# Soliton Molecules and Optical Rogue Waves

Benasque, October 2014

INSTITUT FÜR PHYSIK

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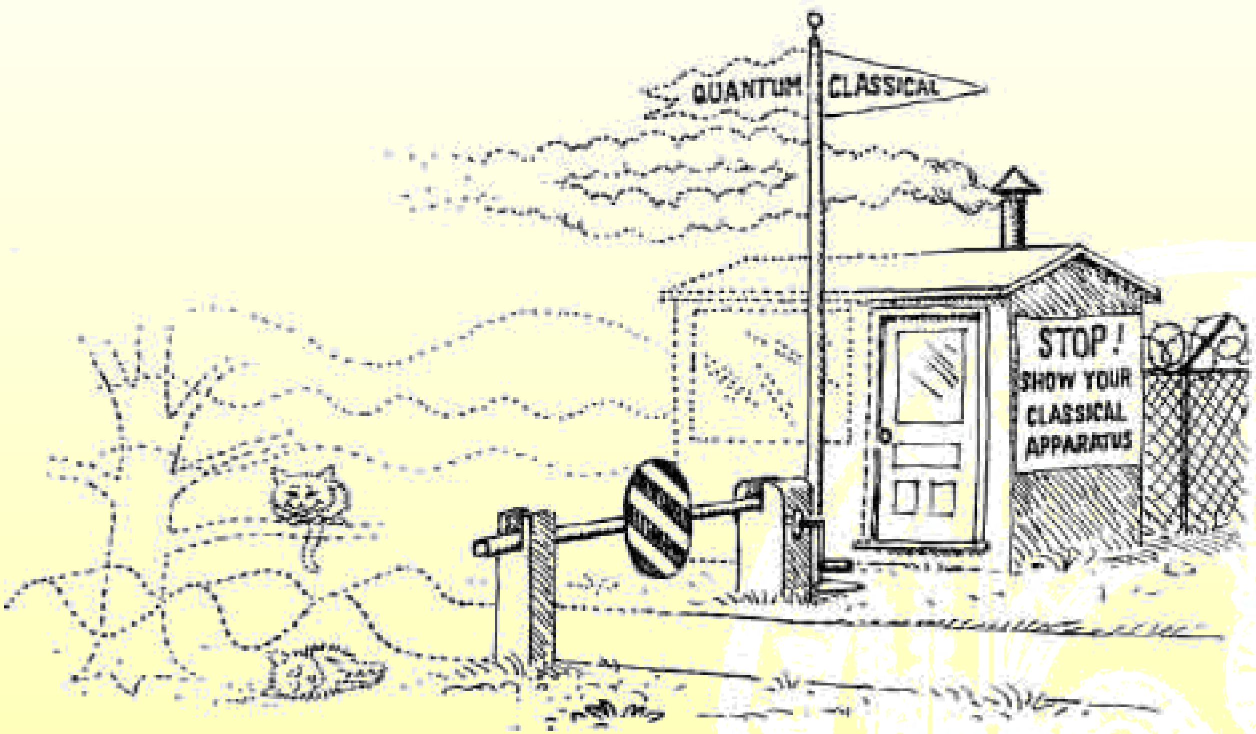



**Benasque School on Quantum Optics and Nonlinear Optics**

## My Program for this week:

I) Optical Fiber – How Does It Work?

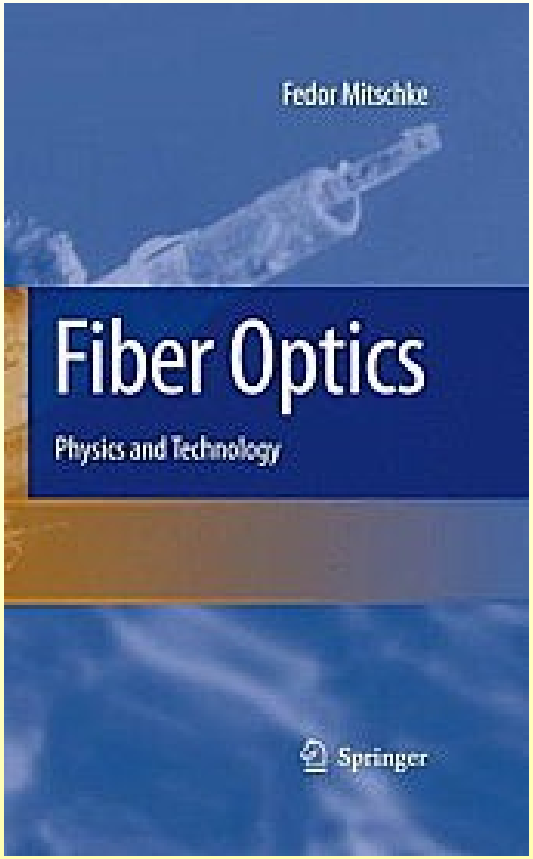
II) Nonlinearity in Fiber Propagation

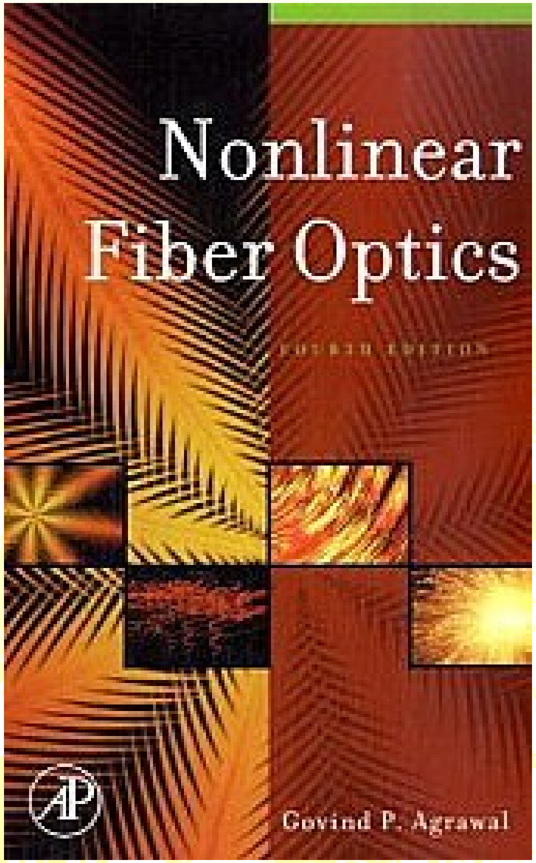
III) Solitons, Breathers, etc., and some applications

IV) State-of-the-art telecommunication and Soliton Molecules

V) Supercontinuum and Rogue Waves

# Literature

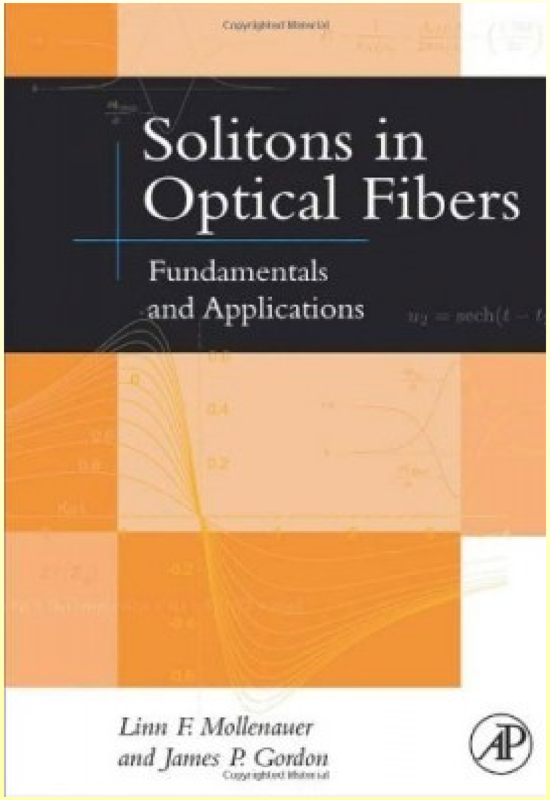


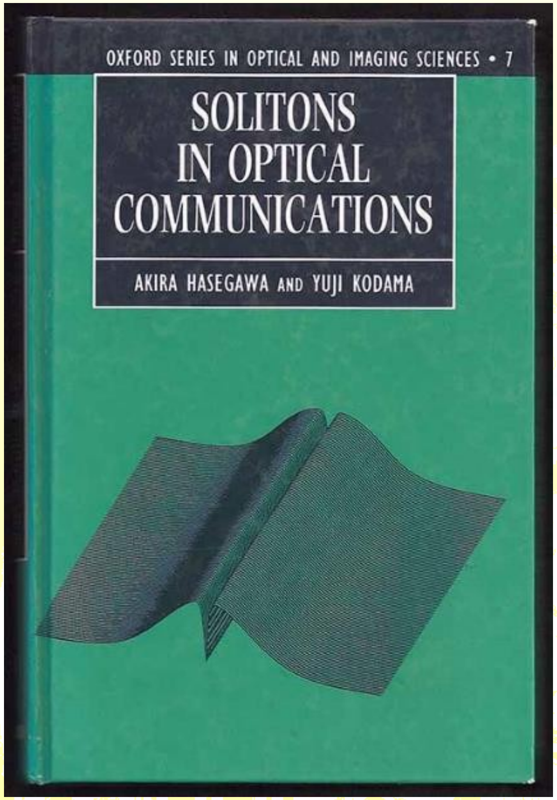




General textbook on Fiber Optics, including both technological aspects and nonlinear phenomena

Fiber Optics, specialized to nonlinear phenomena. Meanwhile, fifth edition!

# Literature




Fiber Optics, in particular
for telecom applications

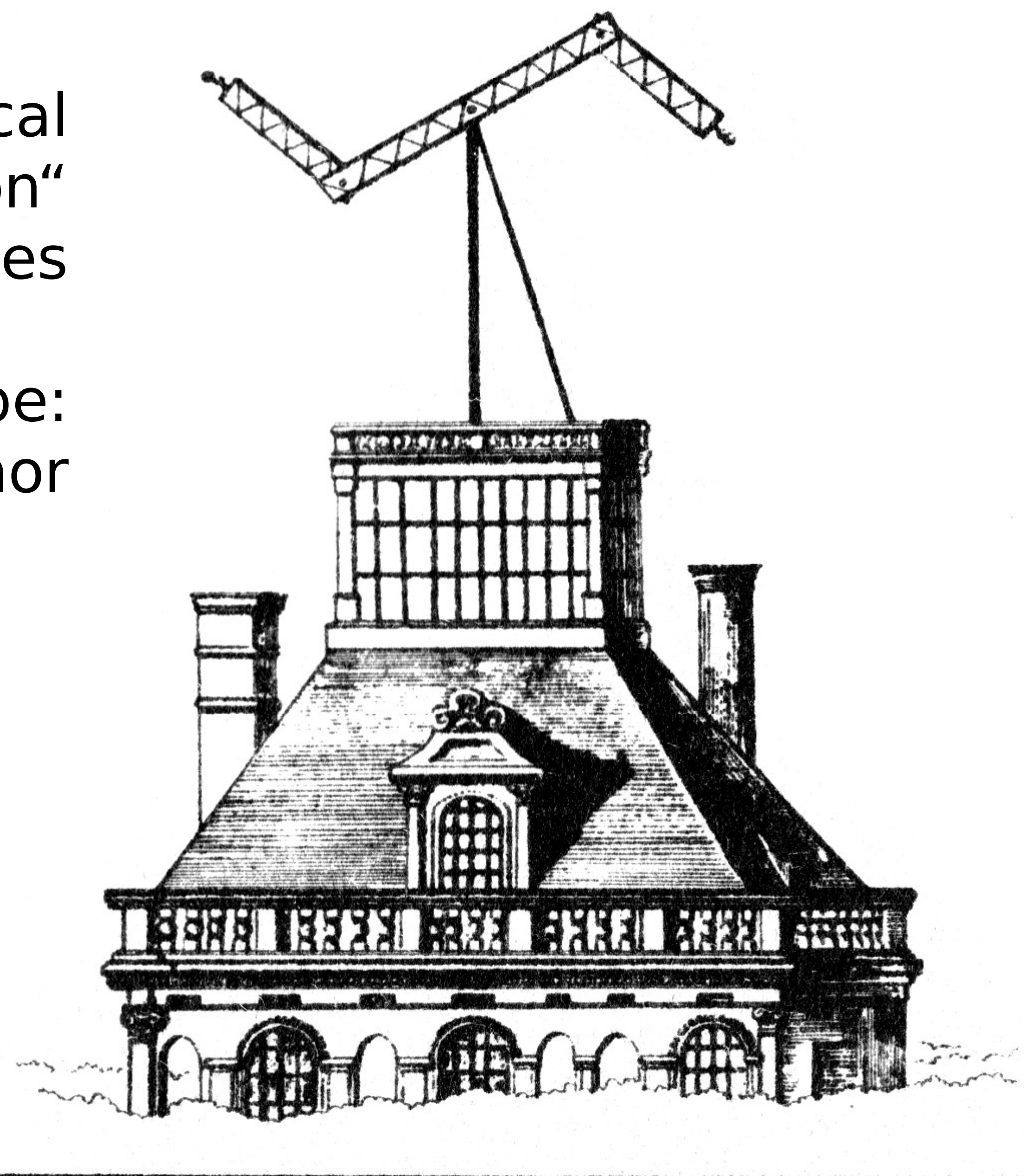
Fiber Optics for telecom applications,
the theorist‘s view

# Part I

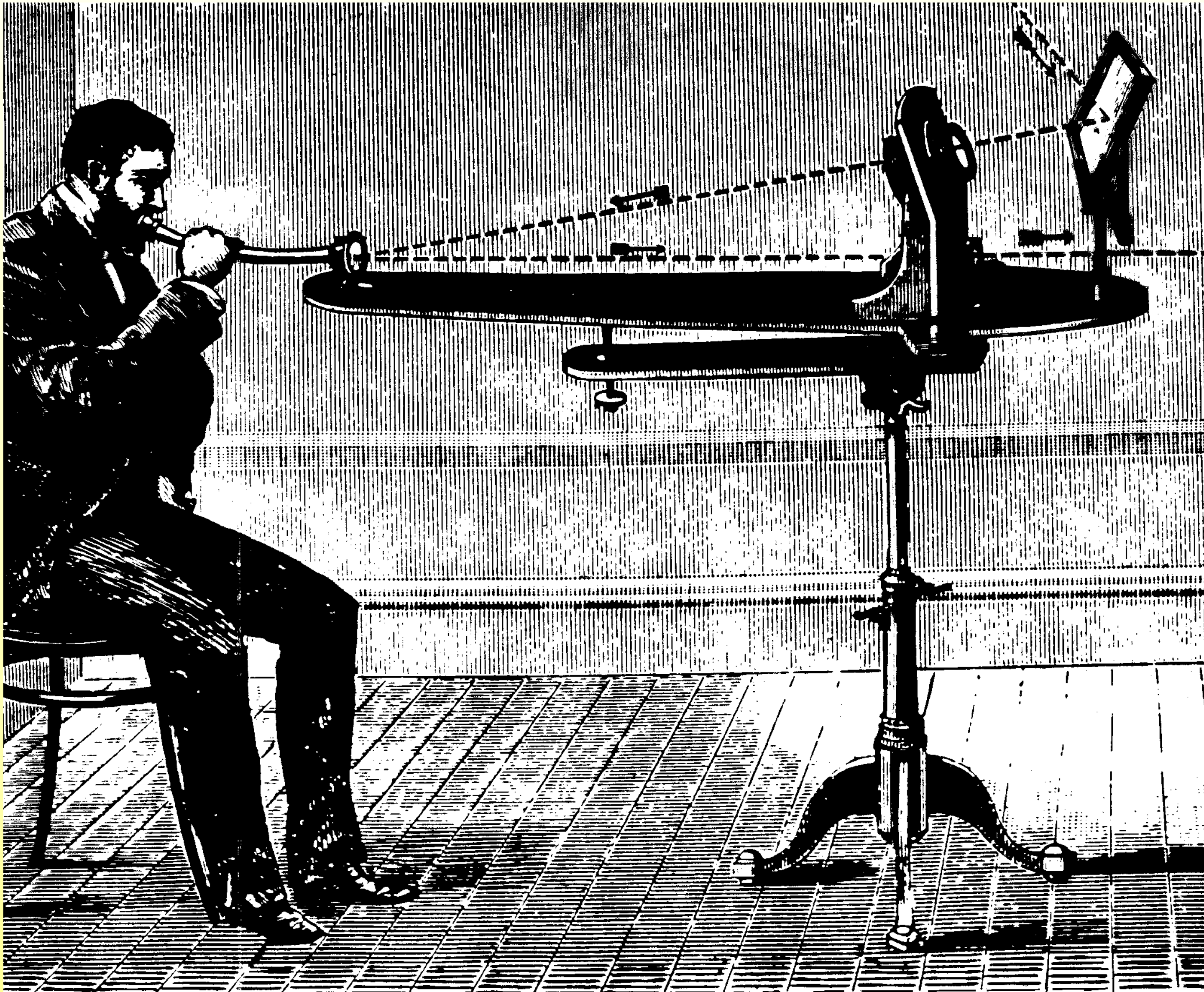
# Optical Fiber:
# Why, What, How

„Optical telecommunication“ in Napoleonic times

Claude Chappe: Semaphor



The „photophone“
by Alexander G. Bell
ca. 1880

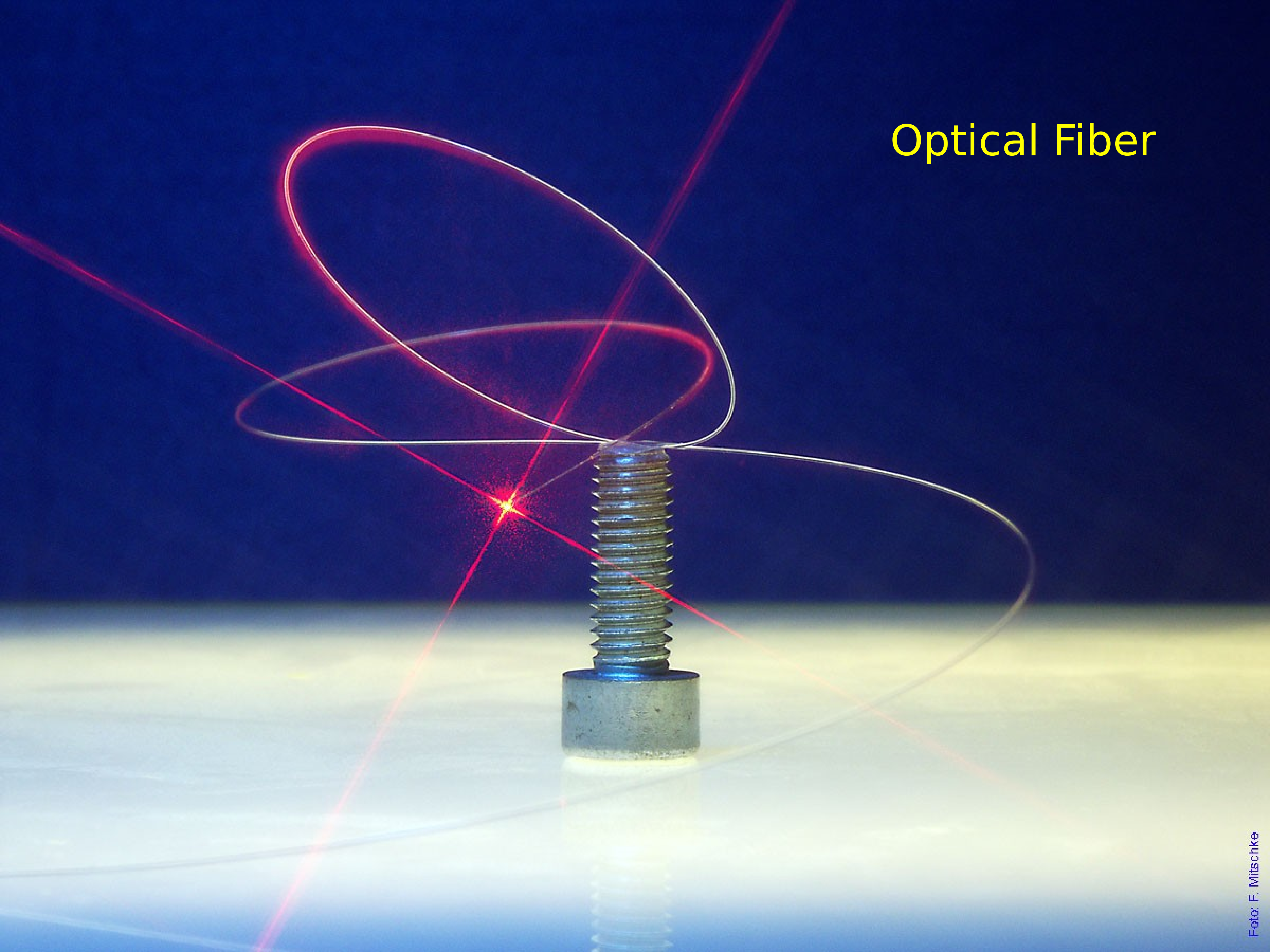


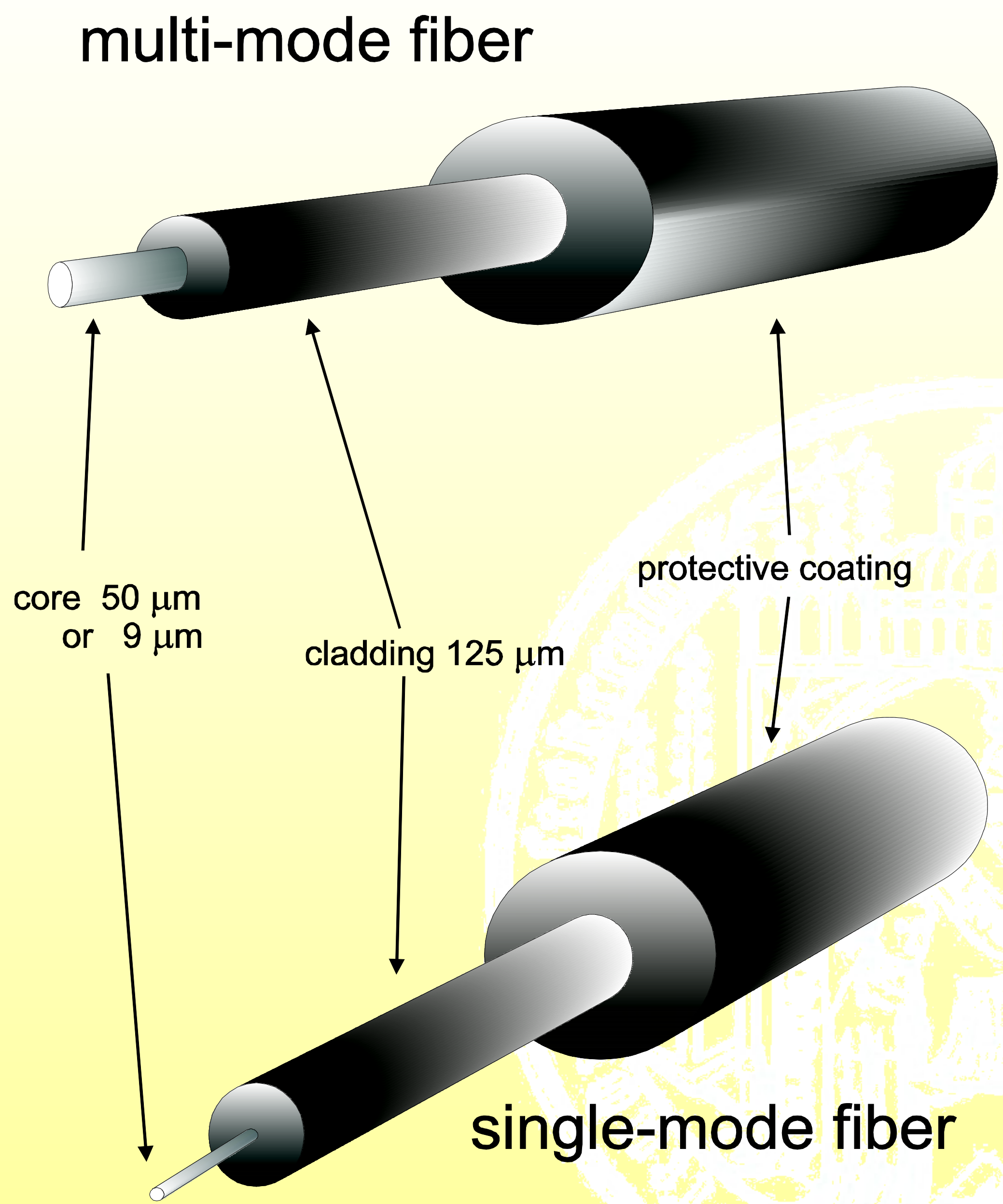
Two items were missing in Bell`s design:

a) Perpetual sunshine
b) Perpetual clear line-of-view from transmitter to rece

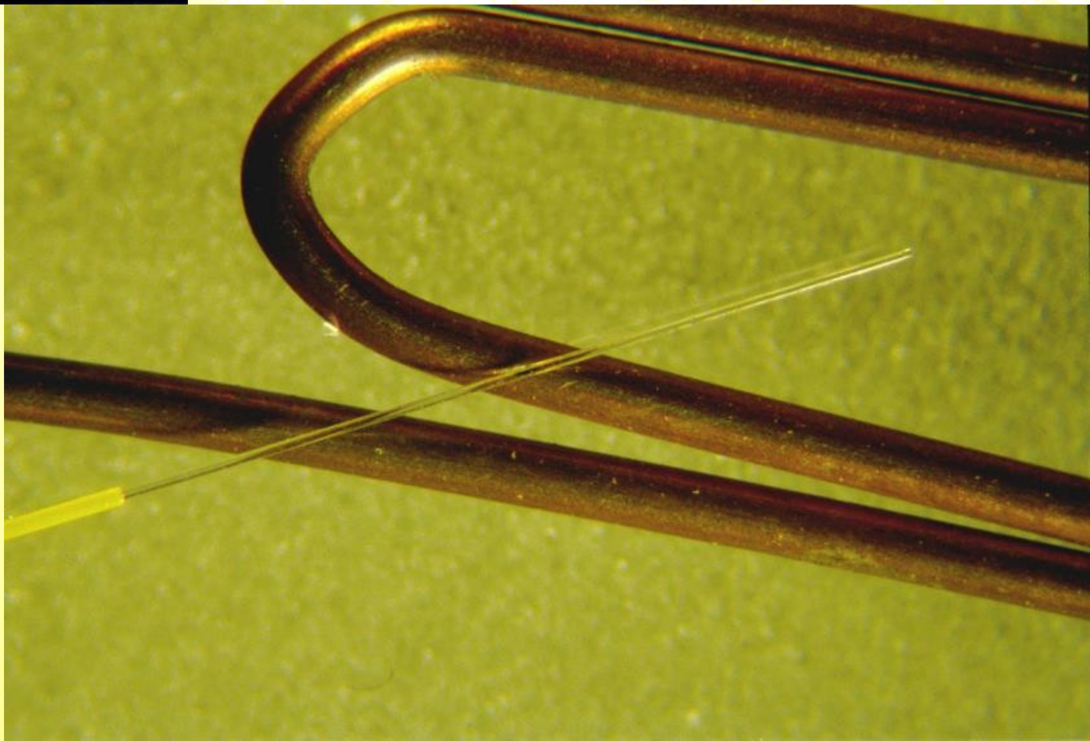
Ad a)

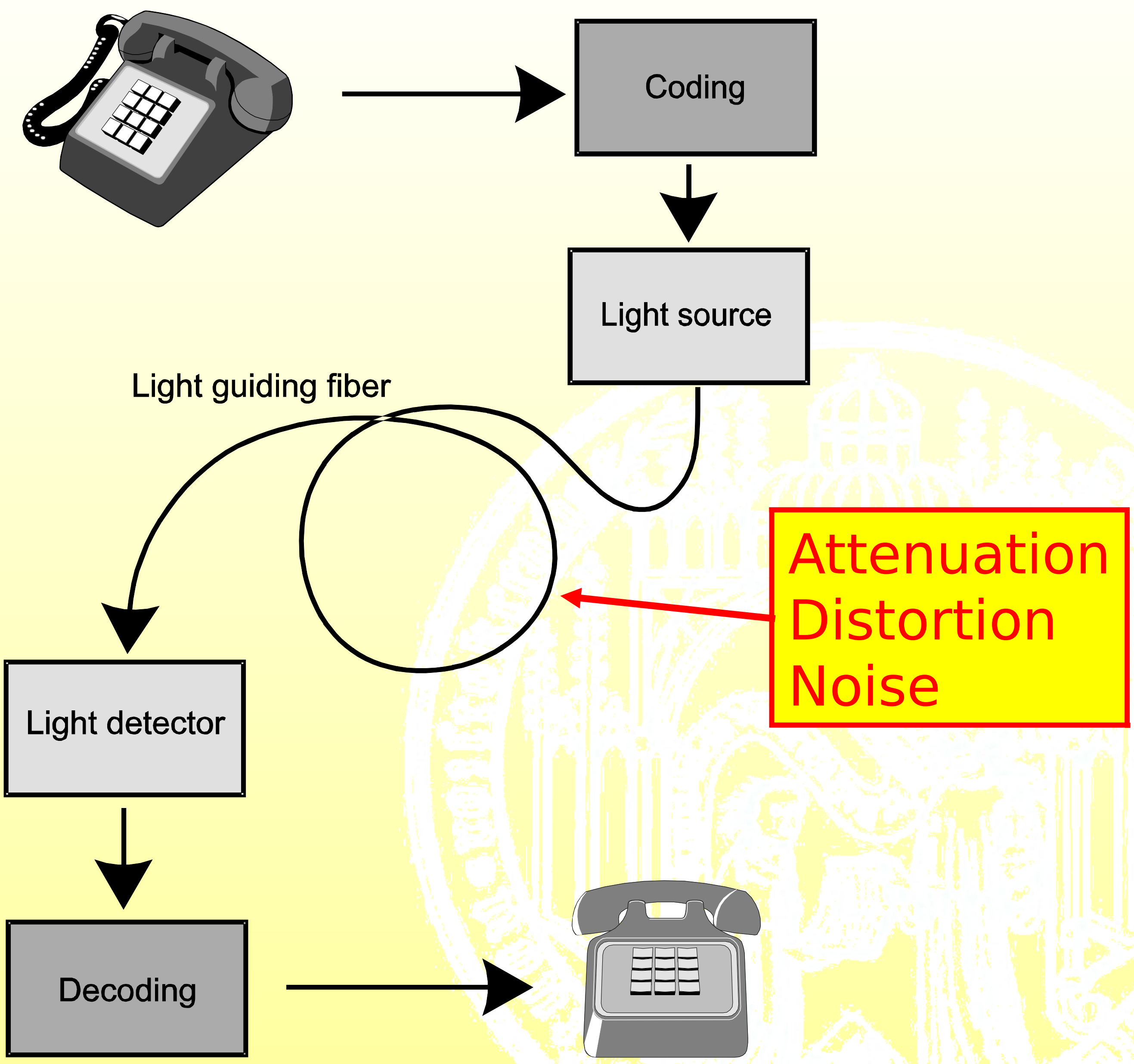

packaged laser diode with monitor, cooler, pigtail


Optical Fiber
Foto: F. Mitschke


multi-mode fiber
core 50 µm
or 9 µm
cladding 125 µm
protective coating
single-mode fiber

Size of an optical fiber
in comparison to matc
paper clip




Coding
Light source
Light guiding fiber
Attenuation
Distortion
Noise
Light detector
Decoding

# Decibel: a logarithmic measure

> One Bel denotes an order of magnitude in the ratio of two quantities both of which are of the dimension of power or energy.

„Bel“ is named after Alexander Graham Bell, „deci“ is the prefix for $10^{-1}$. Bel is used exclusively with prefix deci: dB (pronounced “decibel“ or „dee-bee“).

## Definition

$$\delta[\mathrm{Bel}] = \log_{10}\frac{P_1}{P_0} \qquad (P_i \text{ are powers})$$

Thus,

$$\delta[\mathrm{dB}] = 10\log_{10}\frac{P_1}{P_0}$$

Note: power is always proportional to (amplitude squared). Hence

$$\delta[\mathrm{dB}] = 10\log_{10}\frac{A_1^2}{A_0^2} = 10\log_{10}\left(\frac{A_1}{A_0}\right)^2 = 20\log_{10}\frac{A_1}{A_0} \qquad (A_i \text{ are amplitudes})$$

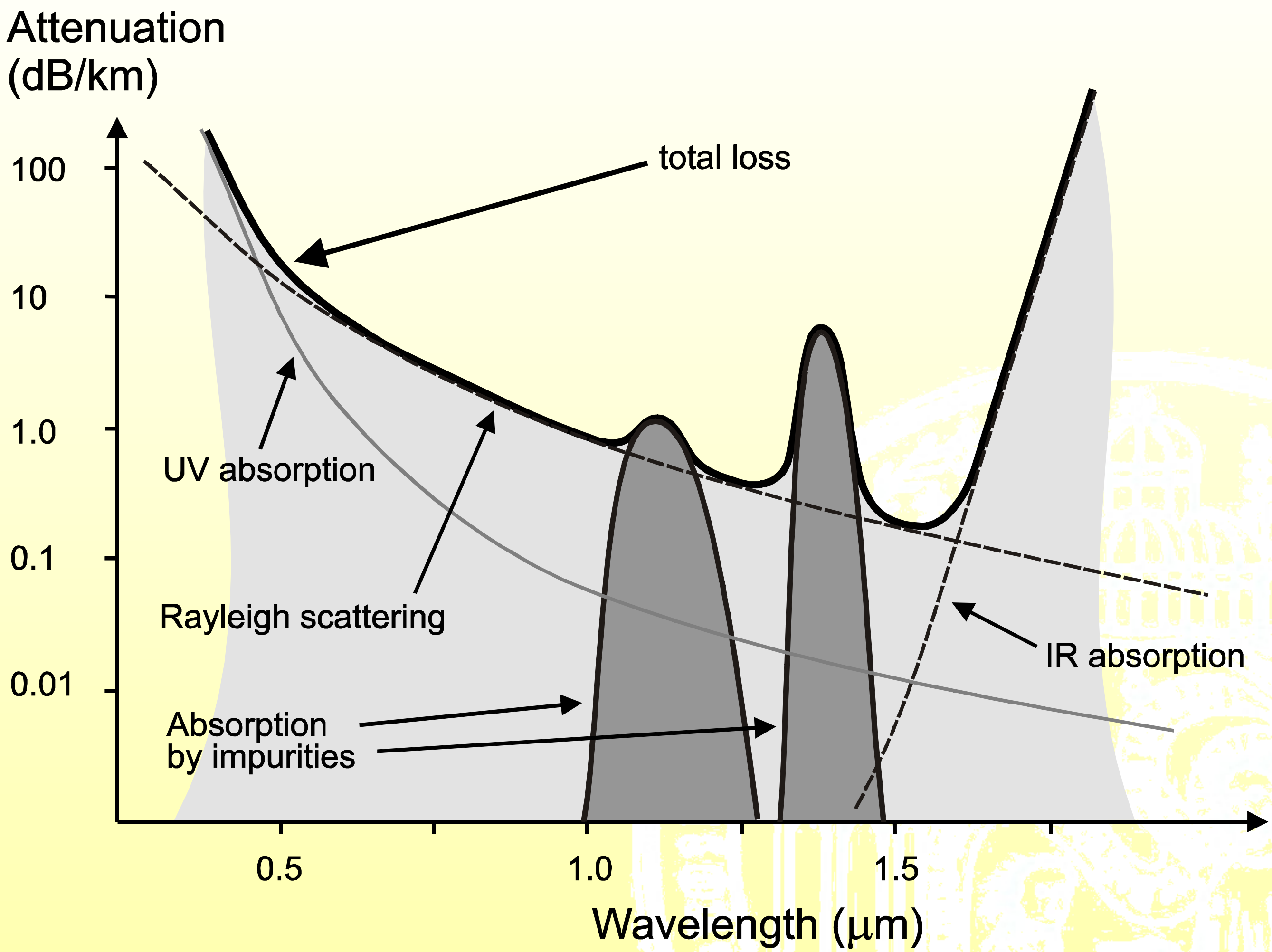
## Examples

**40 dB** correspond to $10^4 : 1$ (power) or $100 : 1$ (amplitude), respectively

**6 dB** correspond pretty closely to $4 : 1$ (power) or $2 : 1$ (amplitude), respectively

**3 dB** correspond pretty closely to $2 : 1$ (power) or $\sqrt{2} : 1$ (amplitude), respectively

A stack of windows, sitting in a hardware store for sale:
Glass is not *perfectly* transparent




Around 1.5 µm, fiber has a loss below 0.2 dB/km.
This is second to none:
Among all solid-state materials this is by far the most transp

# Can you cross the ocean in one go?

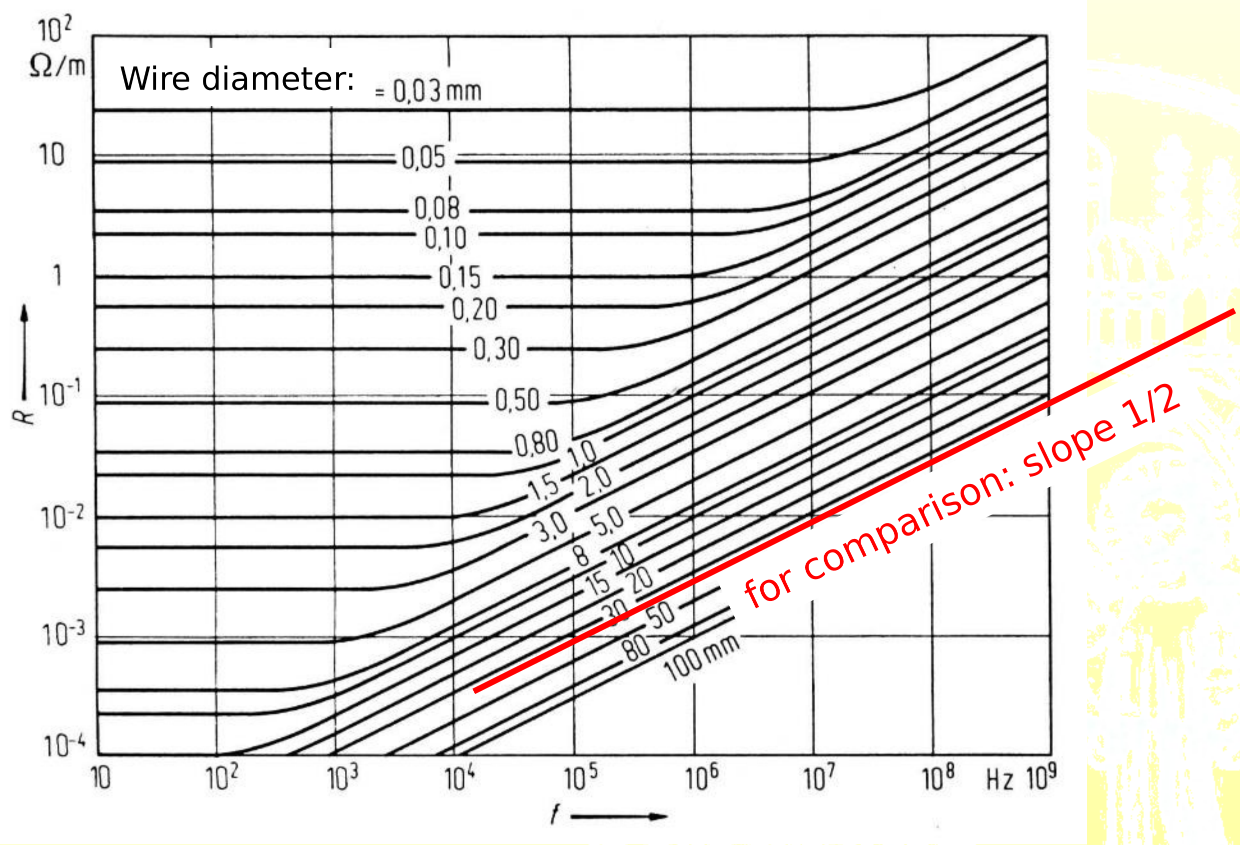
- Assume fiber loss 0.2 dB/km
  distance 5000 km
- total loss 1000 dB
- received power $10^{-100}$ of transmitted power
- to receive a single photon, you must send $10^{100}$ photons
- photon energy $E = h\nu = 6\cdot10^{-34}$ Js $\cdot$ $2\cdot10^{14}$ Hz $\approx 10^{-19}$ J
- it takes $10^{81}$ J to receive a single photon (on average!)
- fiber-optic transmission is quantum limited to much less than 1000 km

  1 J attenuated by $10^{-19}$ down to 1 photon: -190 dB $\Rightarrow$ max. distance 950 km

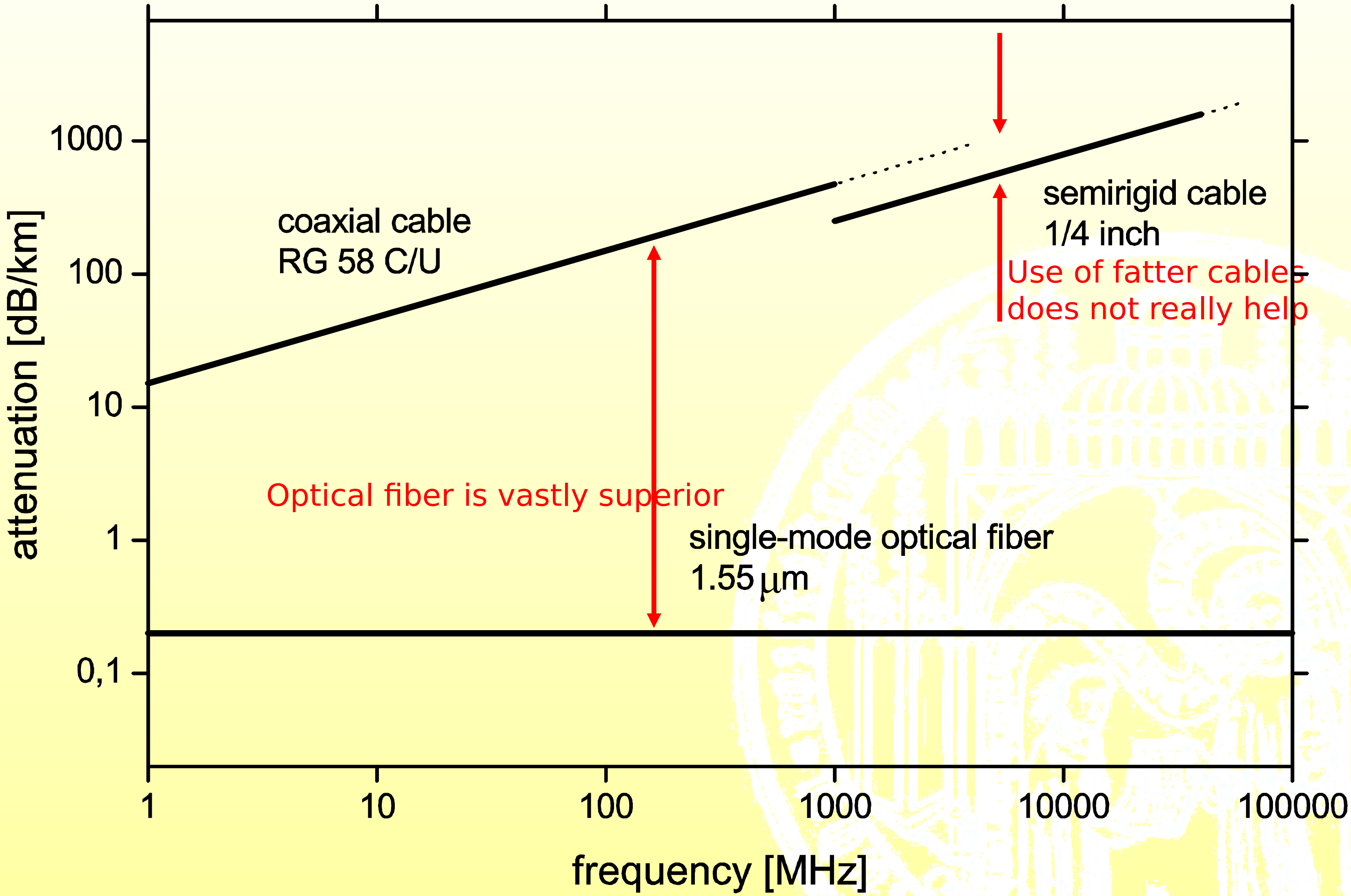
- How come you can hear radio stations from around the globe with shortwave radio?

# Why is fiber so much better than electrical cables?

Skin effect: magnetic fields compress the current density to a thin surface layer




Resistance of copper wire increases with frequenc



coaxial cable
RG 58 C/U
semirigid cable
1/4 inch
Use of fatter cables
does not really help
Optical fiber is vastly superior
single-mode optical fiber
1.55 µm
attenuation [dB/km]
1000
100
10
1
0,1
1
10
100
1000
10000
100000
frequency [MHz]

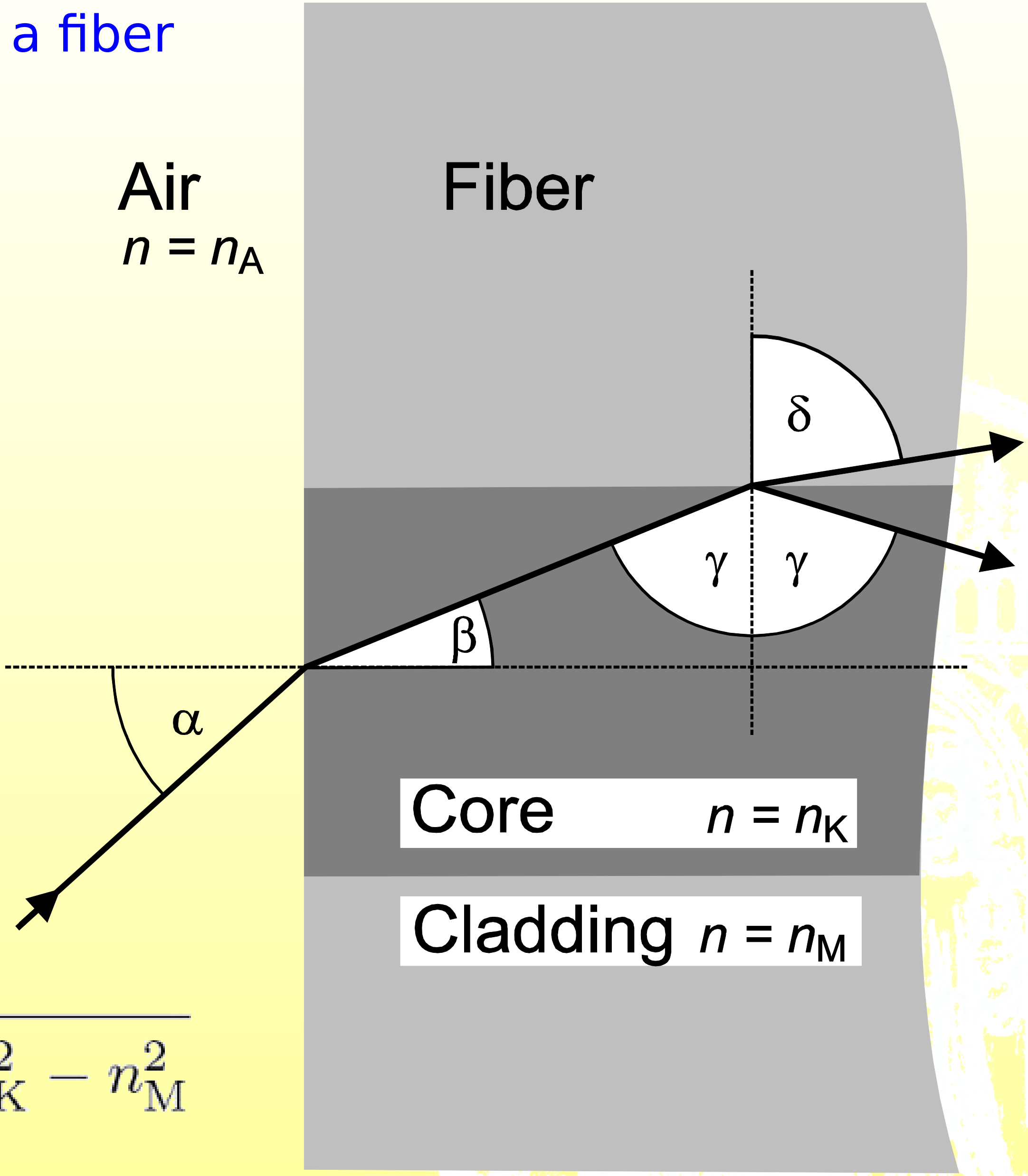
# Light guiding in fibers

The simple-minded approach:
Ray Optics


total internal refraction

$$\alpha_{\mathrm{crit}} = \arcsin\left(\frac{n_2}{n_1}\right)$$

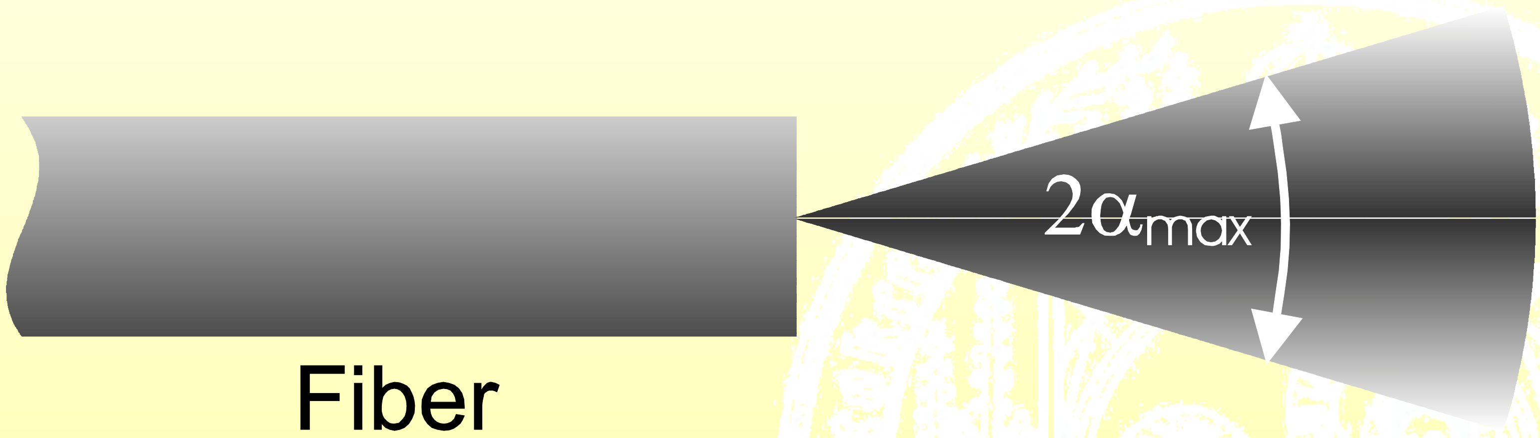
# Entrance of rays into a fiber



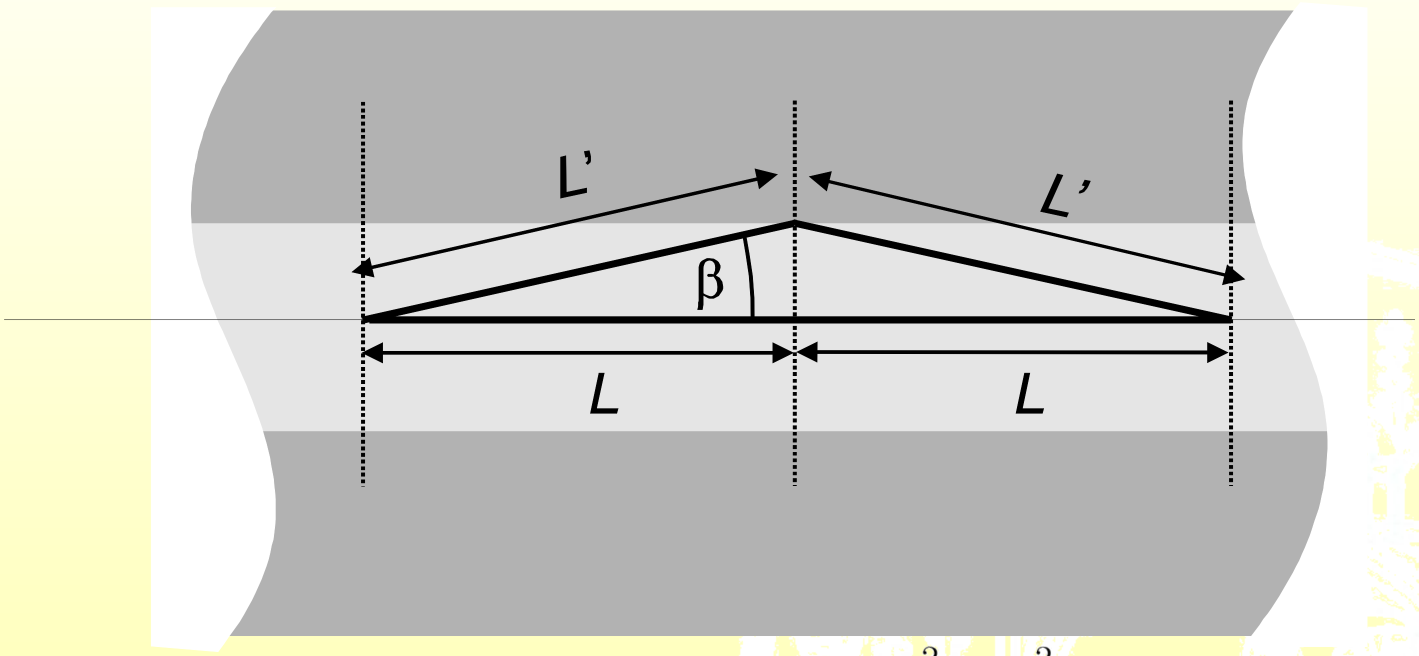

$$\alpha_{\max} = \arcsin\sqrt{n_K^2 - n_M^2}$$

Entrance cone same as exit cone:
limited by maximum angle

Typically ± 7°



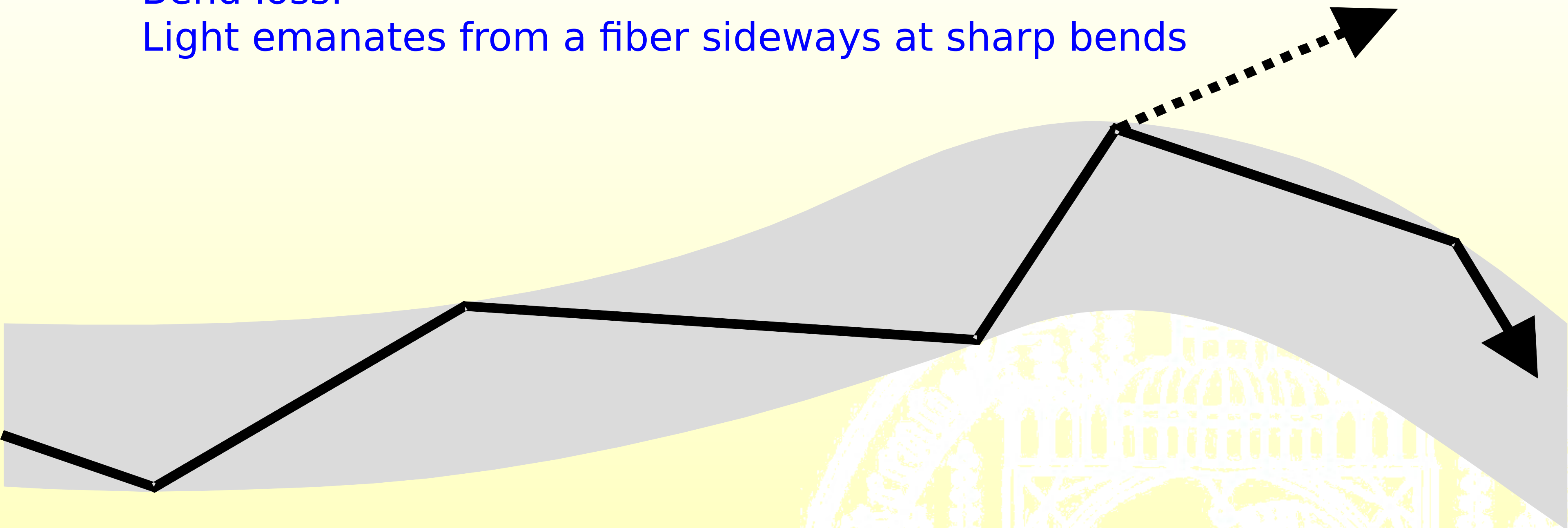
## Some rays travel longer than others




Relative timing difference is ca. $\Delta = \frac{n_{\mathrm{K}}^2 - n_{\mathrm{M}}^2}{2n_{\mathrm{K}}^2} \approx \frac{n_{\mathrm{K}} - n_{\mathrm{M}}}{n_{\mathrm{K}}}$

Typical values: $\Delta$=0.3% so that in 1 km of fiber, arrival times will scatter by 15 n

Bend loss:
Light emanates from a fiber sideways at sharp bends



# Light guiding in fibers

A better approach:
Wave Optics

In MKS units of measurement, Maxwell's equations are

$$
\begin{aligned}
\nabla \cdot \vec{D} &= \rho \\
\nabla \cdot \vec{B} &= 0 \\
\nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t}
\end{aligned}
$$

Here,

| | | |
|---|---|---|
| $\vec{E}$ | electric field strength | V/m |
| $\vec{H}$ | magnetic field strength | A/m |
| $\vec{D}$ | dielectric displacement | As/$m^2$ |
| $\vec{B}$ | magnetic induction | Vs/$m^2$=T |
| $\vec{J}$ | current density | A/$m^2$ |
| $\rho$ | charge density | As/$m^3$ |

$$
\begin{aligned}
\vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\
\vec{B} &= \mu_0 (\vec{H} + \vec{M}) \\
\vec{J} &= \sigma \vec{E} \quad .
\end{aligned}
$$

| | |
|---|---|
| $\vec{P}$ | polarization |
| $\vec{M}$ | magnetization |
| $\sigma$ | conductivity |

where

| | | |
|---|---|---|
| $\epsilon_0$ | vacuum permittivity | (dielectric constant of free space), |
| $\mu_0$ | vacuum permeability | (permeability constant of free space). |

$$
\begin{aligned}
\mu_0 &= \frac{4\pi}{10^7} \frac{\mathrm{Vs}}{\mathrm{Am}} \approx 1.256637 \cdot 10^{-6} \frac{\mathrm{Vs}}{\mathrm{Am}} \\
\epsilon_0 &= \frac{1}{\mu_0 c^2} \approx 8.854188 \cdot 10^{-12} \frac{\mathrm{As}}{\mathrm{Vm}}
\end{aligned}
$$

$\mu_0 \epsilon_0 = 1/c^2$ $\qquad$ $c = 2.99792458 \cdot 10^8 \frac{\mathrm{m}}{\mathrm{s}}$

$\mu_0/\epsilon_0 = Z_0^2$ $\qquad$ $Z_0 = \frac{4\pi c}{10^7} \frac{\mathrm{Vs}}{\mathrm{Am}} \approx 376.7303\,\Omega$

$$\begin{aligned}\vec{P} &= \epsilon_0 \chi_{\mathrm{el}} \vec{E} \\ \vec{M} &= \chi_{\mathrm{mag}} \vec{H} \\ \vec{J} &= \sigma \vec{E} \quad .\end{aligned}$$

All properties of the medium are contained in $\chi_{\mathrm{el}}, \chi_{\mathrm{mag}}, \sigma$

With some simplifications...

material is

- nonconducting,
- nonmagnetic,
- isotropic,
- and responds instantaneously

...we are left with only the polarisation

$$\vec{P} = \epsilon_0 \left( \chi^{(1)} \vec{E} + \chi^{(2)} \vec{E}^2 + \chi^{(3)} \vec{E}^3 + \dots \right)$$

# Writing a wave equation

Applying $\nabla\times$ yields

$$\begin{aligned}
\nabla\times\nabla\times\vec{E} &= \nabla\times\left(-\frac{\partial\vec{B}}{\partial t}\right)\\
\nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E} &= -\frac{\partial}{\partial t}(\nabla\times\vec{B})
\end{aligned}$$

With some substitutions...

$$\begin{aligned}
\nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E} &= -\frac{\partial}{\partial t}\left(\mu_0\frac{\partial\vec{D}}{\partial t}\right)\\
&= -\mu_0\frac{\partial^2}{\partial t^2}\vec{D}\\
&= -\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\vec{E}-\mu_0\frac{\partial^2}{\partial t^2}\vec{P}
\end{aligned}$$

... we have...

$$-\nabla(\nabla\cdot\vec{E})+\nabla^2\vec{E}=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}+\mu_0\frac{\partial^2}{\partial t^2}\vec{P}$$

... and if all vectors are parallel ...

$$\nabla^2\vec{E}=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}+\mu_0\frac{\partial^2}{\partial t^2}\vec{P}$$

For very weak fields,
ncate series expansion after linear term:

$$\vec{P} = \epsilon_0 \chi^{(1)} \vec{E}$$

Then:

$$\vec{D} = \epsilon_0 \vec{E} \left(1 + \chi^{(1)}\right)$$

Relative dielectric constant ε
with index $n$ and absorption coefficient α

$$1 + \chi^{(1)} = \epsilon = \left(n + i \frac{c}{2\omega} \alpha\right)^2$$

For low loss (α≈0) this reduces to

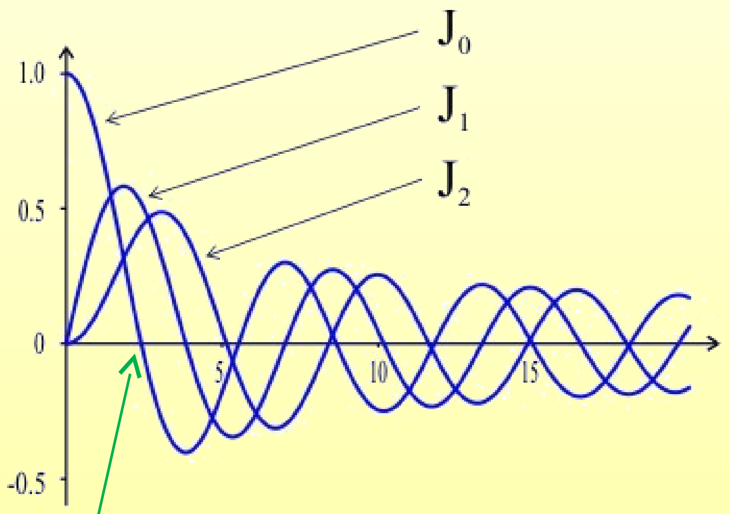
$$\epsilon = n^2$$

and the linear wave equation is

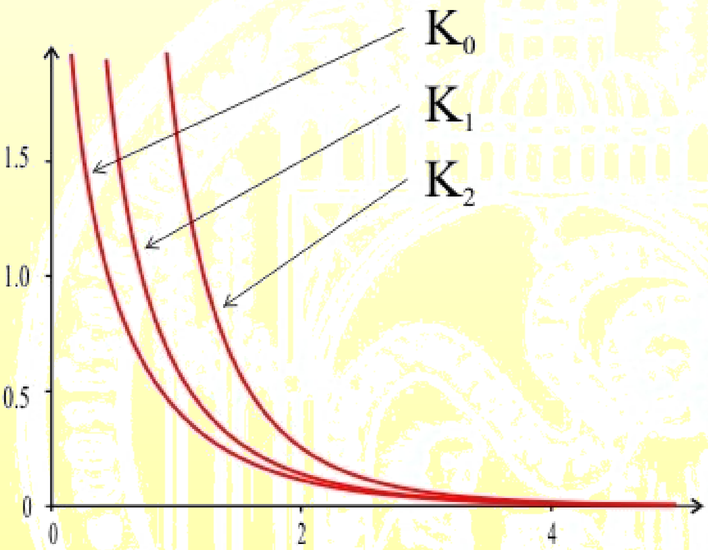
$$\boxed{\nabla^2 \vec{E} = \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} \vec{E}}$$

Writing the wave equation in cylindrical coordinates
and demanding a smooth transition at the core-cladding boundary
leads to a discrete set of field distributions: **The modes**

Circular symmetry invokes Bessel functions for the radial field distribution.




Bessel J functions



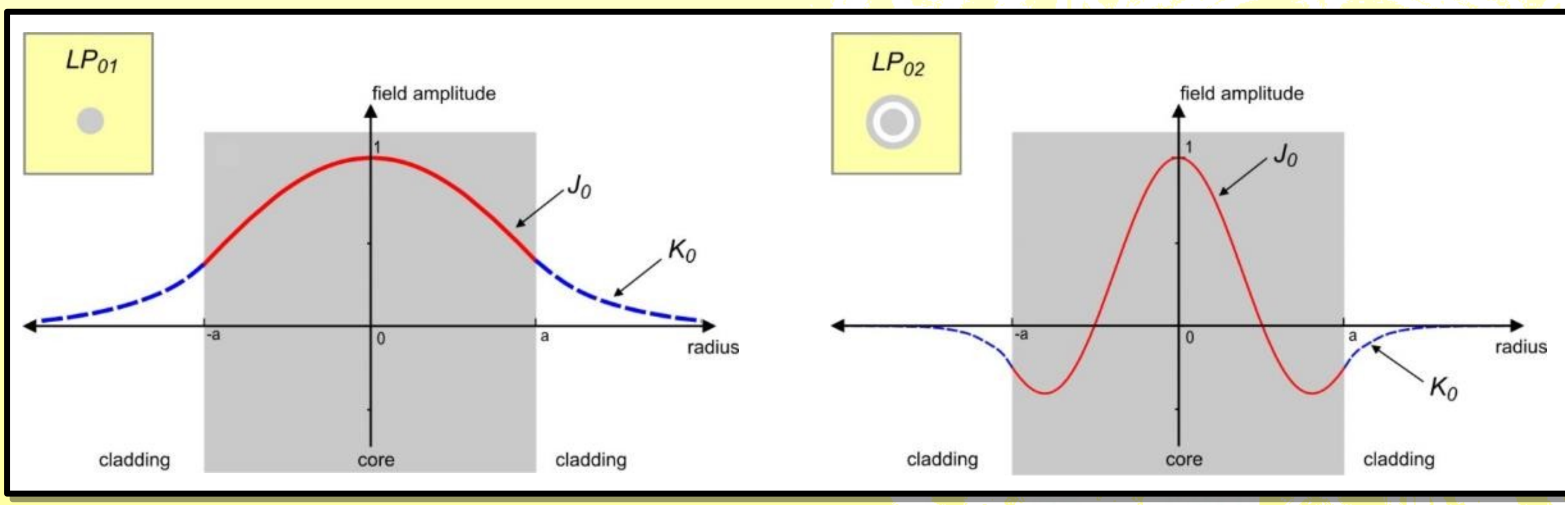

Bessel K functions

First Zero: $J_0(2,4048) = 0$

Writing the wave equation in cylindrical coordinates
and demanding a smooth transition at the core-cladding boundary
leads to a discrete set of field distributions: **The modes**

Circular symmetry invokes Bessel functions for the radial field distribution:
Bessel J in the core, and Bessel K in the cladding.

The azimuthal field distribution follows cos($m\phi$) with $m$

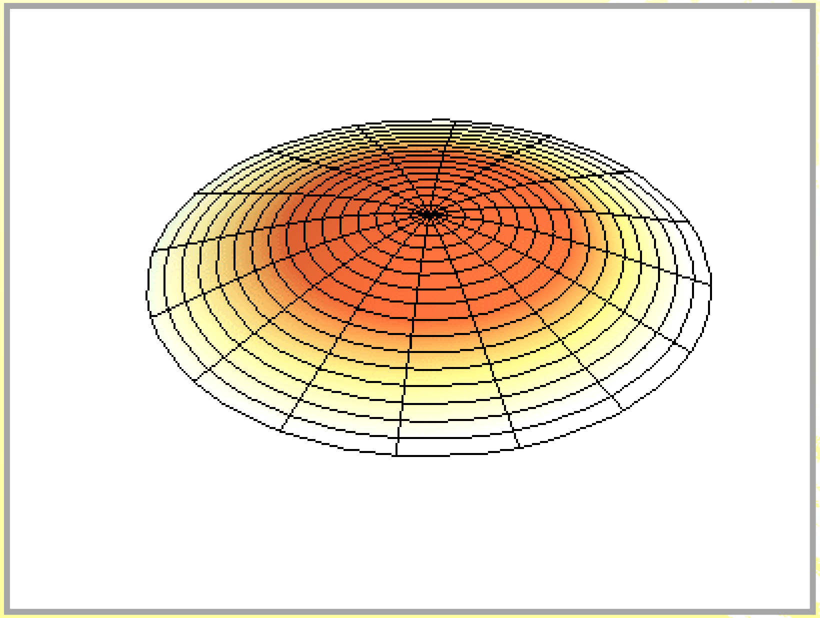



Examples: Radial field distribution for $m$=0 and $p$=1, $p$=2

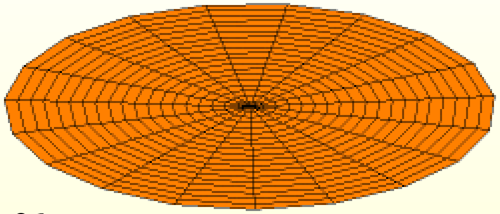
NOTE: These modes are quite similar
to the vibration modes of a circular membrane, fixed at the rim (like a drum head).

Main difference:

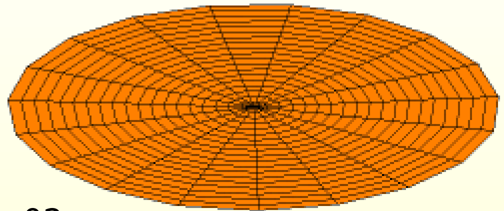
Amplitude of membrane is forced to zero at the rim,
but field is not zero at the boundary in the fiber: stretches somewhat into cladding.



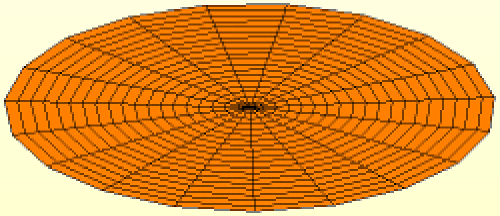
Fundamental oscillation mode of a circular membrane



01



02



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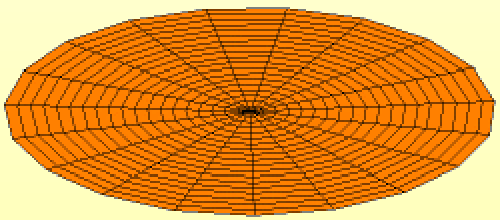
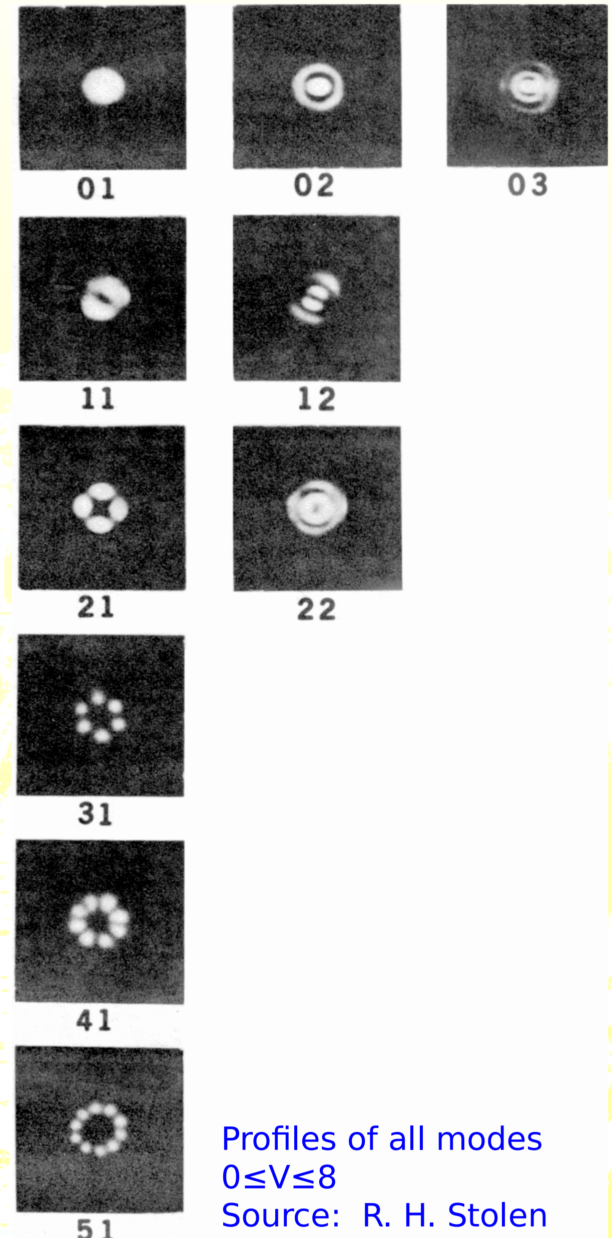


21

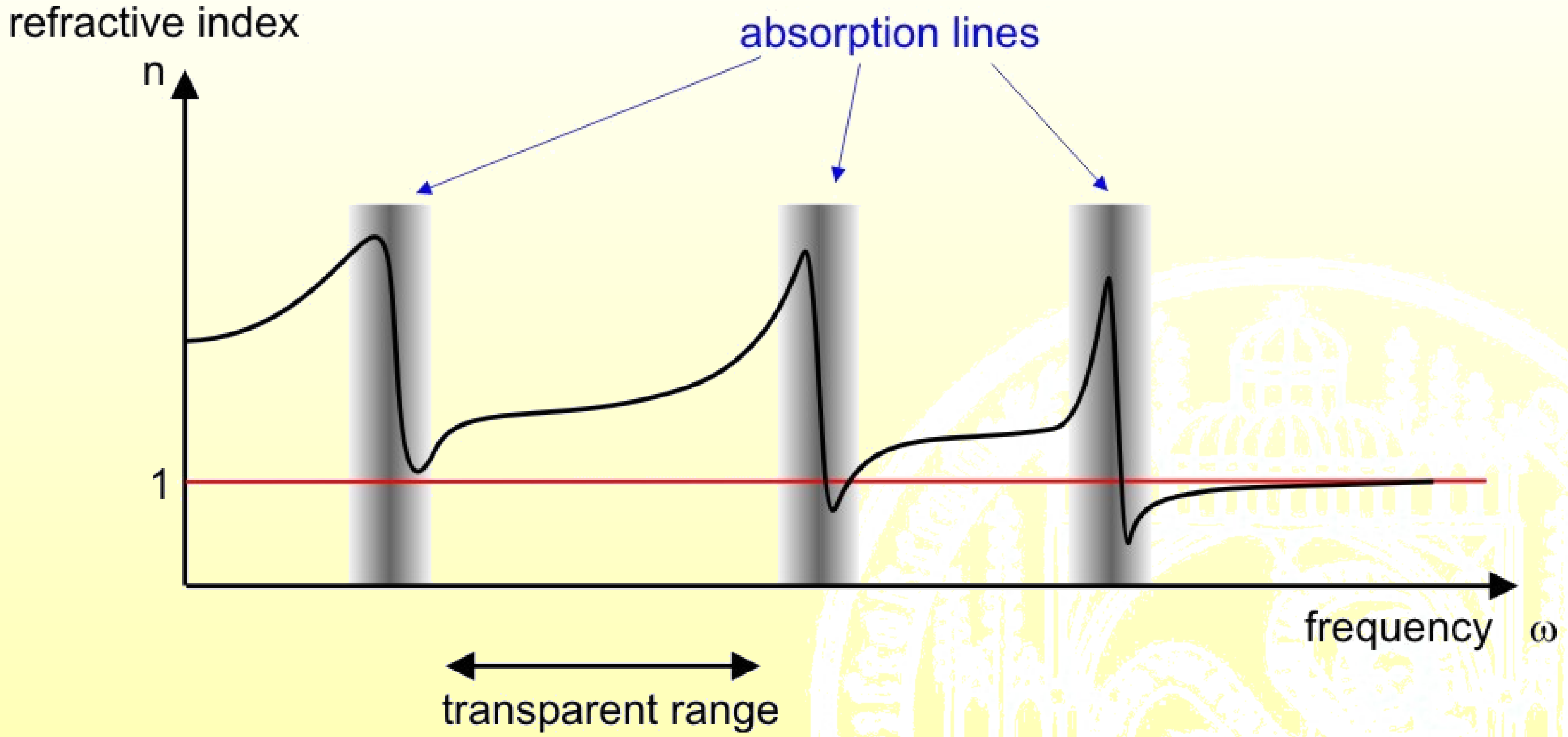
Condition for a fiber to be in the single-mode regime:

$$\lambda \geq \frac{2\pi a}{2.4048}\sqrt{n_{\mathrm{K}}^2 - n_{\mathrm{M}}^2} \approx \frac{2\pi a}{2.4048}\, n_{\mathrm{K}}\sqrt{2\Delta}$$

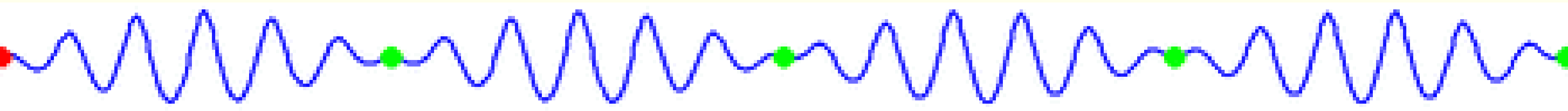



Profiles of all modes
0≤V≤8
Source: R. H. Stolen
1976

# Group velocity dispersion




Refractive index in a wide frequency range



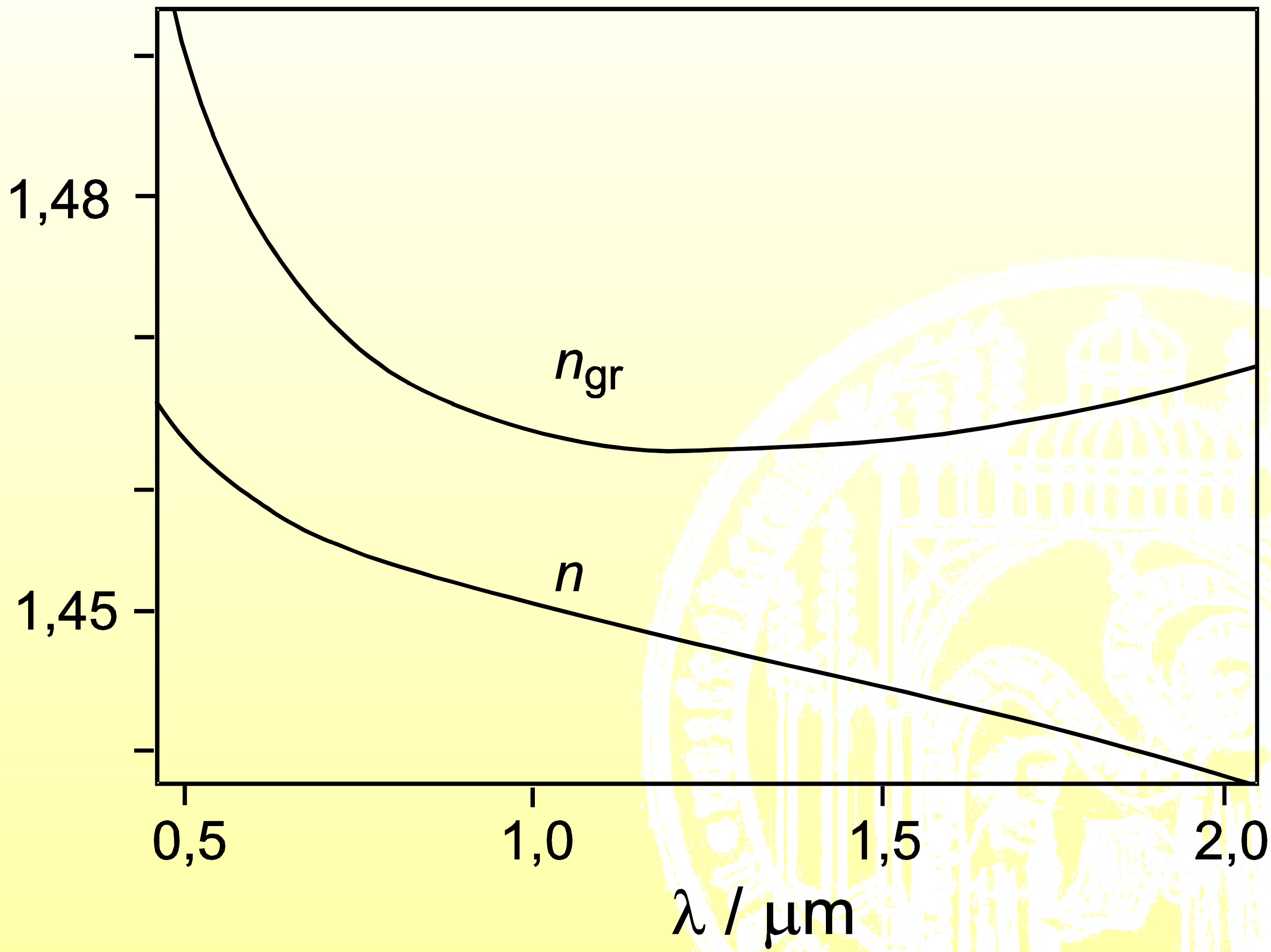
## Distinction between phase velocity and group velo

$$v_{\mathrm{ph}} = \omega/\beta$$
$$v_{\mathrm{gr}} = d\omega/d\beta$$

$$\tau \quad = \quad \frac{L}{v_{\mathrm{gr}}} = \frac{L}{c}\left(n - \lambda\,\frac{dn}{d\lambda}\right)$$

$$n_{\mathrm{gr}} = \left(n - \lambda\,\frac{dn}{d\lambda}\right)$$

*group index*




Distinction between phase index $n$ and group index $n_{gr}$

$$D = \frac{1}{L}\frac{d\tau}{d\lambda} = -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$$

Dispersion coefficient in ps/(nm km)

Alternatively, dispersion may be quantified through

$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \beta_0 + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2 + \ldots$$

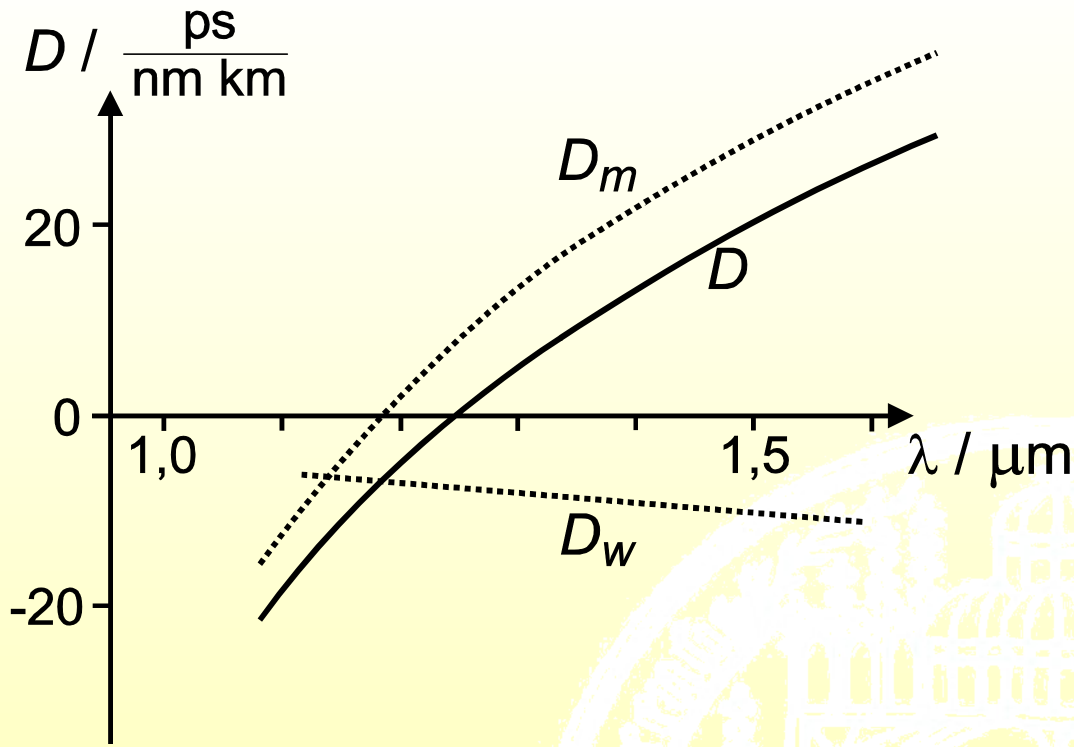
$$\beta_m = \left.\frac{d^m\beta}{d\omega^m}\right|_{\omega=\omega_0}$$

$$\left\{\begin{array}{l} \beta_0 = \beta(\omega=\omega_0) = kn \\ \beta_1 = \dfrac{1}{v_{\mathrm{gr}}} = \dfrac{n_{\mathrm{gr}}}{c} \\ \beta_2 = \dfrac{d\beta_1}{d\omega} = \dfrac{1}{c}\left(2\dfrac{dn}{d\omega} + \omega\dfrac{d^2 n}{d\omega^2}\right) \end{array}\right.$$

GVD parameter in ps²/km

Conversion between both specifications:

$$D_m = -\beta_2\left(\frac{2\pi c}{\lambda^2}\right) = -\frac{\omega}{\lambda}\beta_2$$



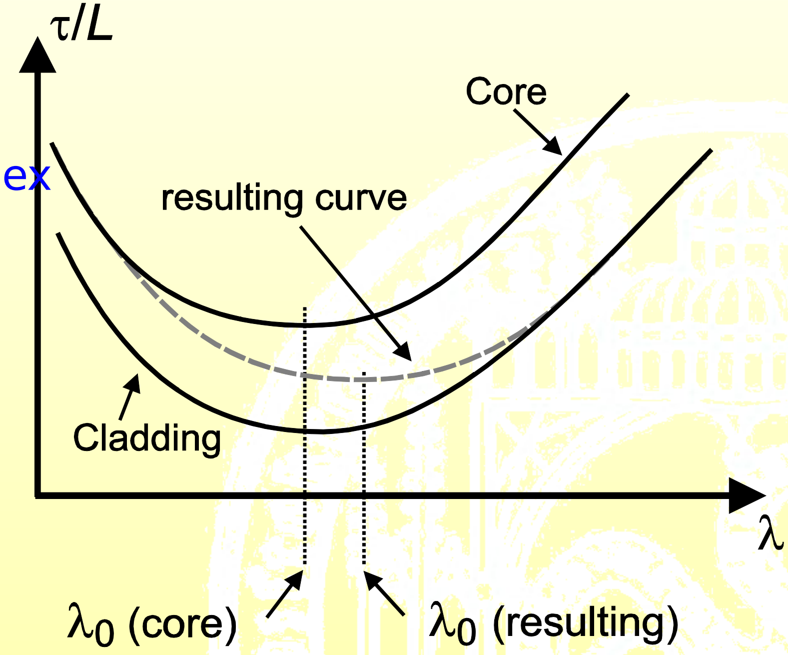

Material dispersion plus modification due to wavegu
gives rise to resulting dispersion

Dispersion has a zero in the near infrared! Distingui
➢ normal dispersion at $D < 0$, $\beta_2 > 0$
➢ anomalous dispersion at $D > 0$, $\beta_2 < 0$
Consider higher-order dispersion!

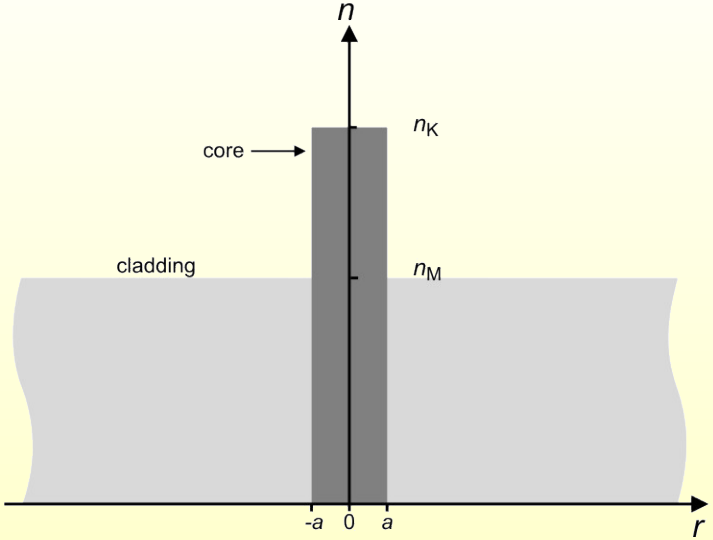
# Explanation of the waveguide contribution to fiber dispersion

The modal field extends somewhat into the cladding:
the longer the wavelength, the more

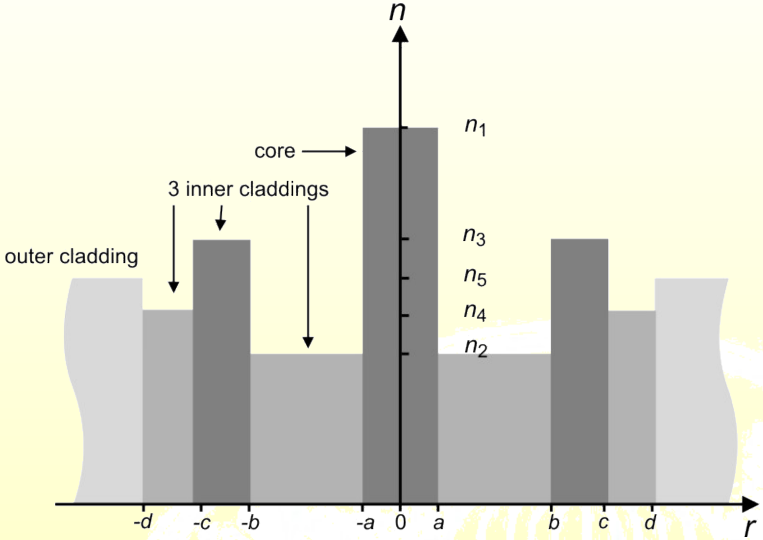
The relevant index
is a weighted average
of core index and cladding index



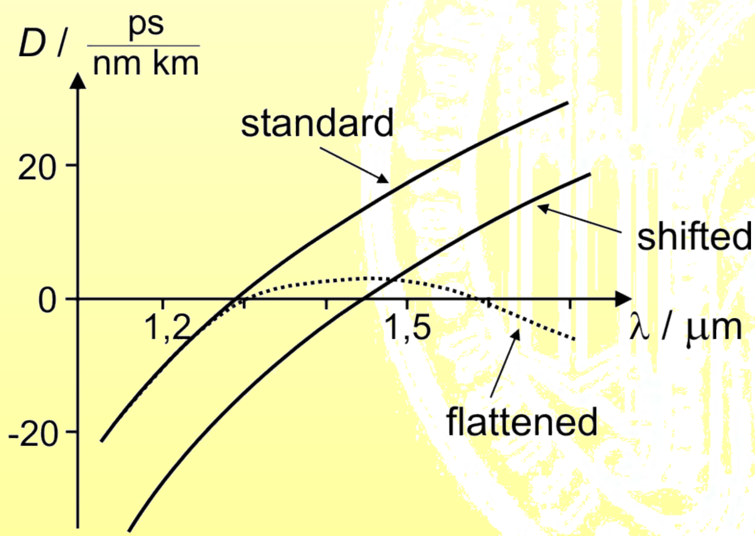
# Radial structure of fibers




Step-index fiber

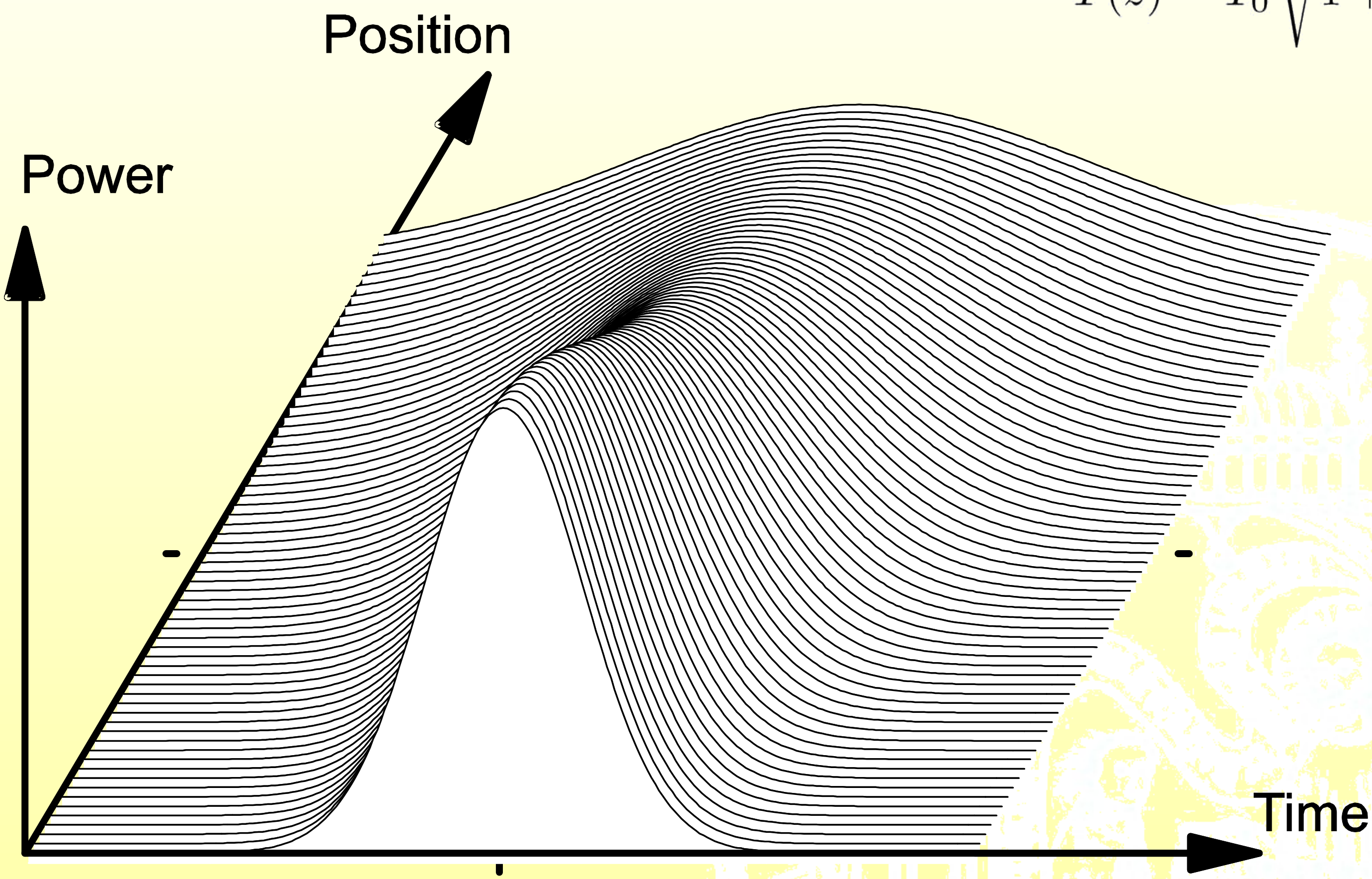



Quadruple-clad fiber

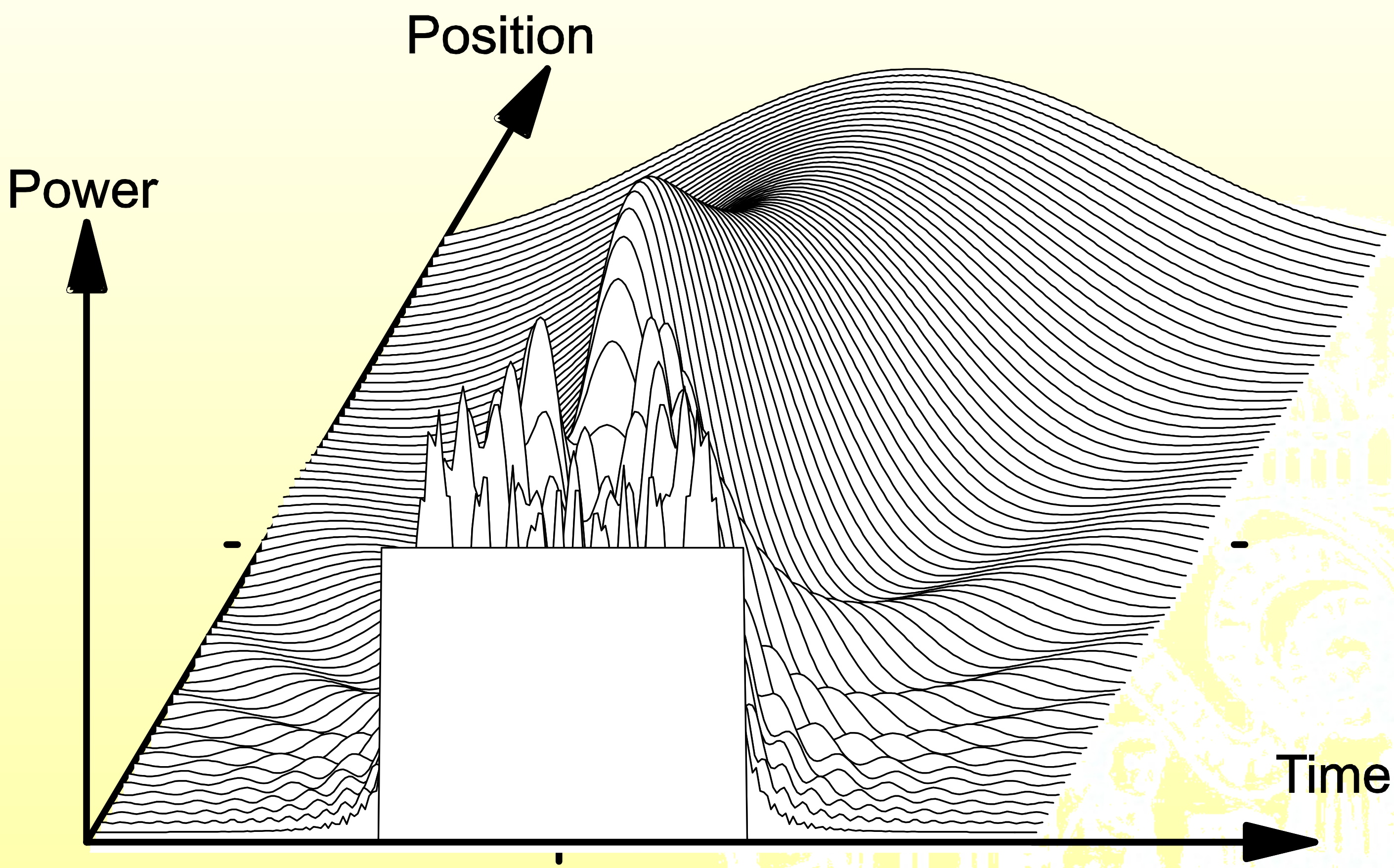


Often used: dispersion length $L_D = T_0^2/|\beta$

$$T(z) = T_0 \sqrt{1 + \left(\frac{z}{L_D}\right)^2}$$

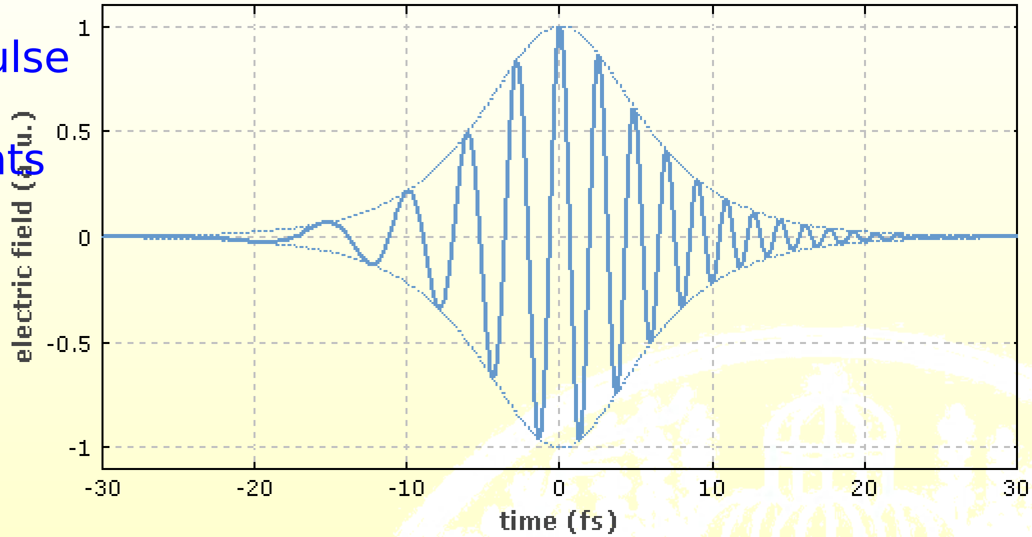



A short light pulse in a dispersive medium will spread out

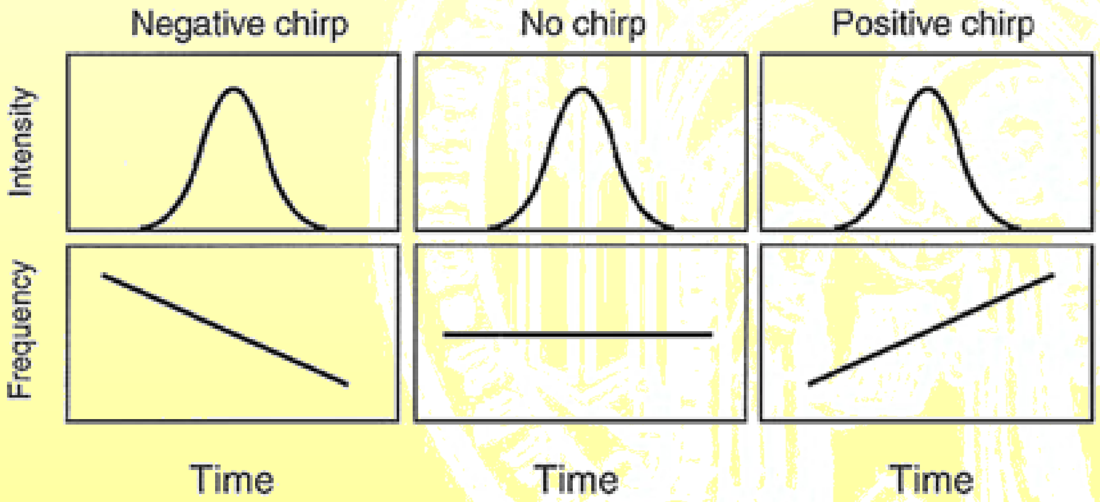



A hypothetical square pulse to demonstrate dispersive effects

Dispersion broadens the pulse
by differential timing
of it's frequency components




A sliding-frequency pulse is called „chirped“




Chirp is invisible in the power profile