Applications of PEPS to condensed matter: some simple examples

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- Constructing topological RVB spin liquids on the kagome lattice
- Gapped spin liquids in a magnetic field (restricted to AKLT liquids)
- Unconventional RVB superconductor

COLLABORATORS

Topological RVB spin liquids

Norbert Schuch, DP, J. Ignacio Cirac, and David Pérez-García,
Phys. Rev. B **86**, 115108 (2012)

DP, Norbert Schuch, David Pérez-García, and J. Ignacio Cirac,
Phys. Rev. B **86**, 014404 (2012)

DP and Norbert Schuch, Phys. Rev. B 87, 140407 (2013)

Norbert Schuch, DP, J. Ignacio Cirac, and David Perez-Garcia
Phys. Rev. Lett. 111, 090501 (2013)

Quantum magnets under magnetic field

DP, Norbert Schuch, J. Ignacio Cirac, Phys. Rev. B 88, 144414 (2013)

Thibaut Picot and DP, in preparation

Unconventional RVB superconductors

DP, Philippe Corboz, Norbert Schuch, and J. Ignacio Cirac,
Phys. Rev. B RC, in press (2014)

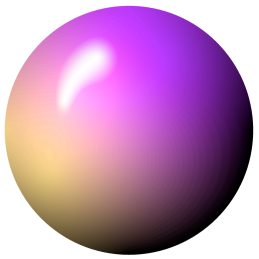
Exotic «spin liquids» beyond the «order parameter» paradigm

* no spontaneous broken symmetry
* no local order
* Topological order

- Do they exist in materials ?
  in simple models ?

- How to detect them ?

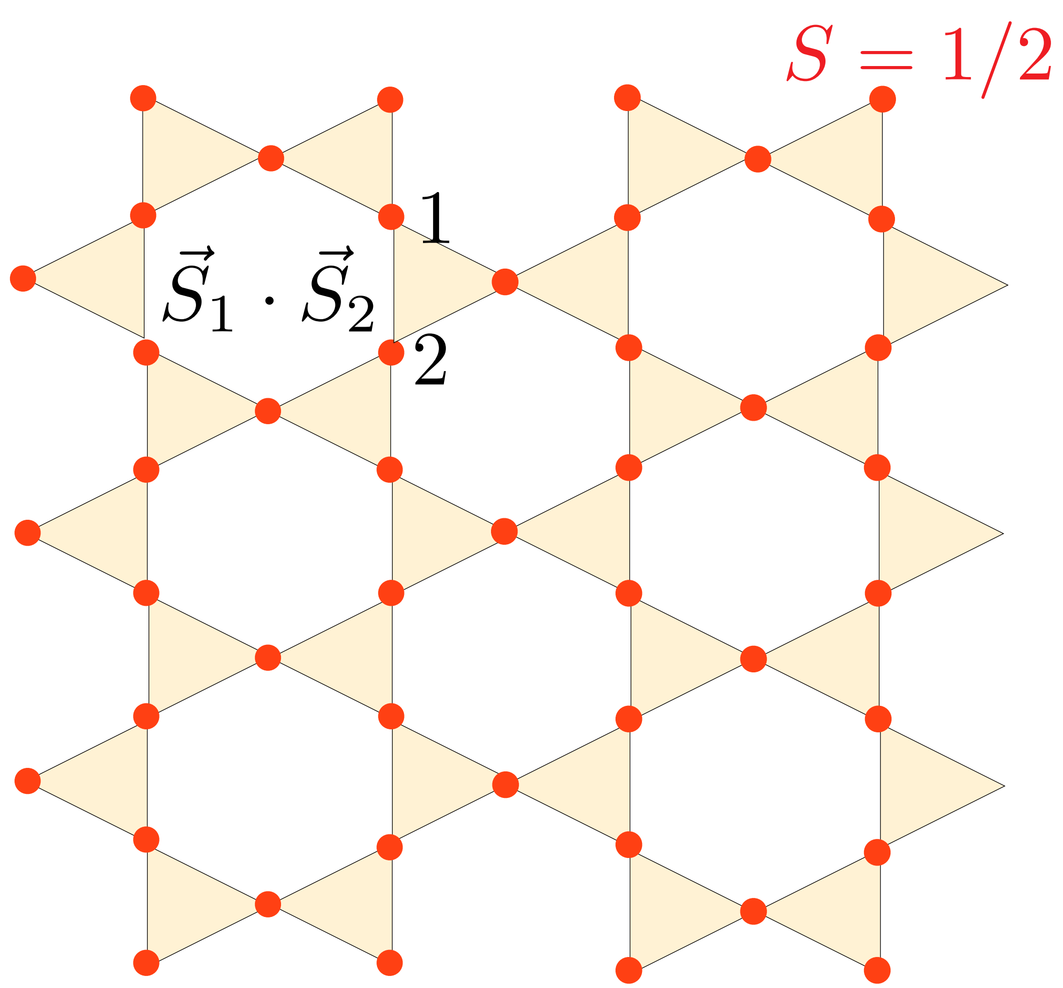
X. G. Wen



GS degeneracy (depends on **topology** of space)

Topological order can also be detected by entanglement measures !

Best candidate : spin-1/2 Heisenberg QAF on the Kagome lattice !



Herbertsmithite: P. Mendels (Orsay) & Z. Hiroi (ISSP)

Numerical «evidence» (DMRG) for a (gapped) spin liquid:

S. Yan, D.A. Huse & S. White, Science 2011
S. Depenbrock, I.P. McCulloch & U. Schollwock, PRL 2012

topological features ?

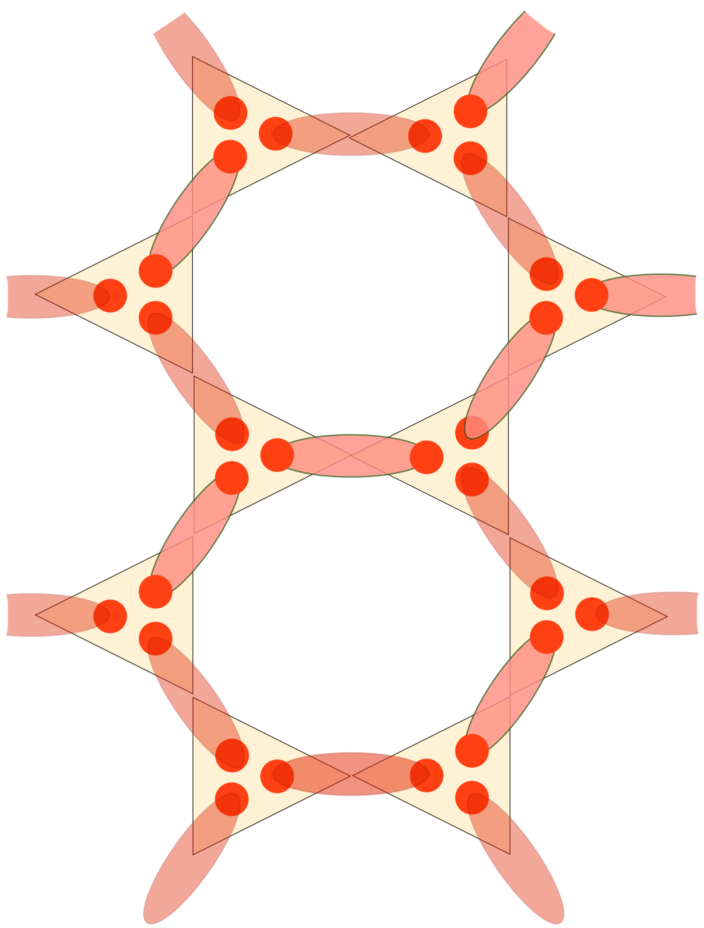
TWO TYPES OF SPIN LIQUIDS:

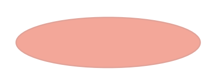
spin-3/2 AKLT

$Bi_3Mn_4O_{12}(NO_3)$ material

J. Lavoie et al., Nat. Phys. 6, 850 (2010)

M. Matsuda et al., Phys. Rev. Lett. 105, 187201 (2010)





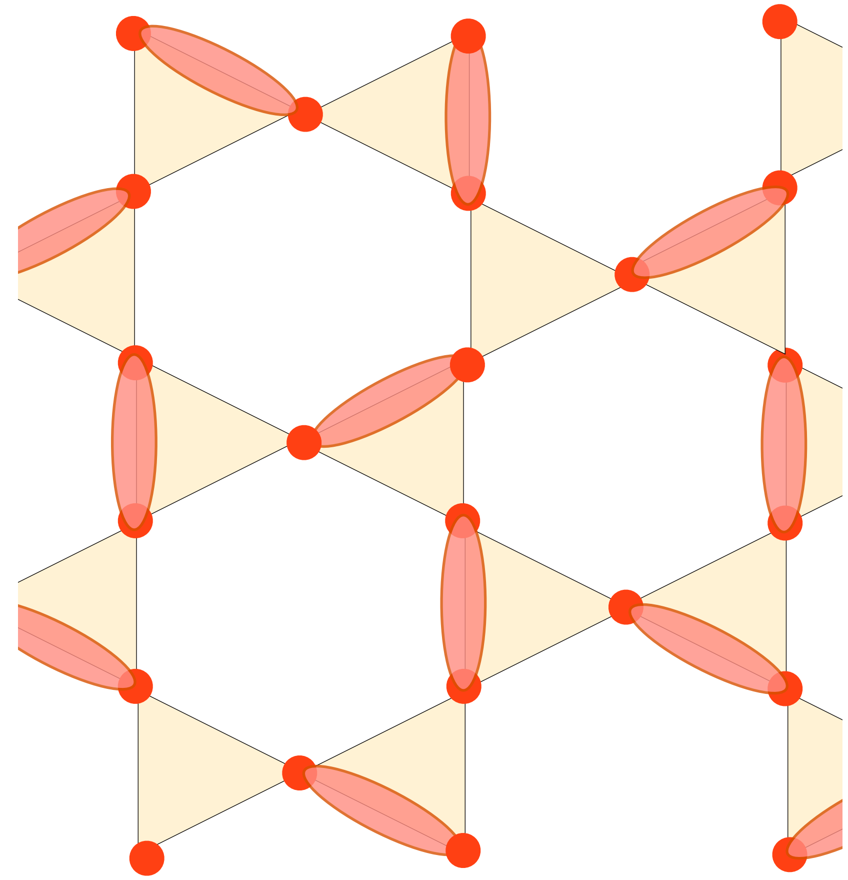
$$S = 0$$

«Trivial» liquid

spin-1/2 RVB

P. Fazekas and P.W. Anderson

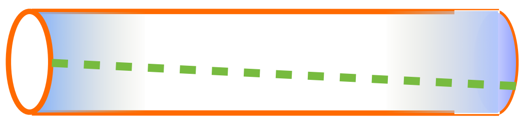
Philosophical Magazine **30**, 423-440 (1974)



Equal-weight superposition of NN singlet coverings

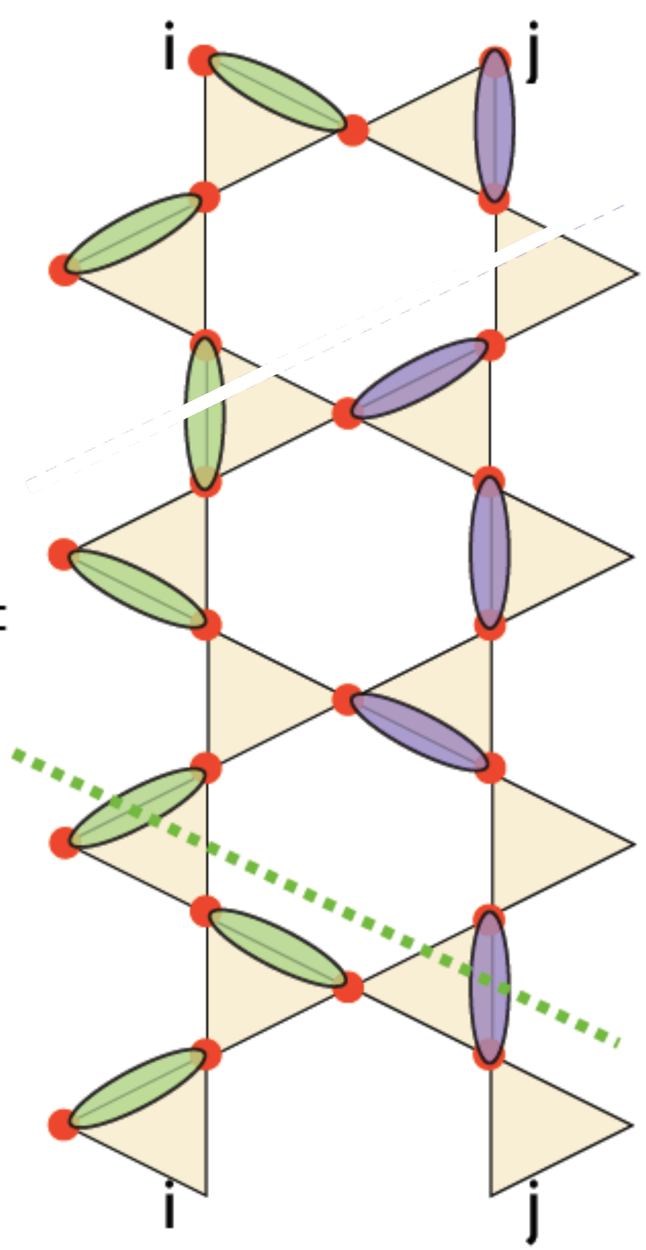
Topological liquid

Eigenstates of a «Wilson loop» operator

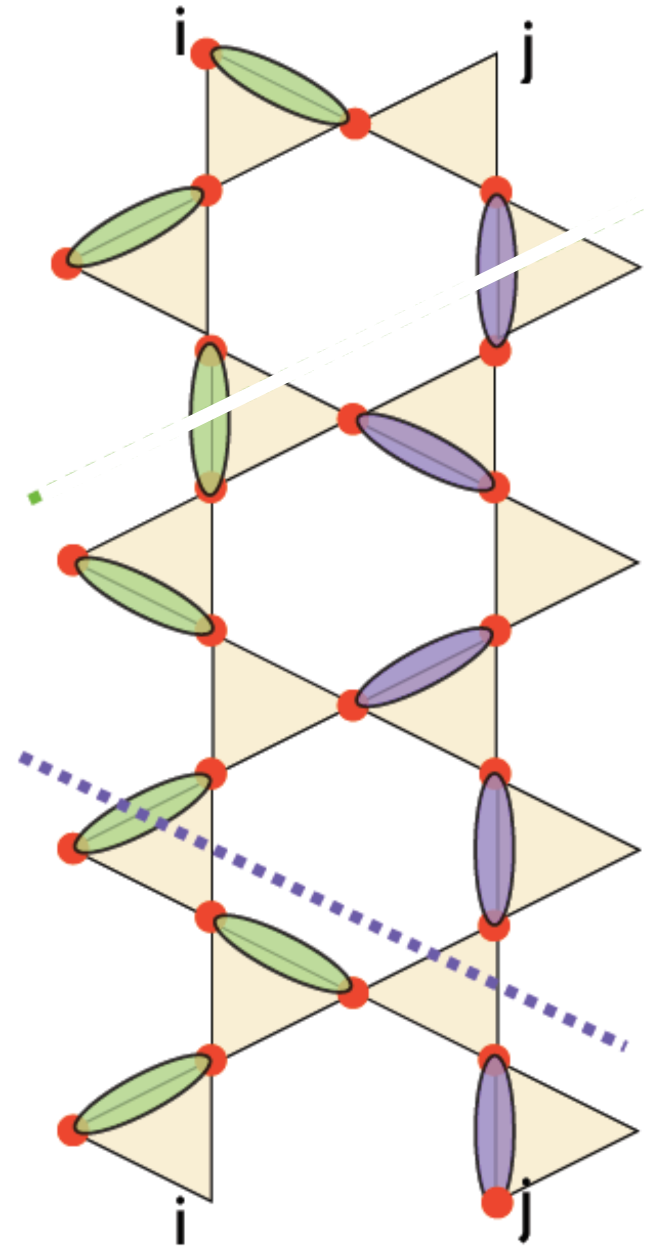


cylinder geometry

$$\Psi^{\pm}_{\mathrm{RVB}} =$$



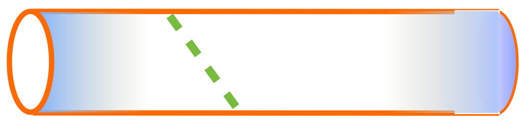
$$\pm$$



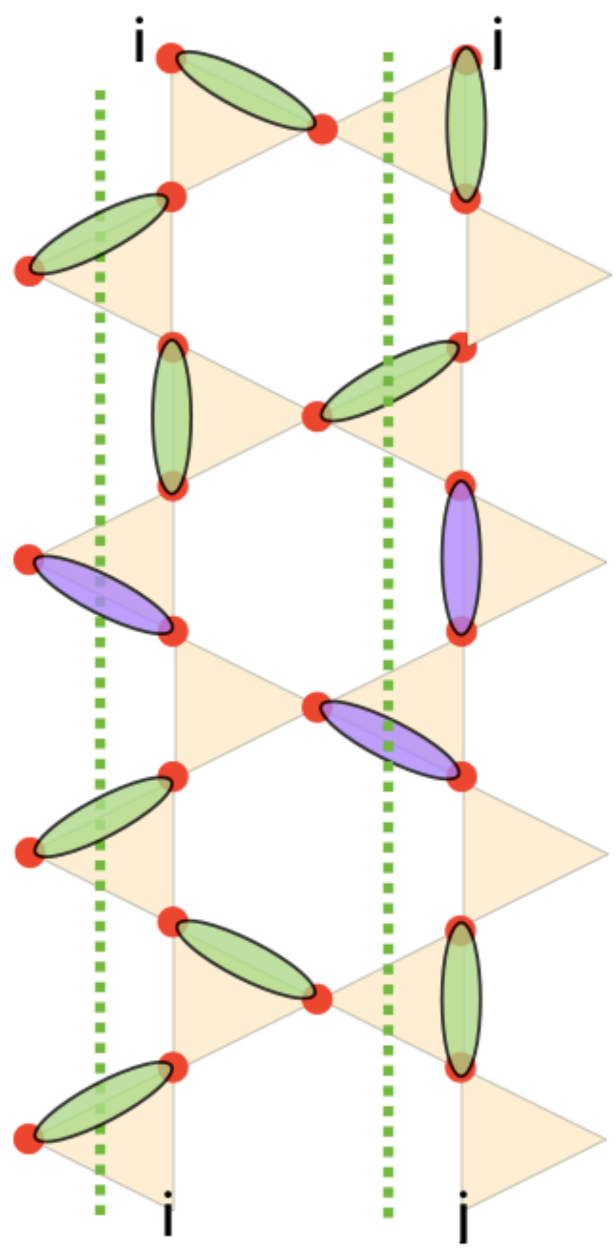
$+$ no vison flux: $w_v = +1$

$-$ $\mathbb{Z}_2$ vison flux: $w_v = -1$

Fix the cylinder boundaries

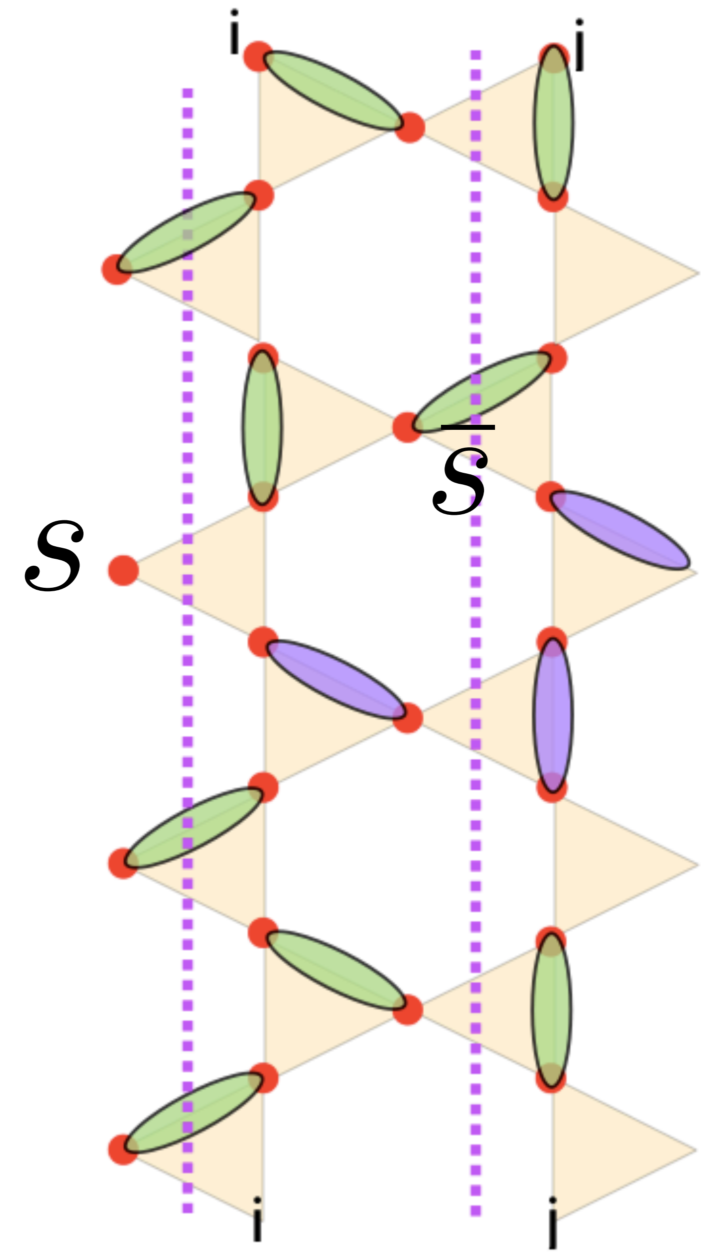


cylinder geometry



«even»

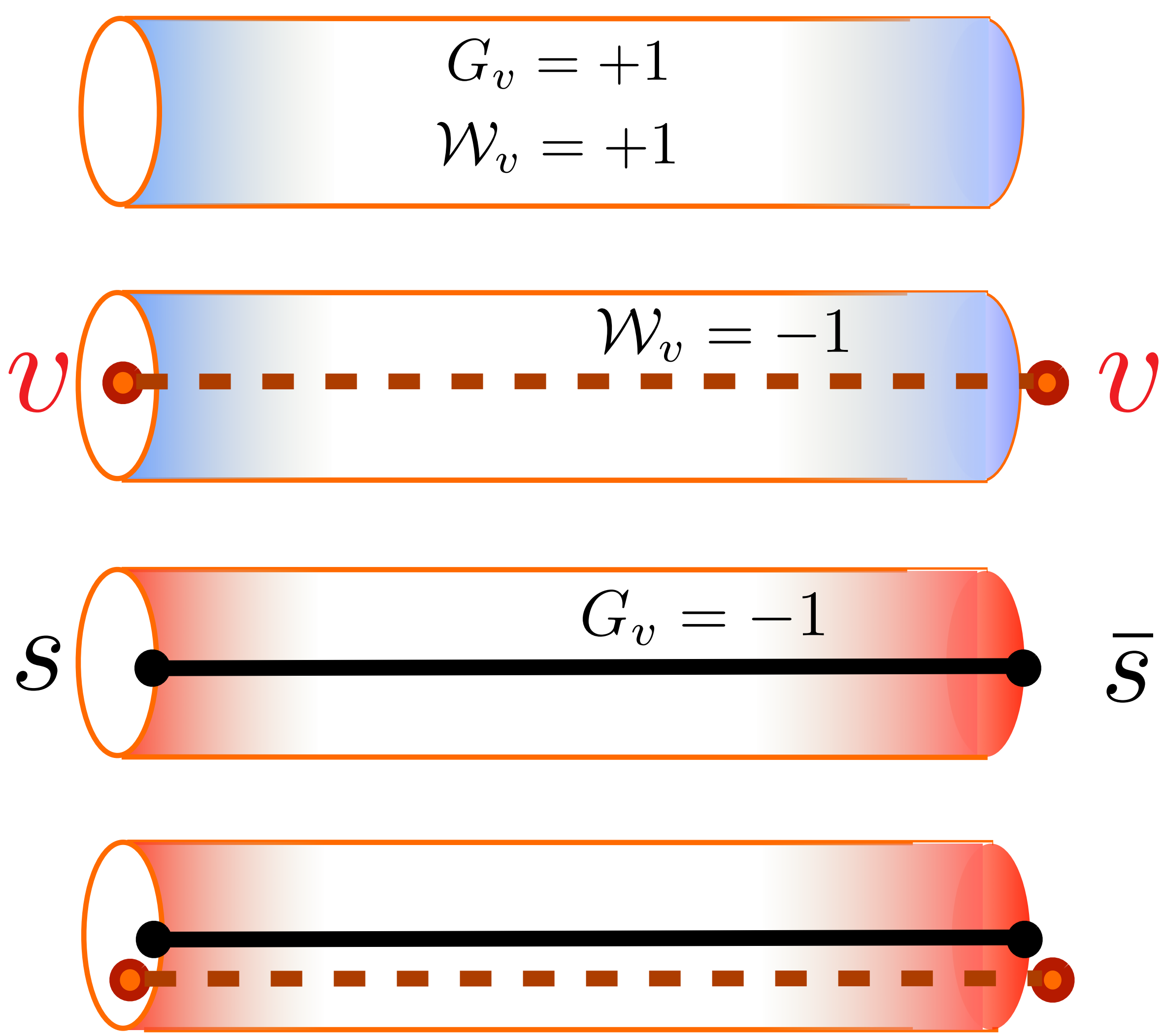
$$G_v = +1$$



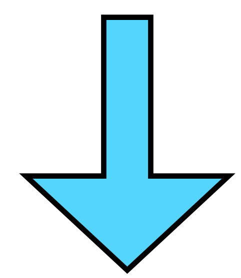
«odd»

$$G_v = -1$$

$\mathbb{Z}_2$ spin liquid :
topological GS inserting «spinons» and «visons»



Kitaev's Toric Code
(fixed point $\xi = 0$)



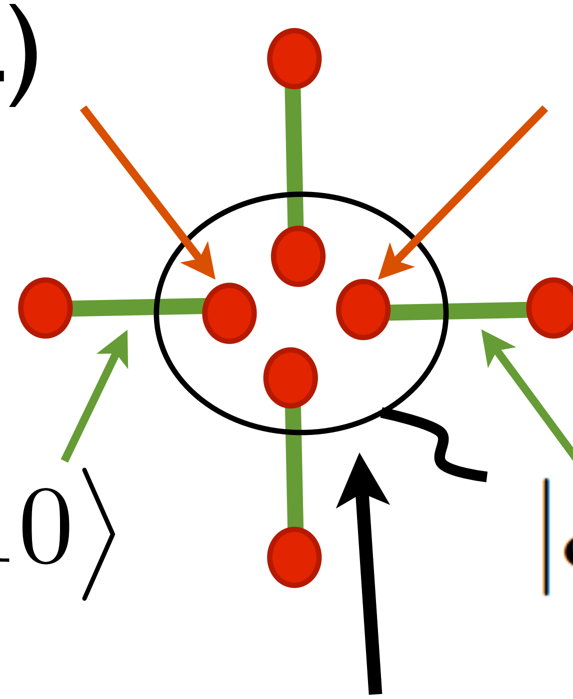
RVB SU(2) spin liquid
(finite ξ ?)

PEPS construction

I. Cirac
F. Verstraete
G. Vidal

S=2 AKLT | spin-1/2 RVB

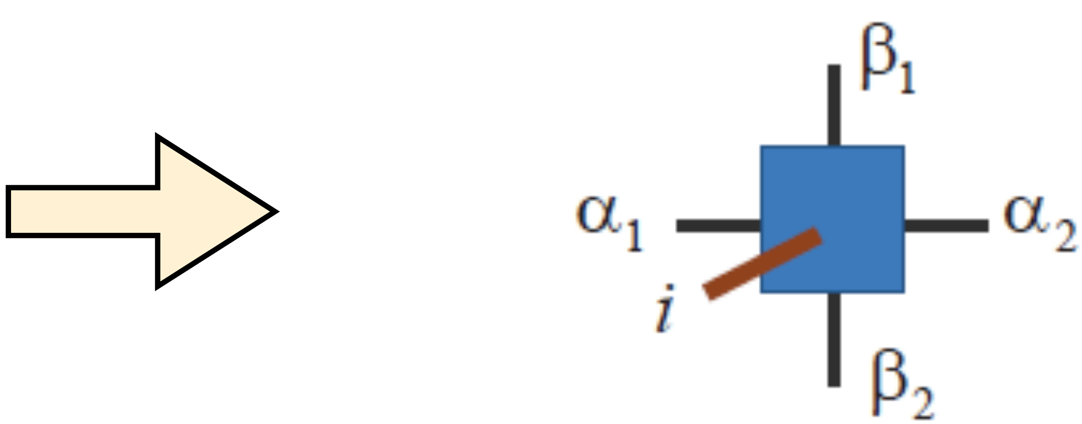
virtual states: $S=1/2$ (D=2) | $1/2 \oplus 0$ (D=3)



$|\mathcal{S}\rangle = |01\rangle - |10\rangle$

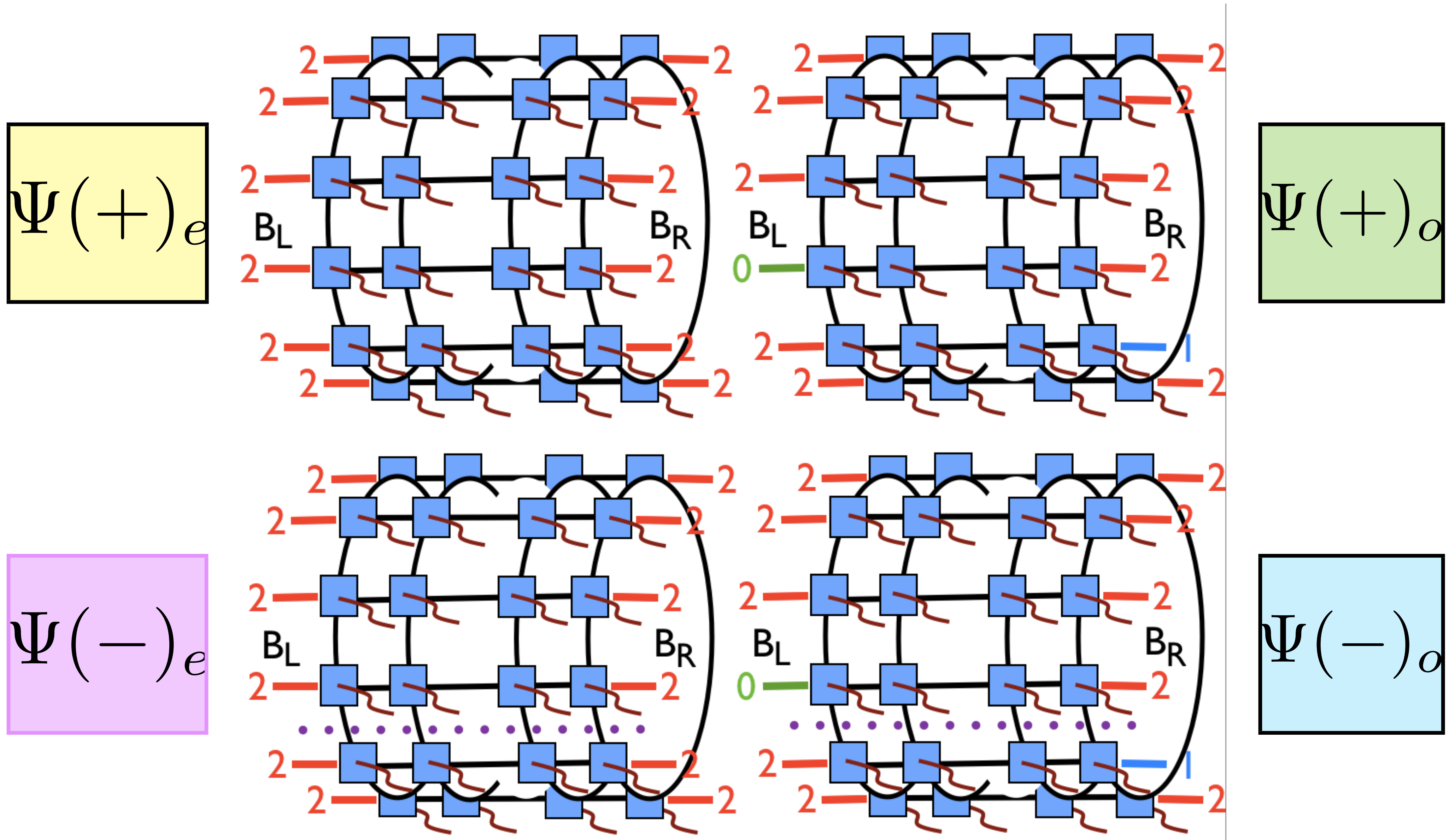
$|\mathcal{S}\rangle = |01\rangle - |10\rangle + |22\rangle$

Project onto physical subspace $d = 2S_{\text{phys}} + 1$:



PEPS tensor

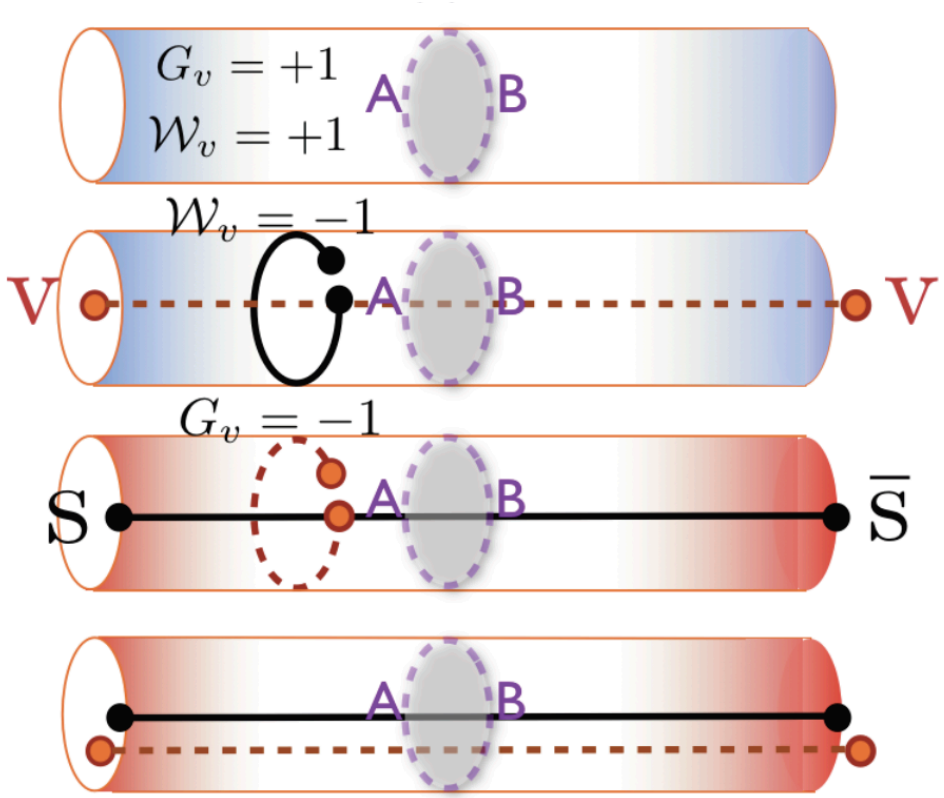
Easy formalism to construct all topological GS !

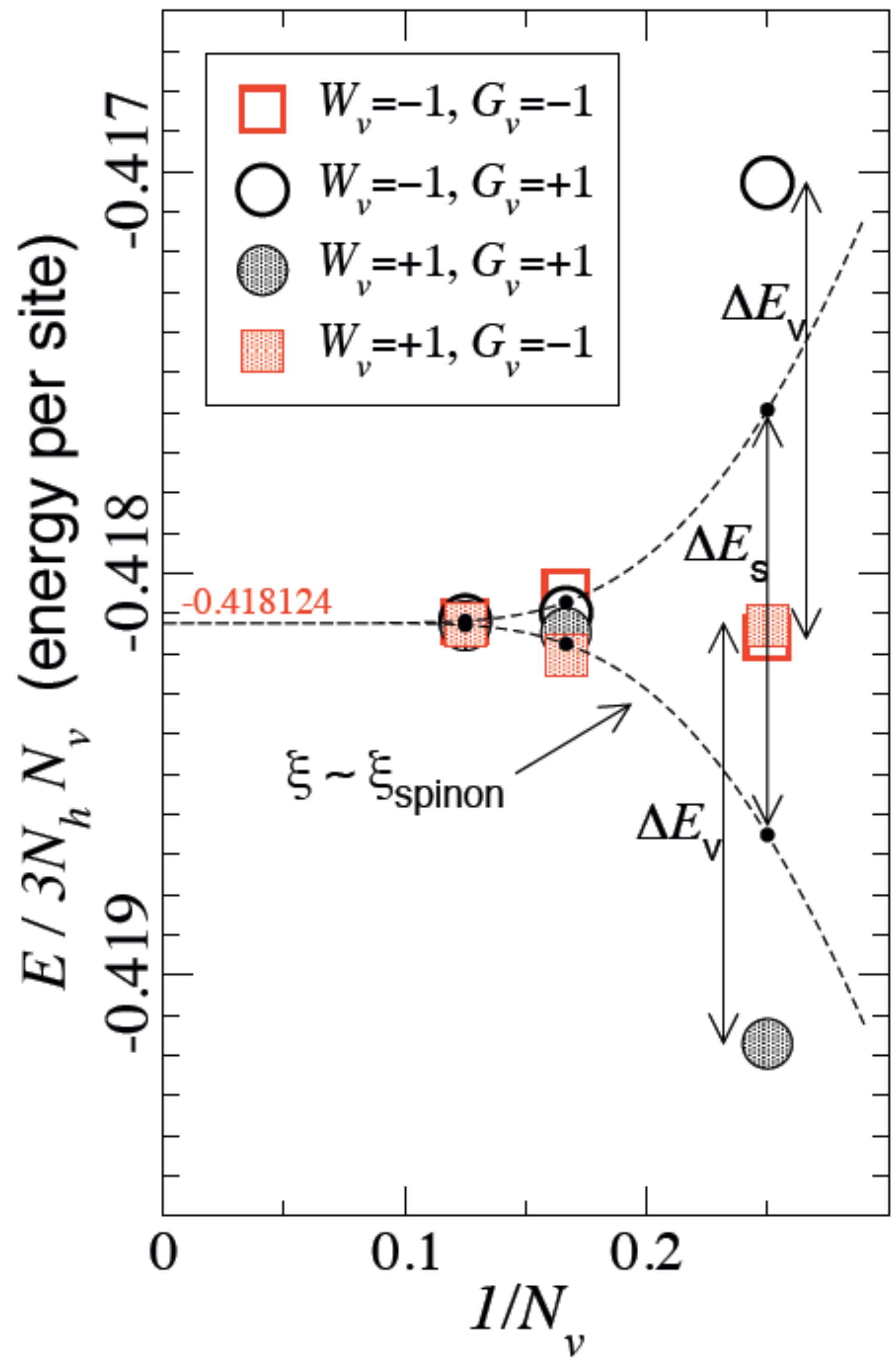


vison line = bond operator diag[1;1;-1]

→ orthogonal in the limit of infinite cylinders $(N_h = \infty)$

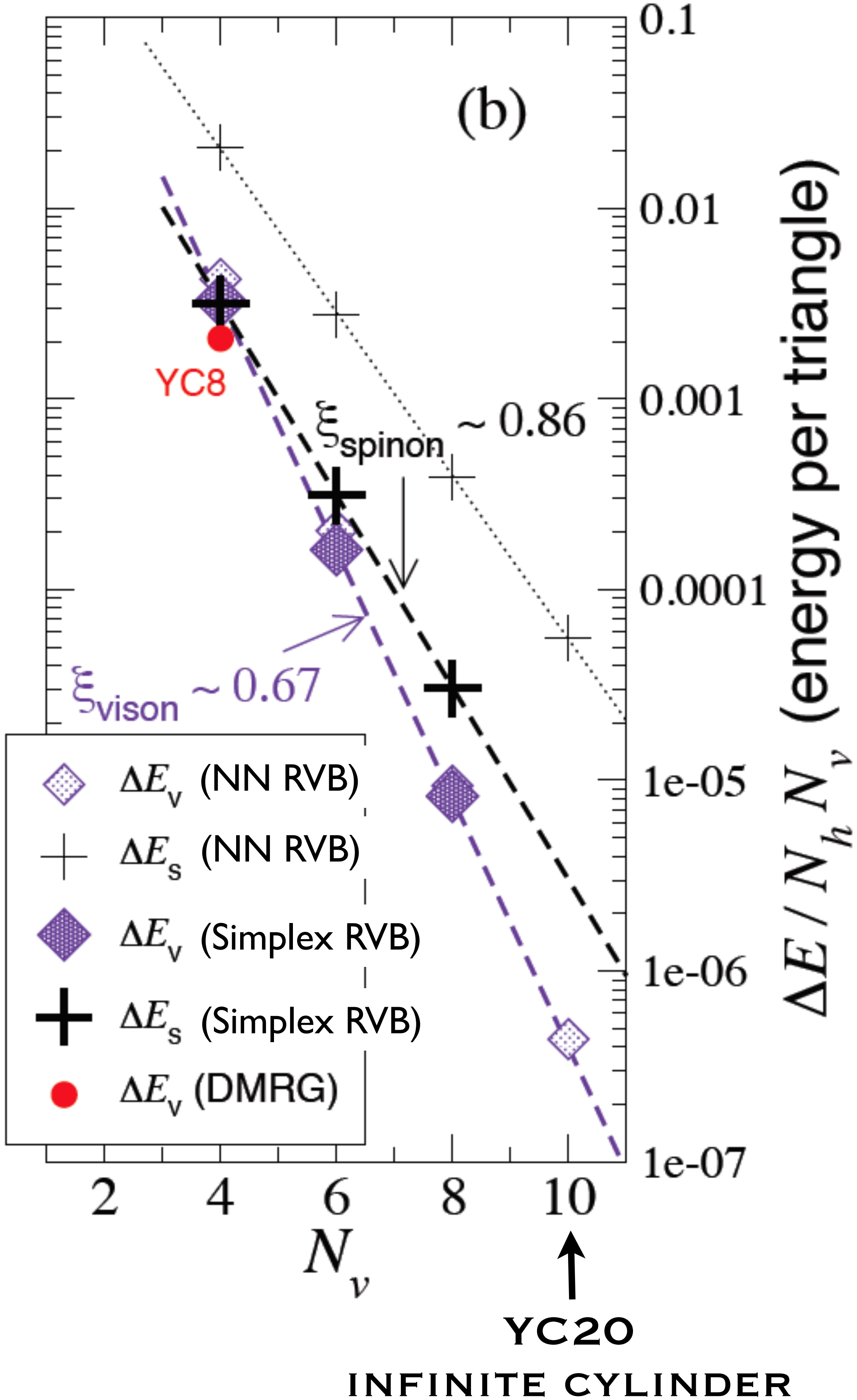
Finite size scaling of RVB energy





YC16 YC12 YC8 cylinders

Numerical data $(N_h \to \infty)$

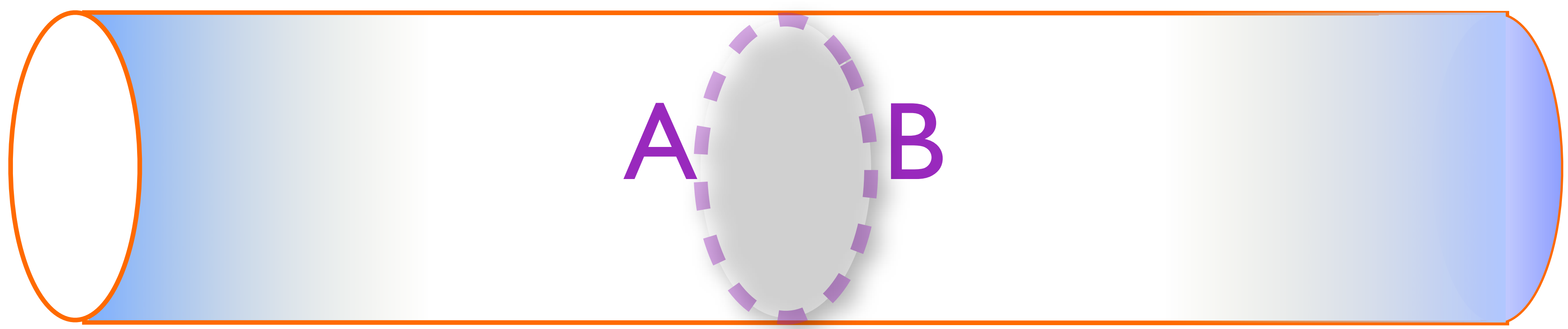


$$\Delta E_{\rm s} = a N_h N_v \exp\left(-N_v/\xi_{\rm spinon}\right),$$
$$\Delta E_{\rm v} = b N_h N_v \exp\left(-N_v/\xi_{\rm vison}\right),$$

Very short coherence lengths

$\xi < 1$ unit cell

Entanglement entropy



$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

Reduced density matrix

$$S_{\mathrm{VN}} = -\mathrm{Tr}\{\rho_A \ln \rho_A\}$$

(Von Neumann)

"area" law

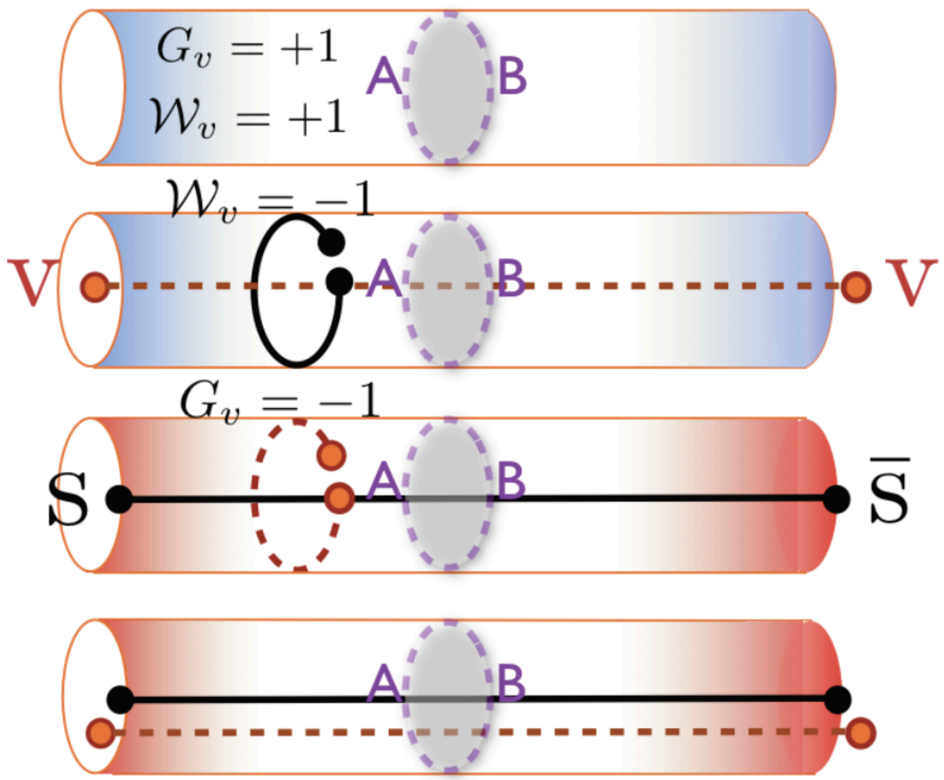
$$S_{\mathrm{VN}} \sim C N_v - \ln D$$

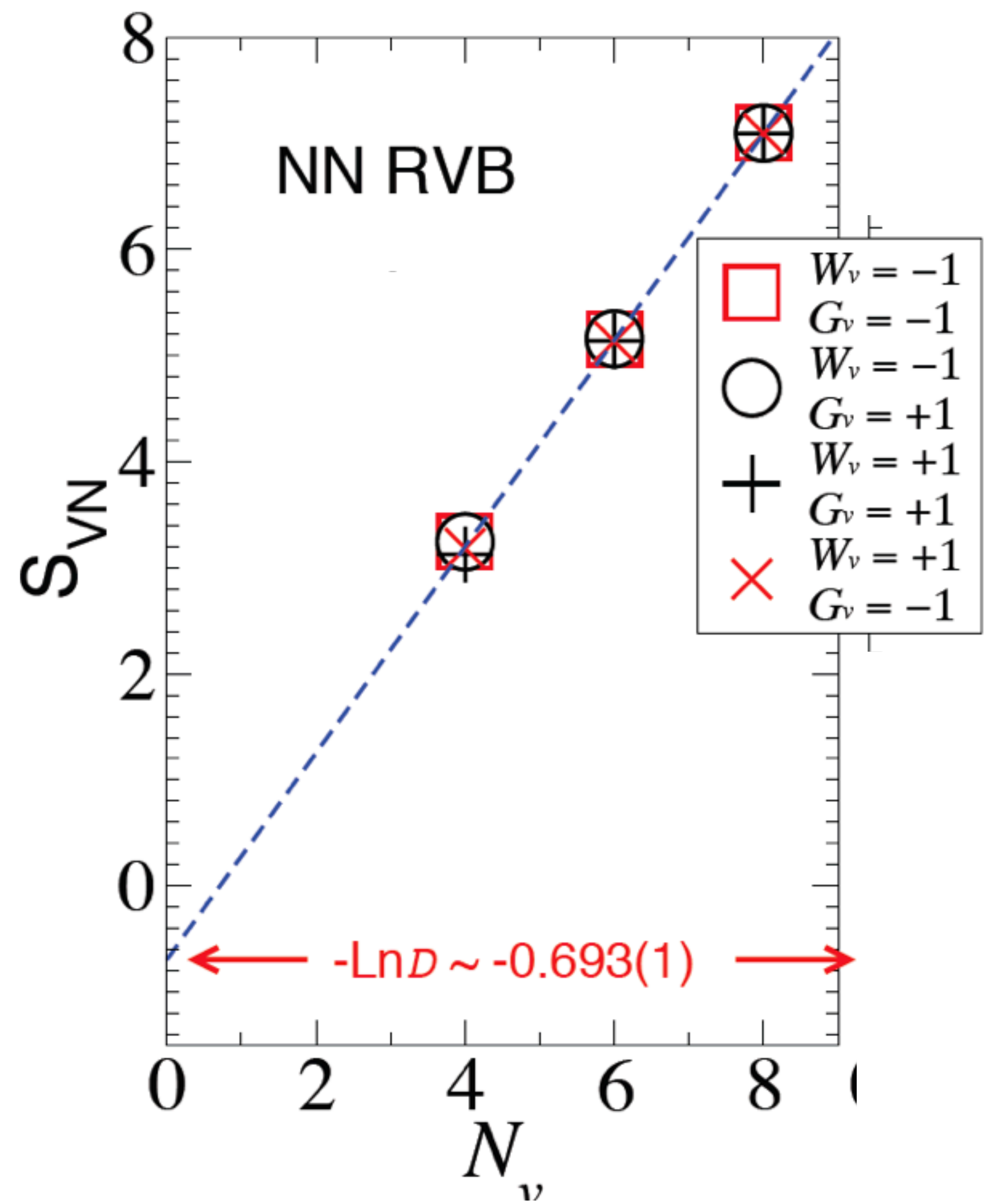
Kitaev & Preskill, 2006

Levin & Wen, 2006

subleading correction to area law: topological entropy

Numerical results





$S_{\mathrm{TE}} \simeq -\ln 2$ → $\mathbb{Z}_2$ spin liquid

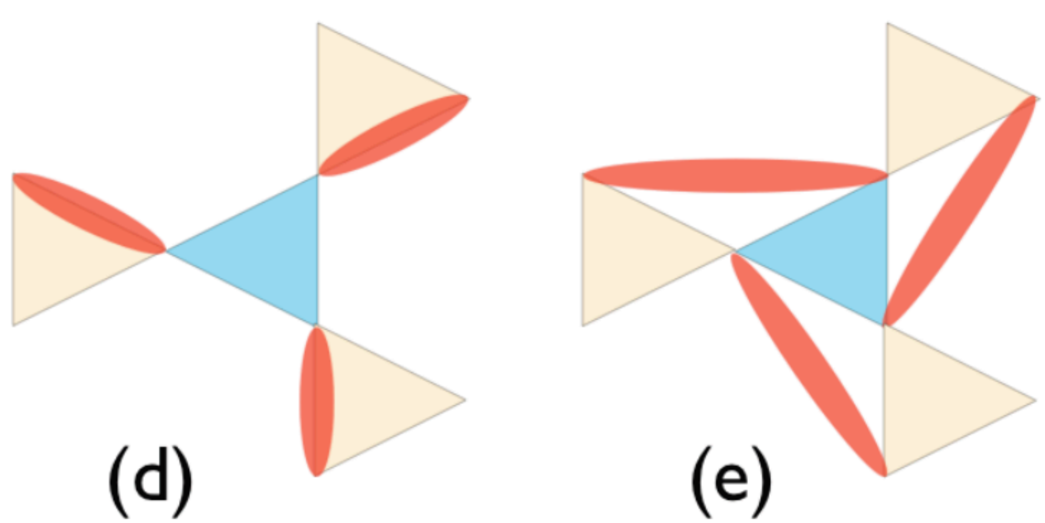
Improving the RVB / PEPS ...

Step 1: The «Simplex RVB»

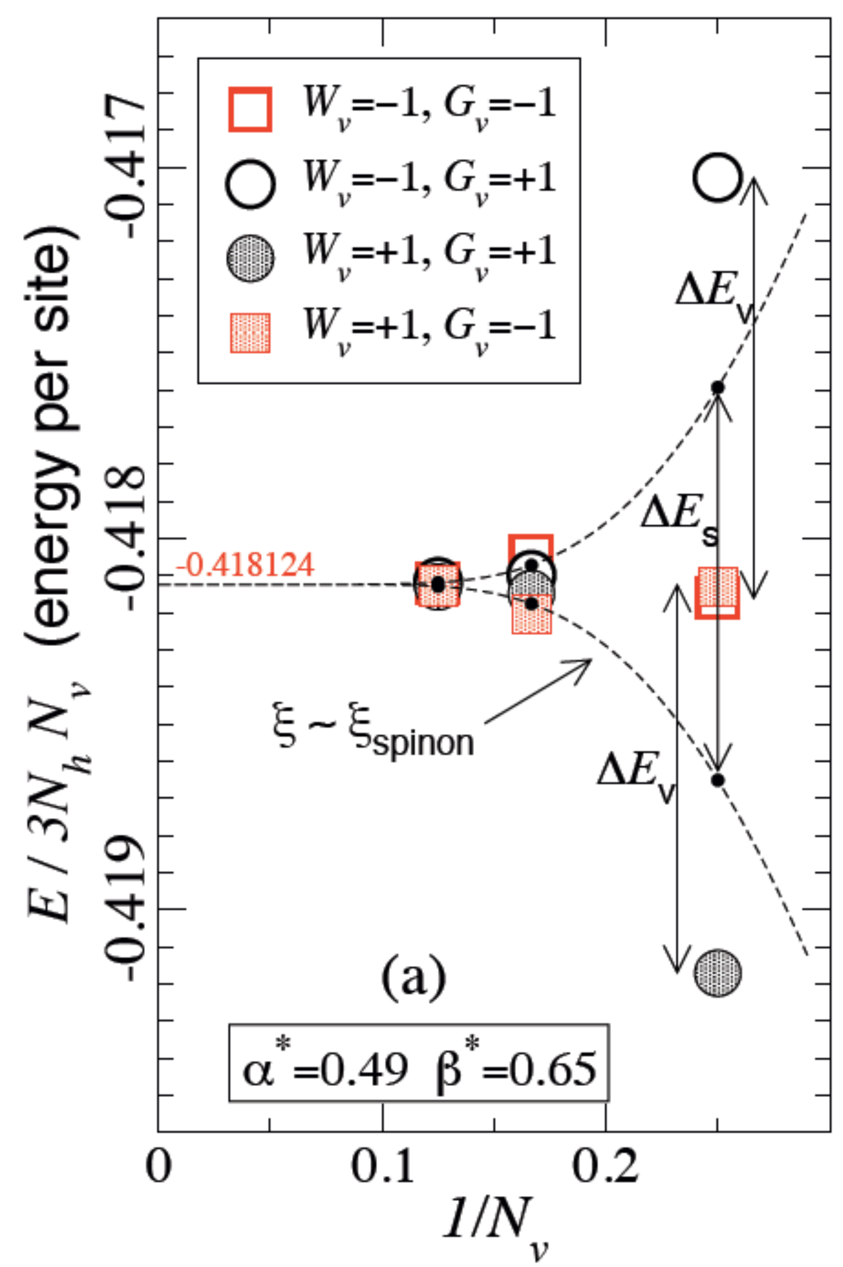
«defect triangles» cost energy !

dressing by introducing NNN singlets

(Zeng & Elser 90')



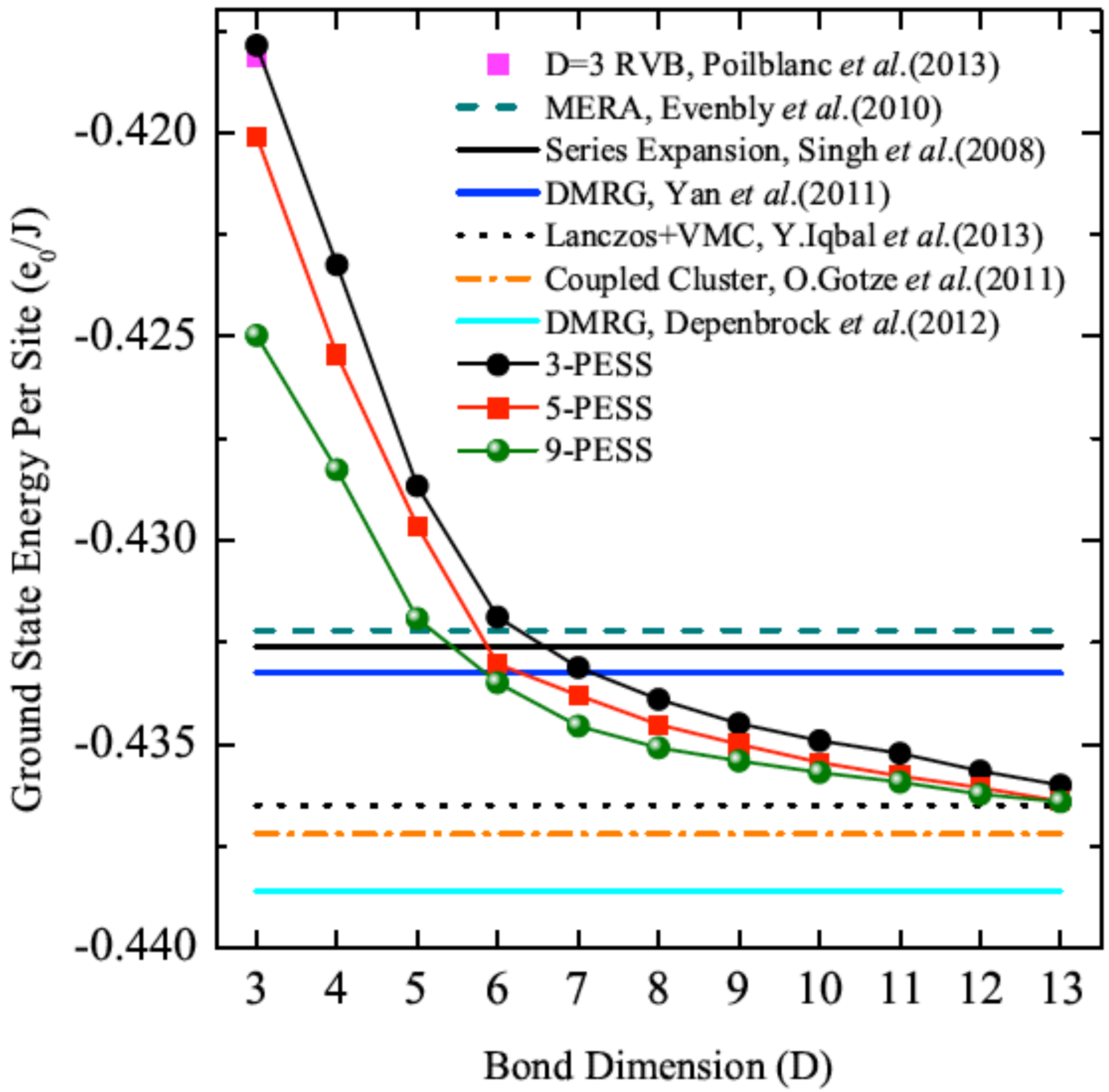
Finite size scaling of energy



Improving the RVB / PEPS ...

«Projected Entangled Simplex»

Z. Y. Xie, J. Chen, J. F. Yu, X. Kong1, B. Normand, and T. Xiang, arXiv:1307.5696



Simple update method
based on imaginary-time evolution

Spin liquids under magnetic field

$$\mathcal{H} = -\gamma H S_Z = -h S_Z$$

Continuous U(1) symmetry

If U(1) symmetry spontaneously broken:

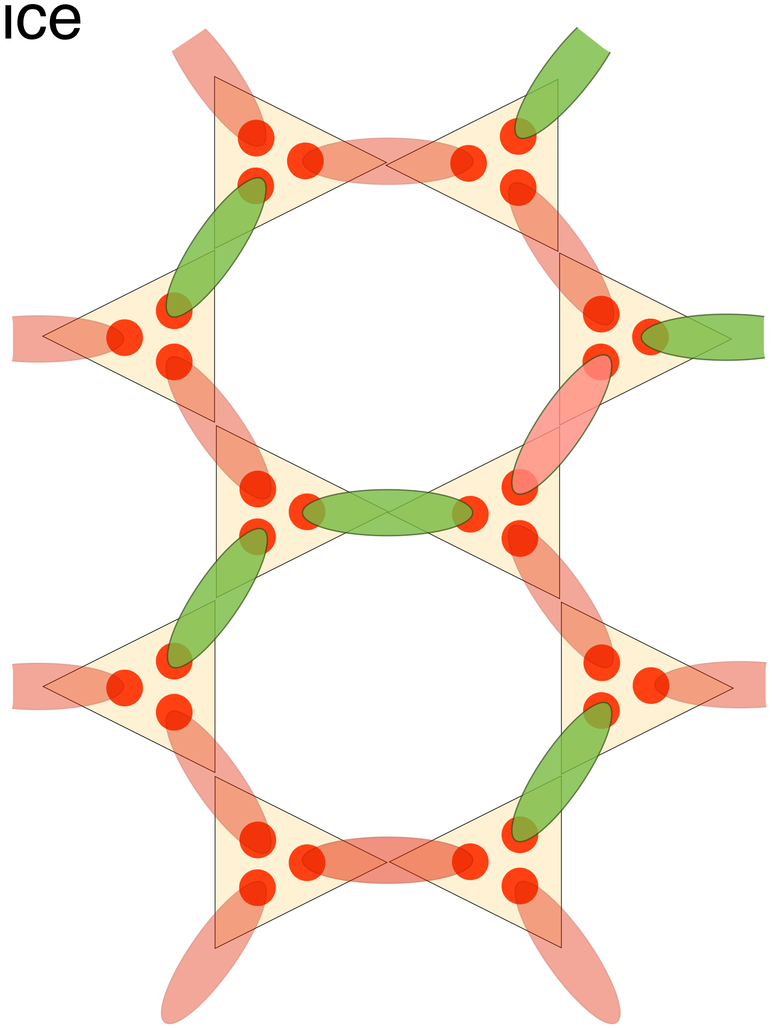
⟹ LR magnetic (tranverse) order

similar to superfluid

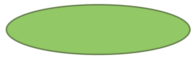
Simple exemple:
spin-3/2 AKLT in a magnetic field

Let's construct a simple ansatz !

Hexagonal lattice



$|22\rangle$



$S_z = +1$

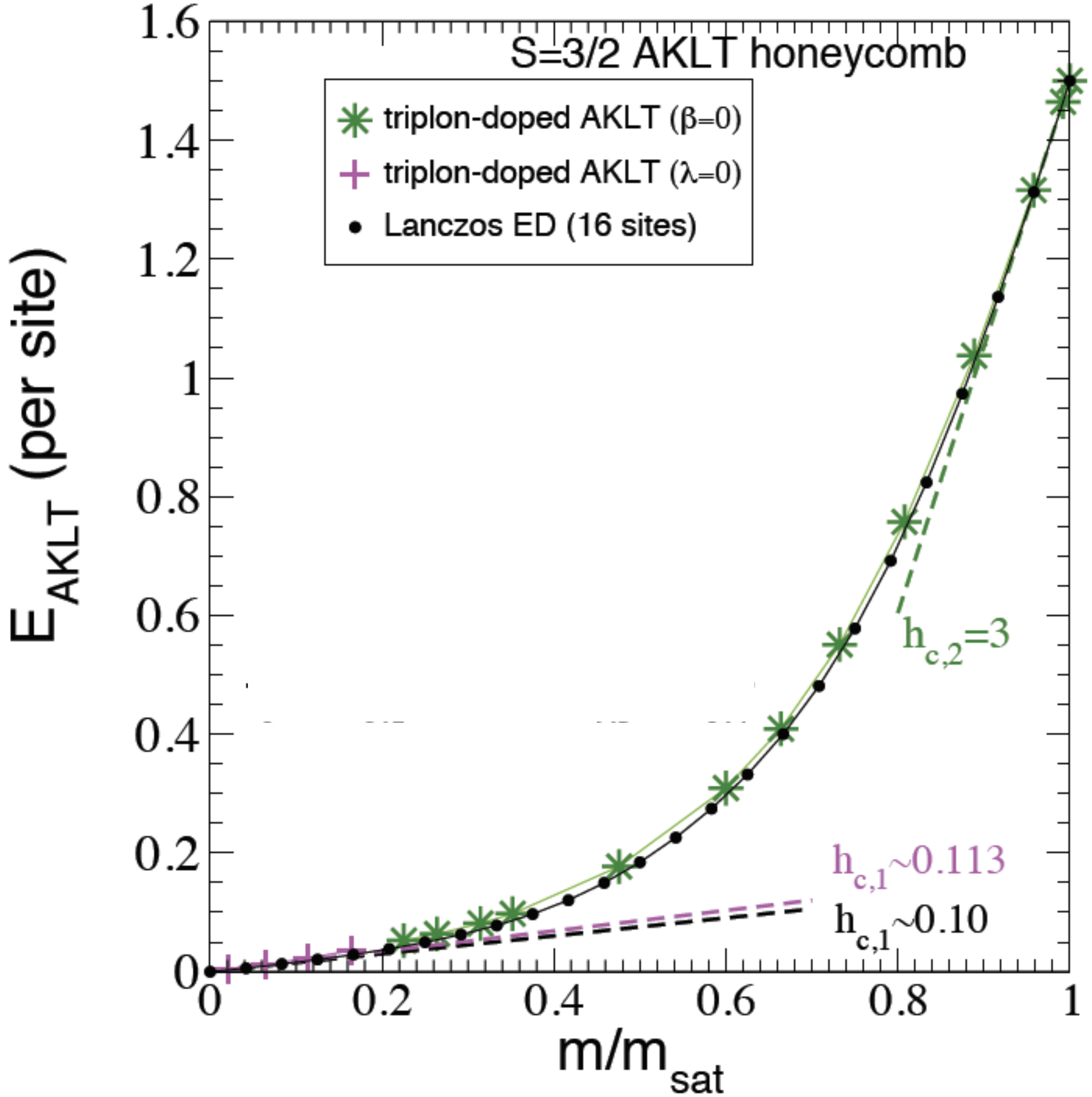
triplons

$$|01\rangle - |10\rangle + \beta|11\rangle$$

2-parameter family of D=3 PEPS

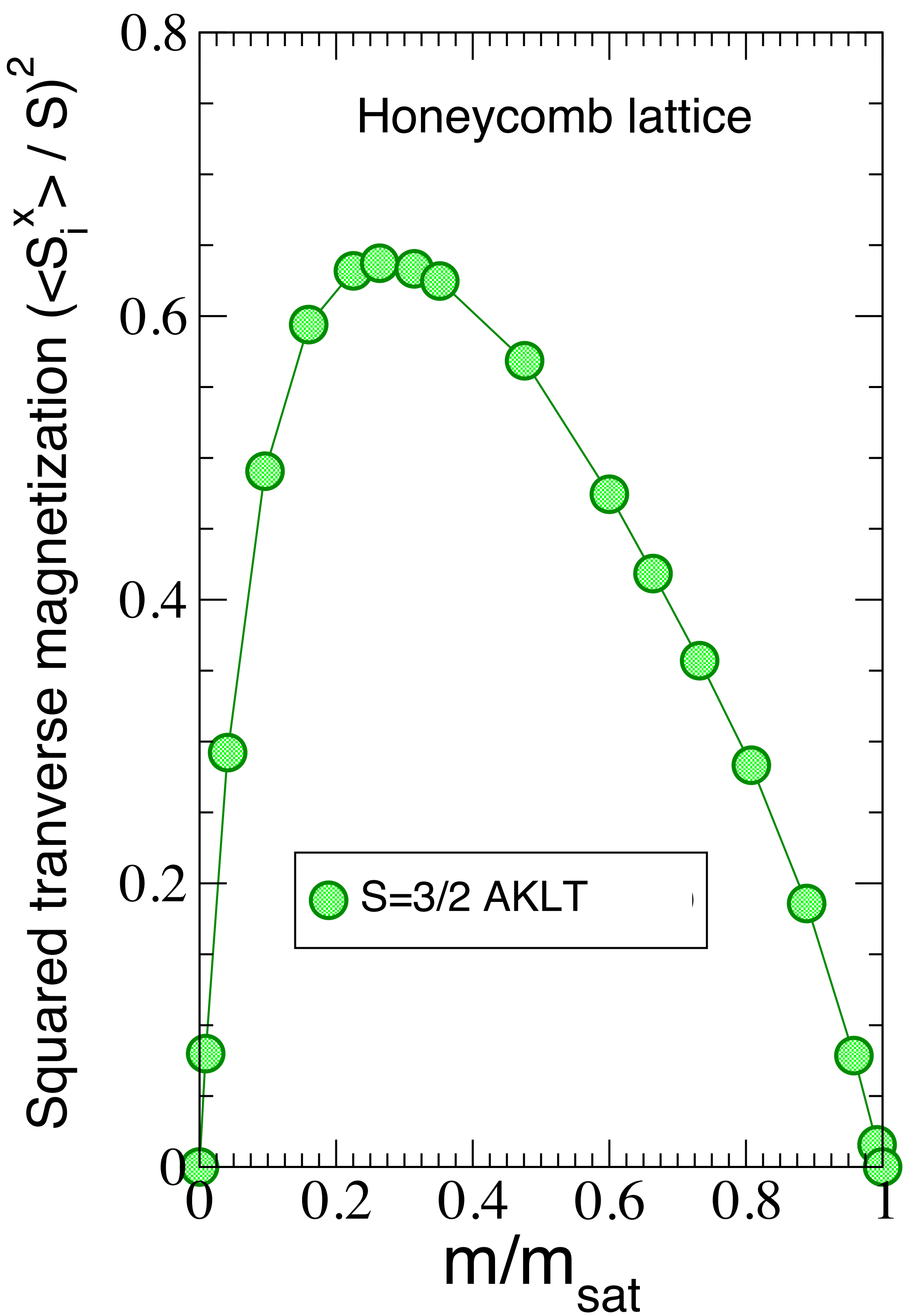
Energetics

Excellent ansatz for AKLT Hamiltonian :



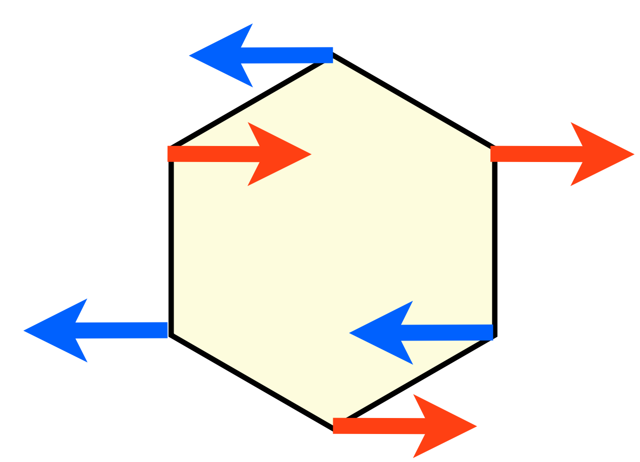
$$h = \partial E / \partial m$$

Spontaneous U(1) symmetry breaking does occur !



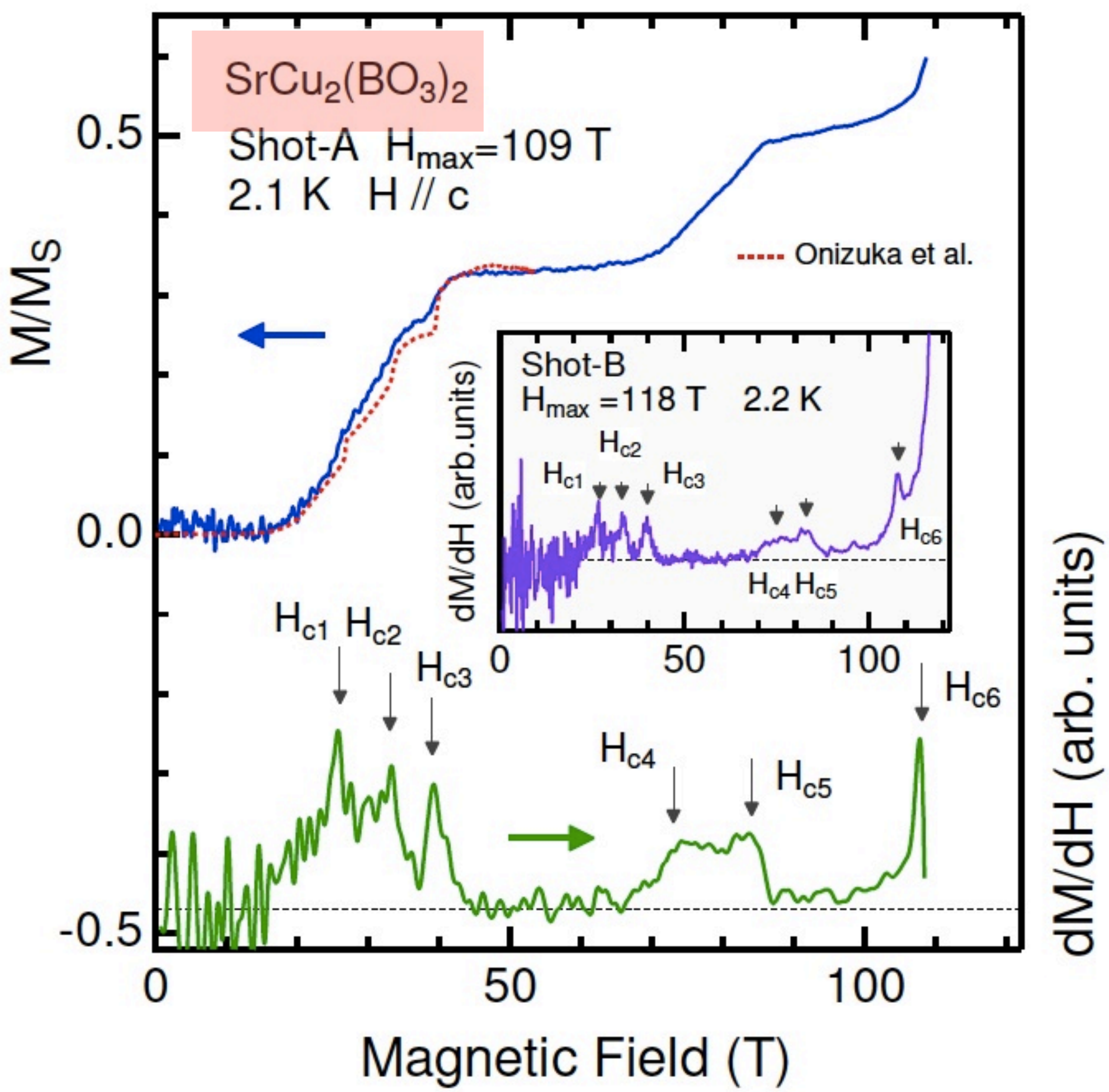
Transverse Néel order

~ superfluid density



Magnetization plateaux

Matsuda et al. PRL 2013



compressibility:

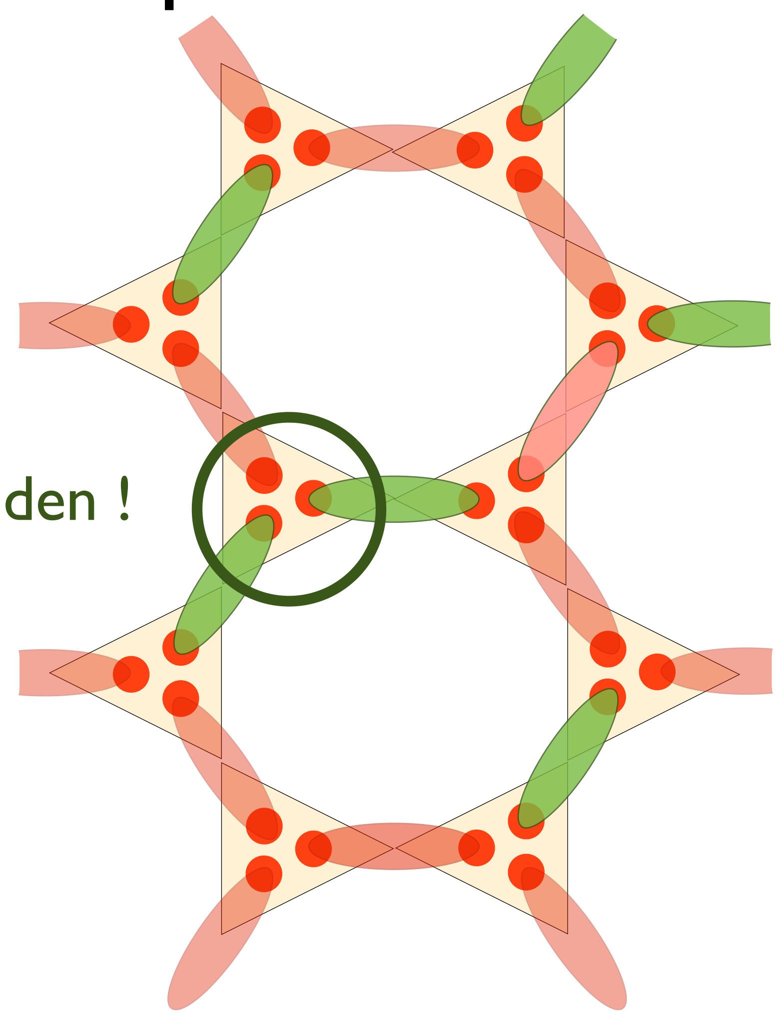
$$\kappa = \partial M / \partial H$$

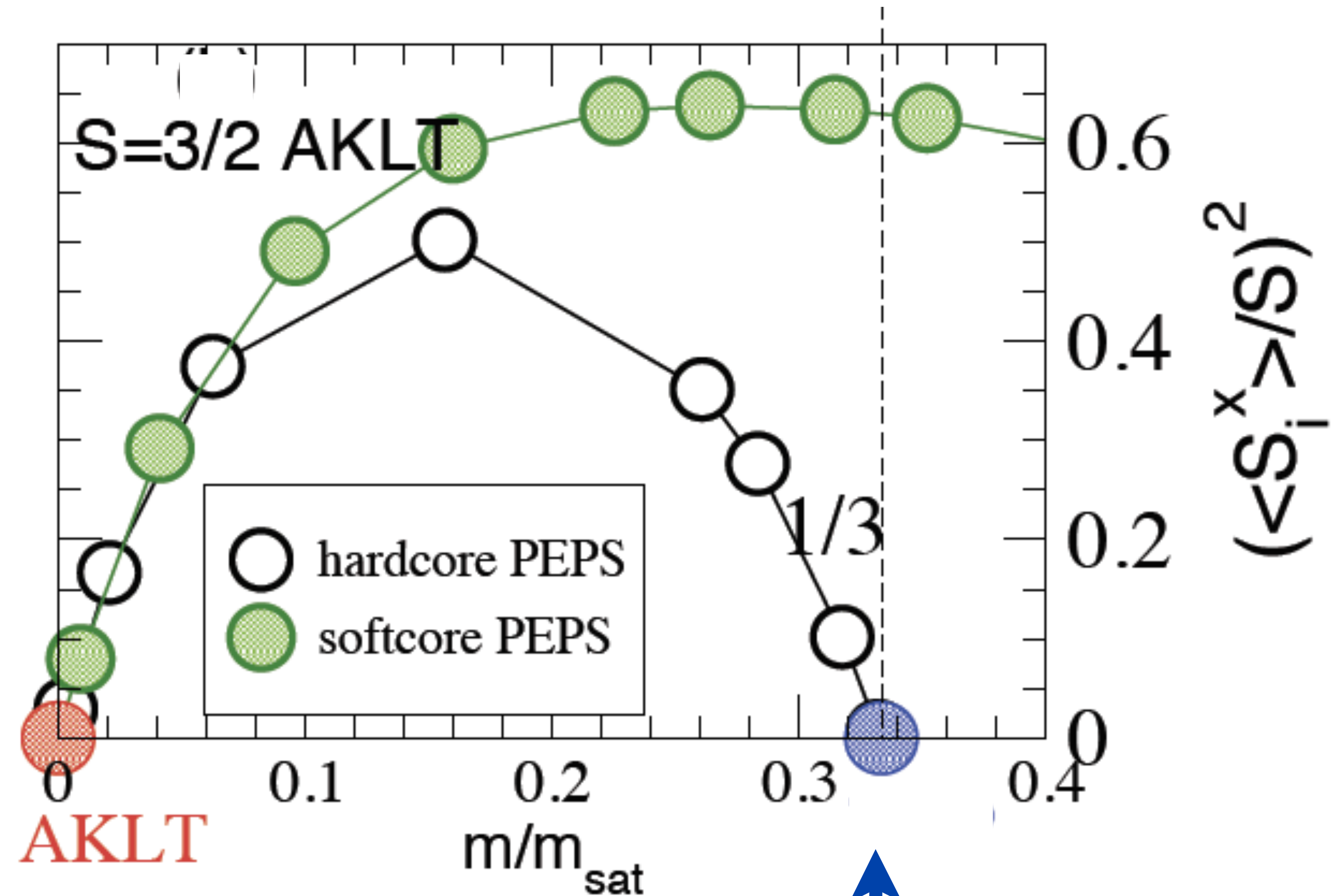
Uncompressible phases with PEPS ?

IDEA : make the triplets HARDCORE

spin-3/2 AKLT

forbidden !





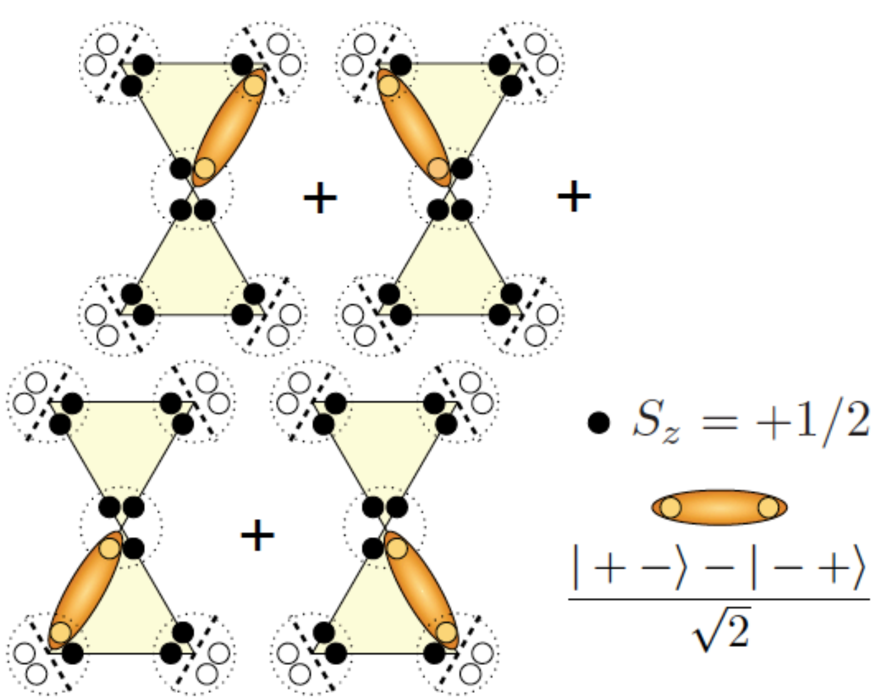
Topological «Resonating Triplet Bond» state

More interesting physics on the kagome lattice !

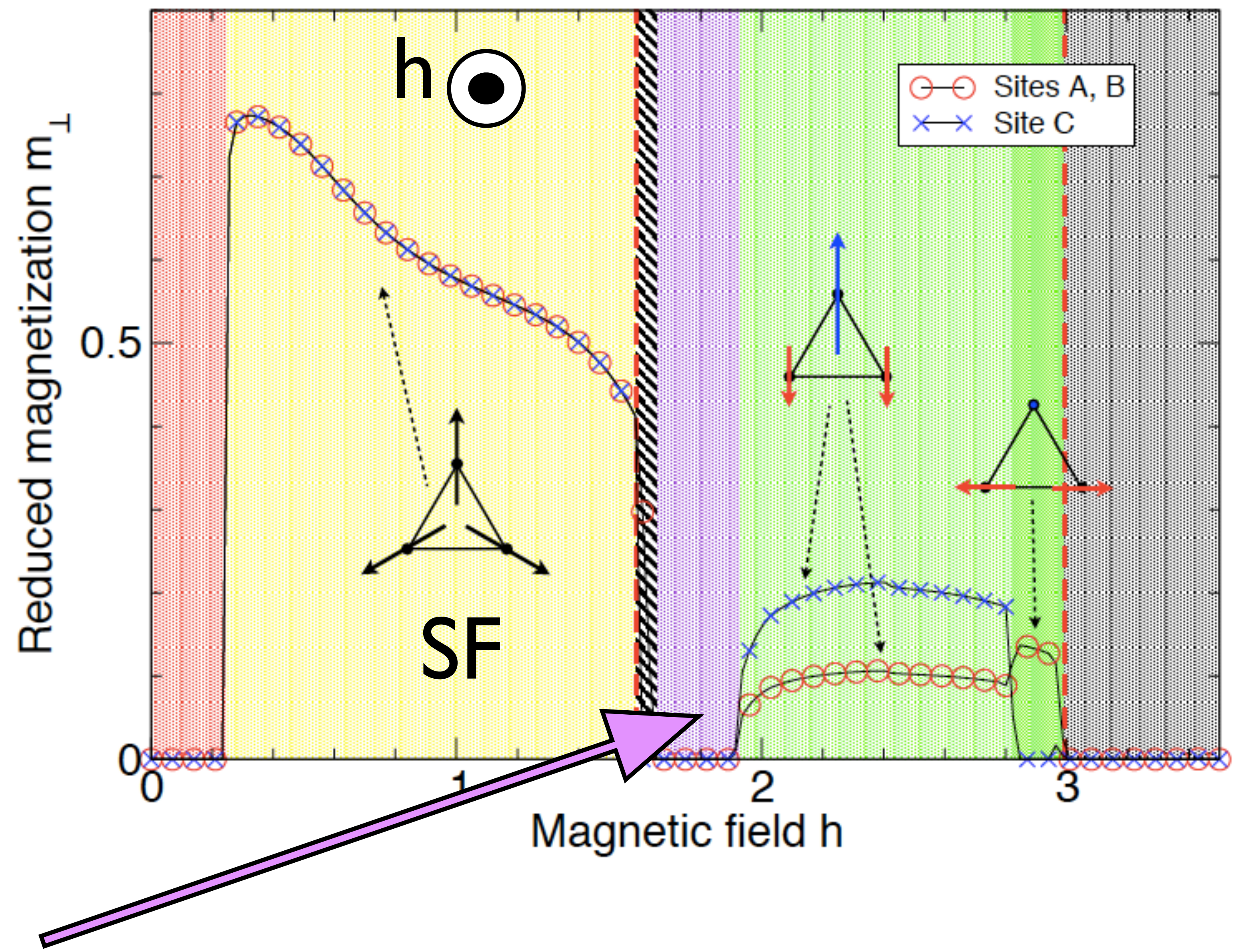
S=2 AKLT

uncompressible «nematic»

$m/m_{\mathrm{sat}} = 5/6$



SF + N

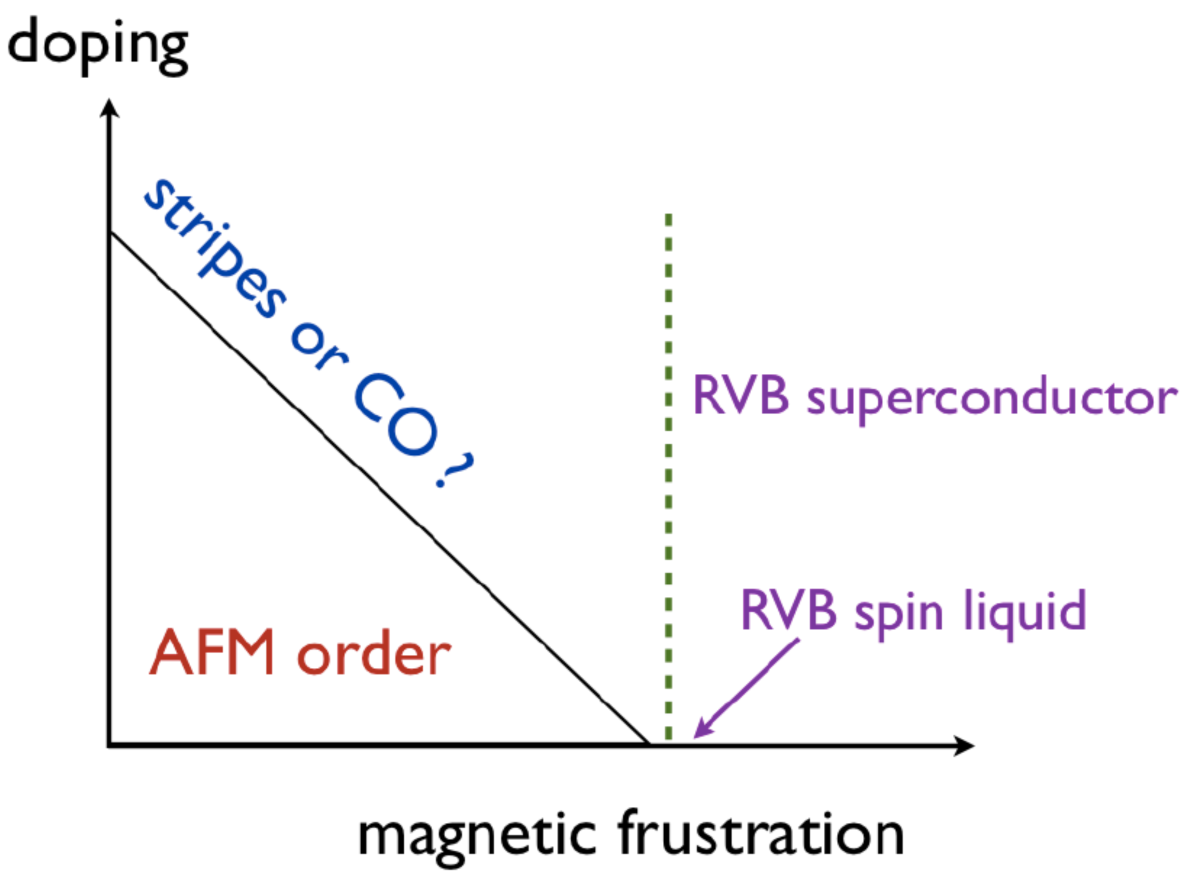


Thibaut Picot and DP, in preparation

(iPEPS + TEBD)

RVB superconductors for high-Tc superconductivity

Idea #1: the RVB spin liquid is the «parent» insulator of the high-Tc superconductor P.W. Anderson, T.M. Rice, etc…



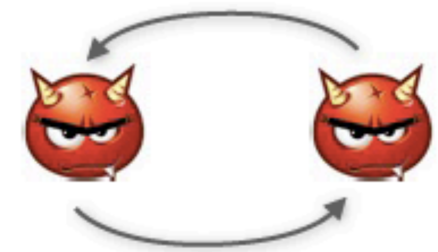
Idea #2: magnetic frustration is the key parameter to stabilize a (RVB) spin liquid on the 2D square lattice

Ling Wang, DP, Zheng-Cheng Gu, Xiao-Gang Wen, Frank Verstraete
PRL **111** 037202 (2013)

Away from half-filling needs fPEPS !

Breakthrough in 2009: Fermions with 2D tensor networks

How to take fermionic statistics into account?



$$\hat{c}_i \hat{c}_j = -\hat{c}_j \hat{c}_i$$

fermionic operators *anti*commute

Different formulations:

P. Corboz, G. Evenbly, F. Verstraete, G. Vidal, Phys. Rev. A 81, 010303(R) (2010)

C. V. Kraus, N. Schuch, F. Verstraete, J. I. Cirac, Phys. Rev. A 81, 052338 (2010)

C. Pineda, T. Barthel, and J. Eisert, Phys. Rev. A 81, 050303(R) (2010)

P. Corboz and G. Vidal, Phys. Rev. B 80, 165129 (2009)

T. Barthel, C. Pineda, and J. Eisert, Phys. Rev. A 80, 042333 (2009)

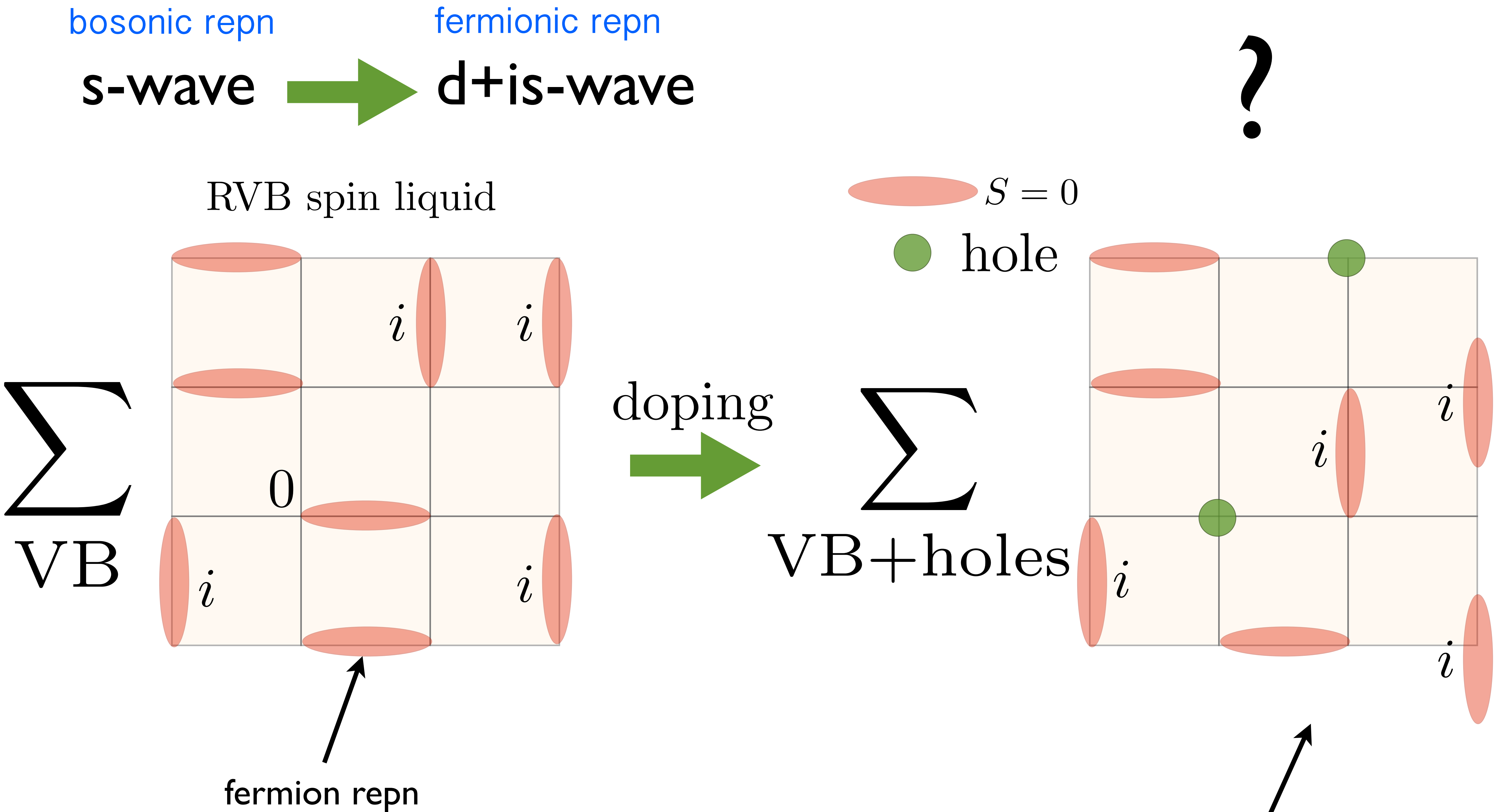
Q.-Q. Shi, S.-H. Li, J.-H. Zhao, and H.-Q. Zhou, arXiv:0907.5520

P. Corboz, R. Orus, B.Bauer, G. Vidal, PRB 81, 165104 (2010)

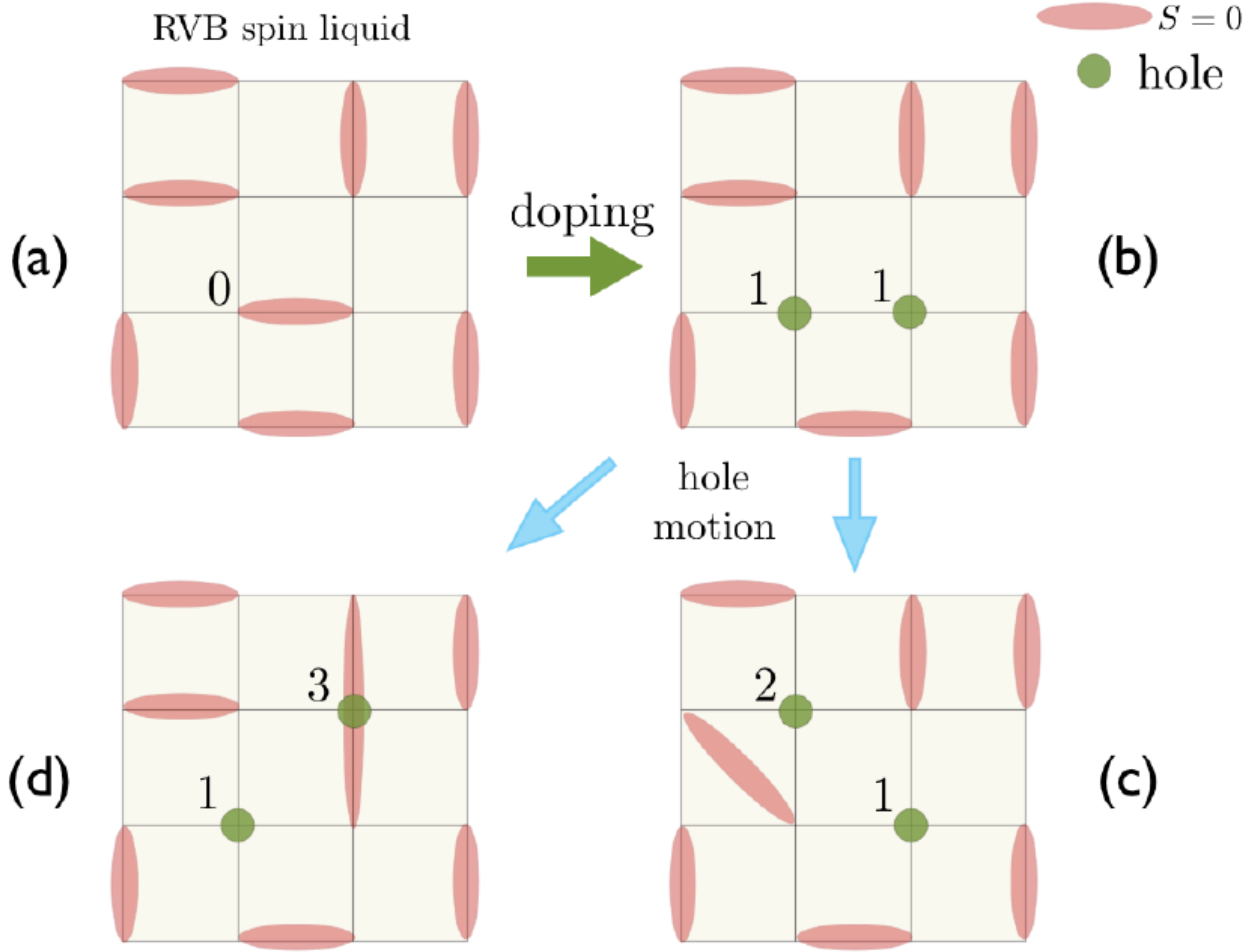
I. Pizorn, F. Verstraete, Phys. Rev. B 81, 245110 (2010)

Z.-C. Gu, F. Verstraete, X.-G. Wen. arXiv:1004.2563

courtesy of P. Corboz



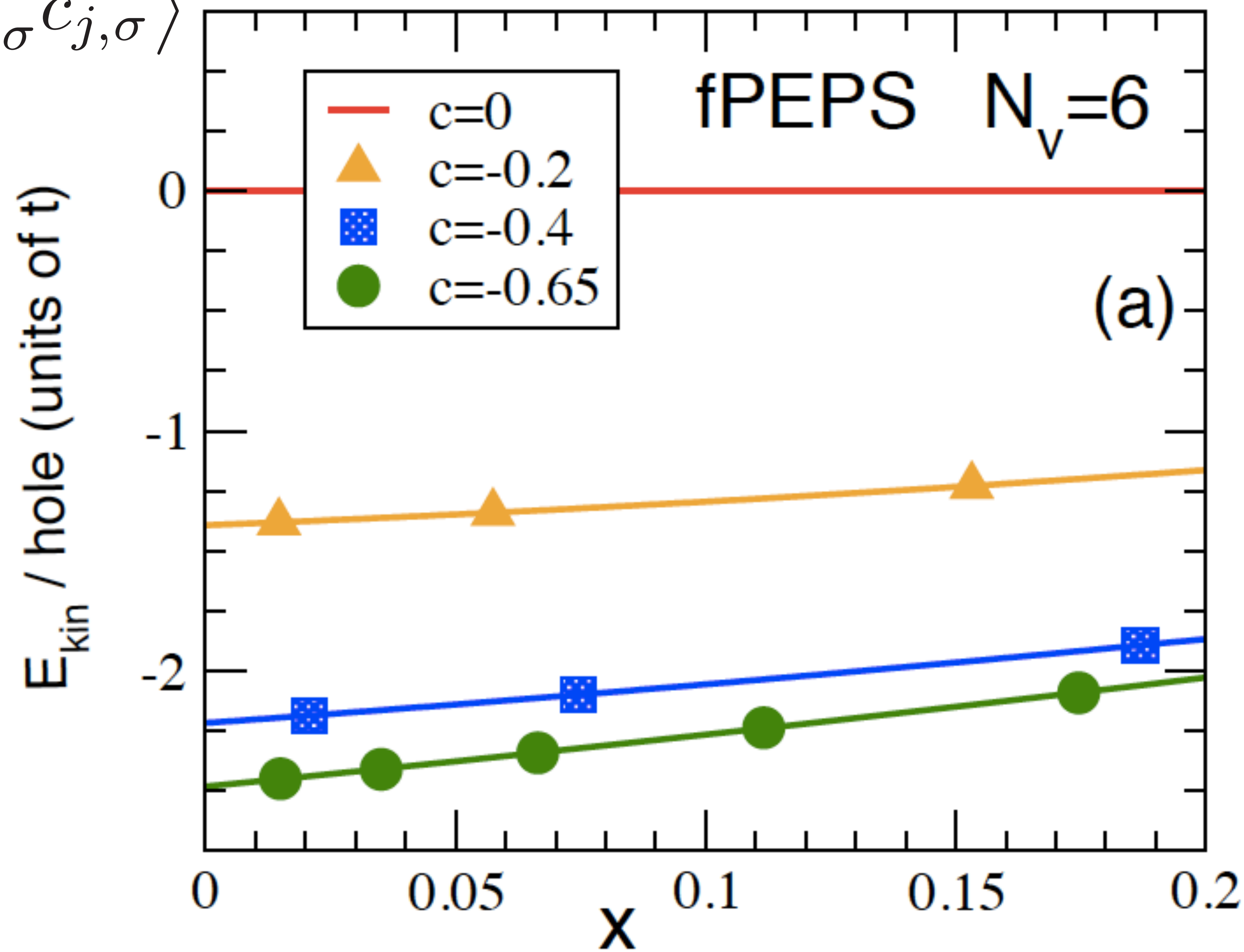
but ... this wf has zero kinetic energy !!



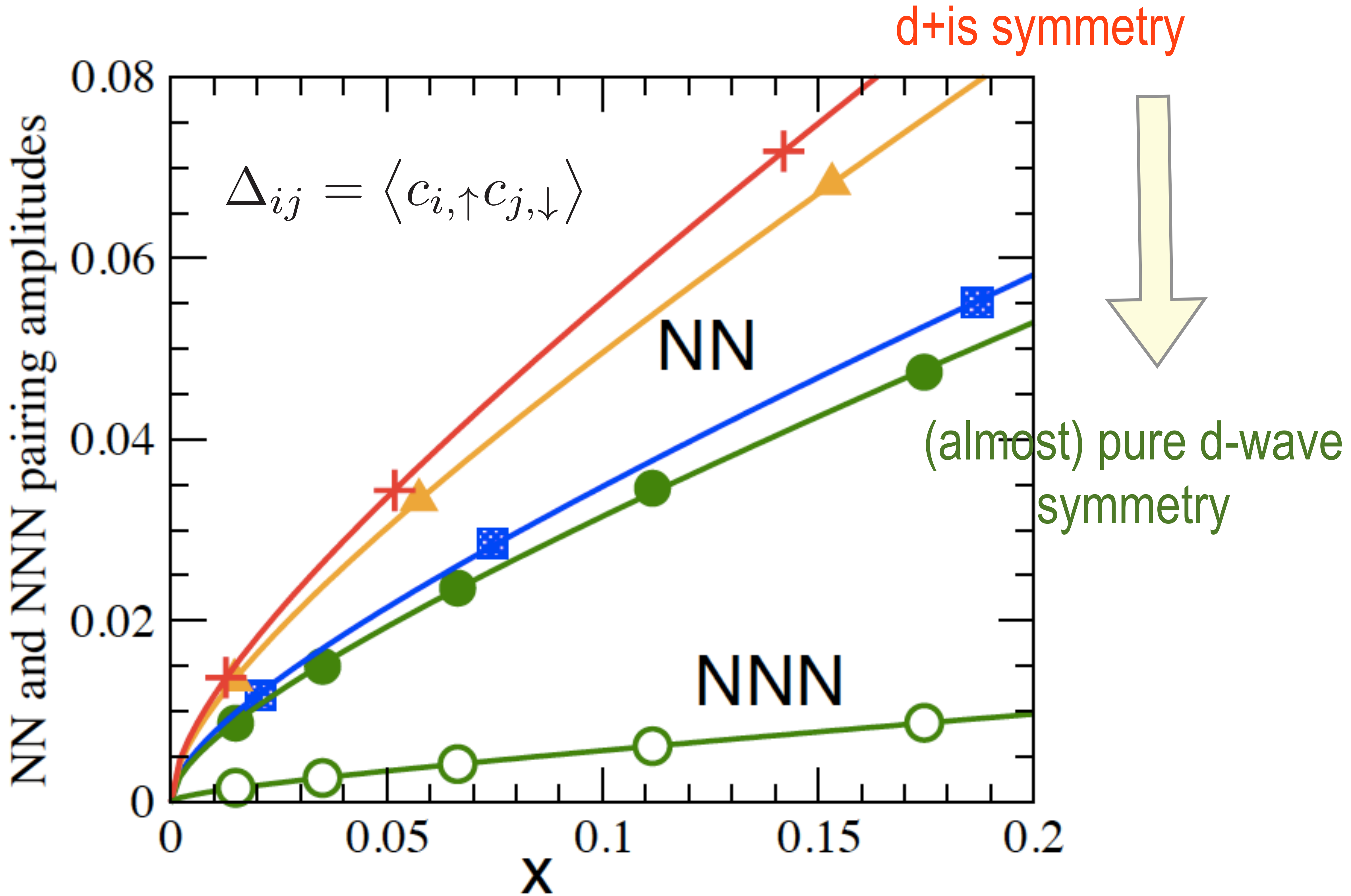
RVB spin liquid
$S = 0$
hole
(a)
0
doping
(b)
1
1
hole
motion
(d)
3
1
(c)
2
1

hole kinetic energy

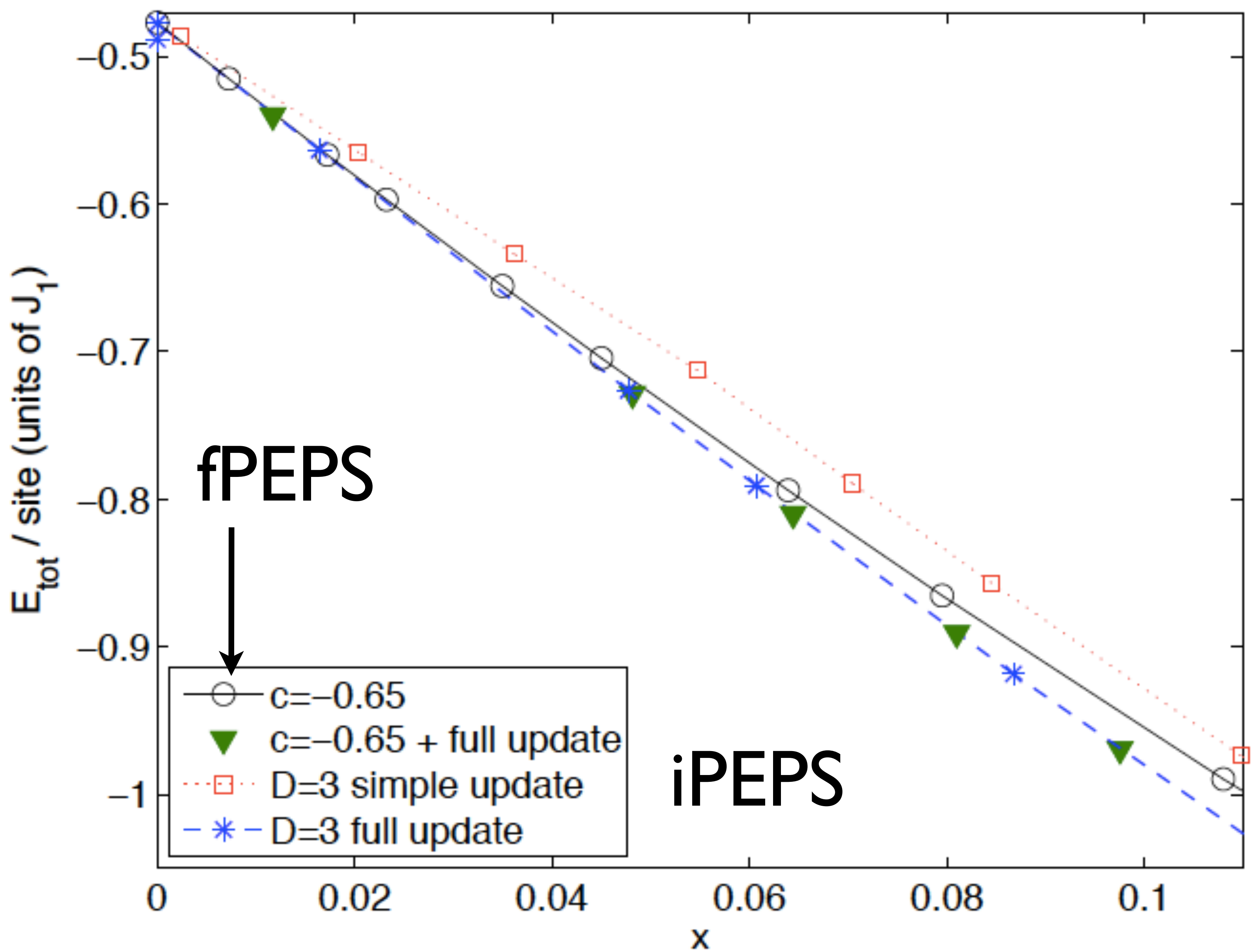
$\langle c_{i,\sigma}^{\dagger} c_{j,\sigma} \rangle$



Superconducting pairing amplitude



Variational energy : fPEPS vs iPEPS



Very good ansatz !

CONCLUSION

* Qualitative understanding of (simple) correlated phases (topo SL, SF, uncompressible phases, SC,...)

* Systematic improvement can be made for physical Hamiltonians : iPEPS

OUTLOOK

* Field induced topo phases ?

* Chiral RVB SL or RVB SC ?