Soliton Molecules and Optical Rogue Waves

Benasque, October 2014

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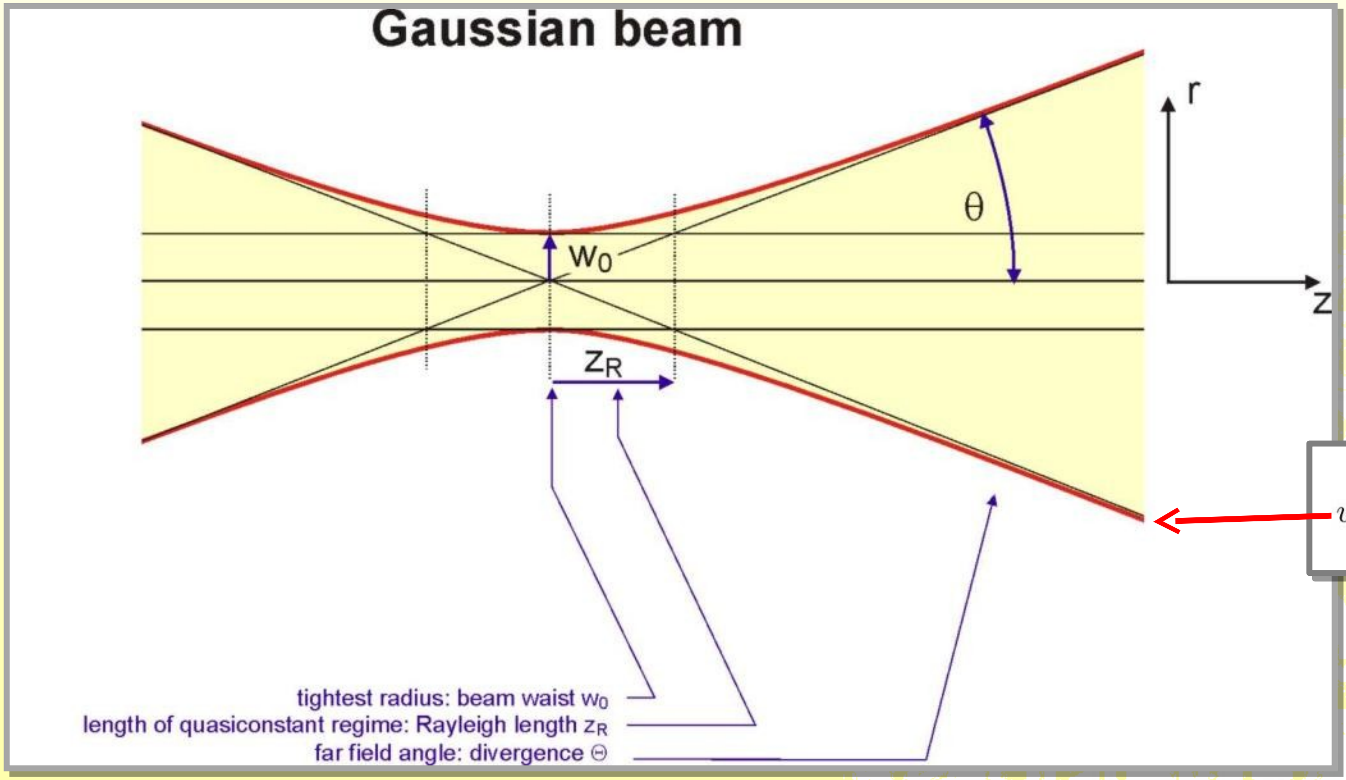
Part II

Fiber Nonlinearity

Why is it that optical nonlinearity is so important in fibe



$$w(z) = w(0)\sqrt{1+\left(\frac{z}{z_R}\right)^2}$$

In free-space optics, a tight focus comes with a short depth-of-focu

leading nonlinear effect in fibers is a modification of the refractive inde

„Optical Kerr effect“

Remember the series expansion $\vec{P} = \epsilon_0 \left(\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E}^2 + \chi^{(3)} \vec{E}^3 + \ldots \right)$

We had truncated after the linear term: $P = \epsilon_0 \chi^{(1)} E$

$$n^2 = \epsilon = 1 + \chi^{(1)}$$

In glass $\chi^{(2)} = 0$

Including the next term yields $P = \epsilon_0 \left\{ \chi^{(1)} + \chi^{(3)} E^2 \right\} E$

$$\begin{aligned} n^2 = \epsilon = 1 + \chi^{(1)} + \chi^{(3)} E^2 &= \epsilon_{\text{linear}} + \chi^{(3)} E^2 \\ &= \epsilon_{\text{linear}} \left(1 + \frac{\chi^{(3)}}{\epsilon_{\text{linear}}} E^2 \right) \end{aligned}$$

and finally $\boxed{n = n_0 + n_2 I}$

with intensity $I = (n_0 / Z_0) E^2$

and nonlinearity coefficient $n_2 = 3 \cdot 10^{-20} \mathrm{m}^2/\mathrm{W}$

The evolving phase of the light wave can be separated into a linear and a no

$$\phi = \frac{2\pi}{\lambda}\,(n_0 + n_2 I)\,L = \frac{2\pi}{\lambda}\,(n_0 + n_2 P/A_{\text{eff}})\,L$$

$$\phi_{\text{lin}} = \frac{2\pi}{\lambda}\,n_0 L$$

$$\phi_{\text{nl}} = \frac{2\pi}{\lambda}\,\frac{n_2}{A_{\text{eff}}}\,PL = \frac{\omega_0 n_2}{c A_{\text{eff}}}\,PL = \gamma P L$$

with $\gamma = \frac{\omega_0 n_2}{c A_{\text{eff}}}$

Often used: nonlinearity length $L_{\text{NL}} = (\gamma P)^{-1}$

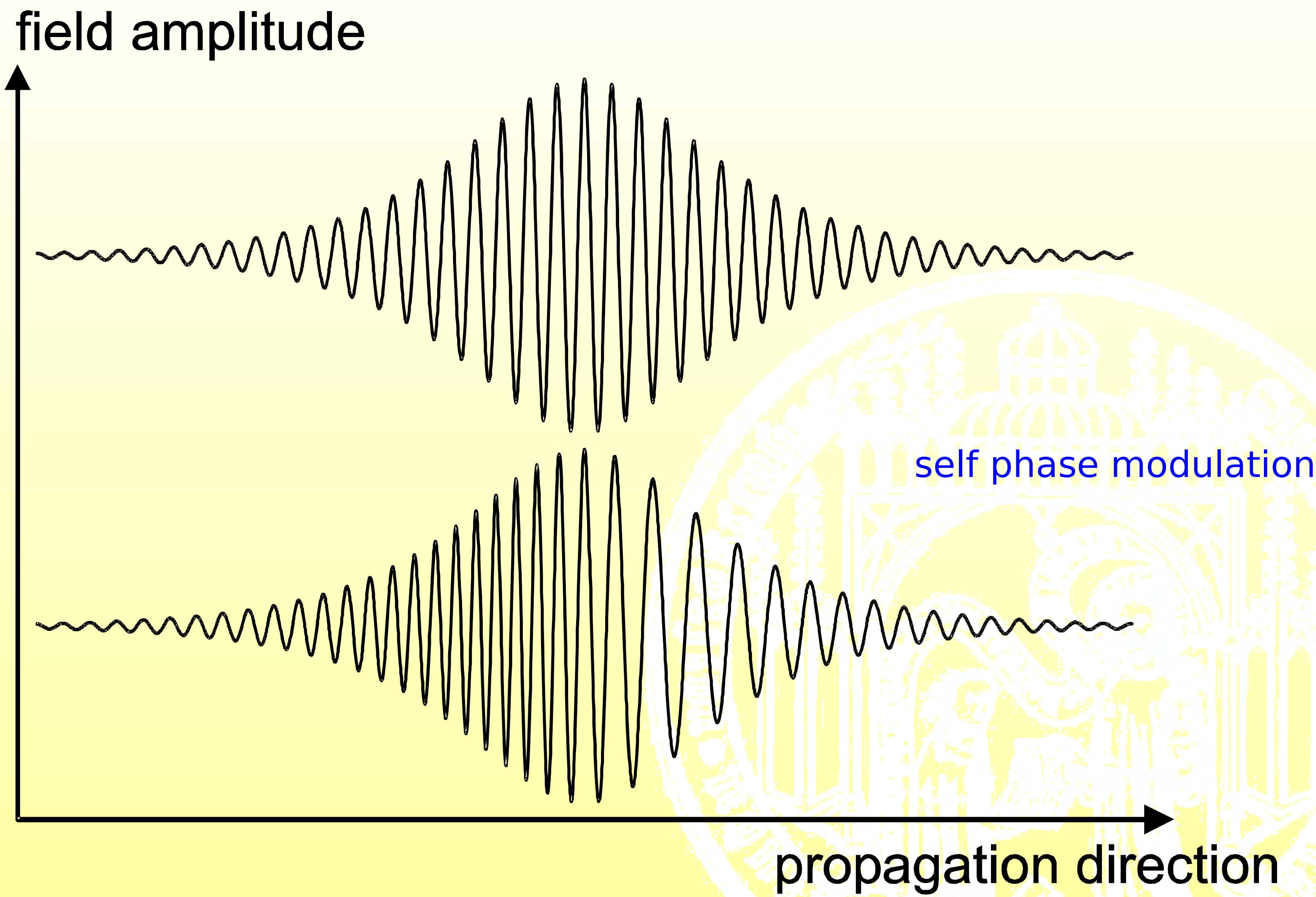
Estimate of typical numerical values:

λ = 1.5 μm $\Rightarrow$ $\omega_o = 2\,\pi \cdot 200$ THz
$n_2 = 3 \cdot 10^{-20}$ m²/W
$c = 3 \cdot 10^8$ m/s
$A_{\text{eff}} = 40$ μm²

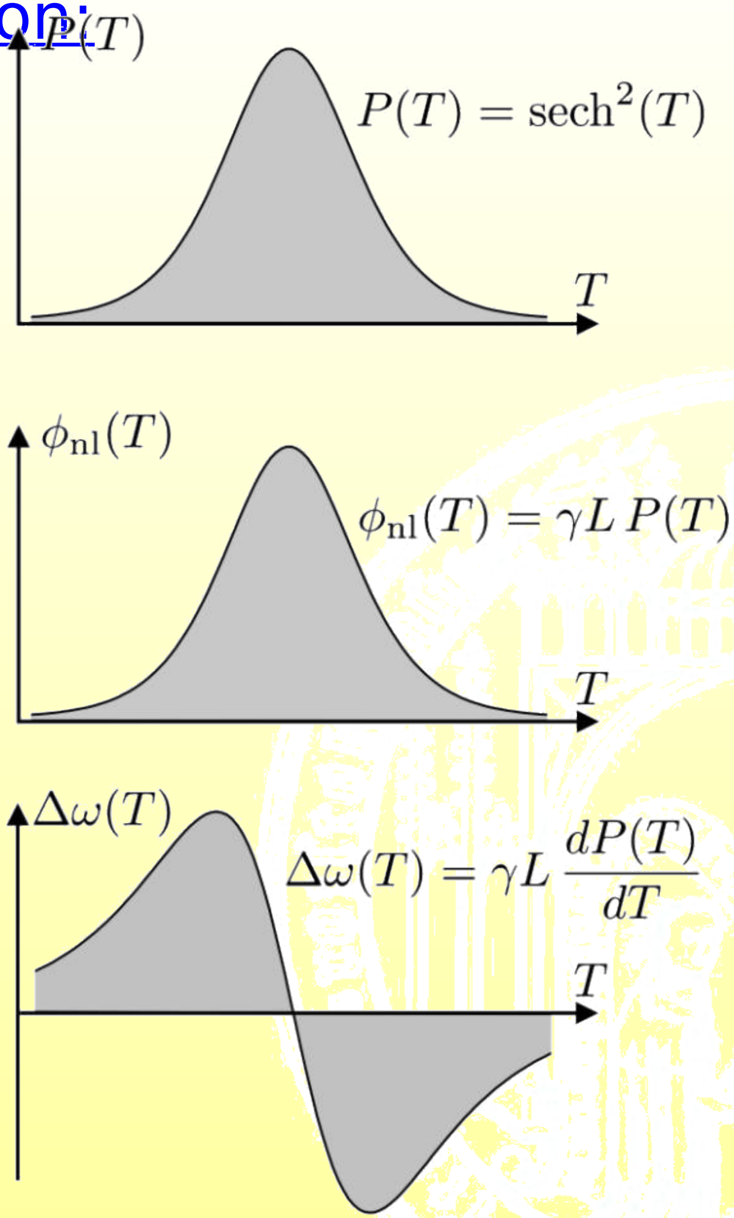
$\gamma = 3.14 \cdot 10^{-3}\,(\text{W m})^{-1}$

Assuming P = 1 W and L = 1 km

$\Rightarrow \quad \phi_{\text{nl}} = 3.14$ rad


field amplitude
self phase modulation
propagation direction

Self Phase Modulation:



nonlinear phase
follows power profile

instantaneous frequency
is modulated, too

Finding a nonlinear wave equation

Linear wave equation:

$$\nabla^2 E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \quad . \tag{1}$$

Ansatz for E:

$$E(x,y,z,t) = A(x,y,z,t)\,\mathrm{e}^{i(\omega_0 t - \beta_0 z)} \quad . \tag{2}$$

Remove oscillating factor at optical frequency $\Rightarrow$ envelope equation for $A(z,t)$.
Introduce dispersion by a Fourier Technique:

$$\Delta\beta = \beta_1 \Delta\omega + \frac{\beta_2}{2}\Delta\omega^2 + \frac{\beta_3}{6}\Delta\omega^3 + \ldots \quad , \tag{3}$$

$$-i\frac{\partial}{\partial z}A = i\beta_1 \frac{\partial}{\partial t}A - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A - i\frac{\beta_3}{6}\frac{\partial^3}{\partial t^3}A + \ldots \quad . \tag{4}$$

Add nonlinear term $\Delta\beta_{\mathrm{NL}} = n_2 I \beta_0$.
Add loss term with $\Delta\beta_{\mathrm{loss}} = i\alpha/2$.

$$\Delta\beta = \beta_1 \Delta\omega + \frac{\beta_2}{2}\Delta\omega^2 + \frac{\beta_3}{6}\Delta\omega^3 + \ldots + \beta_0 n_2 I + i\frac{\alpha}{2} \quad , \tag{5}$$

$$-i\frac{\partial}{\partial z}A = i\beta_1 \frac{\partial}{\partial t}A - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A - i\frac{\beta_3}{6}\frac{\partial^3}{\partial t^3}A + \ldots + \beta_0 n_2 I A + i\frac{\alpha}{2}A \quad . \tag{6}$$

Remove β_1 term: $t \rightarrow t - \beta_1 z$,

$$i\frac{\partial}{\partial z}A - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A - i\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} + \ldots + \beta_0 n_2 I A + i\frac{\alpha}{2}A = 0 \quad . \tag{7}$$

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Choosing $\beta_0 n_2 I = (\omega_0/c)\, n_2(|A|^2/A_{\mathrm{eff}}) = \gamma|A|^2$:

$$\frac{\partial}{\partial z}A + \frac{i}{2}\beta_2\frac{\partial^2}{\partial t^2}A + \frac{\beta_3}{6}\frac{\partial^3}{\partial t^3}A + \ldots - i\gamma|A|^2A + \frac{\alpha}{2}A = 0 \quad . \tag{8}$$

Important special case Neglect third order dispersion and loss:

$$\boxed{i\frac{\partial}{\partial z}A - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}A + \gamma|A|^2A = 0 \quad .} \tag{9}$$

Nonlinear Schrödinger Equation

Solutions of the NLSE

$$\boxed{\frac{\partial}{\partial z}A(z,T) = -\frac{i}{2}\,\beta_2\,\frac{\partial^2 A}{\partial T^2} + i\gamma|A|^2 A}$$

We are mostly concerned with solutions at anomalous dispersion

continuous wave solution:

$$A = \sqrt{P_0}\ \mathrm{e}^{i\gamma P_0 z}$$

This is a stable solution only for normal dispersion; it is unstable in the anomalous dispersion regime.

This is known as Modulation Instability.

Modulation Instability (MI)

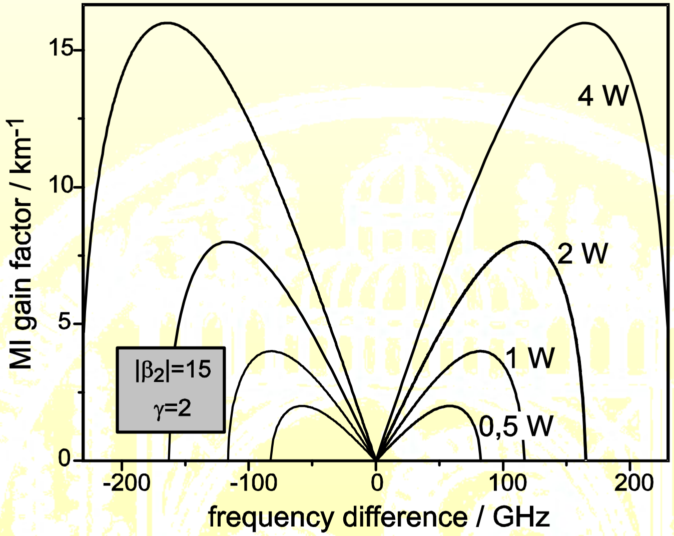
- cw solution of NLSE is unstable for anomalous dispersion
- Stability analysis reveals frequency band of sensitivity to perturbation

MI gain $$g = |\beta_2|\omega\sqrt{\omega_c^2 - \omega^2}$$

with $$\omega_c = \sqrt{\frac{4\gamma P_0}{|\beta_2|}}$$

has maximum at $$g_{\max} = 2\gamma P_0$$

and $$\omega_{\max} = \pm\omega_c/\sqrt{2} = \pm\sqrt{\frac{2\gamma P_0}{|\beta_2|}}$$



- Perturbation grows exponentially; modulates the cw solution

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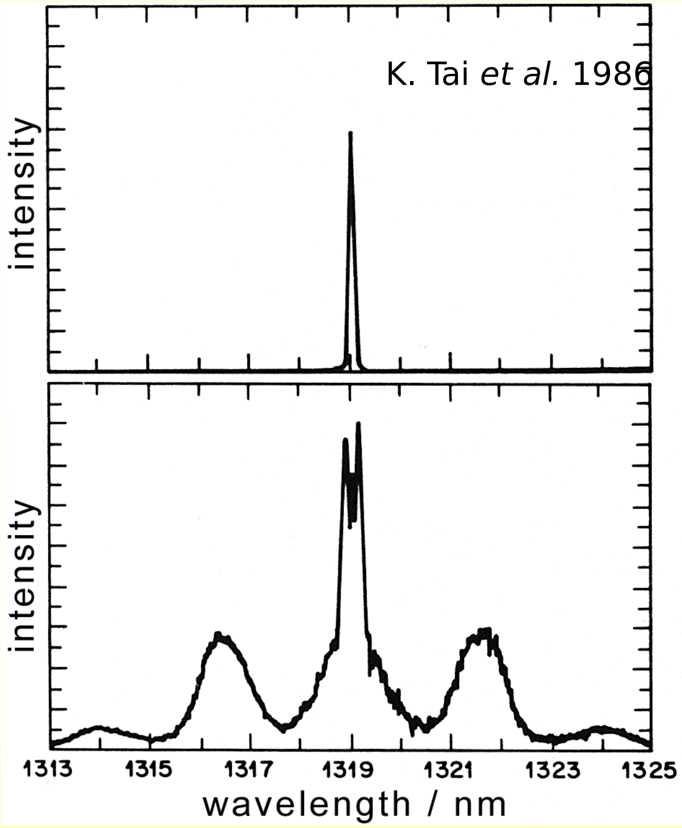

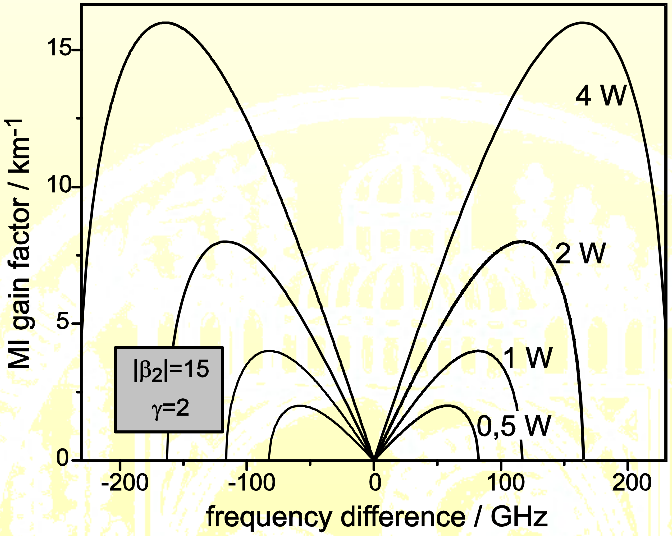

First experimental observation

Ripple marks in sand: periodic structure from uniform agitation

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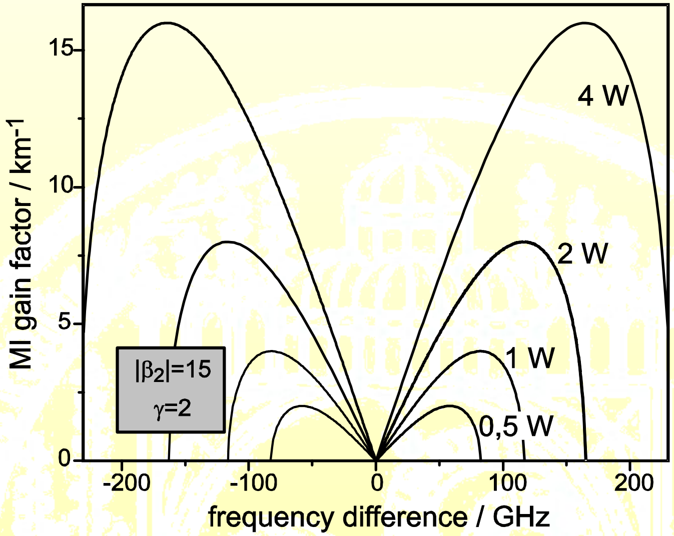


- Perturbation grows exponentially; modulates the cw solution
- Long term evolution? Solution of NLSE for this case found in

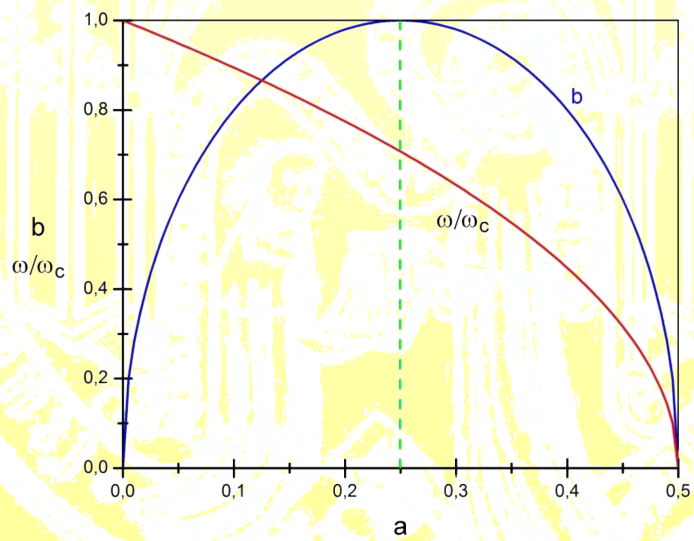
N. Akhmediev, V. I. Korneev, Theor. Math. Phys. 62, 1089 (1986)

- Was considered only recently: Akhmediev breather

Akhmediev Breather

$$A(Z,T) = \sqrt{P_0}\left[1 + \frac{2(1-2a)\cosh(bZ) + ib\sinh(bZ)}{\sqrt{2a}\cos(\omega T) - \cosh(bZ)}\right]\exp(iZ)$$

$$\begin{aligned} 0 \leq \; & a \leq 1/2 \\ b &= \sqrt{8a - 16a^2} \\ \omega &= \omega_c\sqrt{1-2a} \\ \omega_c &= \sqrt{\frac{4\gamma P_0}{|\beta_2|}} \\ Z &= z/L_{\mathrm{NL}} \end{aligned}$$



$\{$ b peaks at $a = ¼$, $b = 1$

$b \rightarrow 0$ for both $a \rightarrow 0$, $a \rightarrow ½$

Discussion of the Akhmediev Breather

$$A(Z,T) = \sqrt{P_0}\left[1 + \frac{2(1-2a)\cosh(bZ) + ib\sinh(bZ)}{\sqrt{2a}\cos(\omega T) - \cosh(bZ)}\right]\exp(iZ)$$

- Propagation in Z with phase factor $\exp(iZ)$

- Modulation on constant background $\sqrt{P_0}$

- Oscillatory in time T due to $\cos\omega T$ term

- Symmetrically exponential in space Z due to hyperbolic functions

Remember: $\cosh(x) = \frac{1}{2}\left(\mathrm{e}^{x} + \mathrm{e}^{-x}\right)$ and $\sinh(x) = \frac{1}{2}\left(\mathrm{e}^{x} - \mathrm{e}^{-x}\right)$

For large Z, hyperbolic functions dominate:

$$\lim_{Z\to\pm\infty} |A|^2 = P_0$$

At $Z = 0$, oscillatory part dominates:

$$A(0,T) = \sqrt{P_0}\left[1 + \frac{\sqrt{\frac{2}{a}}\,(1-2a)}{\cos\omega T - \frac{1}{\sqrt{2a}}}\right]\exp(iZ)$$

How to excite an Akhmediev breather?

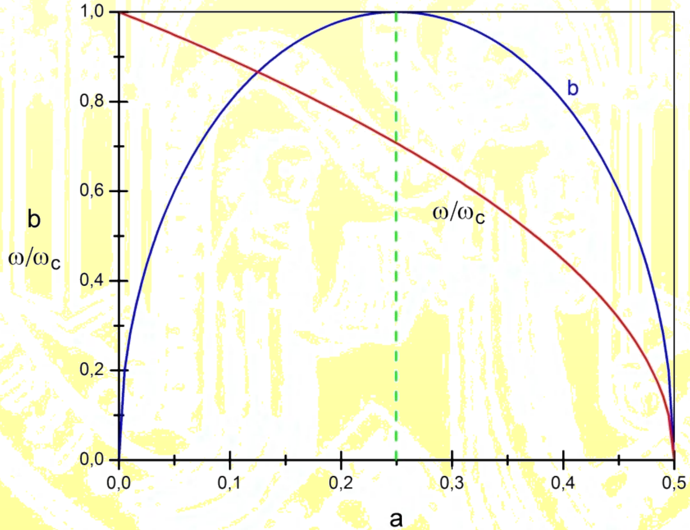
1) Start with cw: infinite wave (in practice, long pulse)

2) Perturb in suitable way:
   * periodic (at a frequency for which there is gain)
   * random (noise with frequency content where there is gain)

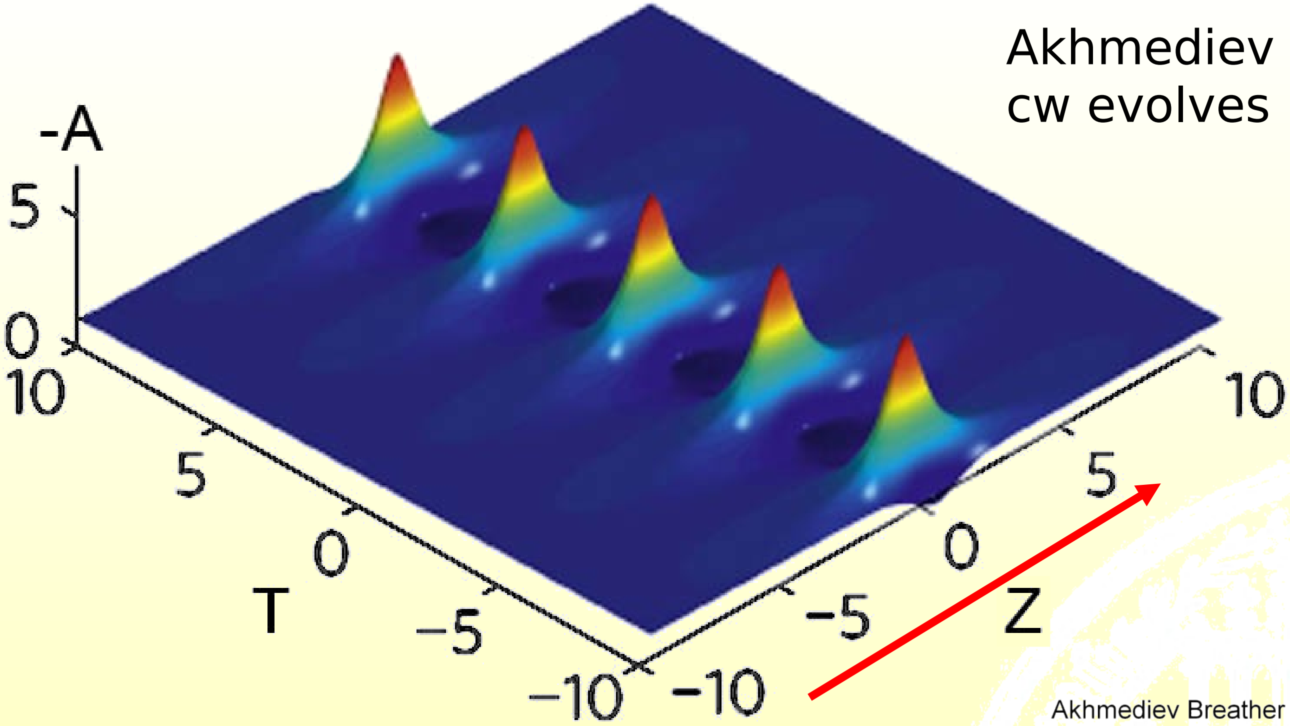
3) Perturbation will grow fastest at frequency of maximum gain

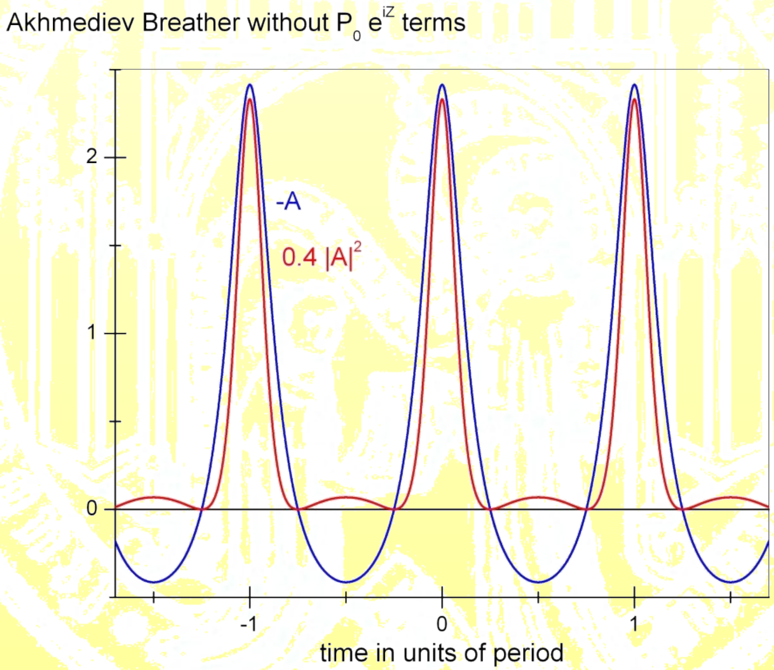
Consider $a = 3$!

$$\begin{cases} b & \text{peaks at} \quad a = 1/4,\ b = 1 \\ b \to 0 & \text{for both} \quad a \to 0,\ a \to 1/2 \end{cases}$$



Akhmediev Breather at *a* = ¼:
cw evolves into pulse train and back





cross section at *Z* = 0:
maximally expressed pulse train
Peak at $|A|^2$ = 5,828

CW with modulation: Related solution types

Remember

$$A(a,T,Z) = \sqrt{P_0}\left[1 + M(a,T,Z)\right]\exp(iZ)$$

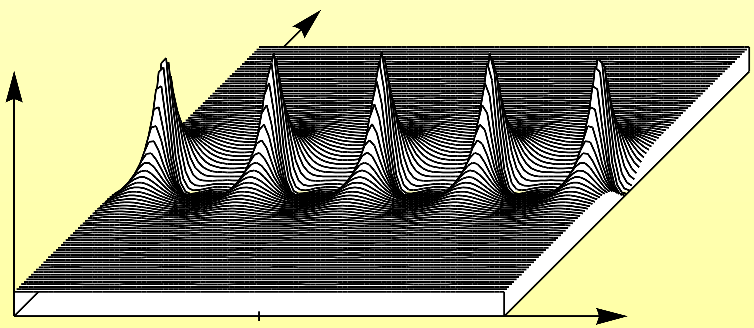
with modulated part

$$M(a,T,Z) = \begin{cases} \dfrac{2(1-2a)\cosh(bZ) + ib\sinh(bZ)}{\sqrt{2a}\,\cos(\omega T) - \cosh(bZ)} & : \quad 0 < a < \frac{1}{2} \\ -\dfrac{4(1+2iZ)}{1+\omega_c^2 T^2 + 4Z^2} & : \quad a = \frac{1}{2} \\ \dfrac{2(1-2a)\cos(|b|Z) - i|b|\sin(bZ)}{\sqrt{2a}\,\cosh(|\omega|T) - \cos(|b|Z)} & : \quad a > \frac{1}{2} \end{cases}$$

Akhmediev Breather

Peregrine Soliton

Kuznetsov-Ma soliton



N. Akhmediev, V. I. Korneev,
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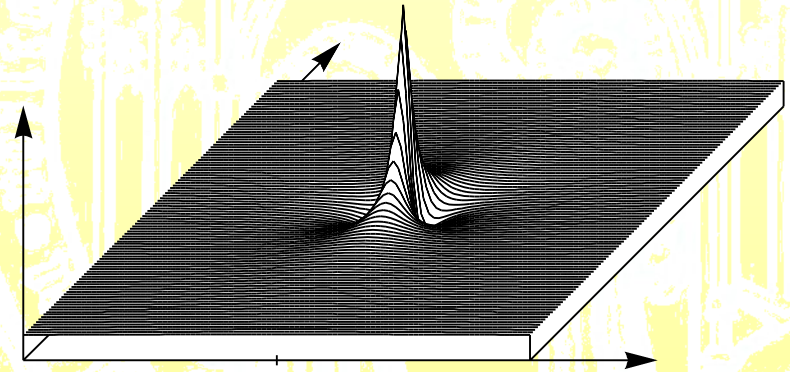

D. H. Peregrine,
J. Aust. Math. Soc. B **25**, 16
(1983)

CW with modulation: Related solution types

Remember

$$A(a,T,Z) = \sqrt{P_0}\left[1 + M(a,T,Z)\right]\exp(iZ)$$

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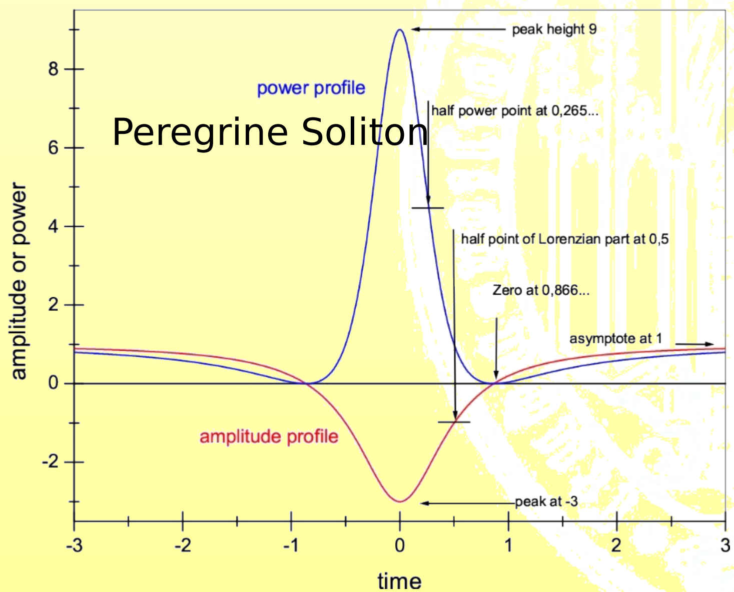
Akhmediev Breather

Peregrine Soliton

Kuznetsov-Ma soliton



Arguably the most important solution of the NLSE:
The (fundamental) *soliton*

The word ‚soliton‘ refers to a well-defined concept
to which the Peregrine and Kuznetsov-Ma solitons do not belong

The (fundamental) *soliton*

$\gamma > 0, \; \beta_2 < 0$

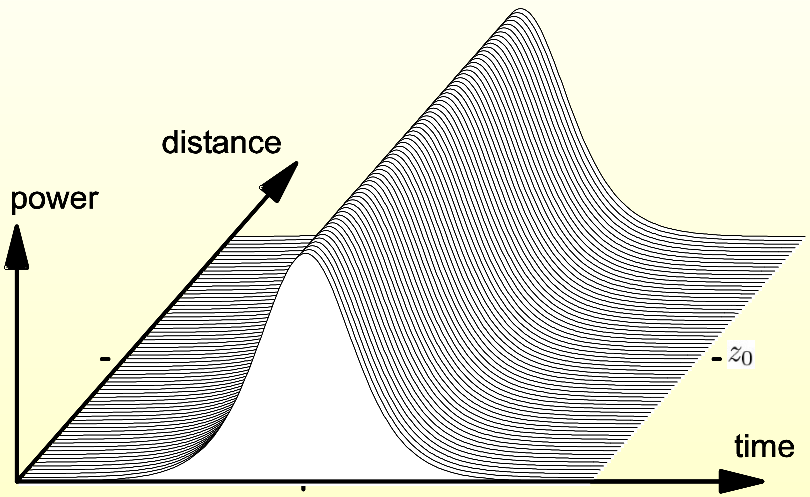
$$A(z,T) = \sqrt{P_1}\,\operatorname{sech}\left(\frac{T}{T_0}\right)\, e^{i\gamma P_1 z/2} \qquad \text{with} \qquad P_1 T_0^2 = \frac{|\beta_2|}{\gamma}$$

(up to trivial constant shifts in time, position, phase)

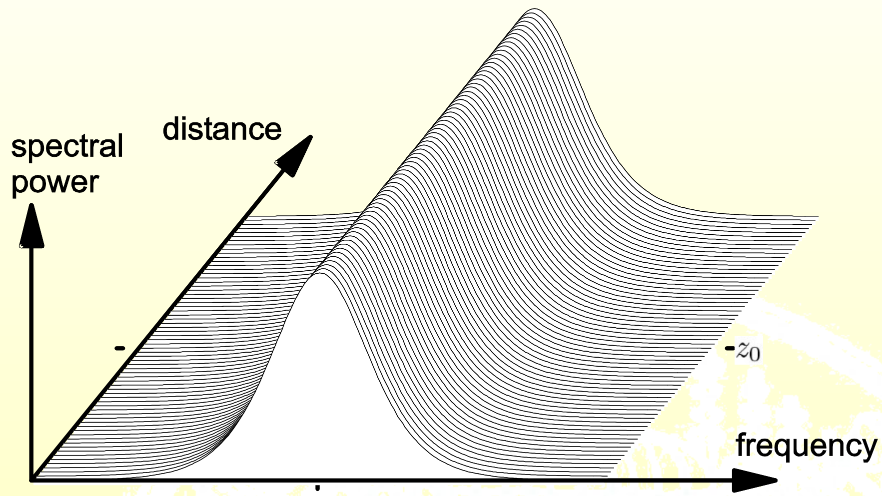
$A(z,T)$	envelope of electric field
z	position
T	time (in comoving frame)
β_2	coefficient of group velocity dispersion
γ	coefficient of Kerr nonlinearity (contains n_2)
P_1	peak power
T_0	pulse duration

Pulses of invariant shape, stable solutions of wave equation:
Solitons are the natural bits for telecom

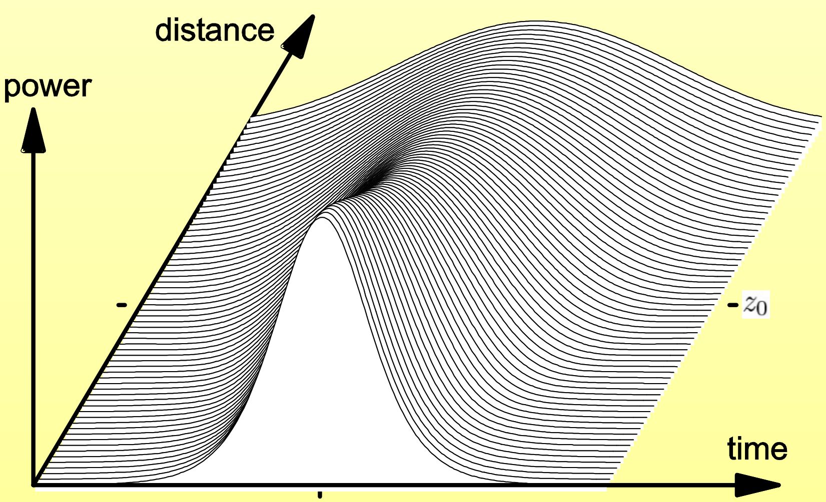
a soliton propagates without change of shape



temporal profile



spectral profile



Length scale $z_0 = \frac{\pi}{2} L_{\mathrm{D}} = \frac{\pi}{2} \frac{T_0^2}{|\beta_2|}$

For comparison, temporal profile
of same sech² pulse
in the absence of nonlinearity

Scaling of the soliton

$$P_1 T_0^2 = \frac{|\beta_2|}{\gamma}$$

τ	$\hat{P}$	z_0	E_1	n_{phot}
1 ns	22.4 μW	28 100 km	25.4 fJ	$1.92 \cdot 10^5$
100 ps	2.24 mW	281 km	254 fJ	$1.92 \cdot 10^6$
10 ps	224 mW	2810 m	2.54 pJ	$1.92 \cdot 10^7$
1 ps	22.4 W	28.1 m	25.4 pJ	$1.92 \cdot 10^8$
100 fs	2.24 kW	281 mm	254 pJ	$1.92 \cdot 10^9$

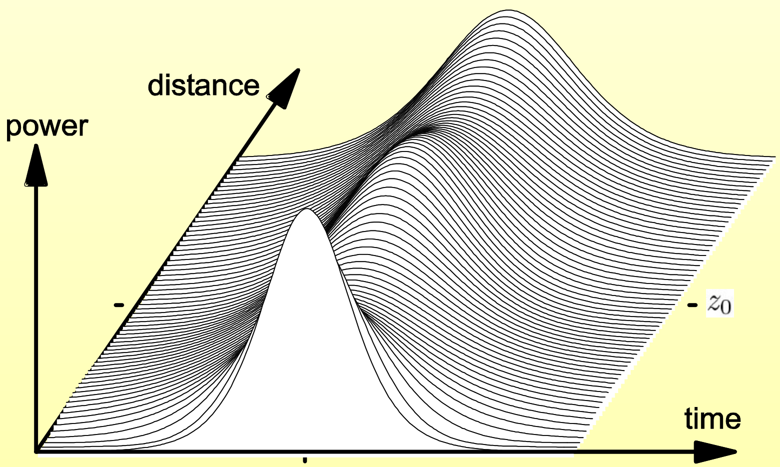
Note FWHM: $\tau = 2\, T_0 \cosh^{-1}\sqrt{2} = 1.763\, T_0$

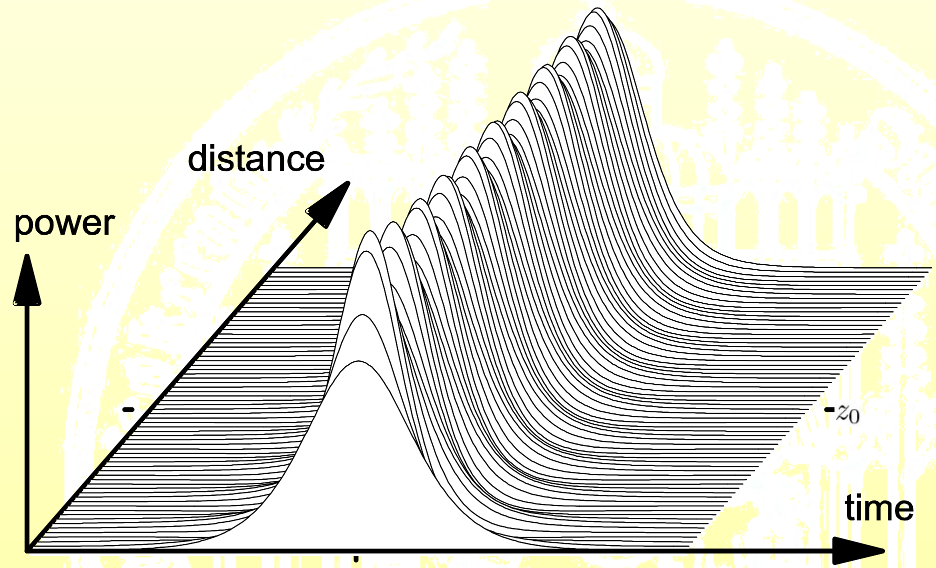
Typical orders of magnitude of characteristic soliton parameters.

Assumed are a wavelength of 1.5 µm, a fiber dispersion of $\beta_2 = -18\,\mathrm{ps}^2/\mathrm{km}$ corresponding to ca. $D = 15\,\mathrm{ps/(nm\,km)}$, and a nonlinearity coefficient $\gamma = 2.5 \cdot 10^{-3}\,\mathrm{W}^{-1}\mathrm{m}^{-1}$ corresponding to $n_2 = 3 \cdot 10^{-20}\,\mathrm{m}^2/\mathrm{W}$, and $A_{\text{eff}} \approx 50\,\mu\mathrm{m}^2$. The table gives the peak power $\hat{P}$, the soliton period z_0, its energy, and the photon number, always rounded to three significant digits. In all cases the action $W = |\beta_2/\gamma| = 7.2 \cdot 10^{-24}\,\mathrm{W\,s}^2$.

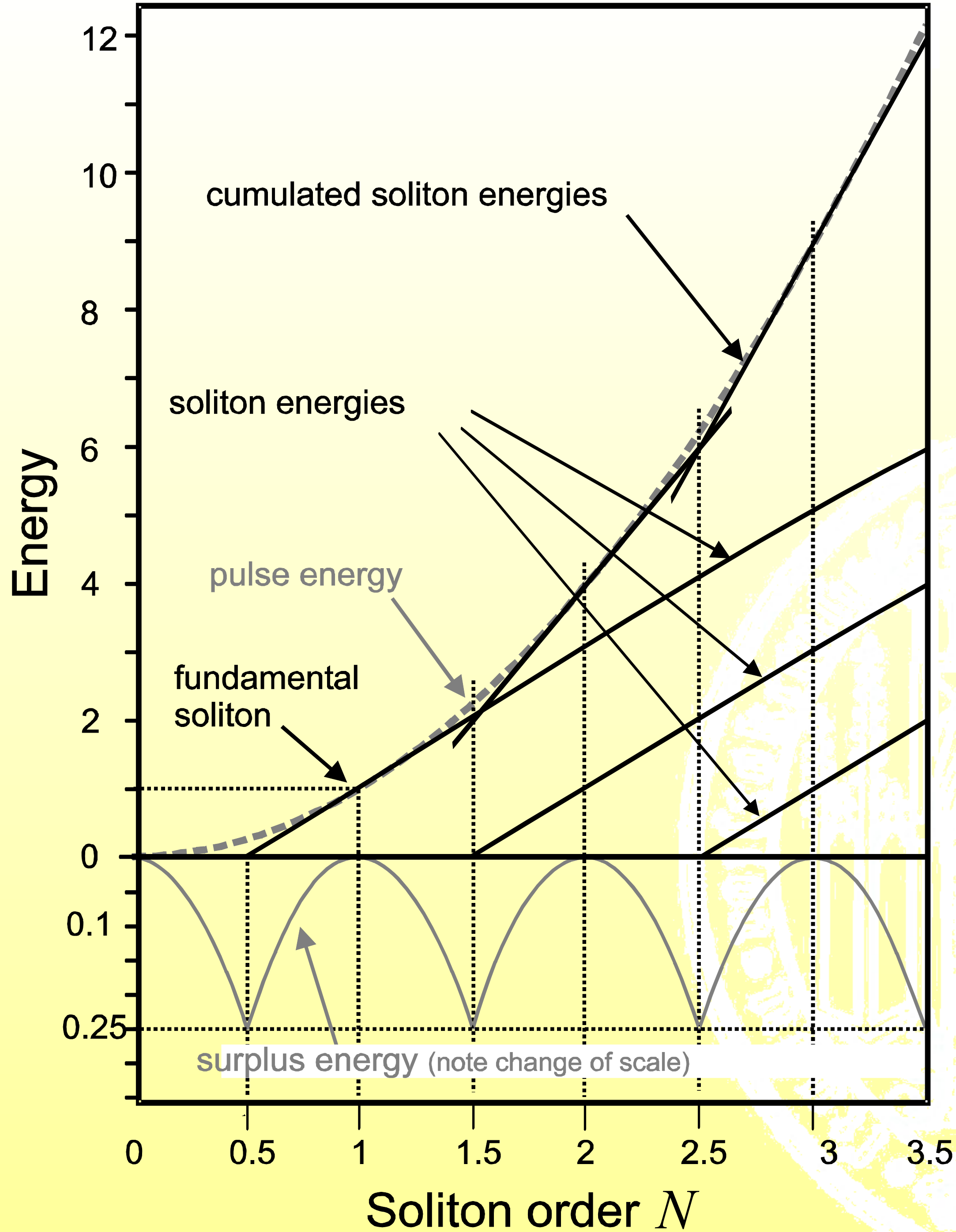
How to excite a soliton?

Suppose you launch $P_1 T_0^2 = N^2 \frac{|\beta_2|}{\gamma}$





Pulse launched with $N = 0.8$ Pulse launched with $N = 1.2$



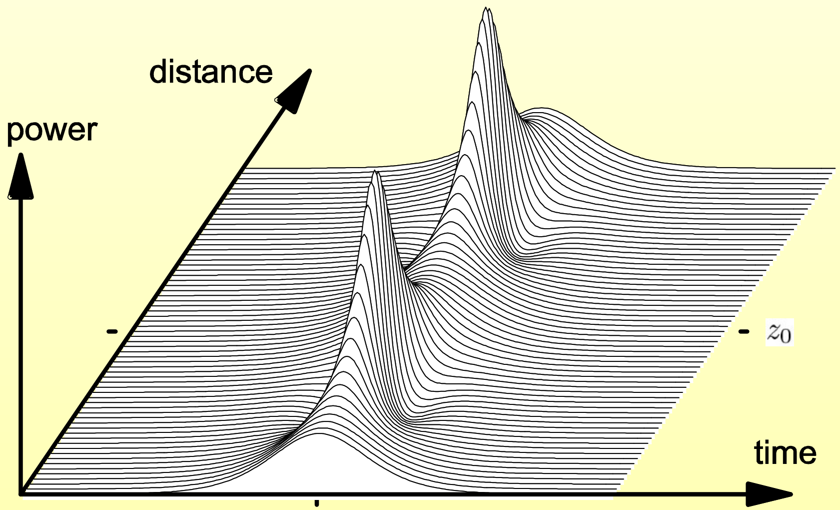
For integer N,
N solitons are formed.

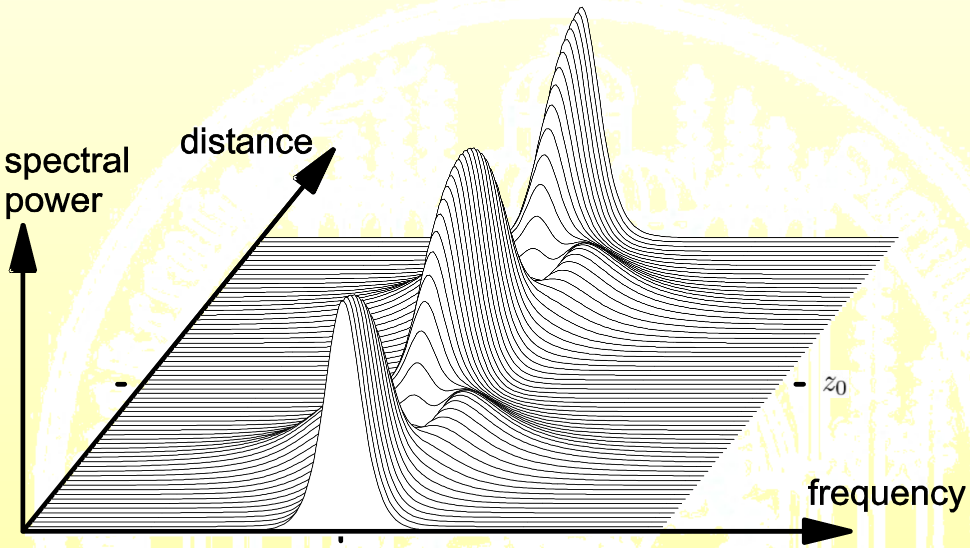
For non-integer N,
some energy is radiated aw

higher-order solitons

$$P_1 T_0^2 = N^2 \frac{|\beta_2|}{\gamma}$$

$N = 2$

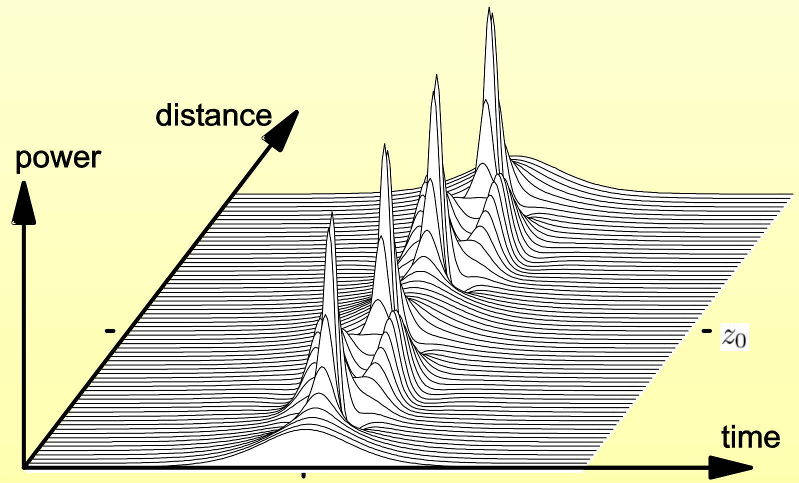


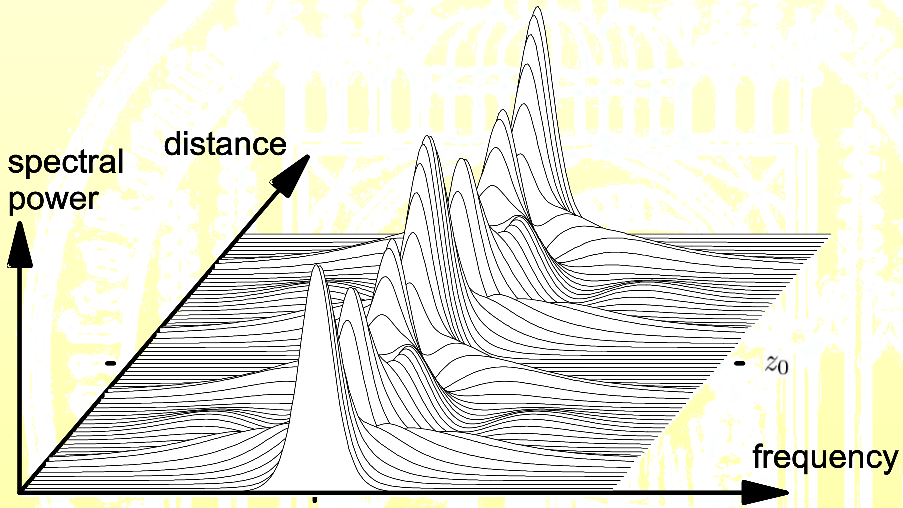


higher-order solitons

$$P_1 T_0^2 = N^2 \frac{|\beta_2|}{\gamma}$$

$N = 3$

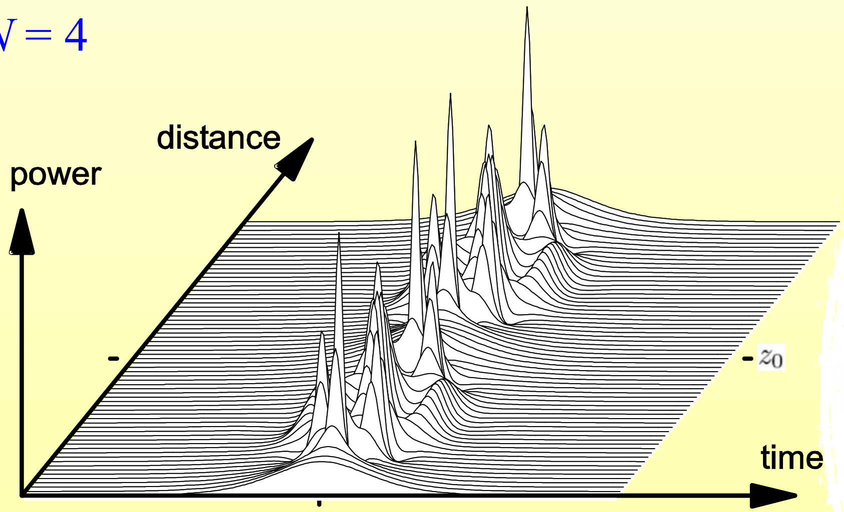


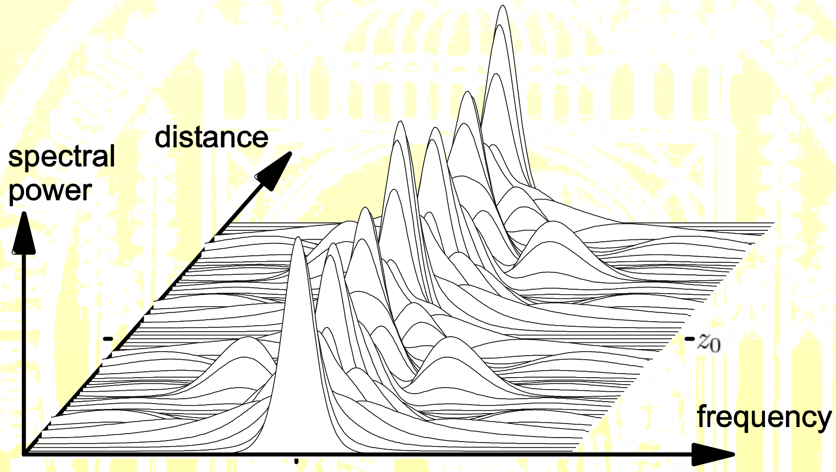


higher-order solitons

$$P_1 T_0^2 = N^2 \frac{|\beta_2|}{\gamma}$$

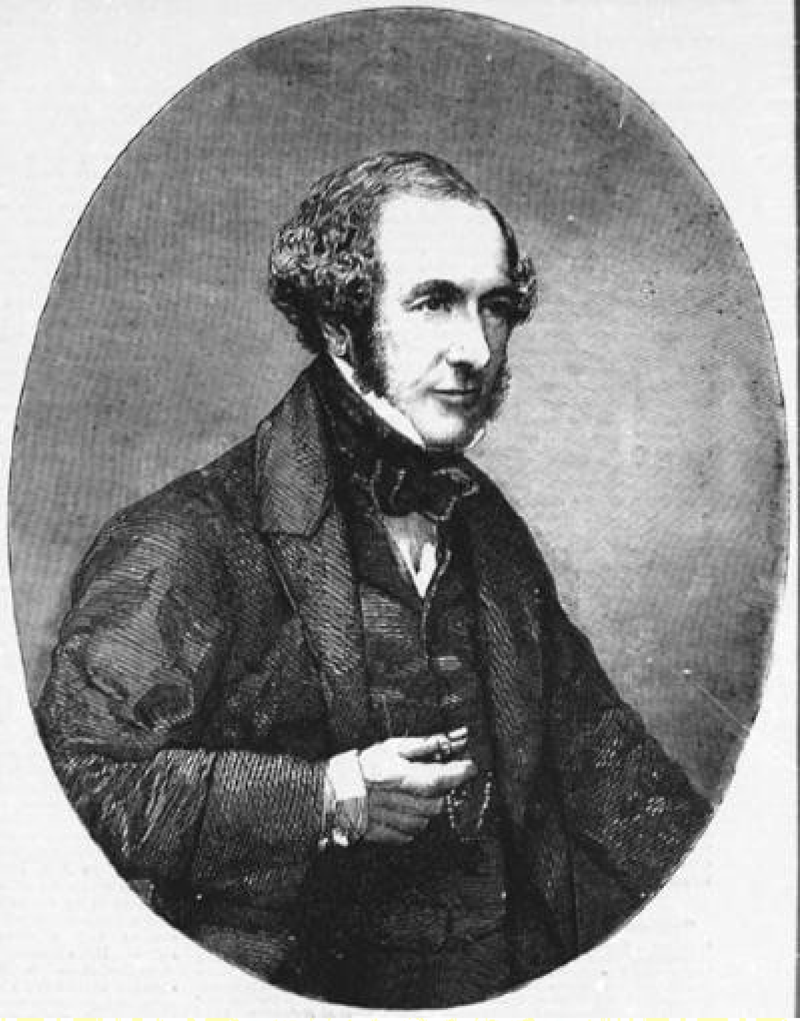
$N = 4$





runners on a mattress
illustrate the principle of fiber solitons

John Scott Russell, 1844:



I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed.

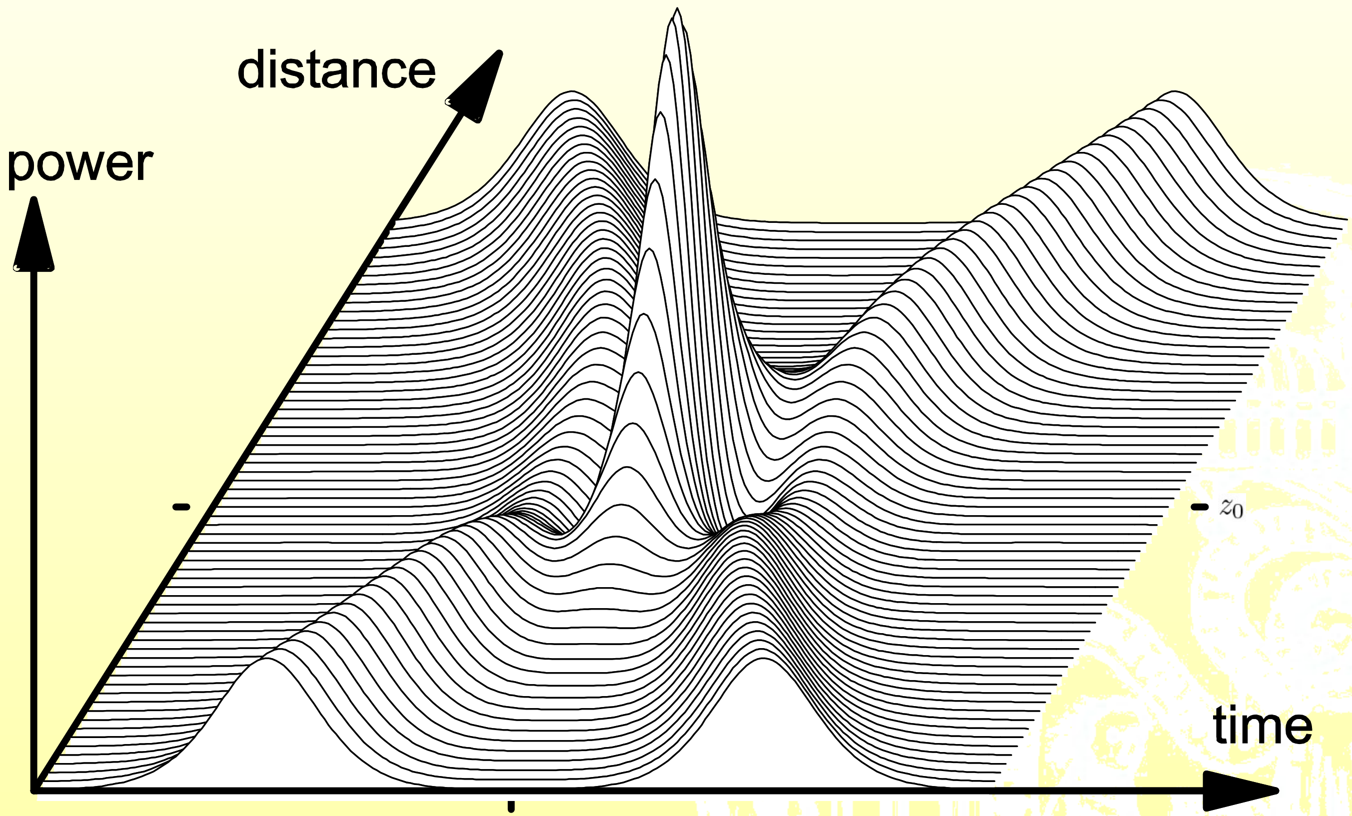
I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height.

Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel.

Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation.

Soliton on the Scott Russell Aqueduct on the Union Canal, 12 July 1995.

Photo from Nature 376, 3 Aug 1995, pg 373



soliton – soliton collision

In the normal dispersion regime, a different type of solution arises!

$\gamma > 0, \beta_2 < 0$ (anomalous dispersion)

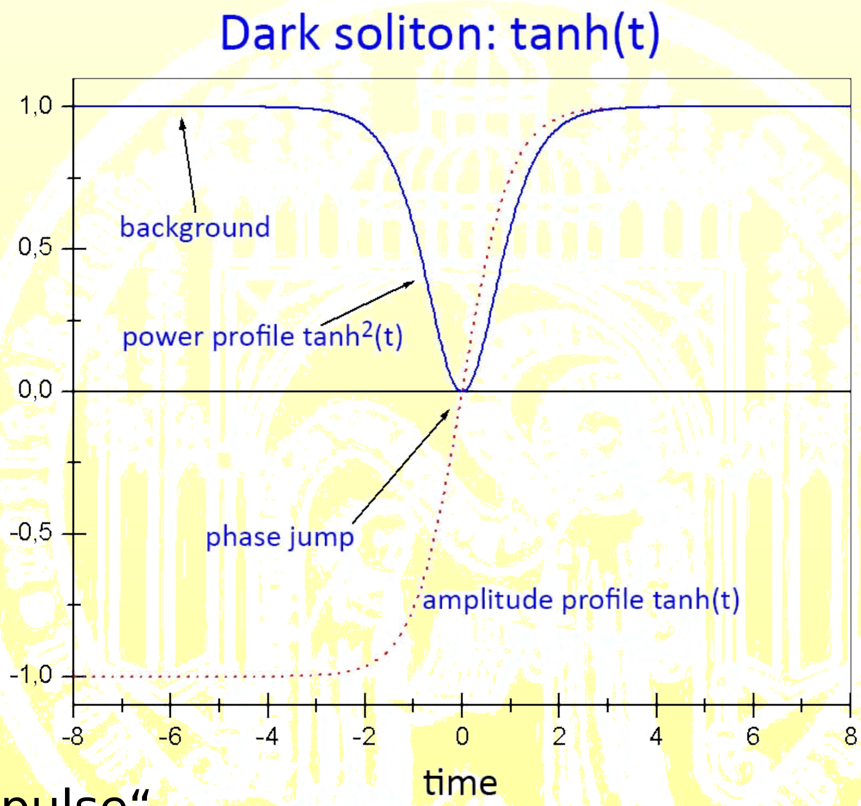
$$A(z,t) = \sqrt{P_1}\ \mathrm{sech}\left(\frac{T}{T_0}\right)\ e^{i\gamma P_1 z/2}$$

bright soliton

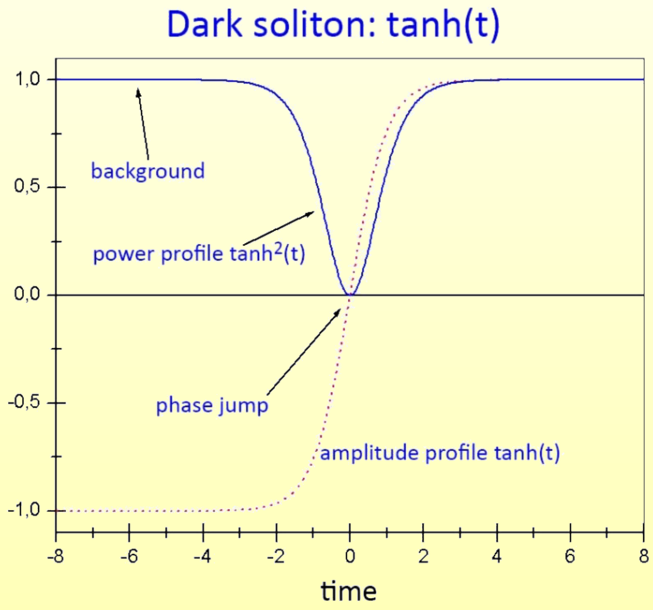
$\gamma > 0, \beta_2 > 0$ (normal dispersion)

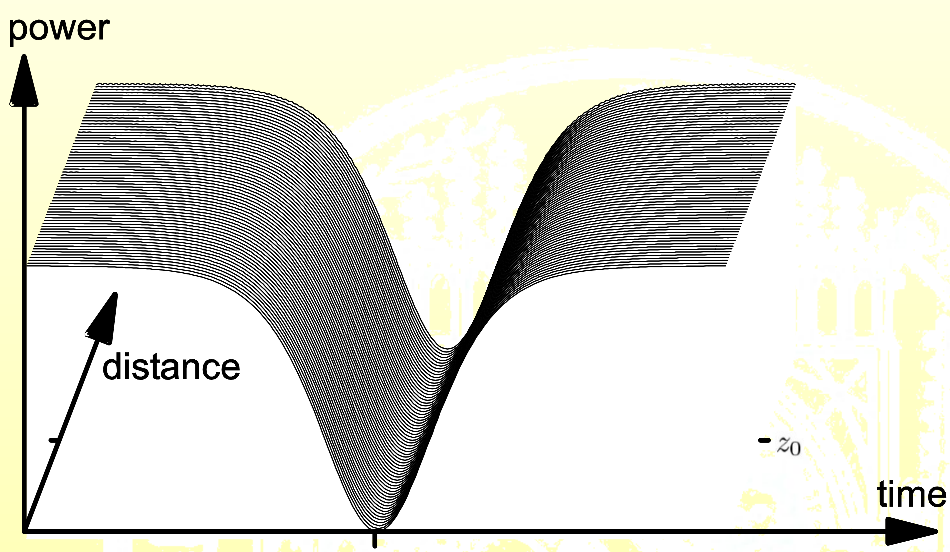
$$A(z,t) = \sqrt{P_1}\ \tanh\left(\frac{T}{T_0}\right)\ e^{i\gamma P_1 z}$$

dark (black) soliton



Remember that $\tanh^2(\tau) = 1 - \mathrm{sech}^2(\tau)$
to see that a dark soliton is a „dark pulse“


Dark soliton: tanh(t)
1,0
0,5
0,0
-0,5
-1,0
background
power profile tanh²(t)
phase jump
amplitude profile tanh(t)
-8
-6
-4
-2
0
2
4
6
8
time


power
distance
z_0
time

We consider a few more nonlinear effects:

- Cross phase modulation
- Four-wave mixing
- Inelastic scattering (Brillouin, Raman)

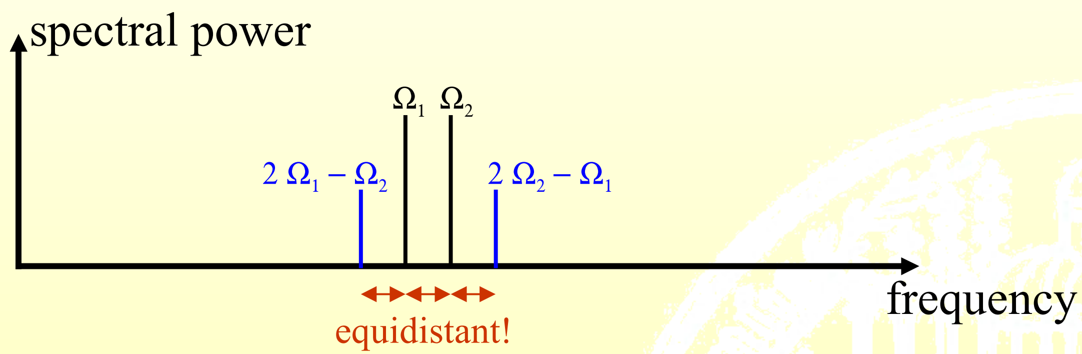
Cross phase modulation

Two coupled NLSE‘s

$$
\begin{aligned}
\frac{\partial A_1}{\partial z} &= -\frac{i}{2}\beta_{21}\,\frac{\partial^2 A_1}{\partial T^2} + i\gamma_1(|A_1|^2 + 2|A_2|^2)A_1 \\
\frac{\partial A_2}{\partial z} &= -\frac{i}{2}\beta_{22}\,\frac{\partial^2 A_2}{\partial T^2} + i\gamma_2(|A_2|^2 + 2|A_1|^2)A_2
\end{aligned}
$$

Four wave mixing

$$\begin{aligned} \omega_1 + \omega_2 &= \omega_3 + \omega_4 \\ k_1 + k_2 &= k_3 + k_4 \quad \text{where} \quad k_i = n_i \omega_i / c \end{aligned}$$



Implication for data transmission on several wavelength channels:
Channel cross talk

Degenerate four wave mixing

The two pump frequencies coincide

Phase matching

In the presence of dispersion, different frequency components may have different wa

⇒ Relative phase varies, energy transfer thwarted

⇒ To reduce channel cross talk, employ strong dispersion

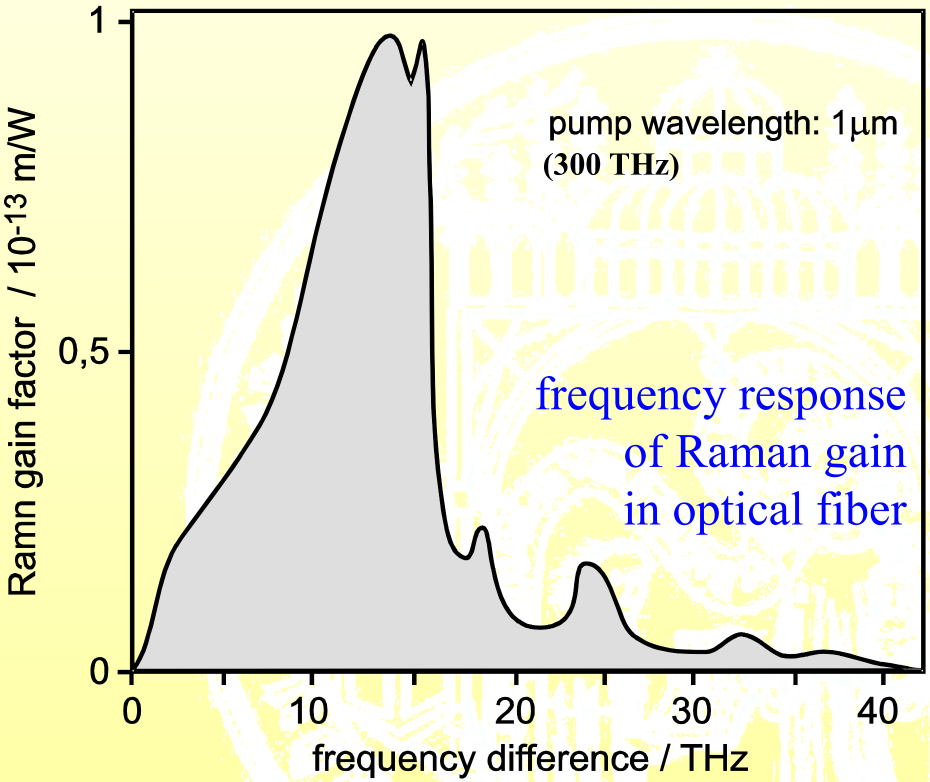
⇒ To facilitate sideband buildup, minimize differential phase evolution

$$\Delta k = \Delta k_{\mathrm{fiber}} + \Delta k_{\mathrm{NL}} \stackrel{!}{=} 0$$

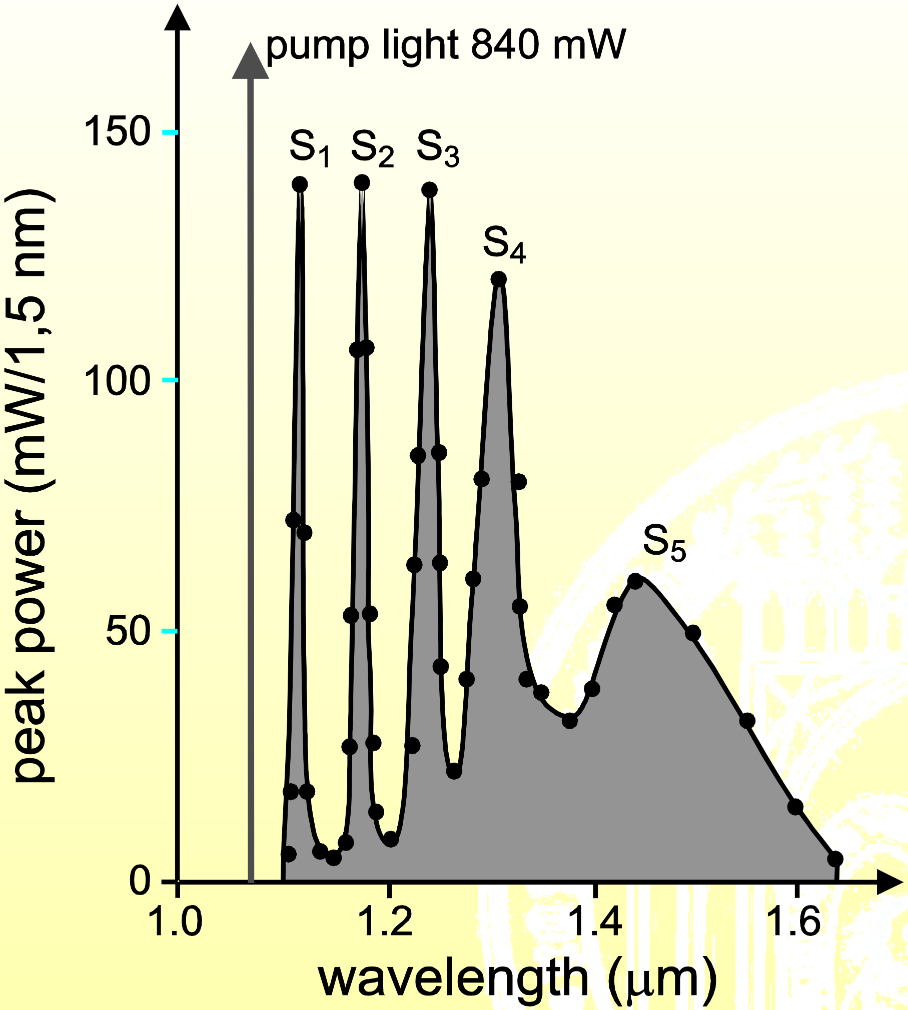
Inelastic scattering processes

Brillouin Scattering:
Electrostriction creates sound wave which acts as moving grating

Raman Scattering:



- Causes signals to shift their frequency
- Can be used to provide gain: lasers, amplifiers



Raman scattering spectrum with five scattering orders

Corrections to the propagation equation for a non-idealized situation

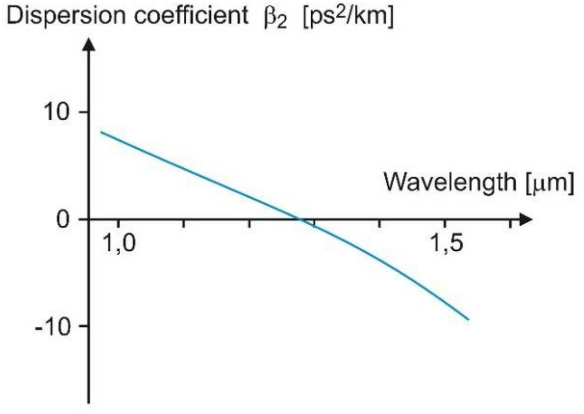
In reality several corrections may apply:
Some effects are not captured in the NLSE,
but may be described by additional terms

NLSE

$$
\begin{aligned}
i\,\frac{\partial A}{\partial z} \;=\; & \frac{1}{2}\,\beta_2\,\frac{\partial^2 A}{\partial T^2} + \frac{i}{6}\,\beta_3\,\frac{\partial^3 A}{\partial T^3} - \frac{1}{24}\,\beta_4\,\frac{\partial^4 A}{\partial T^4} + \ldots \\
& - \gamma\,|A|^2 A
\end{aligned}
$$

series expansion of dispersion

Dispersion:
Series expansion around operating wavelength.
Close to the zero, higher order terms gain importance



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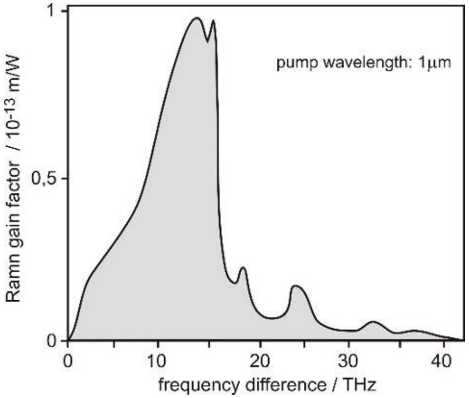
&\quad - \gamma\,|A|^2 A - \frac{i\gamma}{\omega_0}\,\frac{\partial}{\partial T}\left(|A|^2 A\right) + T_{\mathrm{R}}\gamma A\frac{\partial}{\partial T}|A|^2
\end{aligned}
$$

series expansion of dispersion

other nonlinear terms

Raman scattering:
Energy is continuously transferred from the short-wave to the long-wave side – there is a continuous shift of the central frequency of optical signals which scales with τ^{-4}.
(May be neglected for τ>5 ps)

On other hand: Possibility of amplification with pump wave.



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& -\gamma\,|A|^2 A - \frac{i\gamma}{\omega_0}\,\frac{\partial}{\partial T}\left(|A|^2 A\right) + T_{\mathrm{R}}\gamma A \frac{\partial}{\partial T}|A|^2 \\
& -\frac{i\alpha}{2} A
\end{aligned}
$$

series expansion of dispersion

other nonlinear terms

loss

Losses
can be compensated by amplifiers
(e.g. with Er-doped fiber, typically every 50-100 km)