Institut de Ciències Fotòniques

Lattice gauge theories with Tensor Networks

Luca Tagliacozzo

Based on:

L. Tagliacozzo G. Vidal
“Entanglement renormalization and gauge symmetry”
Phys. Rev. B 83, 115127 (2011)

L. Tagliacozzo, A. Celi, M. Lewenstein “Tensor Networks for Lattice Gauge Theories with continuous groups”, arXiv:1405.4811

Time-line

- Byrnes, T. M., Sriganesh, P., Bursill, R. J. & Hamer, C. J. Density matrix renormalization group approach to the massive Schwinger model. *Phys. Rev. D* 66, 13002 (2002).
- Sugihara, T. Matrix product representation of gauge invariant states in a Bbb Z2 lattice gauge theory. *J. High Energy Phys.* **2005**, 022 (2005)
-
- LT, G. Vidal 2010
-
- Bañuls, M. C., Cichy, K., Cirac, J. I., Jansen, K. & Saito, H. Matrix Product States for Lattice Field Theories. *ArXiv:1310.4118* (2013).
- Dittrich, B., Martín-Benito, M. & Schnetter, E. Coarse graining of spin net models: dynamics of intertwiners. *New J. Phys.* **15**, 103004 (2013).
- Bañuls, M. C., Cichy, K., Cirac, J. I. & Jansen, K. The mass spectrum of the Schwinger model with matrix product states. *J. High Energy Phys.* **11**, 158 (2013).
- Buyens, B., Haegeman, J., Van Acoleyen, K., Verschelde, H. & Verstraete, F. Matrix product states for gauge field theories. *ArXiv:1312.6654*
- Liu, Y. *et al.* Exact blocking formulas for spin and gauge models. *Phys. Rev. D* 88, (2013).
- Shimizu, Y. & Kuramashi, Y. Grassmann tensor renormalization group approach to one-flavor lattice Schwinger model. *Phys. Rev. D* 90, 14508 (2014).
- Rico, E., Pichler, T., Dalmonte, M., Zoller, P. & Montangero, S. Tensor Networks for Lattice Gauge Theories and Atomic Quantum Simulation. *Phys. Rev. Lett.* **112**, 201601 (2014)
- Silvi, P., Rico, E., Calarco, T. & Montangero, S. Lattice Gauge Tensor Networks. *ArXiv:1404.7439 2014*
- LT, A. Celi, M. Lewenstein (2014)
- Haegeman, J., Van Acoleyen, K., Schuch, N., Cirac, J. I. & Verstraete, F. Gauging quantum states. *ArXiv.1407:1025 2014*
- ……..
-

Introduction

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Connection with quantum doubles

LGT from TN

Toolbox, group theory

The gauge invariant Hilbert space

Gauge invariant operators

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Variational ansatz

Model building

Spectra

Order parameters

Topological phase transition

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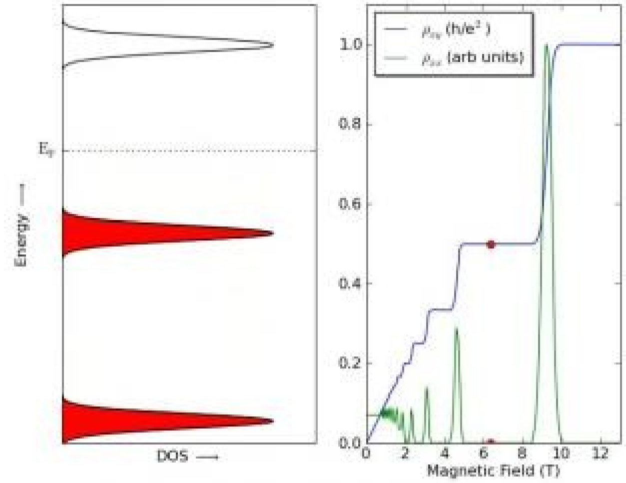
Topological states of matter

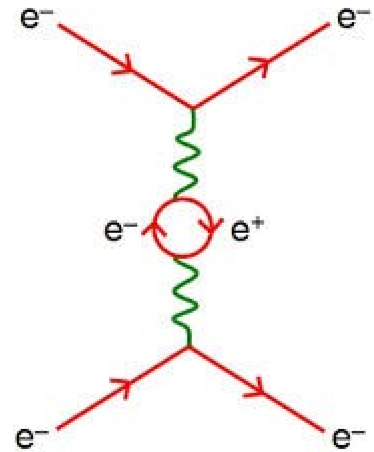
- Electrons coupled to strong magnetic fields gives rise to topological states



- What about photons alone?
- What about non-Abelian photons, gauge bosons?

Gauge theories

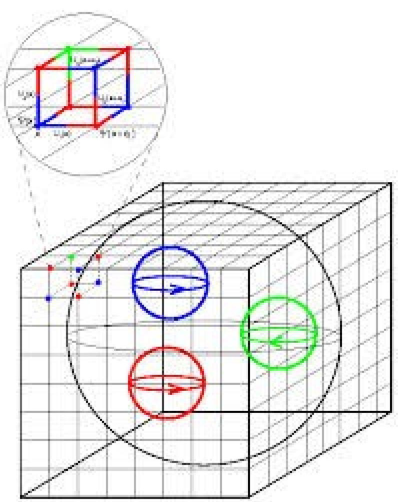
Gauge Theories



→ HEP, form QED, QCD, Standard Model, elementary gauge bosons

→ COND-MAT spin liquids, dimers (electrons in a material), emerging gauge bosons

→ Lattice allows for non-perturbative formulation of QCD

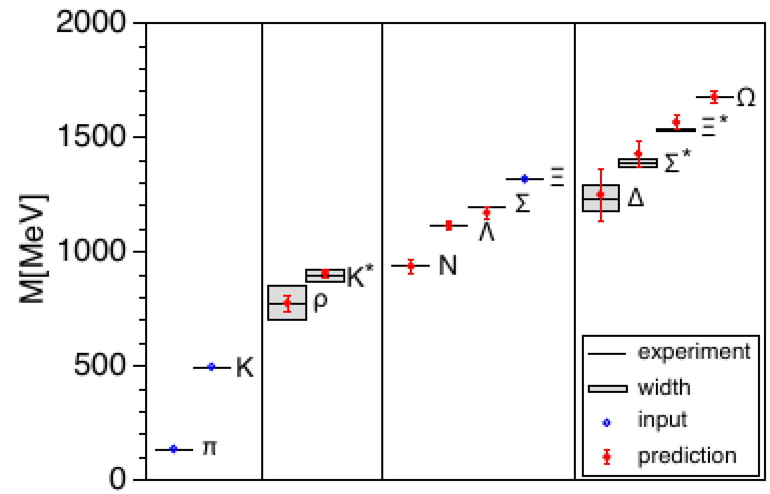


Wilson, K. G. Confinement of quarks. *Phys. Rev. D* **10**, 2445–2459 (1974).

Achievements LGT

- Evidences of mass-gap in Yang Mills from first principles.

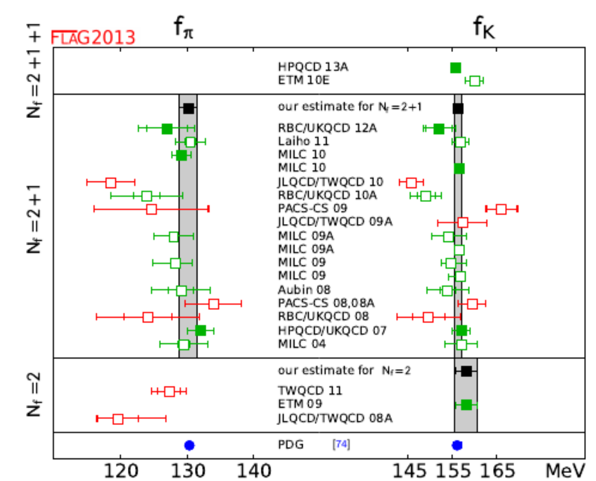
- Precise determination of the lowest excitations (agreement with experiments)



Fodor, Z. & Hoelbling, C.
Light Hadron Masses from Lattice QCD.
Rev. Mod. Phys. 84, 449-495 (2012).

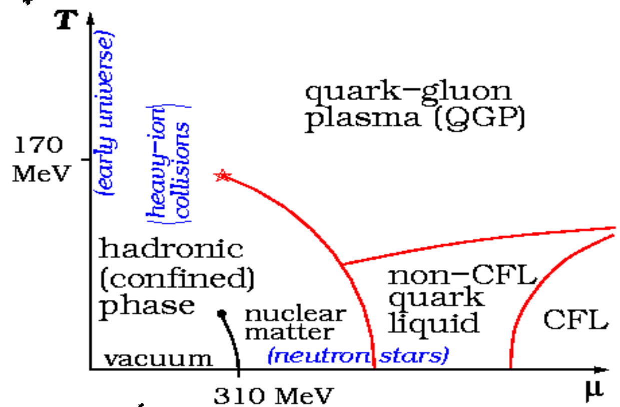
- Matrix elements input for phenomenology of Standard model

Aoki, S. *et al.*
Review of lattice results concerning
low energy particle physics. *ArXiv:1310.8555*

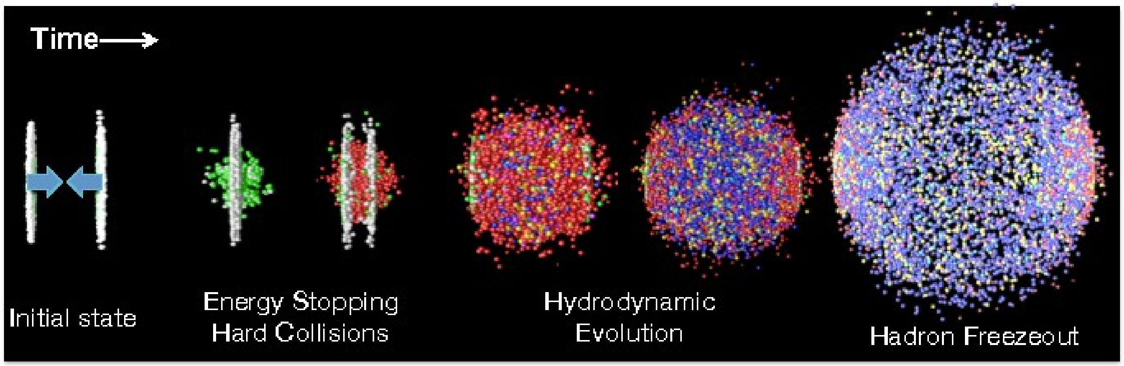


Limitations LGT

- QCD at non-zero temperature and density (nuclear matter)?



- Real time dynamics (experiments at RICH and CERN)



- Classification of phases in presence of dynamical matter

Achievements in TN/Quantum Many Body

- Study of frustrated and fermionic systems

Corboz, P., Evenbly, G., Verstraete, F. & Vidal, G.
Simulation of interacting fermions with entanglement renormalization.
Phys. Rev. A **81**, 010303 (2010).

- Out of equilibrium dynamics

- Vidal, G. Efficient Classical Simulation of Slightly Entangled Quantum Computations.
Phys. Rev. Lett. **91**, 147902 (2003).
- White, S. R. & Feiguin, A. E.
Real time evolution using the density matrix renormalization group.
Phys. Rev. Lett. **93**, (2004).

- Characterization of topological phases

- Kitaev, A. & Preskill, J.
Topological Entanglement Entropy. *Phys. Rev. Lett.* **96**, 110404 (2006).
- Levin, M. & Wen, X.-G.
Detecting Topological Order in a Ground State Wave Function.
Phys. Rev. Lett. **96**, 110405 (2006).

Quantum simulations

- Proposal for experiments with
  - Trapped ions
  - Super conducting qubits
  - Cold atoms, Rydberg atoms
- Characterize the static and dynamics of lattice gauge theories
- Both Abelian/non-Abelian

- Weimer, et al. A Rydberg quantum simulator. *Nat Phys* **6,** 382–388 (2010).
- Tagliacozzo, L. et al. Optical Abelian lattice gauge theories. *Ann. Phys.* **330,** 160–191 (2013).
- Banerjee, D. *et al.* Atomic Quantum Simulation ... *Phys. Rev. Lett.* **109,** 175302 (2012).
- Hauke, P., et al. Quantum Simulation of a Lattice Schwinger Model *Phys. Rev. X* **3,** 041018 (2013).
- Tagliacozzo, L. et al. Simulation of non-Abelian *Nat Commun* **4,** (2013).
- Zohar, E et al. Cold-Atom ... SU(2) *Phys. Rev. Lett.* **110,** 125304 (2013).
- Rico, E., et al. *Phys. Rev. Lett.* **112,** 201601 (2014).
- Kühn et al. Quantum simulation of the Schwinger model: A study of feasibility. *ArXiv:1407.4995*

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Connection between LGT and topological models

- Toric code

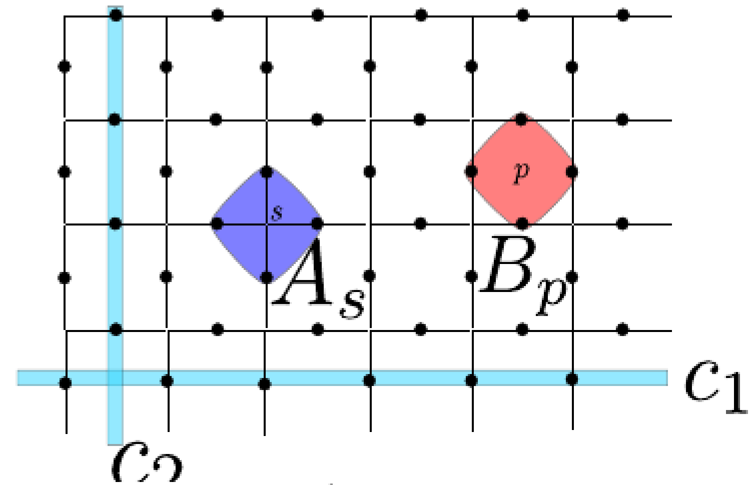
$$H = -J_e \sum_s A_s - J_m \sum_p B_p$$

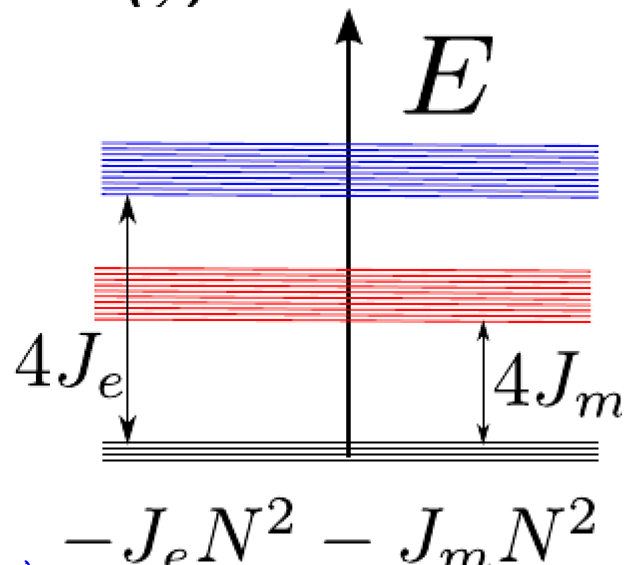
$$A_s = \prod_{s \in l} \sigma_x^l \qquad B_p = \prod_{l \in p} \sigma_z^l$$

$$[A_s, B_p] = 0$$

$$A_s |\Omega\rangle = |\Omega\rangle$$

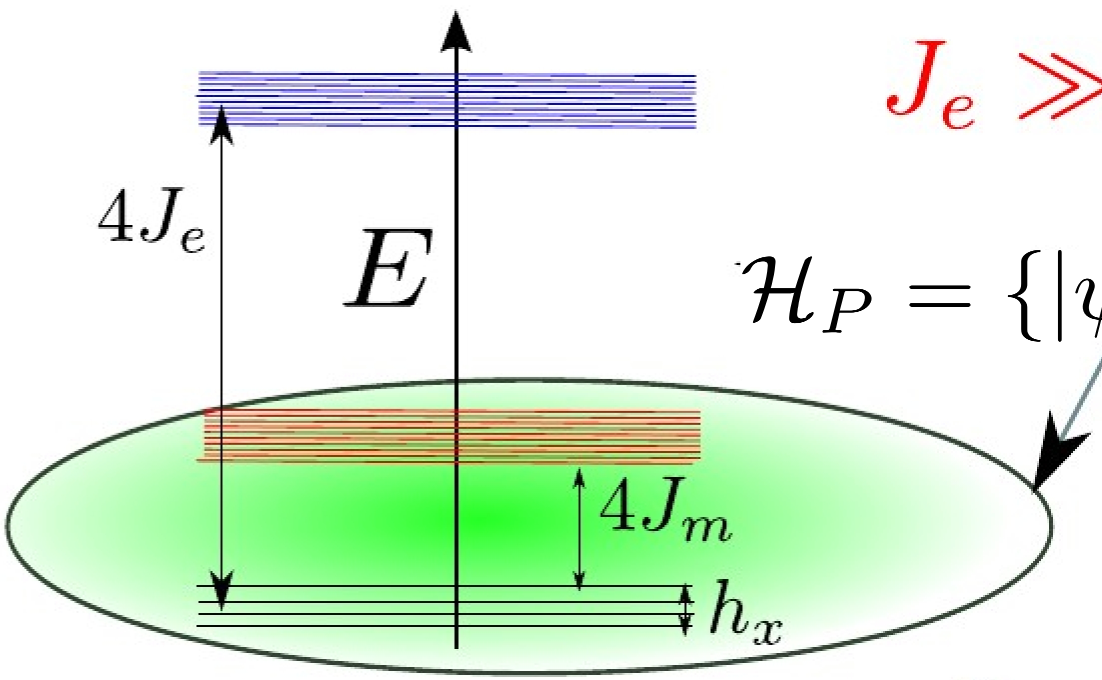
$$B_p |\Omega\rangle = |\Omega\rangle$$





Kitaev, A. Y.
Ann. Phys. **303**, 2–30 (2003).

The gauge invariant Hilbert space



$J_e \gg J_m$ $\quad J_m = 1$

$$\mathcal{H}_P = \{|\psi\rangle : A_s|\psi\rangle = |\psi\rangle\}$$

$$H \equiv -\sum_p B_p - h_x \sum_k \sigma_k^x$$

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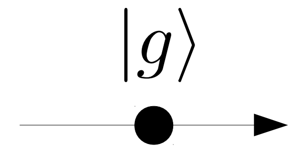
Constructing a LGT

Notion of symmetry

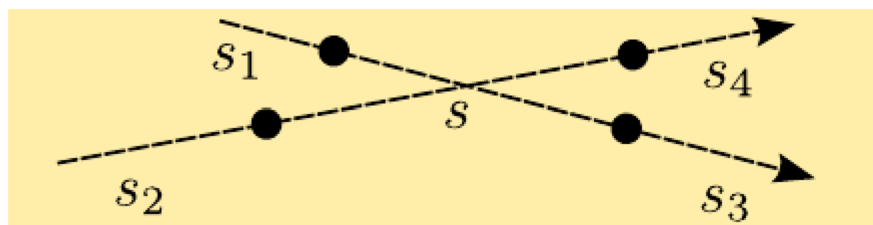
$$h, g, k \in \mathcal{G}$$



- Constituents on links

- Local symmetry operators

$$A_s(h)|\psi\rangle = |\psi\rangle$$



$$A_s(h) = R(h)_{s_1} \otimes R(h)_{s_2} \otimes L(h^{-1})_{s_3} \otimes L(h^{-1})_{s_4}$$

- Left right rotations of the state

$$L(h^{-1})R(k)\,|g\rangle \equiv |h^{-1}gk\rangle$$

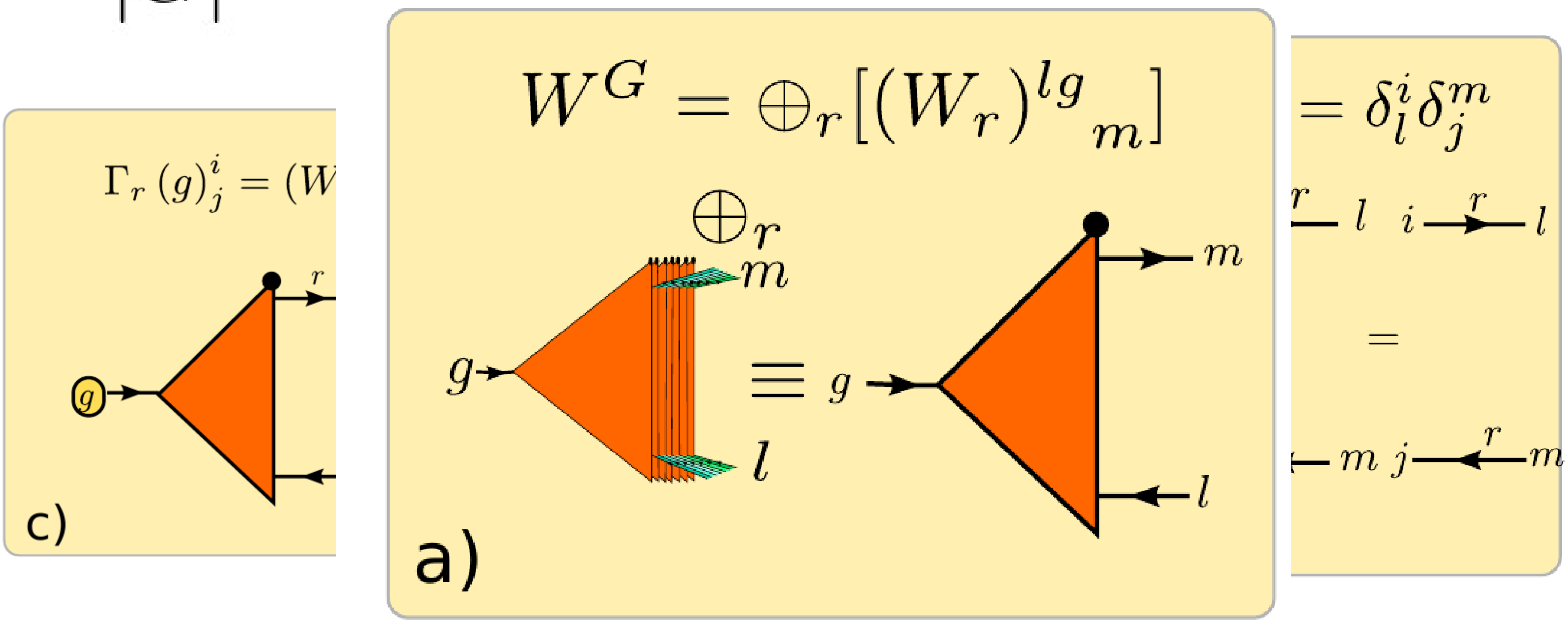
Tagliacozzo, L., Celi, A. & Lewenstein, M.
TN for LGT with continuous groups.
ArXiv:1405.4811

Orthogonality theorem

Serre, J.-P. *Linear representations of finite groups.* (Springer-Verlag, 1977).

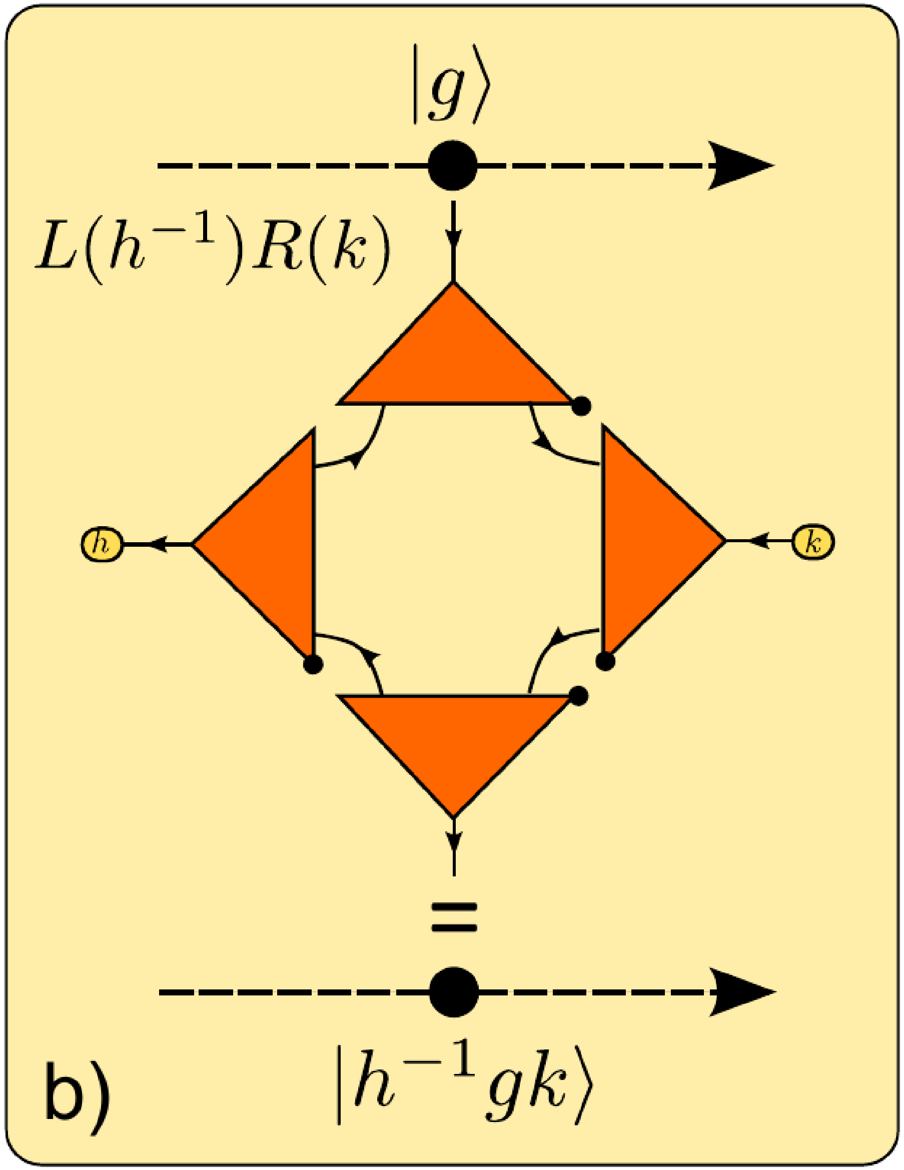
Matrix representation of g in irrep r: $\Gamma_r(g)$

$$\frac{\sqrt{n_r n_{r'}}}{|G|}\sum \Gamma_r(g^{-1})^i_j \Gamma_{r'}(g)^l_k = \delta^i_k \delta^j_l \delta(r,r')$$



LR multiplication

$$L(h^{-1})R(k)\,|g\rangle \equiv |h^{-1}gk\rangle$$



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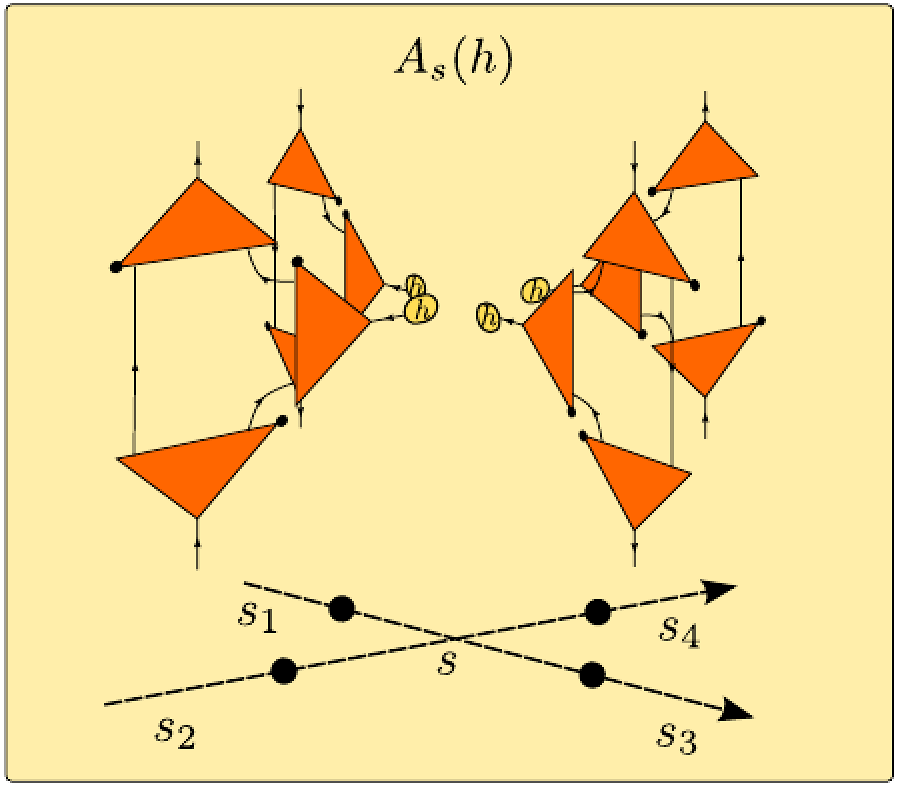
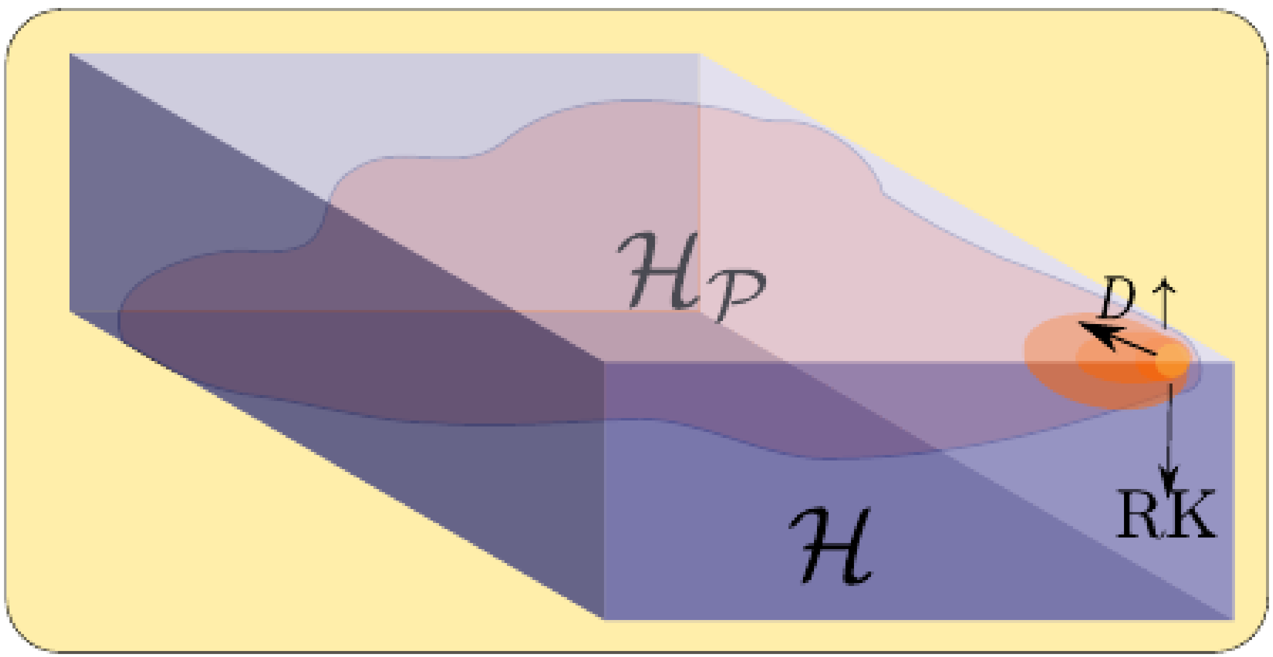
Generalized cross operators

$$A_s(h) = R(h)_{s_1} \otimes R(h)_{s_2} \otimes L(h^{-1})_{s_3} \otimes L(h^{-1})_{s_4}$$



Gauge invariant Hilbert space

$$\mathcal{H}_P = \{|\psi\rangle : A_s(h)|\psi\rangle = |\psi\rangle \, \forall s, h\}$$



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Dynamic on Hp

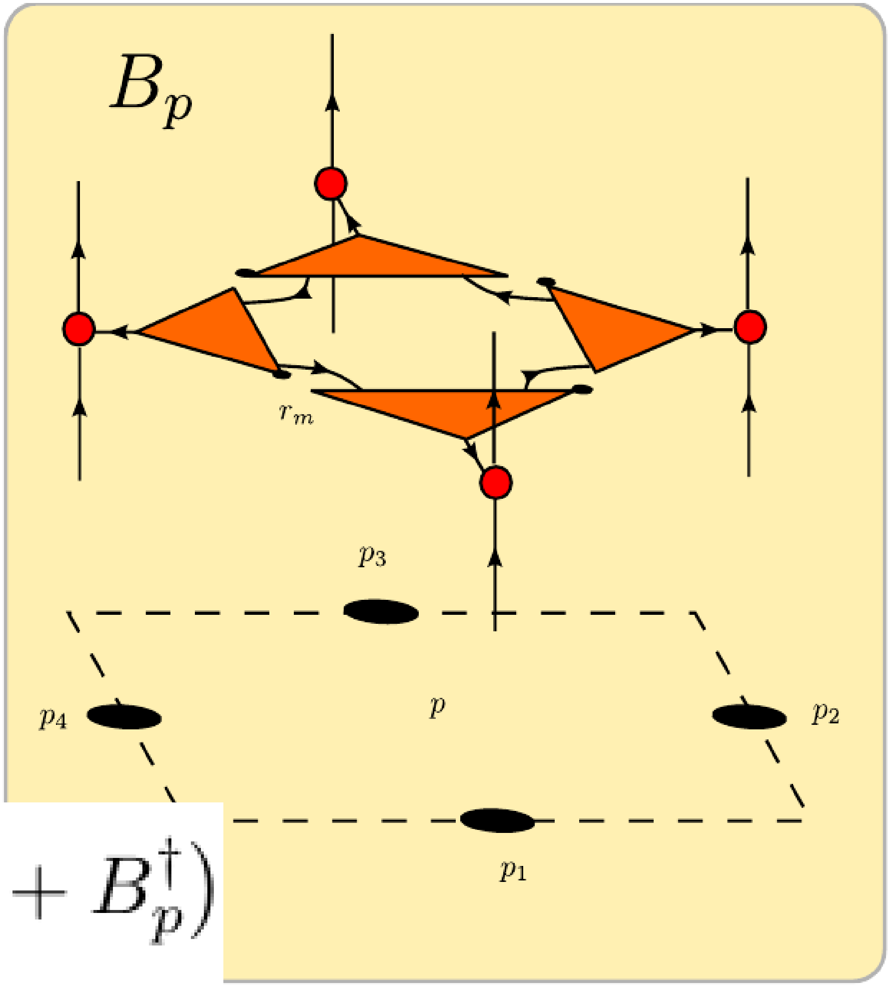
Kogut, J. & Susskind, L. *Phys. Rev. D* **11**, 395–408 (1975).
Creutz, M. *Phys. Rev. D* **15**, 1128 (1977).

$$H = E^2 + B^2$$

$$\mathcal{E}^2_{s_n} = \left[W_G^\dagger \oplus_r \left[c^r\,\mathbb{I}_r \otimes \mathbb{I}_{\bar{r}}\right] W_G^\dagger\right]_{s_n}$$

$$U_{s_n} = \sum_g |g\rangle\langle g|_{s_n} \otimes \Gamma_{r_m}(g)^i_j$$

$$B_p = \mathrm{tr}_{r_m}\left(U_{p_1} U_{p_2} U^\dagger_{p_3} U^\dagger_{p_4}\right)$$



$$H_{LGT} = \sum_l \mathcal{E}^2_l + \frac{1}{\alpha^2}\sum_p \left(B_p + B_p^\dagger\right)$$

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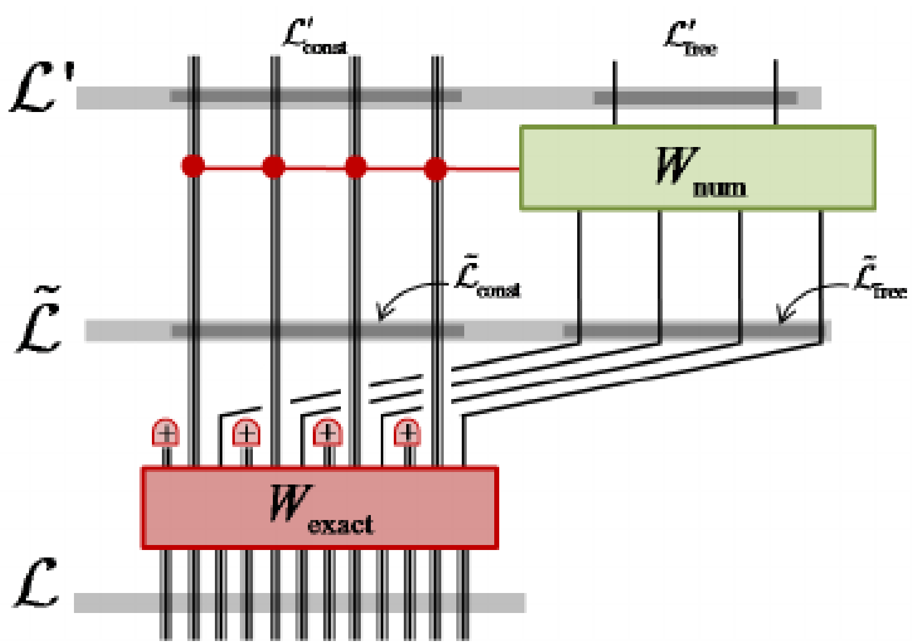
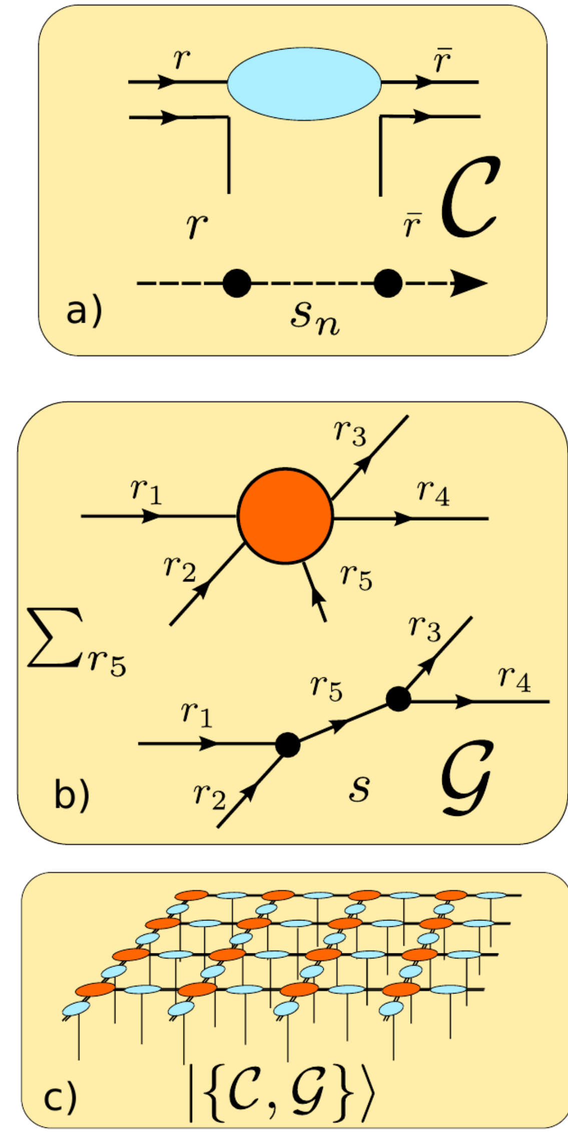
The two ways



MERA,Hierarchical TN

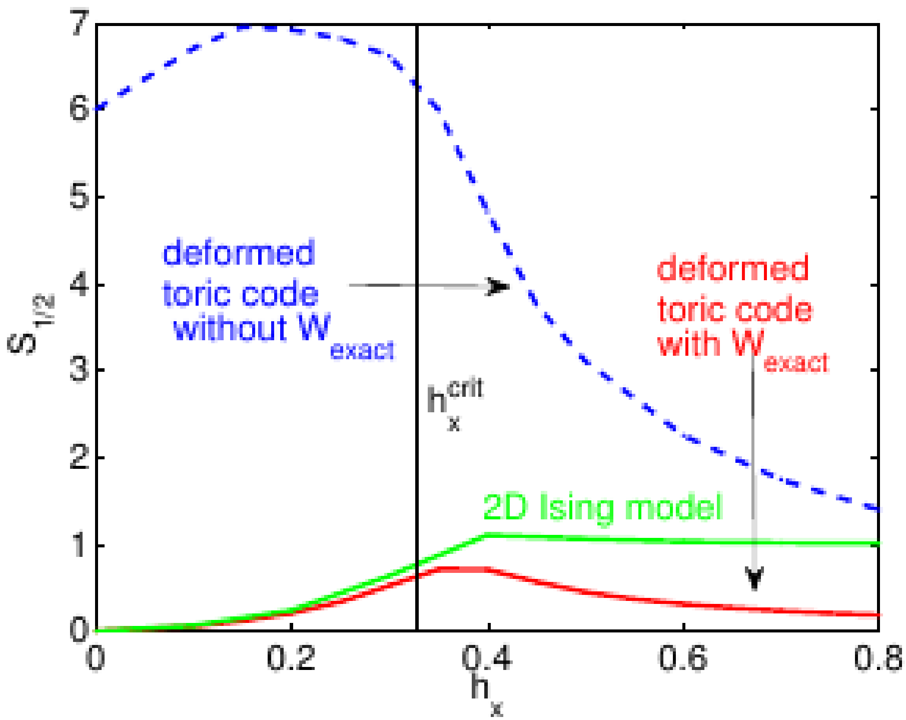


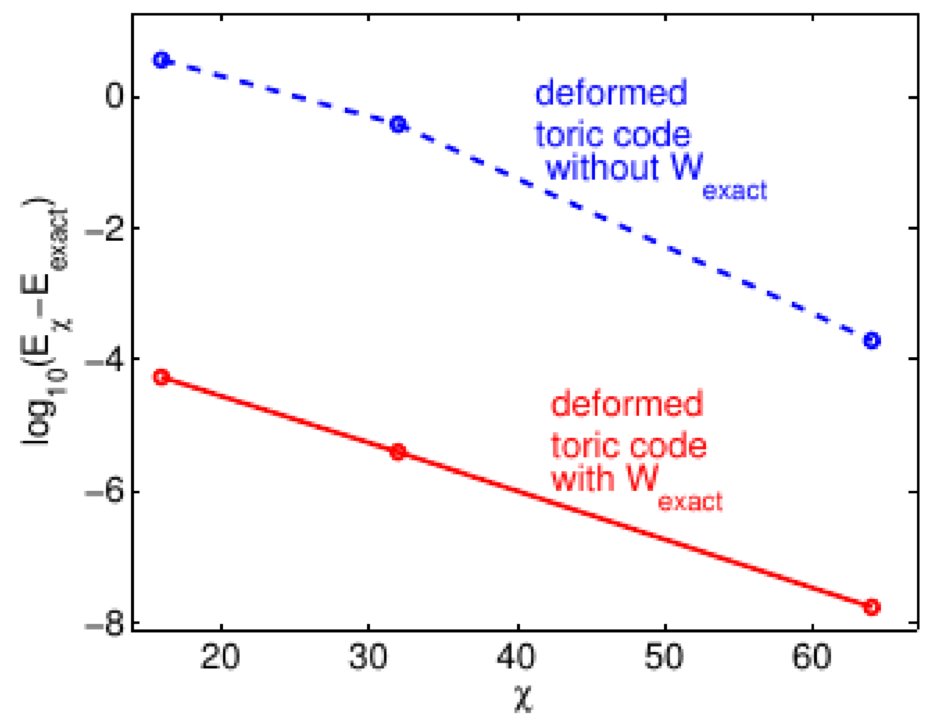
TPS/PEPS

Tagliacozzo, L. & Vidal, G.
Phys. Rev. B **83**, 115127 (2011)

Tagliacozzo, L., Celi, A. & Lewenstein, M.
ArXiv:1405.4811

Variational Ansatz for gauge invariant states





Phys. Rev. B 83, 115127 (2011)

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Truncated KS, continuous groups on finite spaces

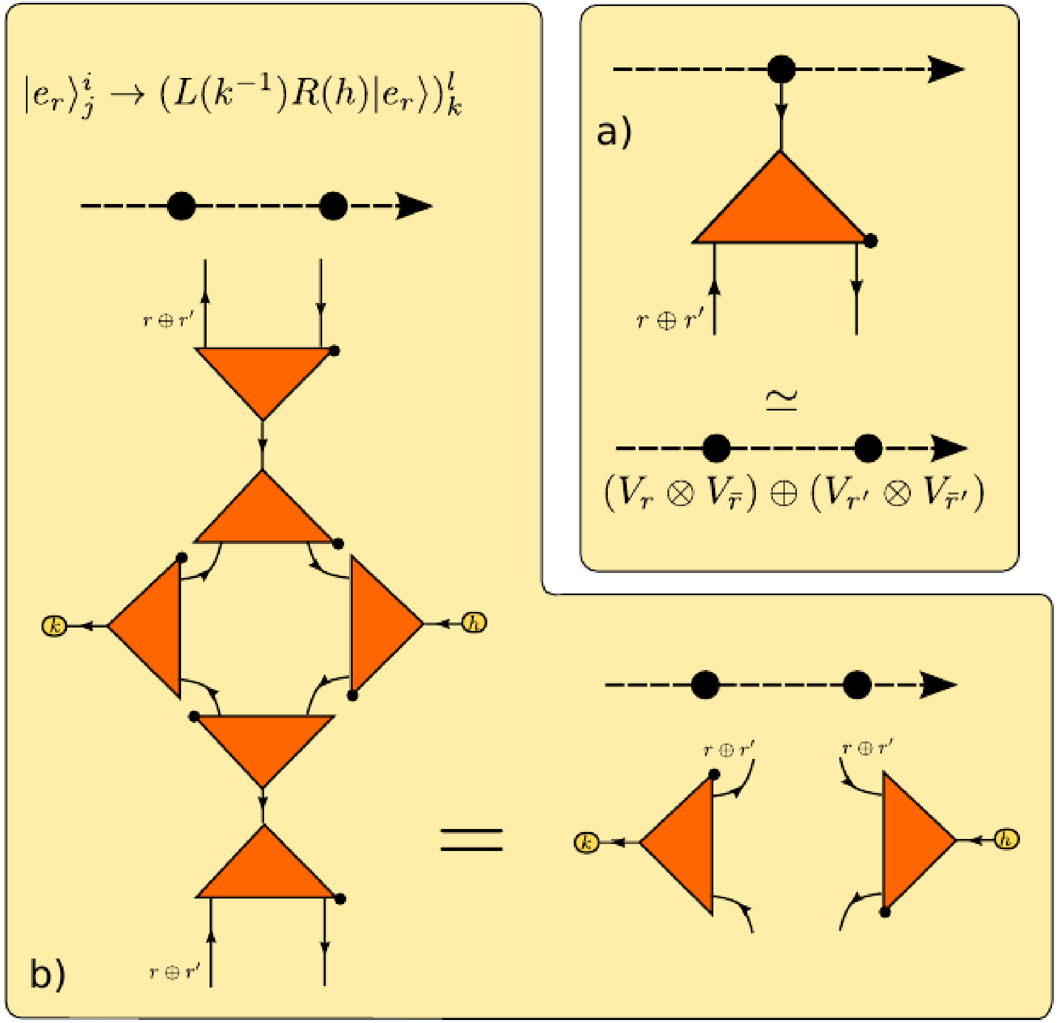
Hilbert space on link
when G continuous
is Infinite dim.

We can express it
as direct sum of irrep

Keep only few →

Finite dim. Hilbert
Space

Finite bond dim TN.



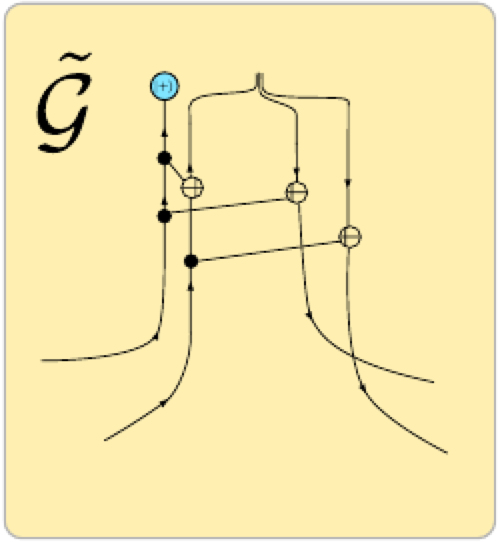
- Horn, D. *PLB* **100**, 149–151 (1981).
- Orland, P. & Rohrlich, D.
Nucl. Phys. B **338**, 647–672 (1990).
- Chandrasekharan, S. & Wiese, U.-J Nucl.Phys. B492 (1997) 455-474.
ArXiv:1405.4811

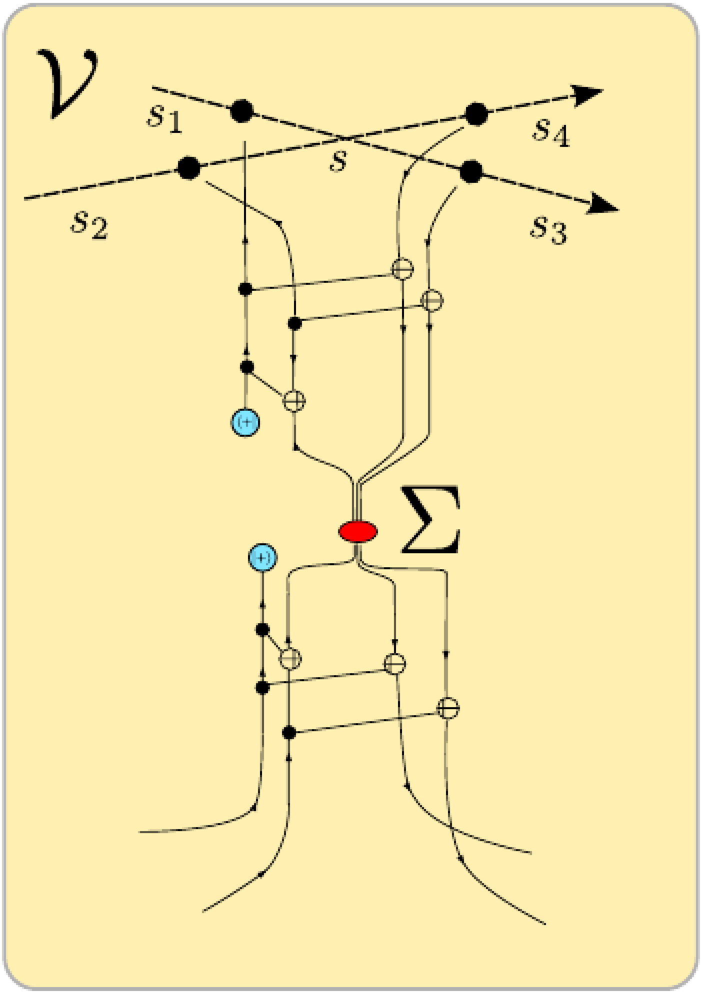
Vertex operators

Co-diagonal operators to
Symmetry constraints

$$\mathcal{V} = \tilde{G}\Sigma\tilde{G}^\dagger$$

Can be used to extend
the LGT Hamiltonian





Ardonne, E., Fendley, P. & Fradkin, E.
Ann. Phys. ***310,*** *493–551 (2004).*

ArXiv:1405.4811

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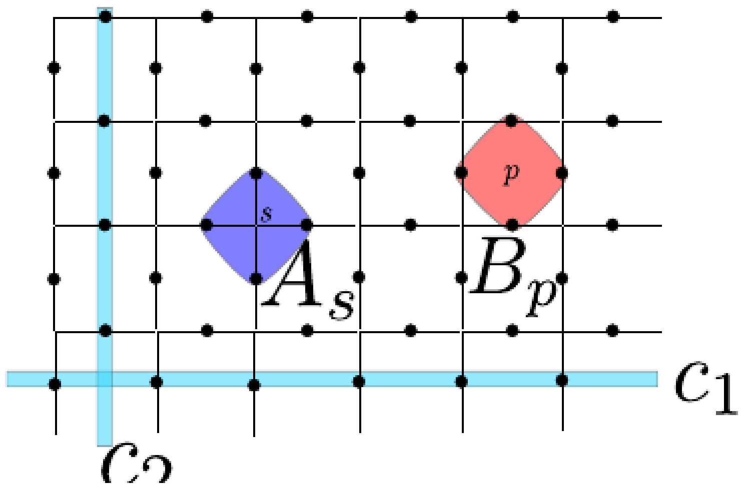
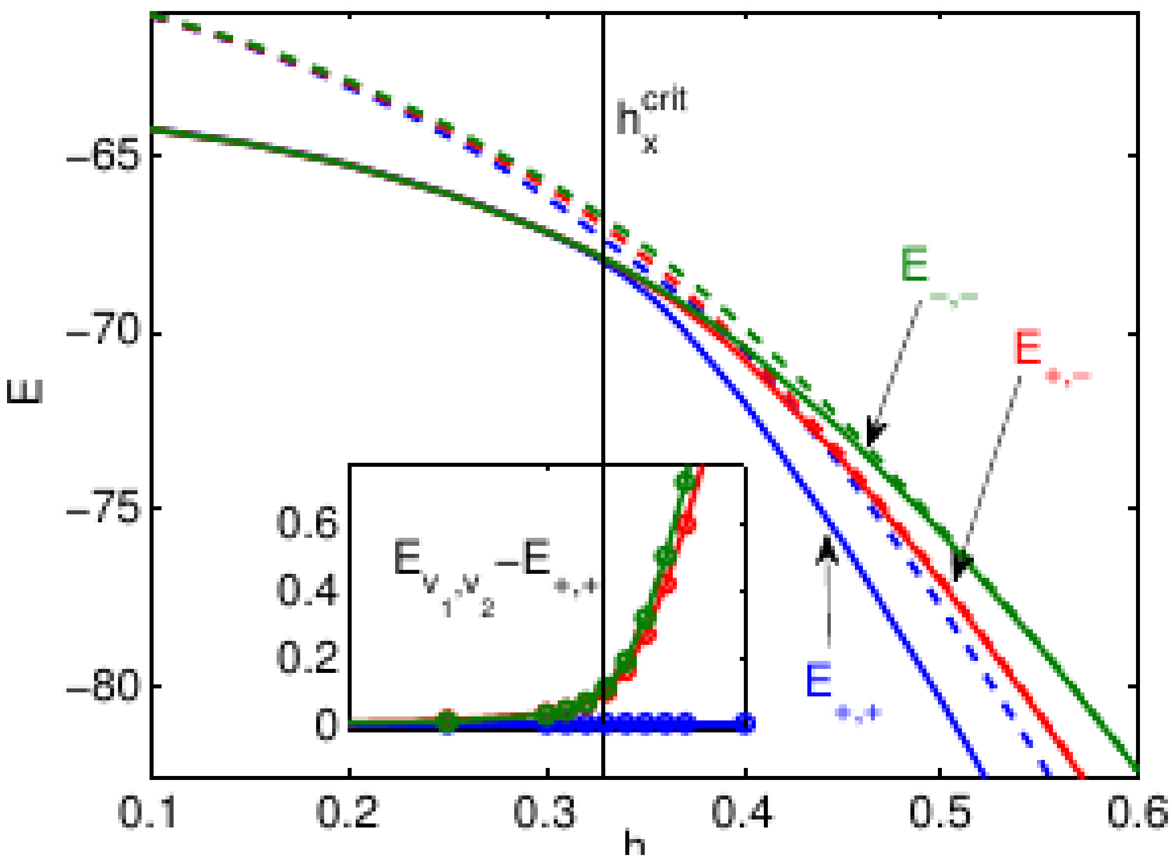
Low energy spectrum MERA

Z2 LGT 8x8 torus





Phys. Rev. B **83**, 115127 (2011)

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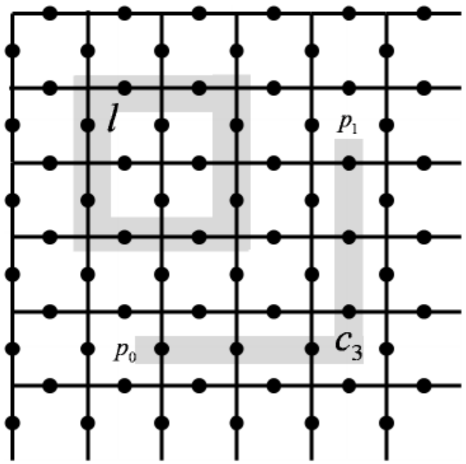
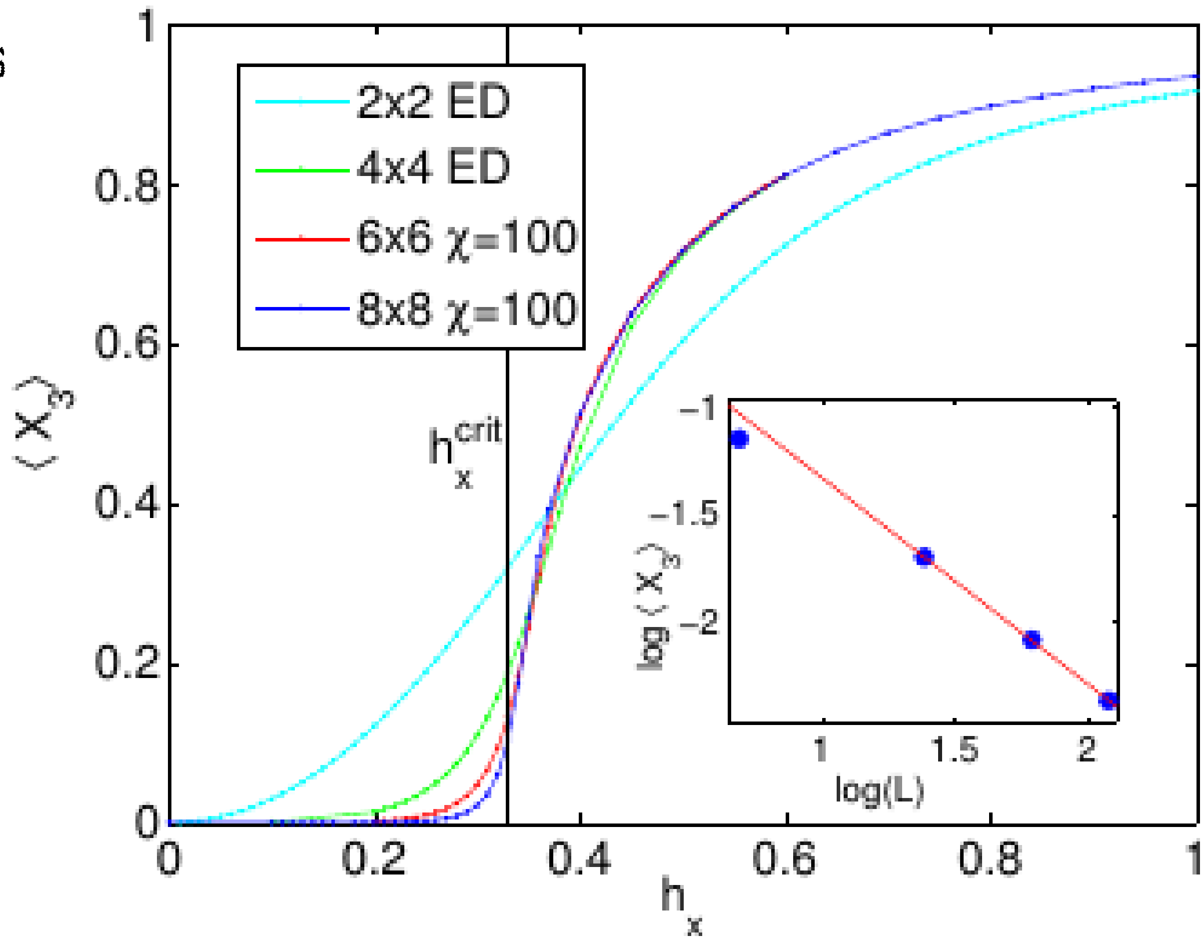
Disorder parameter MERA

Z2 LGT 8x8 torus

$$X_3 \equiv \prod_{j \in c_3} \sigma_j^x ,$$

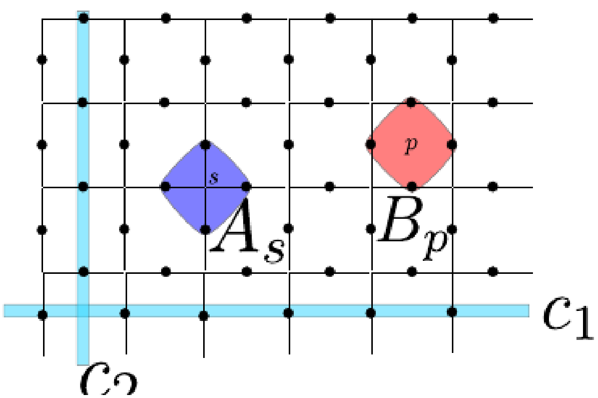


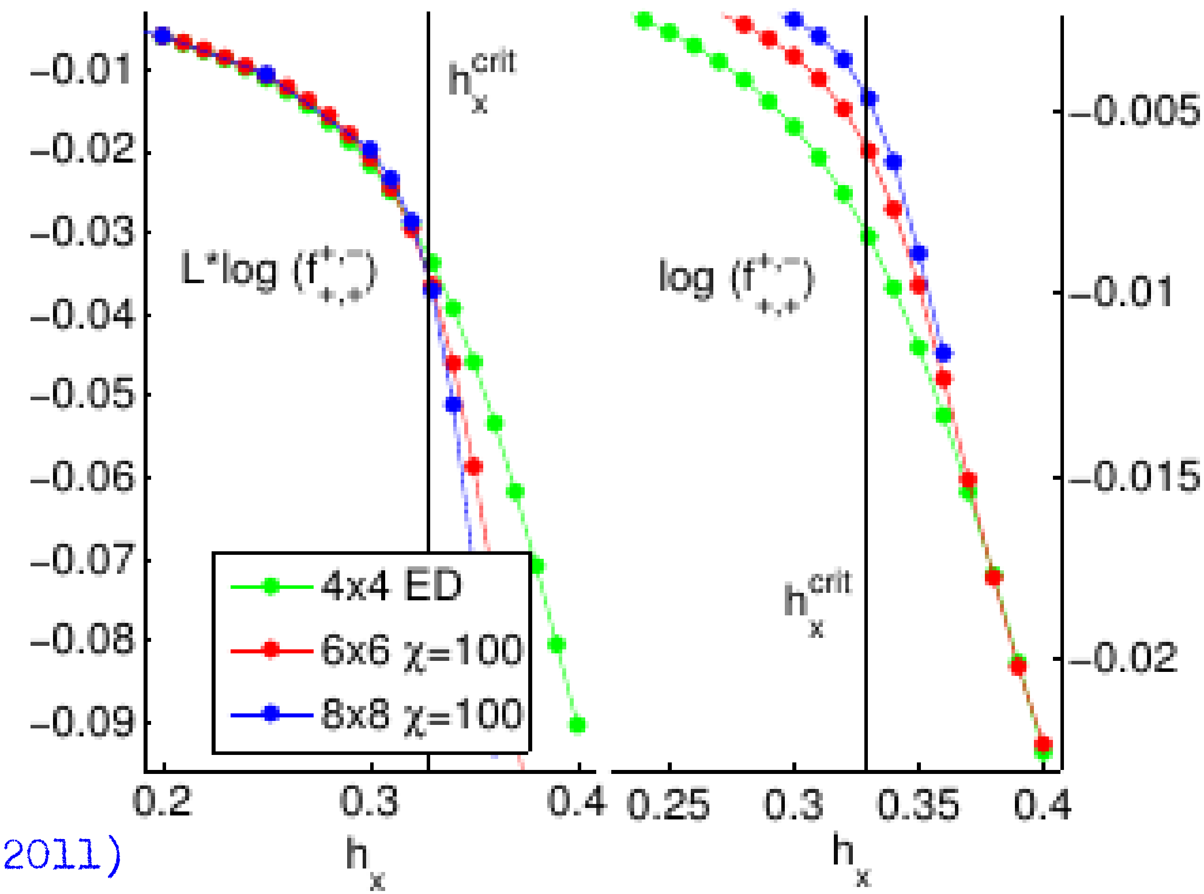


Phys. Rev. B **83**, 115127 (2011)

Topological fidelities
MERA

$$\log(f_{+,+}^{+,-}) \equiv \frac{1}{L^2}\log\left(\langle\Phi_{+,+}|Z_2|\Phi_{+,-}\rangle\right)$$





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Topological QPT with TPS

From the ground state of

$$H_{TC} = \sum_s A_s + \sum_p B_p$$

to the ground state of

$$H_{GM} = \sum_p \left[\left(a_{p_1} a_{p_2} a^\dagger_{p_3} a^\dagger_{p_4} + H.c. \right) - \left(a_{p_1} a_{p_2} a^\dagger_{p_3} a^\dagger_{p_4} + H.c. \right)^2 \right]$$

Through a wave function modification

$$|\psi(\theta)\rangle$$

$$|\psi(0)\rangle = |\Omega_{TC}\rangle \longrightarrow |\psi(\pi/2)\rangle = |\Omega_{GM}\rangle$$

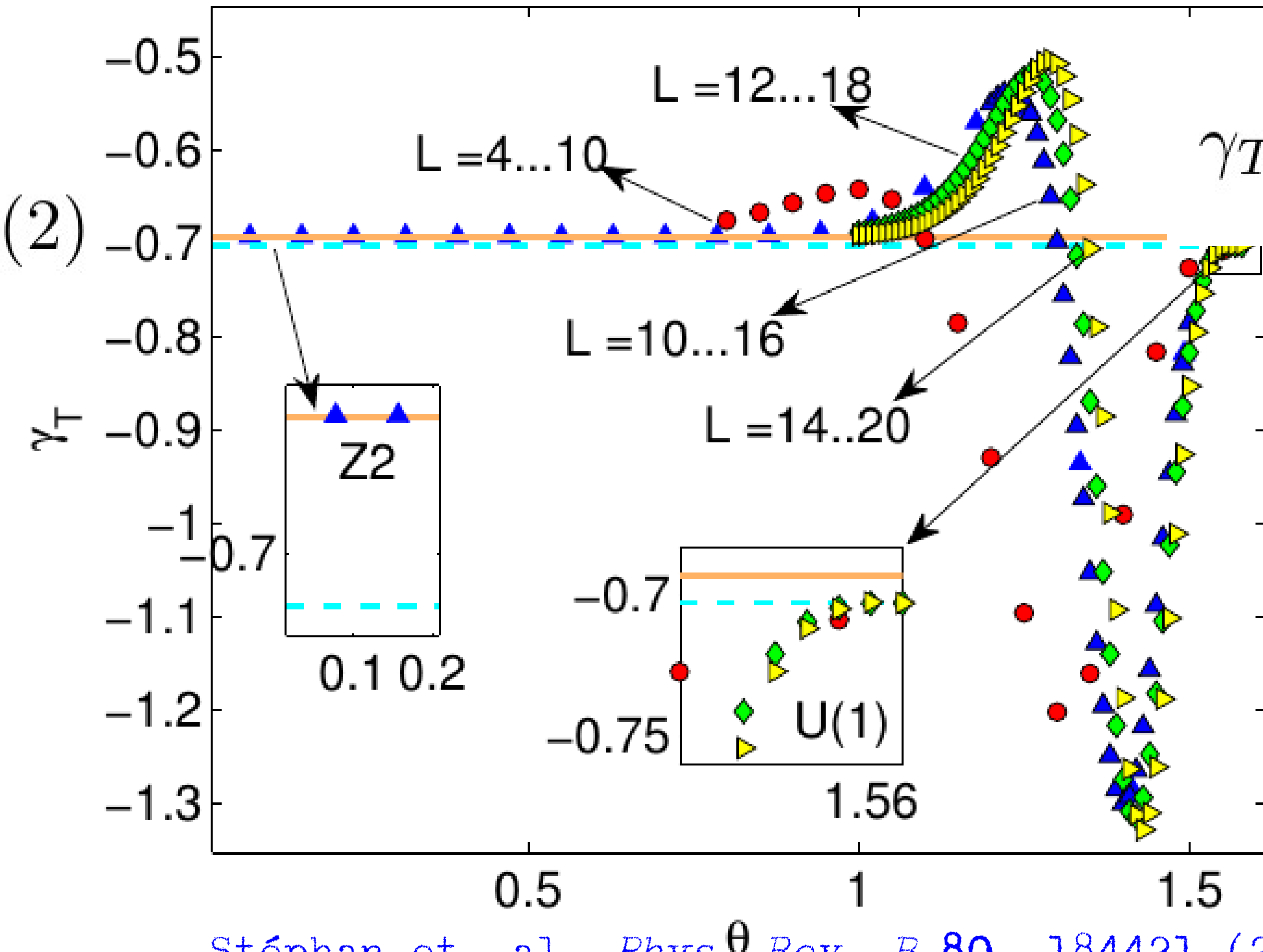
$$H(\theta) = H_{TC} + \mathcal{V}(\theta)$$

$$S = c_1 L + \gamma_T + c_2/L + \ldots$$

ArXiv:1405.4811

Topological Entropy

$$\gamma_T = -\log(2)$$



$$\gamma_T = \log(R) - \frac{1}{2}$$

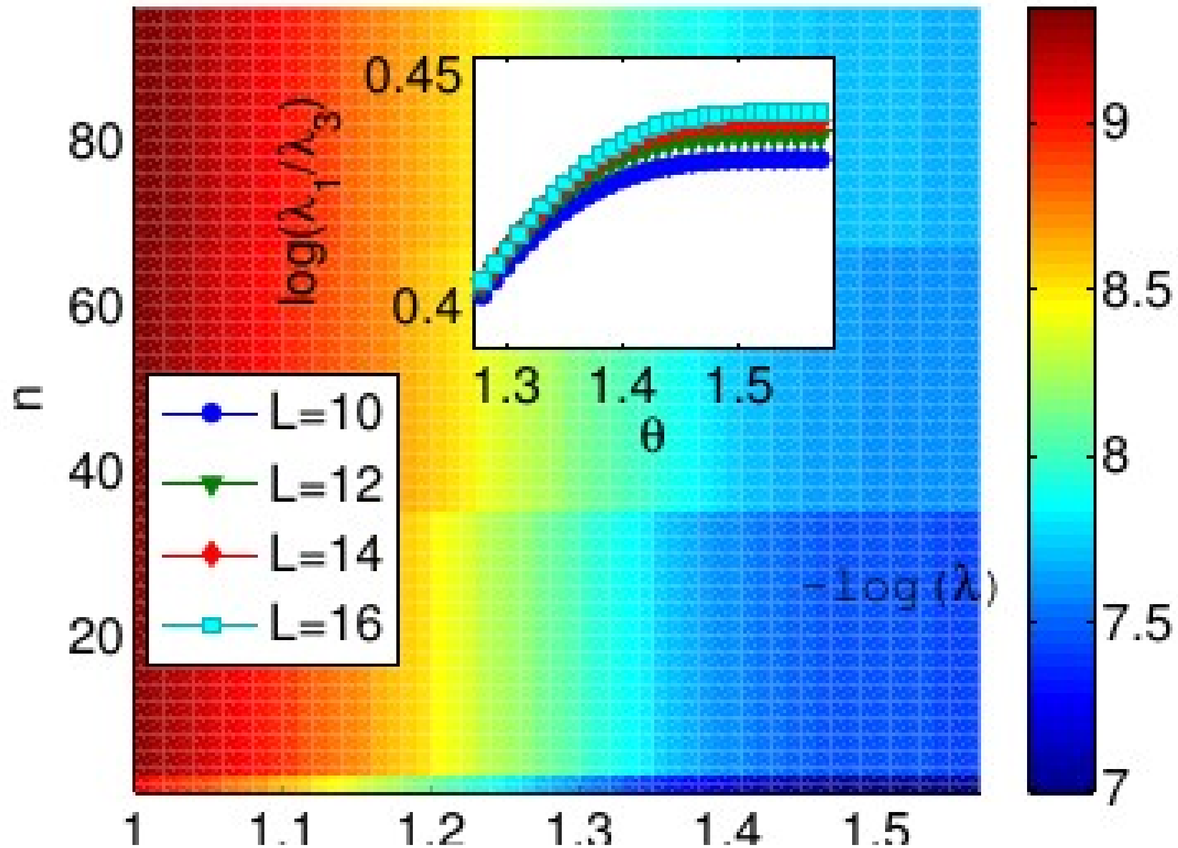
Stéphan et. al. *Phys. Rev. B* **80,** 184421 (2009).
Stéphan et. al. *J. Stat* **2012,** P02003 (2012).

ArXiv:1405.4811

Schmidt-gap

Li, H. & Haldane, F. D. M. *Phys. Rev. Lett.* **101,** 010504 (2008).
De Chiara et. al *Phys. Rev. Lett.* **109,** (2012).
A. Läuchli, arXiv:1303.0741

$$\rho_A = \sum_\alpha \lambda_\alpha |\alpha\rangle\langle\alpha|$$



Does not detect the topological phase transition

ArXiv:1405.4811

Luitz, D. et al. *J. Stat.* **2014,** P08007 (2014).

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Conclusions

- We have presented a TN framework to analyze LGT
- It is suted both for theoretical analysis and to design numerical ansatz
- Discrete, Continous Abelian and Non-Abelian model can be considered
- Both hierarchical TN and TPS/PEPS
- Already have benchmark numerical results in 2D
- Easily extended to include matter
- Interesting time to come...

THANKS FOR THE ATTENTION !!!