Numerical studies of entanglement properties of quantum Ising and XXZ spin-1/2 models in one and two dimensions

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A major topic of interest in modern physics is the study of many-body systems at zero temperature, where thermal fluctuations disappear and quantum fluctuations are dominant. Quantum phase transitions (QPT) occur in many interesting physical systems and attract much attention of scientists in modern condensed-matter physics.

In practice, only a few quantum models are exactly solvable. Usually one needs to implement well-tuned numerical methods to determine physical properties of the systems. Because of the exponential growth of the Hilbert space needed to describe an ensemble of particles and because of the limitations of modern (classical) computing devices high precision numerical studies are mostly focused on 1D systems (chains and rings), and only recently (since about 10 years) 2D and 3D geometries could be tackled.

Among different numerical algorithms, which simulate strongly correlated systems, a class of tensor network (TN) methods became highly popular in last few years. TN can handle systems in different dimensions, of finite and infinite size, with different boundary conditions and symmetries. The family of matrix product states (MPS) is the most famous among TN states. Two-dimensional generalization of the matrix product states are projected entangled pair states (PEPS).

At some stage it was realized that many-body systems can be analyzed from quantum information theory perspective with its tools. Thus, QPT could be characterized by the change of the quantum entanglement.

Having a big variety of numerical (including TN) methods on the market, and also a set of different quantities which characterize the state (order parameters, critical exponents, entanglement measures), two questions arise: (1) which numerical methods are better for modeling the ground states of particular types of models? (2) working within particular method, which quantities are 'better' for the ground state characterization? Here 'better' means relatively low computational cost and relatively high 'characterization power' of the physics of the studied system.

We aim to contribute answers to these two wide questions. For our analysis we choose two models: the quantum Ising in a transverse field and XXZ model in 1D and 2D geometries.

To address first question we choose particular numerical methods able to effectively treat infinite models: translationally invariant MPS [Östlund, Rommer, PRL 75, 3537 (1995); Schollwöck, Ann. Phys. (NY) 326, 96 (2011)] in 1D; Tensor Entanglement Renormalization Group (TERG) [Levin et al., PRB 78, 205116 (2008)] and Corner Transfer Matrix Renormalization Group (CTMRG) [Orus, Vidal, PRB 80, 094403 (2009)] in 2D. We use the imaginary-time evolution to find the approximate ground state for the studied models. Exploiting the translational invariance of the ground states makes the imaginary-time evolution and MPS, TERG, CTMRG algorithms tolerant in computation resources requirements. Our results are compared to the results from other studies based on different techniques.

Concerning second question, we calculate a set of different entanglement measures, such as one-site entanglement entropy, one-tangle, concurrence of formation and assistance, bonds on localizable entanglement, local entanglement, negativity and entanglement per bond and compare their 'characterization power' of the system state, in particular the possibility of different entanglement measures to determine the critical point of the model. Also by comparing entanglement measures dependencies in 1D and 2D for the same models we study more about the entanglement properties in different geometries, in particular about monogamy of entanglement.

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