c.m.}} \sim 6\ \mu\mathrm{K}$

Extrapolating cooling curve: $\longrightarrow P_{\mathrm{gas}} = 10^{-10}$ mbar

Problem : Signal (area of Lorentzian lineshape) $\propto T_{\mathrm{c.m.}}$

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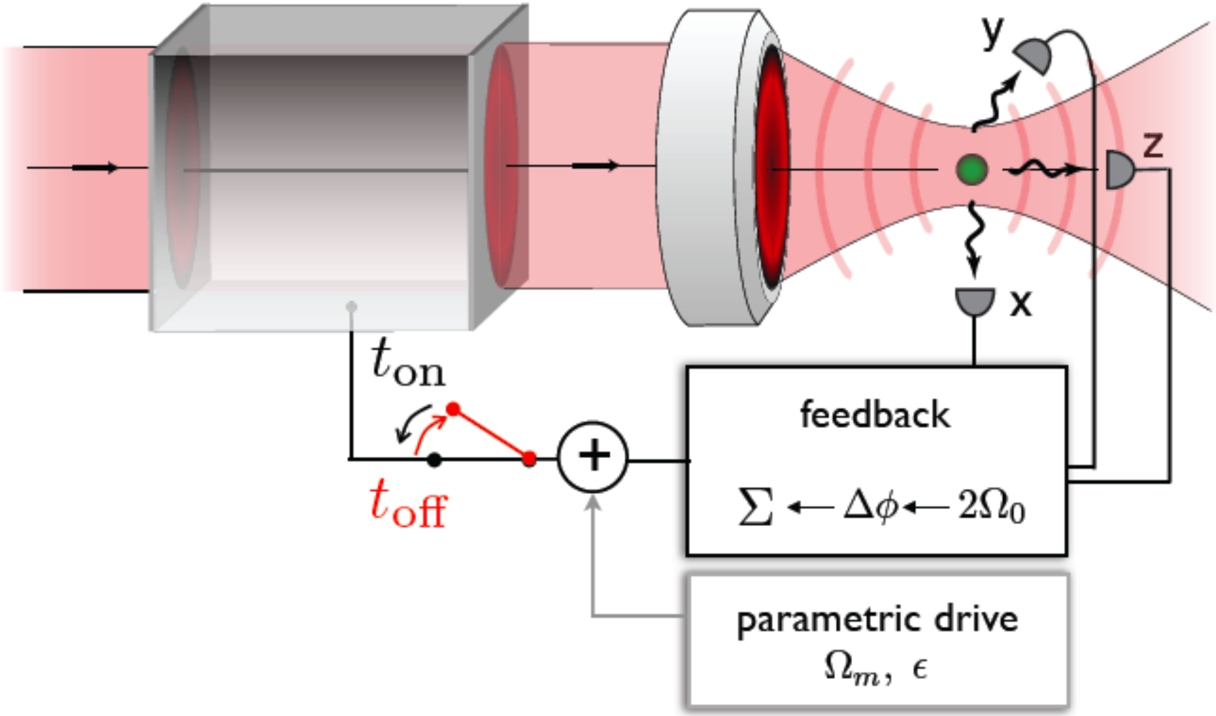
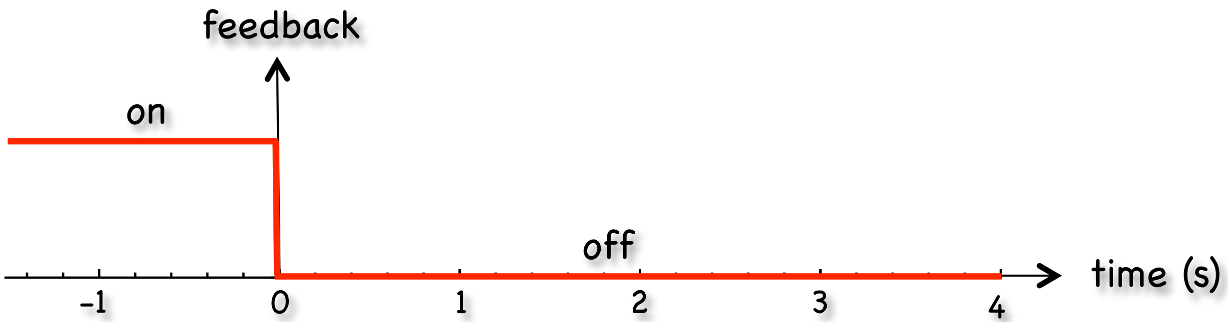
NONEQUILIBRIUM DYNAMICS

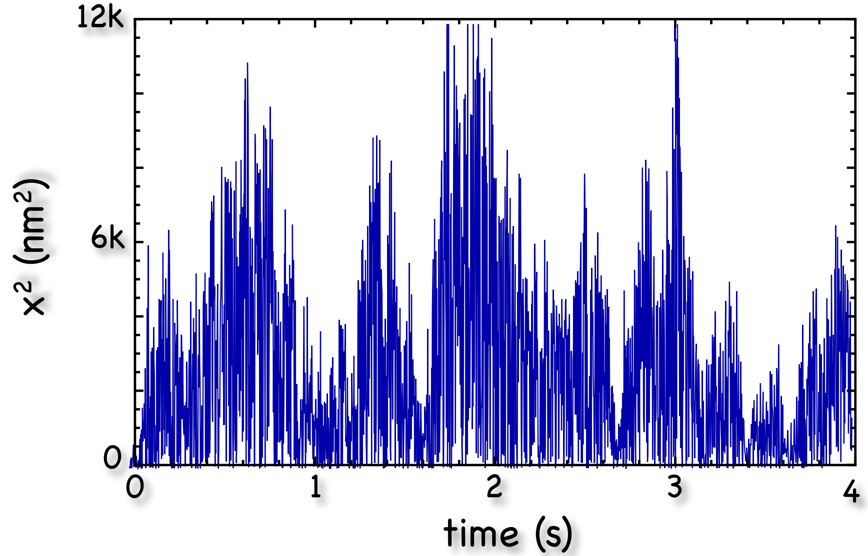


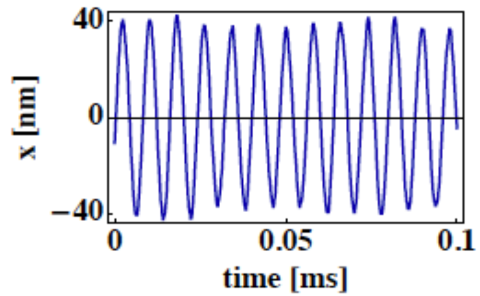
Pressure: 3 10^{-4} mBar

Nature Nanotechnology, in print (2014)

NONEQUILIBRIUM DYNAMICS



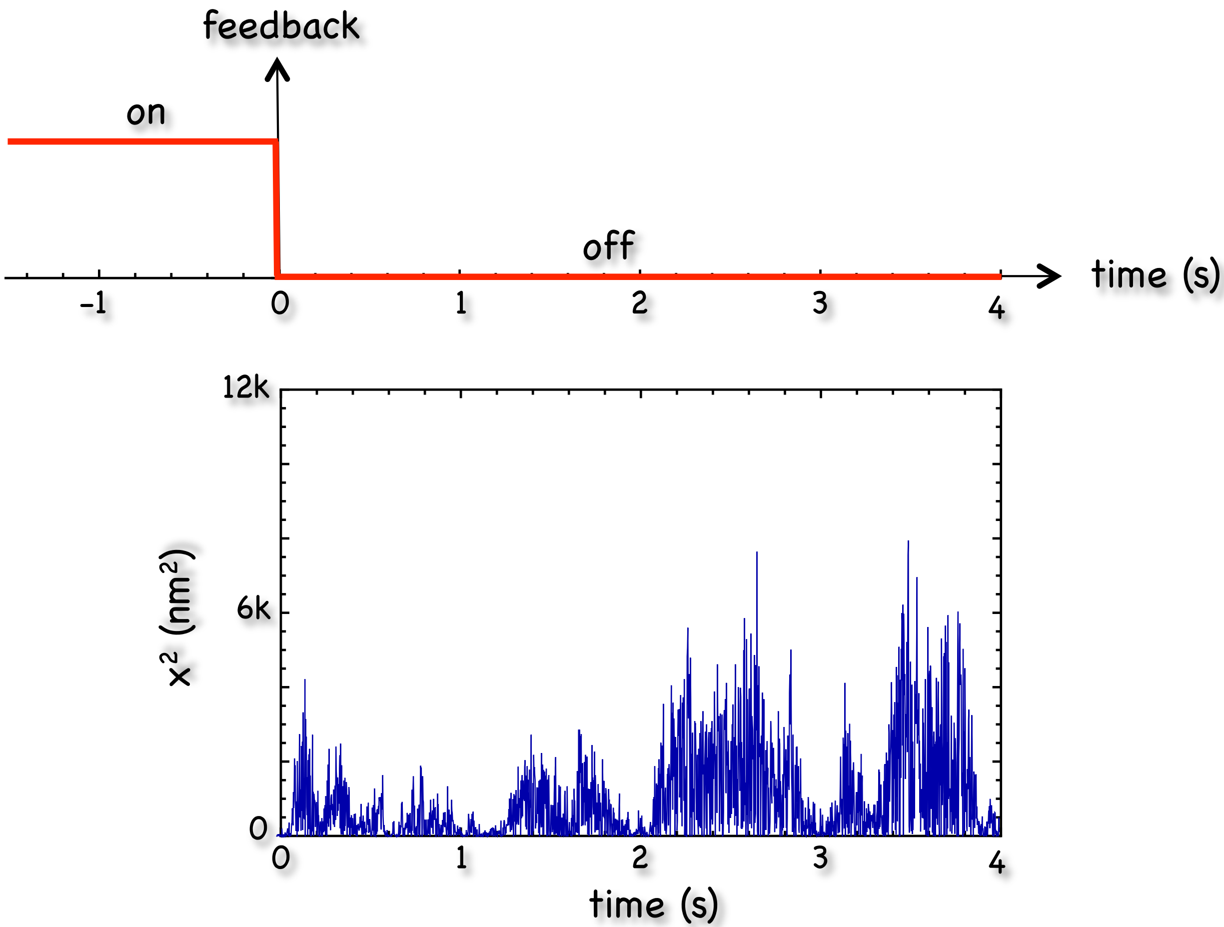




$$E(t) = \frac{1}{2} m \Omega_0^2 \bar{x}(t)^2$$

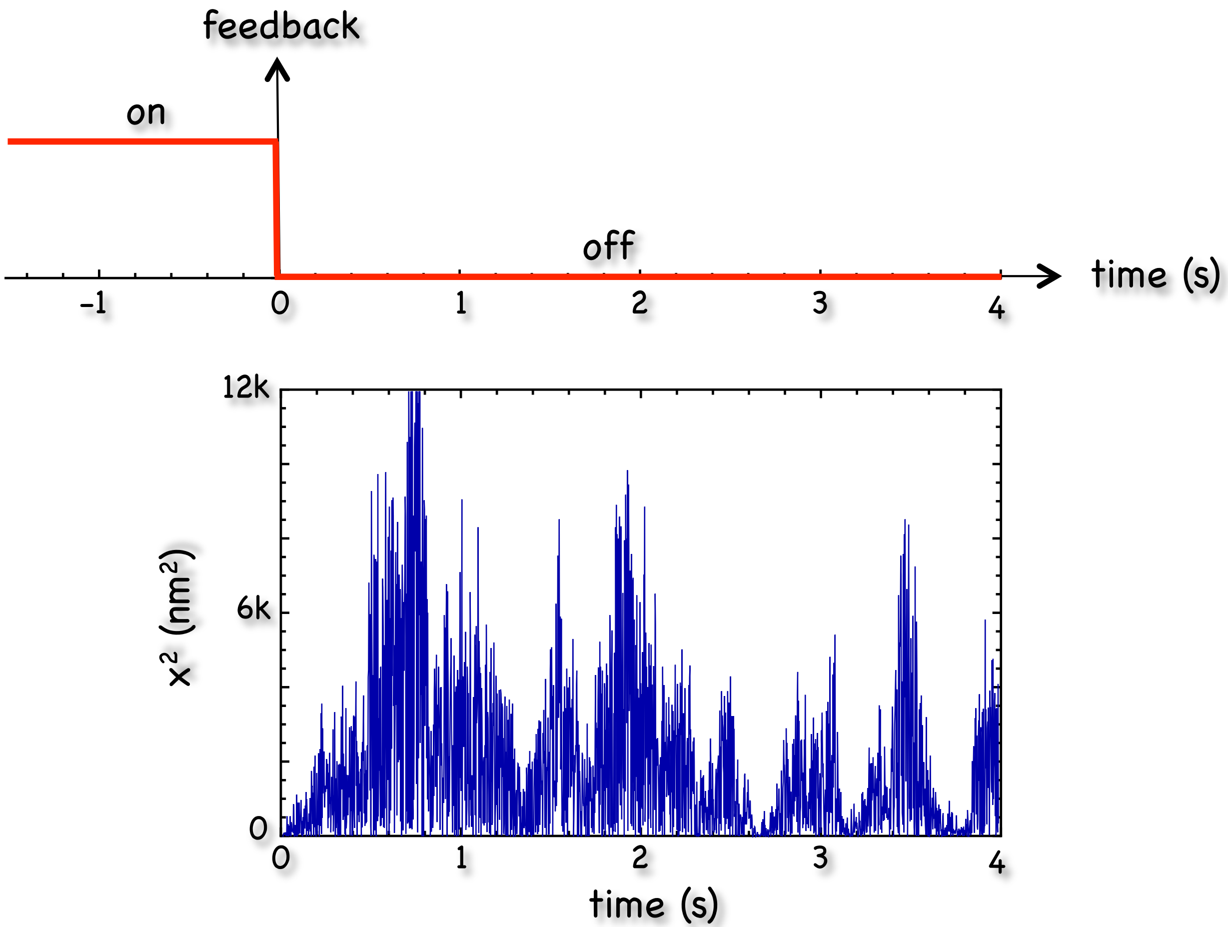
Pressure: 3 10^{-4} mBar

NONEQUILIBRIUM DYNAMICS



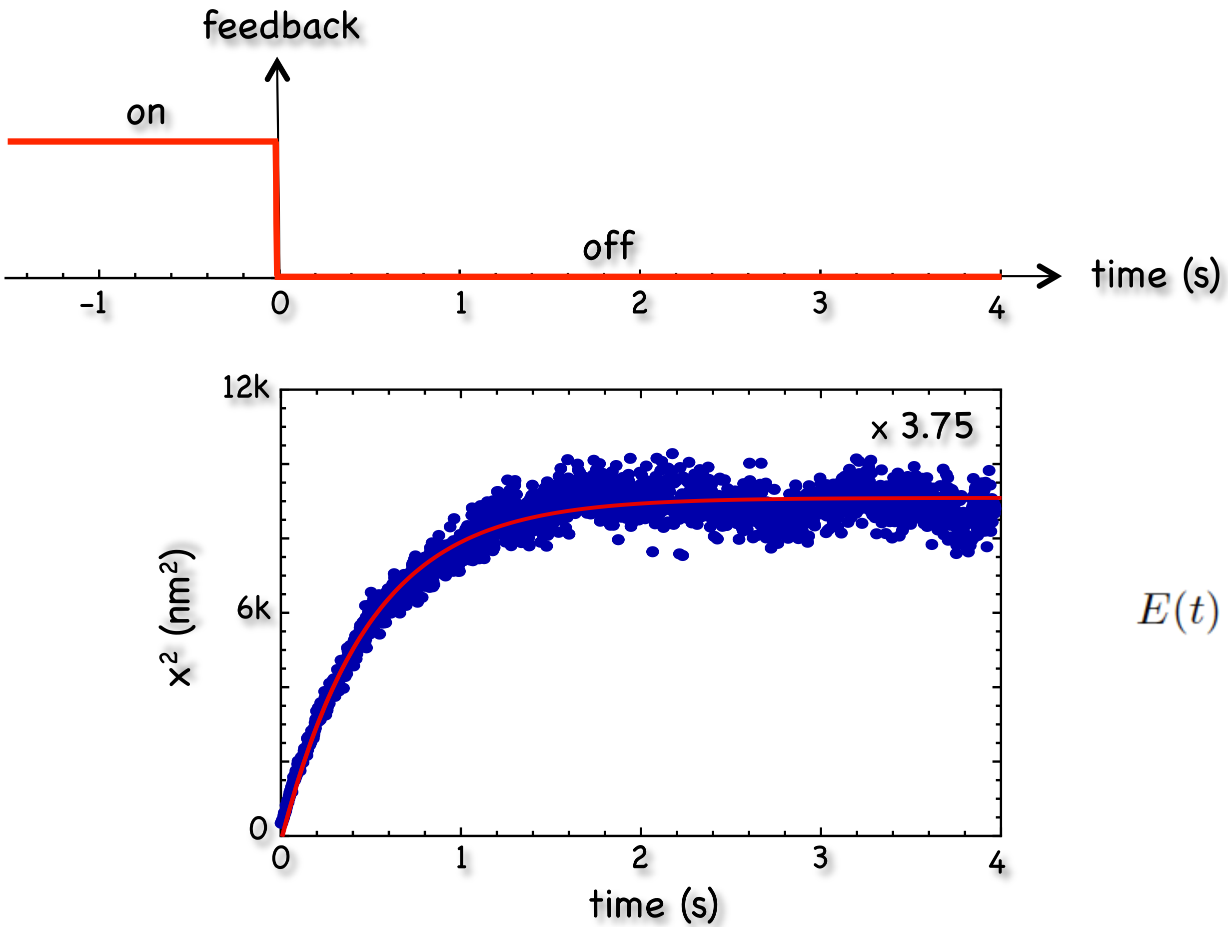
Pressure: 3 10^{-4} mBar

NONEQUILIBRIUM DYNAMICS



Pressure: 3 10^{-4} mBar

FLUCTUATION THEOREM



$$E(t) = \frac{1}{2} m \Omega_0^2 \bar{x}(t)^2$$

$$\langle E(t) \rangle = k_B T_0 + k_B (T_{\text{eff}} - T_0) e^{-\Gamma_0 t}$$

Pressure: $3\ 10^{-4}$ mBar

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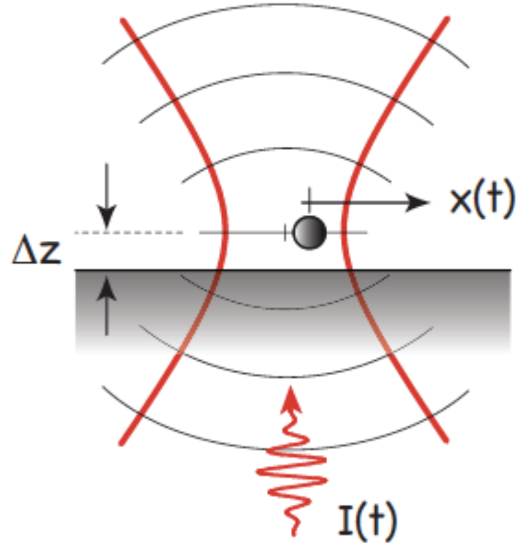
FORCE SENSITIVITY



$$S_x(\Omega) = \int_{-\infty}^{\infty} \langle x(t)x(t-t')\rangle e^{-i\Omega t'}\, dt' \quad = \frac{k_B T}{\pi m}\, \frac{\Gamma_0}{(\Omega_0^2 - \Omega^2)^2 + \Omega^2 \Gamma_0^2}$$

For $\Omega = \Omega_0$ and $S_f^{1/2} = k_{trap}\, S_x^{1/2}$:

$$S_f^{1/2} = \sqrt{2\, k_{trap}^2\, x_{rms}^2\, / (\pi Q f_0)}$$

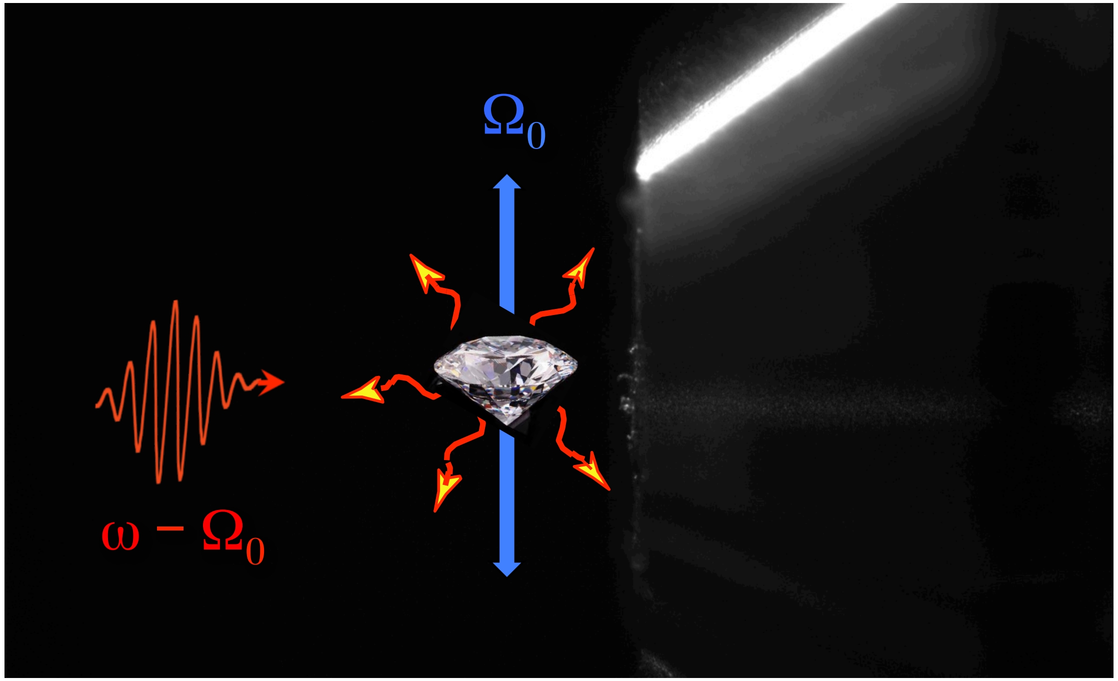
Minimum detectable force in bandwidth B :

$$F_{min} = S_f^{1/2} B^{1/2}$$
$$= \sqrt{2\, B\, k_{trap}\, k_B T\, / (\pi Q f_0)}$$

For $P_{gas} = 10^{-9}\,\mathrm{mbar}$: $\boxed{F_{min} \approx 10^{-20}\,\mathrm{N}}$ in 1 sec

T. D. Stowe *et al.*, Appl. Phys. Lett. **71**, 288 (1997).

COUPLING OF COM AND INTERNAL (SPIN) DEGREES OF FREEDOM



c.f. S. Kolkowitz *et al.*, *Science* **335**, 1603 (2012)
D. Rugar *et al.*, *Nature* **430**, 329 (2004)

Opt. Lett. **38**, 2976 (2013)

SUMMARY

$$\ddot{q} + \Gamma_0 \dot{q} + \Omega_0^2 \left[1 + \underbrace{\epsilon \cos(\Omega_{\mathrm{m}} t)}_{\text{parametric drive}} + \underbrace{\Omega_0^{-1} \eta q \dot{q}}_{\text{feedback}} + \underbrace{\xi q^2}_{\text{Duffing term}} \right] q = \frac{\mathcal{F}_{\text{fluct}}}{m} \approx 0$$

- Trapping and cooling with a single laser beam

- Parametric active feedback (amplification and cooling)

- Compression ratio of 10^4 (< 50 mK)

- Ultrahigh force sensitivity

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