Symmetries and boundary theories for chiral Projected Entangled Pair States

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28 August 2014

- S. Yang
- H.-H. Tu
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- J. I. Cirac

- QCCC Elitenetzwerk Bayern
- Alexander von Humboldt Foundation
- EU Integrated Project SIQS

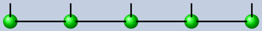
T. B. Wahl, H.-H. Tu, N. Schuch, and J. I. Cirac, Phys. Rev. Lett. **111**, 236805 (2013).

T. B. Wahl, S. T. Haßler, H.-H. Tu, J. I. Cirac, and N. Schuch, arXiv:1405.0447.

Tensor Network States

1D: Matrix Product States (MPS)

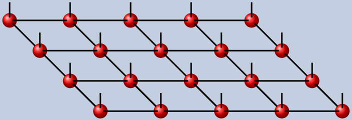
- efficient approximation of local Hamiltonians in 1D $\rightarrow$ DMRG
  F. Verstraete, and J. I. Cirac, Phys. Rev. B 73, 094423 (2006)



- classification of all phases in 1D
  F. Pollmann, A. M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B 81, 064439 (2010)
  X. Chen, Z.-C. Gu, and X.-G. Wen, Phys. Rev. B 83, 035107 (2011)
  N. Schuch, D. Pérez-García, and J. I. Cirac, Phys. Rev. B 84, 165139 (2011)

2D: Projected Entangled-Pair States (PEPS)

- presumably efficient approximation in 2D, proven for finite temperatures
  M. B. Hastings, Phys. Rev. B 76, 035114 (2007)

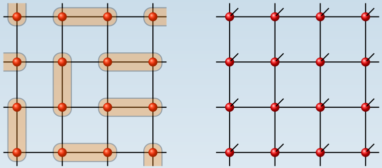


- no sign problem

Topological PEPS

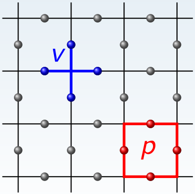
Resonating valence bond states

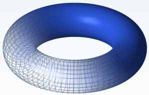
P. W. Anderson, Mater. Res. Bull. 8, 153 (1973)



Toric code

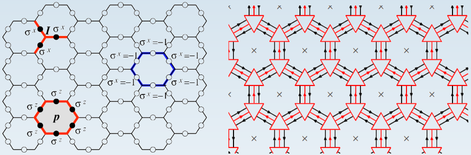
A. Kitaev, Ann. Phys. 303, 2 (2003)





String net models

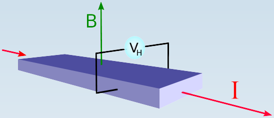
M. A. Levin, and X.-G. Wen, Phys. Rev. B 71, 045110 (2005)

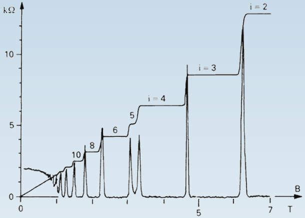


O. Buerschaper, M. Aguado, and G. Vidal, Phys. Rev. B 79, 085119 (2009)

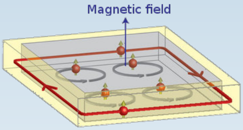
Chiral topological phases

1. Quantum Hall Effect





K. v. Klitzing, G. Dorda, M. Pepper, Phys. Rev. Lett. 45, 494 (1980)



D. Kong, and Y. Cui, Nature Chemistry 3, 845 (2011)

2. lattice systems: Chern insulator, $p + ip$ model

Recent discovery of chiral topological PEPS

J. Dubail, and N. Read, arXiv:1307.7726.

T. B. Wahl, H.-H. Tu, N. Schuch, and J. I. Cirac, Phys. Rev. Lett. 111, 236805 (2013).

Goals

- properties of chiral PEPS
- characterize all low-rank chiral PEPS

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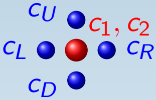
PEPS construction

$$|\Psi_1\rangle = (\alpha_0 + \sum_{n=L,R,U,D} \alpha_n c_n a^\dagger)|\mathrm{vac}\rangle$$



Majorana modes: $c^\dagger = c$

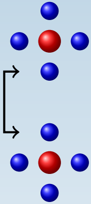
$\frac{1}{2}(c_1 - ic_2) = a$

$\frac{1}{2}(c_1 + ic_2) = a^\dagger$

PEPS construction

$|\Psi_1\rangle =$

$\langle 1|_{D1,U2}$



Majorana modes: $c^\dagger = c$

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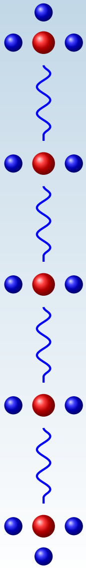
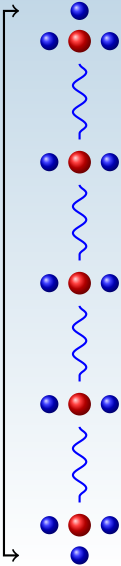
PEPS construction

$|\Psi_2\rangle =$

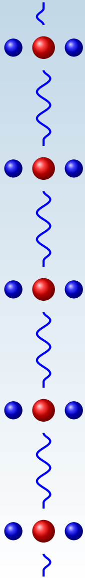
PEPS construction



PEPS construction



PEPS construction



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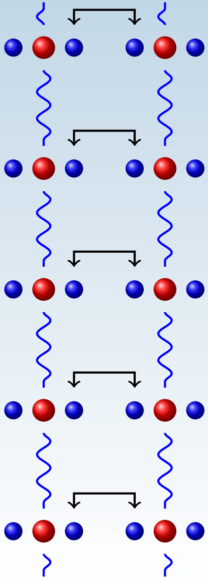
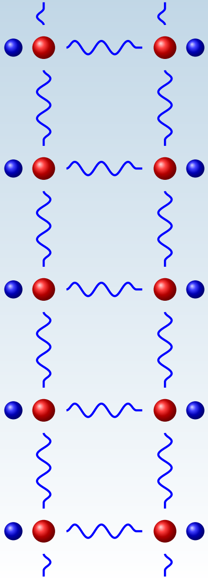


PEPS construction



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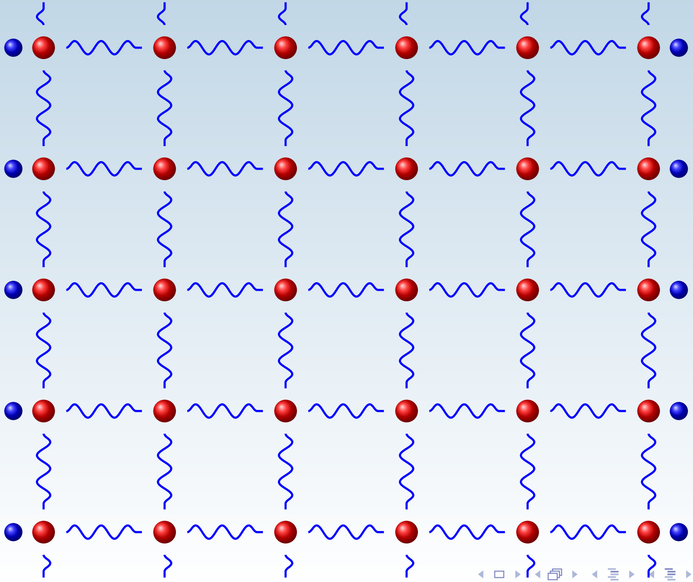
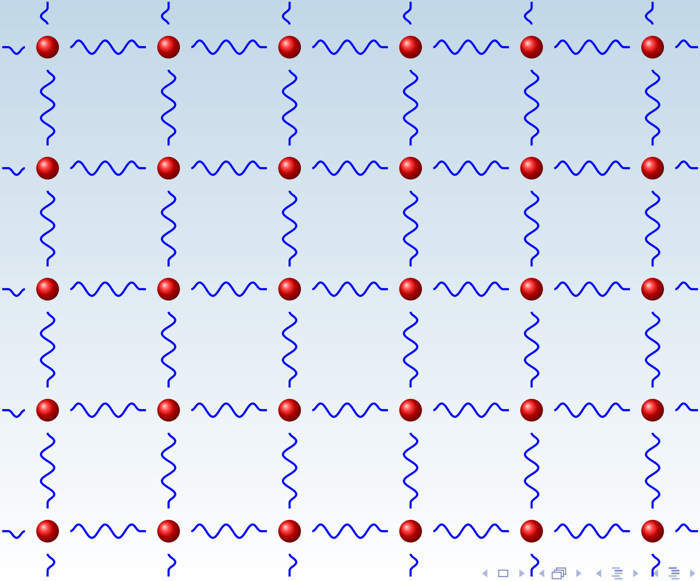


PEPS construction

$|\Phi_{\mathrm{PEPS}}\rangle =$

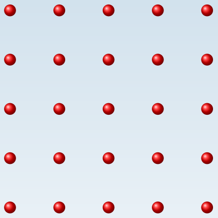


Three ways to detect chiral edge modes

Free fermion systems

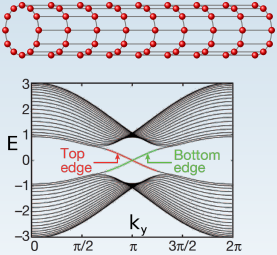
$$H = \sum_{i,j} T_{i,j} a_i^\dagger a_j + \Delta_{i,j} a_i^\dagger a_j^\dagger + \overline{\Delta}_{i,j} a_j a_i$$

1. Chern number *(mathematical)*



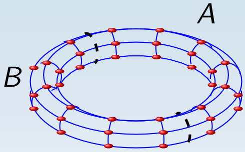
$$C = \int \mathrm{d}k_x \mathrm{d}k_y \mathcal{F}(k_x, k_y)$$

2. energy spectrum *(physical)*



Rechtsman et al., Nature 496, 196 (2013)

3. entanglement spectrum *(mathematical)*



$$\rho_A = \mathrm{tr}_B(|\Phi\rangle\langle\Phi|) \propto e^{-H_{\mathrm{ent}}}$$

For free fermion systems all are equivalent!

A. Kitaev, Ann. Phys. 321, 2 (2006).
L. Fidkowski, Phys. Rev. Lett. 104, 130502 (2010).

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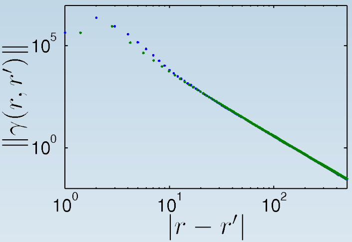
Family of topological Superconductors

$$|\Psi_1\rangle = [\sqrt{1-\lambda} + \sqrt{\lambda} a^\dagger b^\dagger] |\text{vac}\rangle$$

$$b = \frac{1}{\sqrt{8}}(e^{i\frac{\pi}{4}}(c_L + ic_R) - (c_U + ic_D))$$

Chern number:
$C = -1$

2-point correlations



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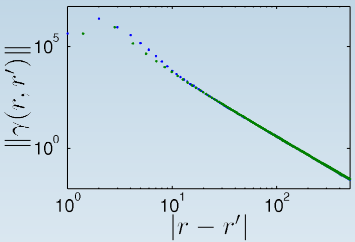


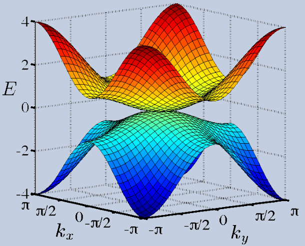
Flat band Hamiltonian

- long-range
- stable to perturbations

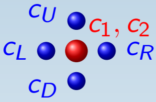
Frustration free Hamiltonian

$H_{\mathrm{ff}} = \sum_j h_j$

- local
- gapless

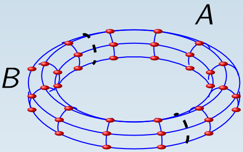


A short Summary ...



$$|\Psi_1\rangle = (\alpha_0 + \sum_{n=L,R,U,D} \alpha_n c_n a^\dagger)|\text{vac}\rangle$$

1. Chern number
2. diagonalize Hamiltonian on cylinder
3. entanglement spectrum



$$\rho_A = \mathrm{tr}_B(|\Phi\rangle\langle\Phi|) \propto e^{-H_{\mathrm{ent}}}$$

Free fermion chiral PEPS

- long-range correlations
- long-range flat band Hamiltonians
- local, gapless Hamiltonians

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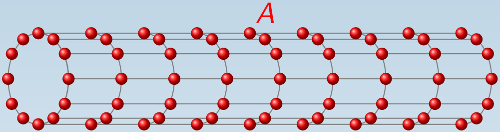
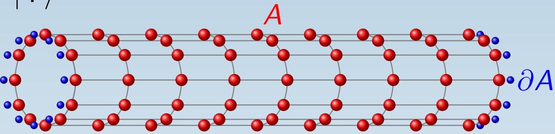
Spectra of the boundary Hamiltonian



$$\rho_A = \mathrm{tr}_B(|\Phi_{\mathrm{PEPS}}\rangle\langle\Phi_{\mathrm{PEPS}}|) \propto e^{-H_{\mathrm{ent}}}$$

Spectra of the boundary Hamiltonian

$|\Psi\rangle =$



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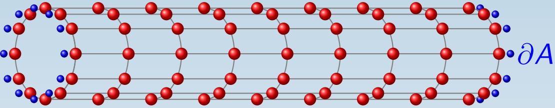
$$= \mathrm{tr}_{\partial A}(|\Psi\rangle\langle\Psi|) \leftrightarrow \sigma_{\partial A} = \mathrm{tr}_A(|\Psi\rangle\langle\Psi|) \propto e^{-H_{\mathrm{b}}}$$

Spectra of the boundary Hamiltonian

$|\Psi\rangle =$



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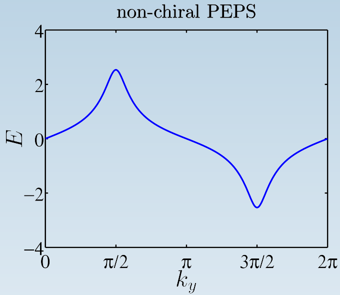

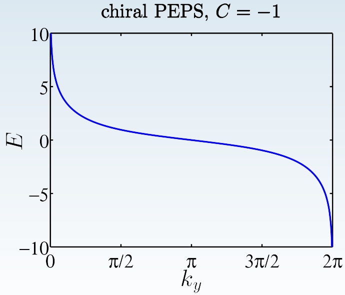

Spectra of the boundary Hamiltonian

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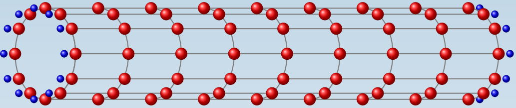

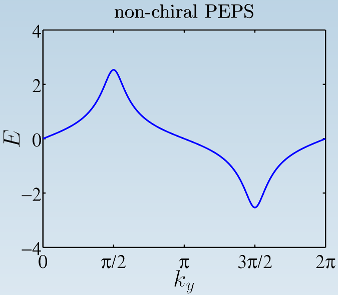

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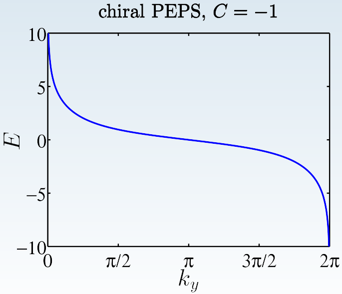
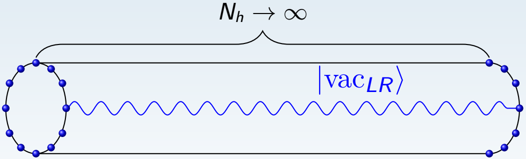
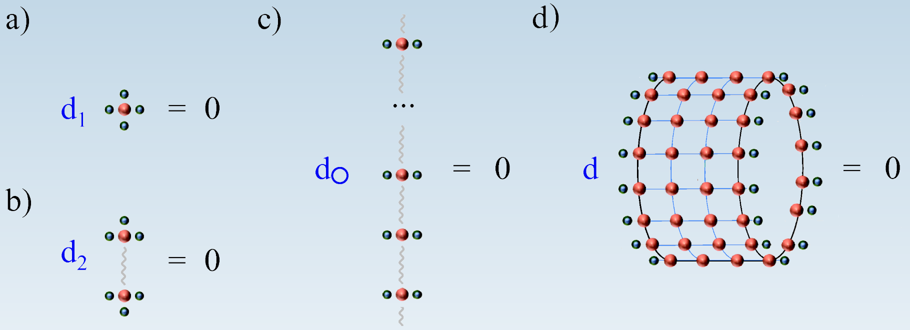




$c_L(k_y=0) = \frac{1}{\sqrt{N_v}} \sum_y c_{y,L}$ $\quad c_R(k_y=0)$

$$d|\Psi\rangle = d|\mathrm{vac}_{LR}\psi_{\mathrm{rest}}\rangle = 0$$

Local vs. global Symmetries



Necessary and sufficient condition for $C = \pm 1$

there exists exactly **one** $d_1|\Psi_1\rangle = 0$ with
$d_1 = \alpha(c_L \pm ic_R) + \beta(c_U \pm ic_D)$

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Rényi entropies

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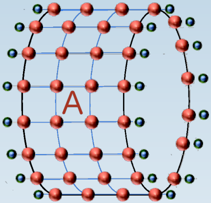
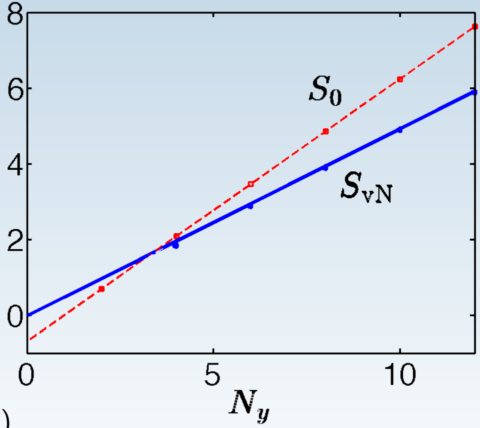


= 0



Rényi entropies:

zero Rényi entropy:

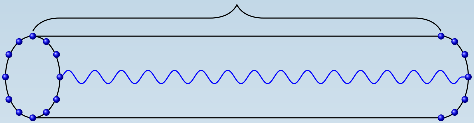
$S_0 = \log(\mathrm{rank}(\rho_A))$

von Neumann entropy:

$S_{\mathrm{vN}} = S_1 = -\mathrm{tr}(\rho_A \log(\rho_A))$

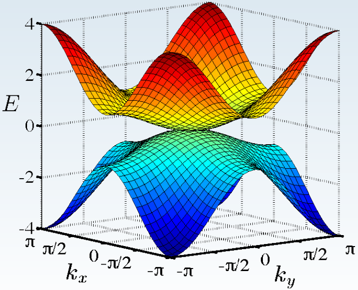
Constructing the ground states

$N_h \to \infty$

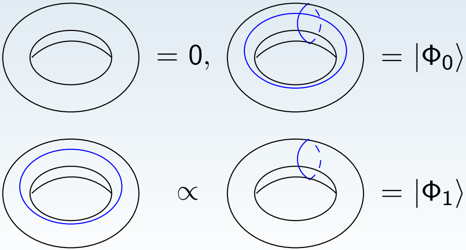


$c_L(k_y\!=\!0)\!=\!\frac{1}{\sqrt{N_v}}\sum_y c_{y,L}$ $c_R(k_y\!=\!0)$

gapless frustration free parent Hamiltonian $H_{\text{ff}} = \sum_j h_j$:

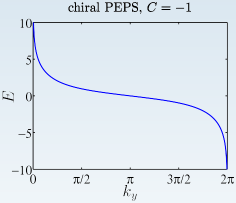


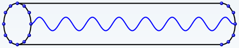
insert string operators $c_L(k_y = 0)$:

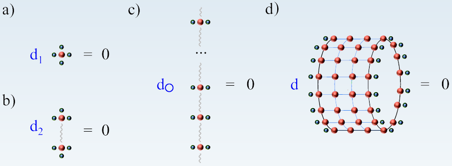


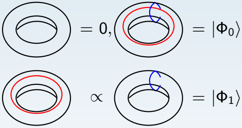
Summary and Outlook

1. flat band Hamiltonian: long-range, chiral
   frustration free Hamiltonian: gapless
2. read off Chern number from boundary Hamiltonian
3. chiral PEPS form pure vacuum modes between the edges
4. local symmetry $d_1 \rightarrow$ global symmetry





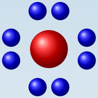




Outlook: Interacting chiral systems: Gutzwiller projection

Chiral example with $C = 2$

$|\Psi_1\rangle =$



c_1, c_2, c_3, c_4

Chiral example with $C = 2$

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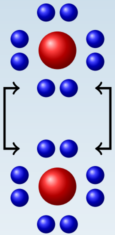
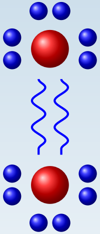


Chiral example with $C = 2$

$|\Psi_2\rangle =$



Chiral example with $C = 2$

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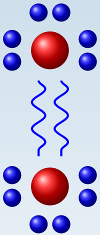
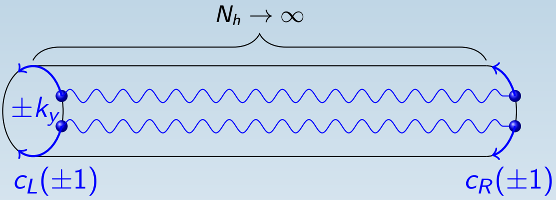
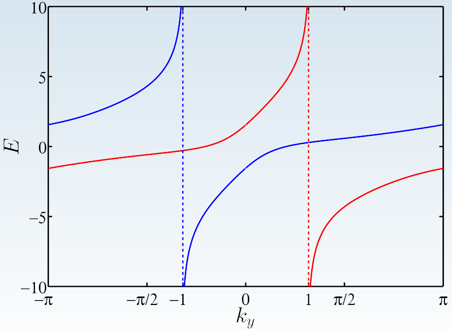


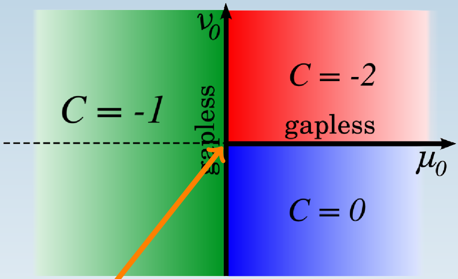
$d_{1,\pm}|\Psi_1\rangle = 0$

$d_{2,\pm}|\Psi_2\rangle = 0$ etc.





Frustration free Hamiltonian



PEPS