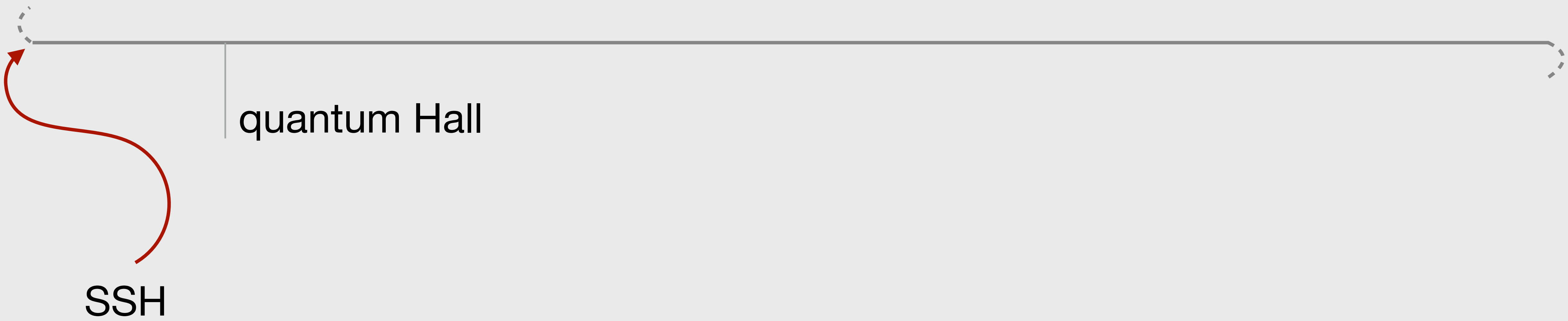
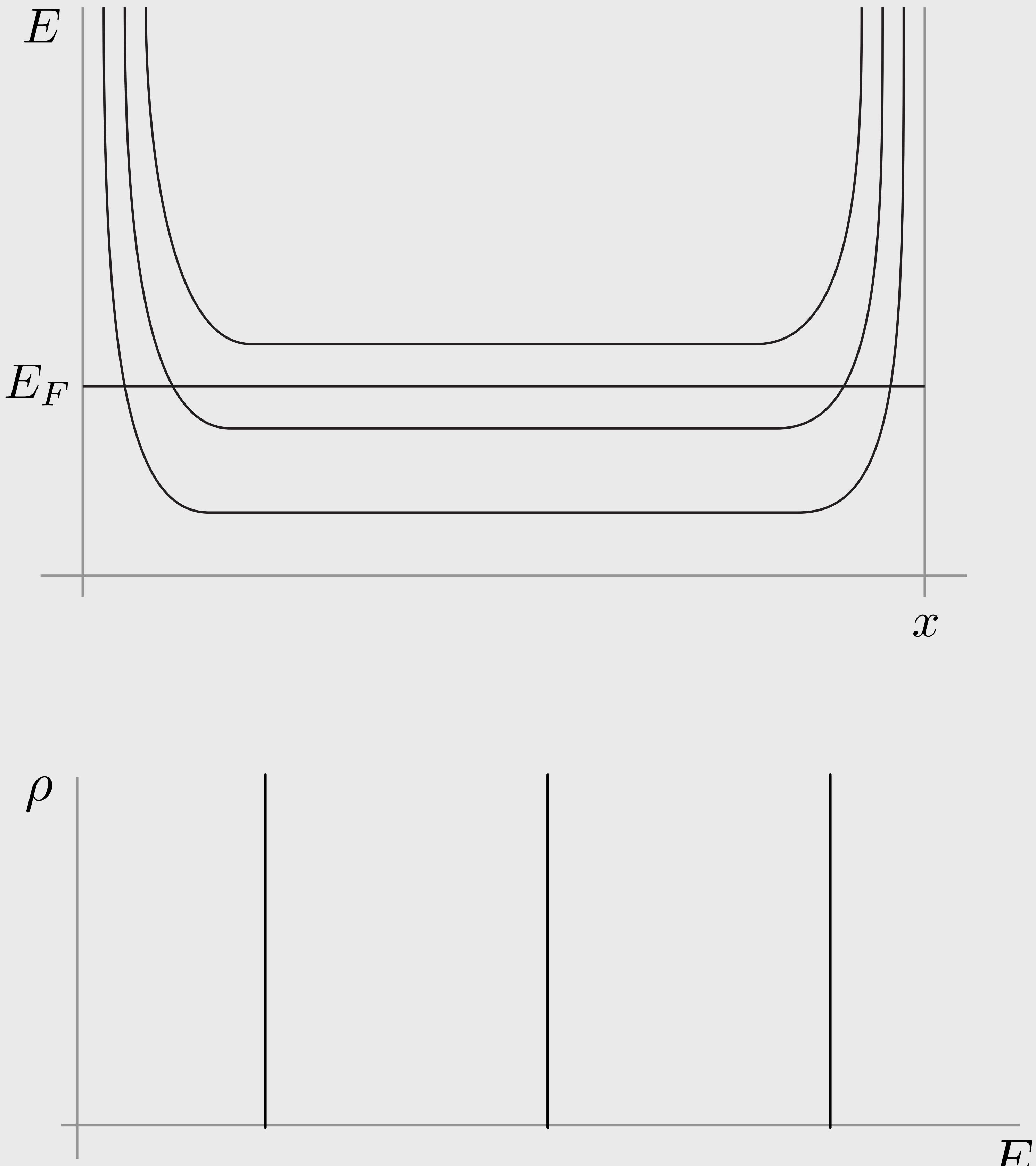
disordered topological matter — time line

disordered topological matter — time line

80s



quantum Hall effect (class A)



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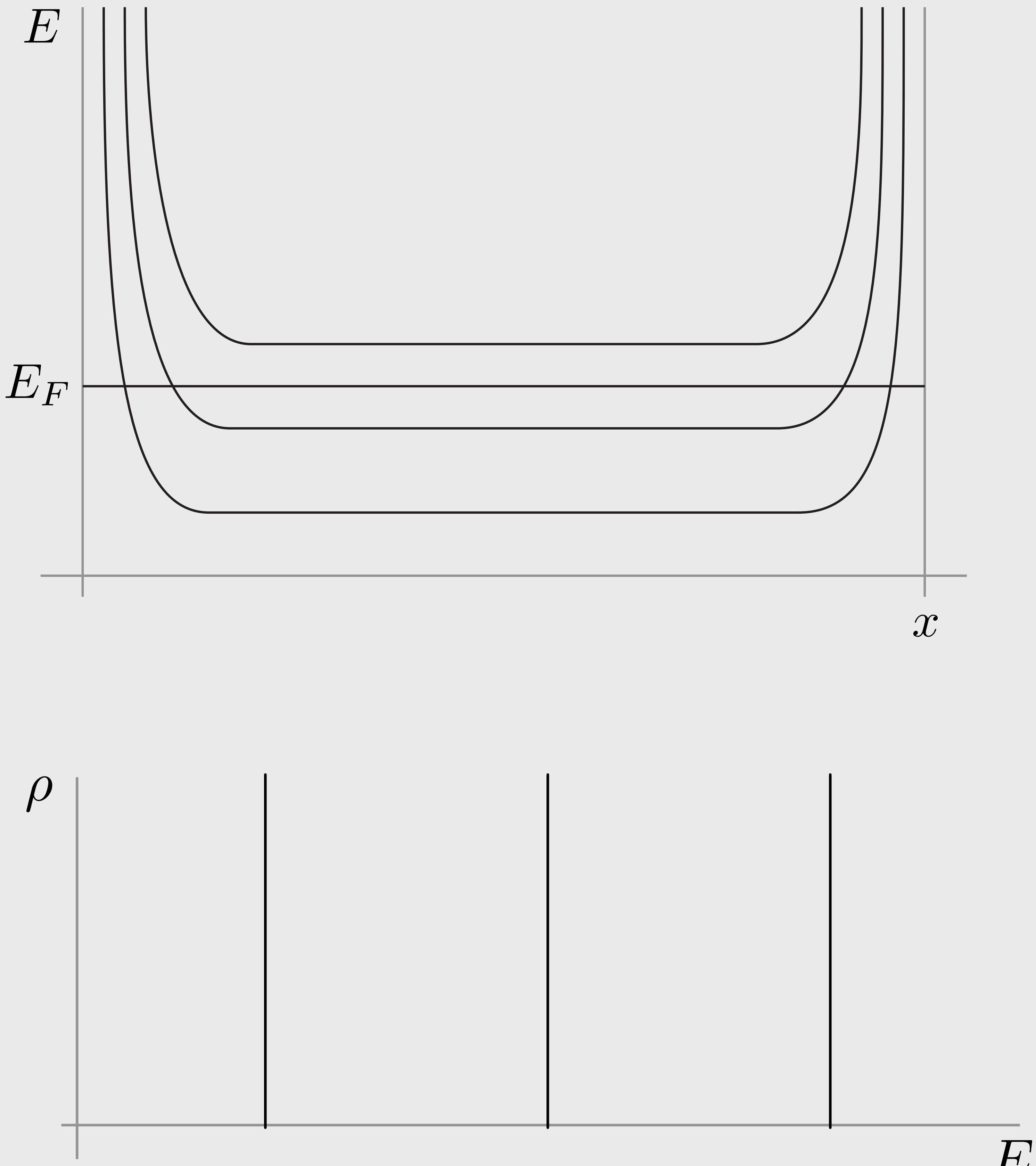
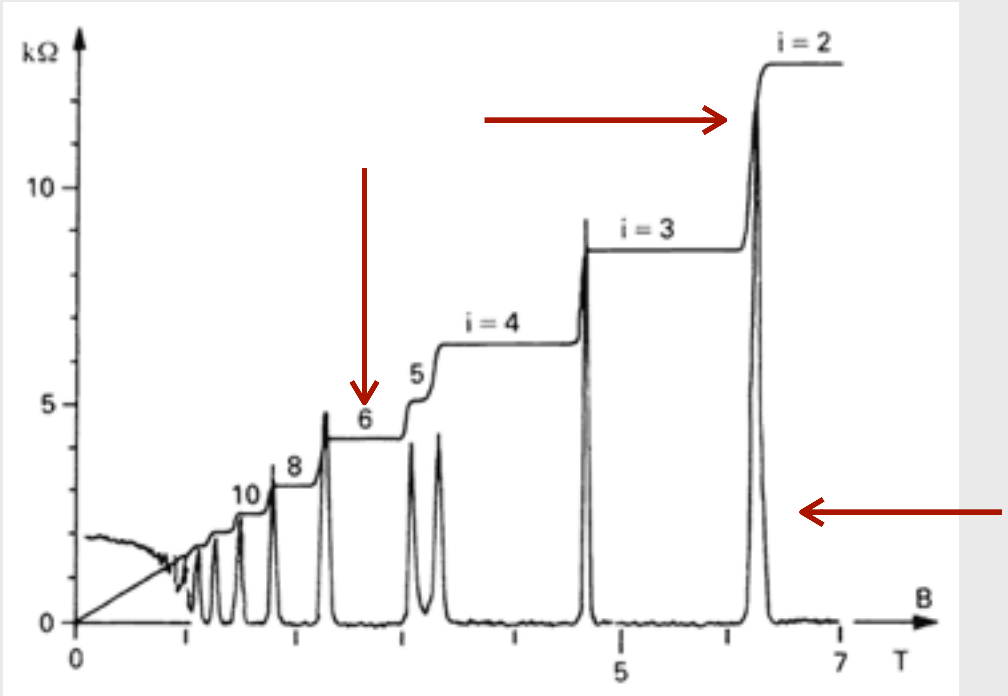
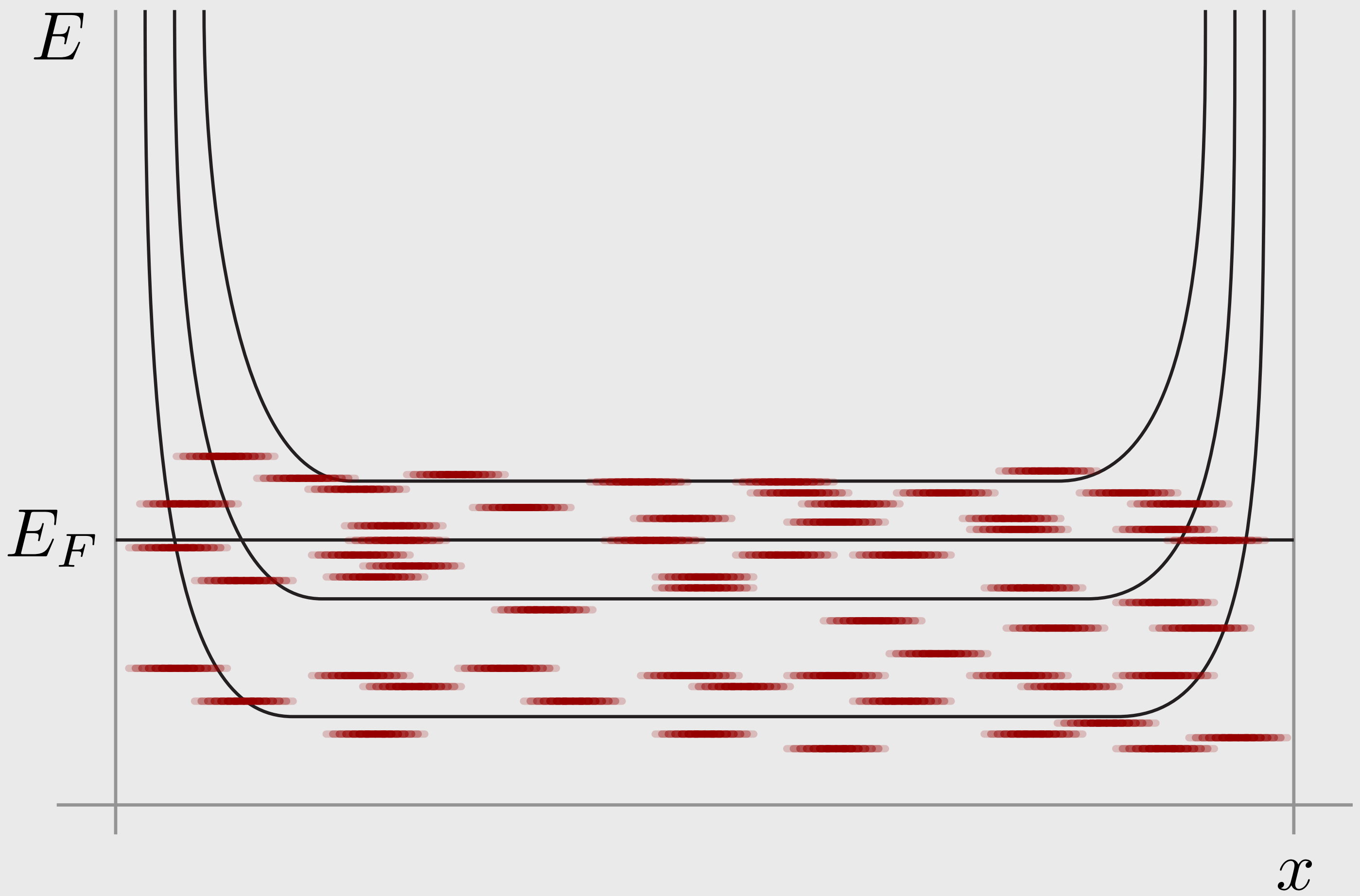
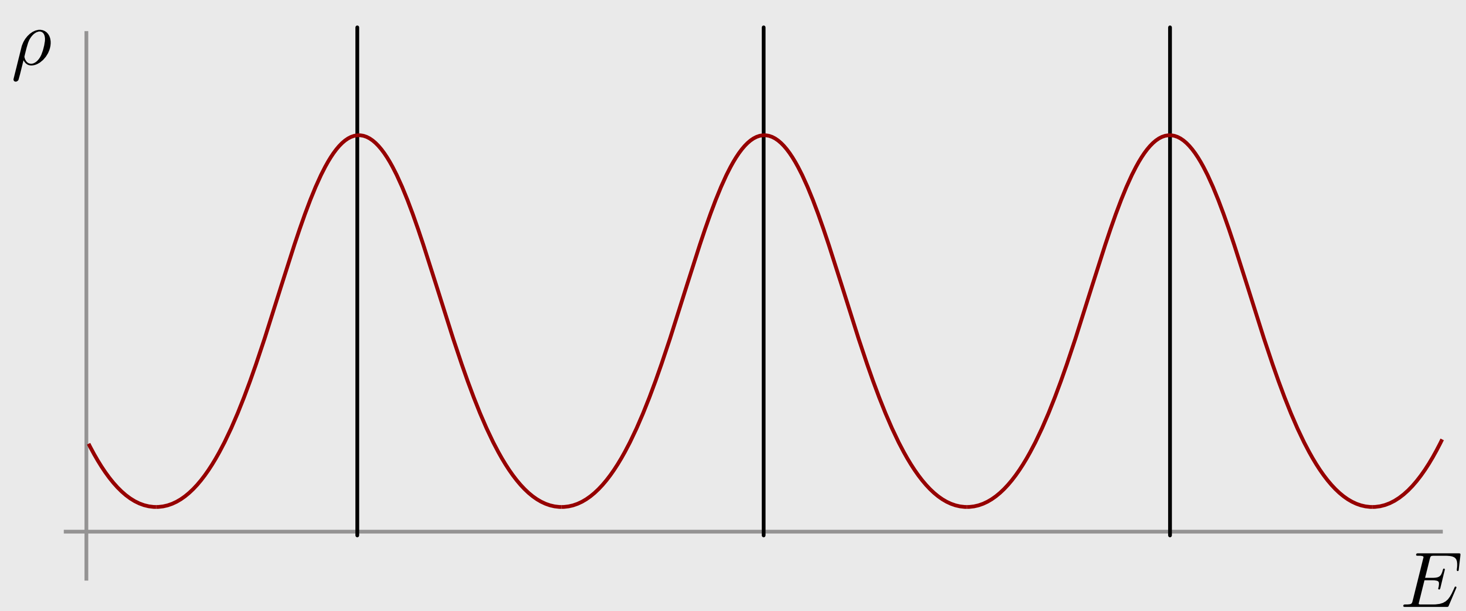


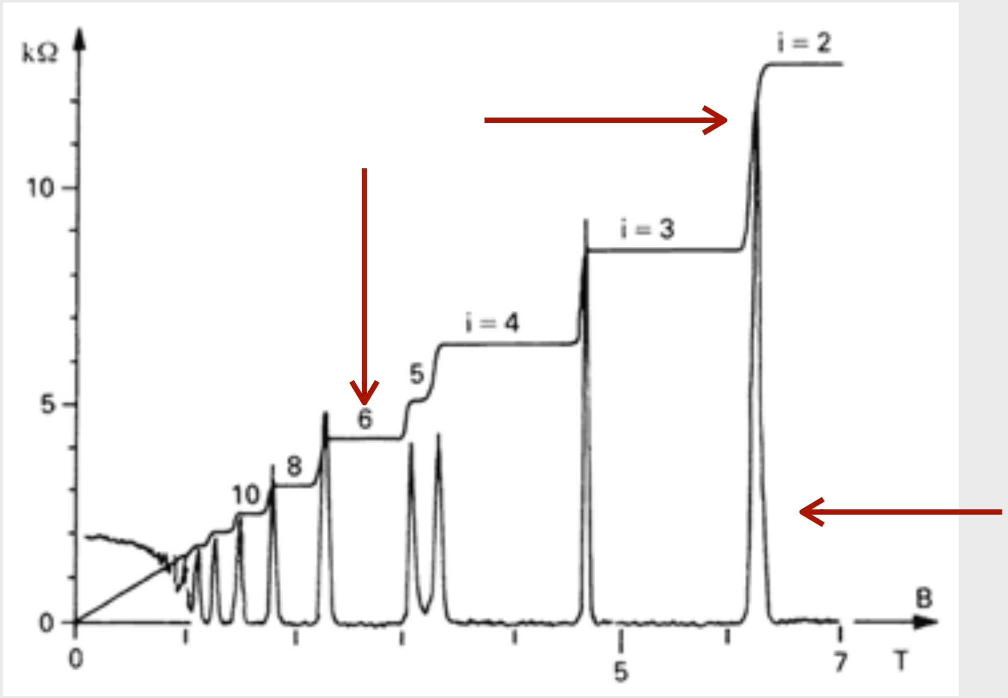


1998 Nobel prize press release

quantum Hall effect (class A)







1998 Nobel prize press release

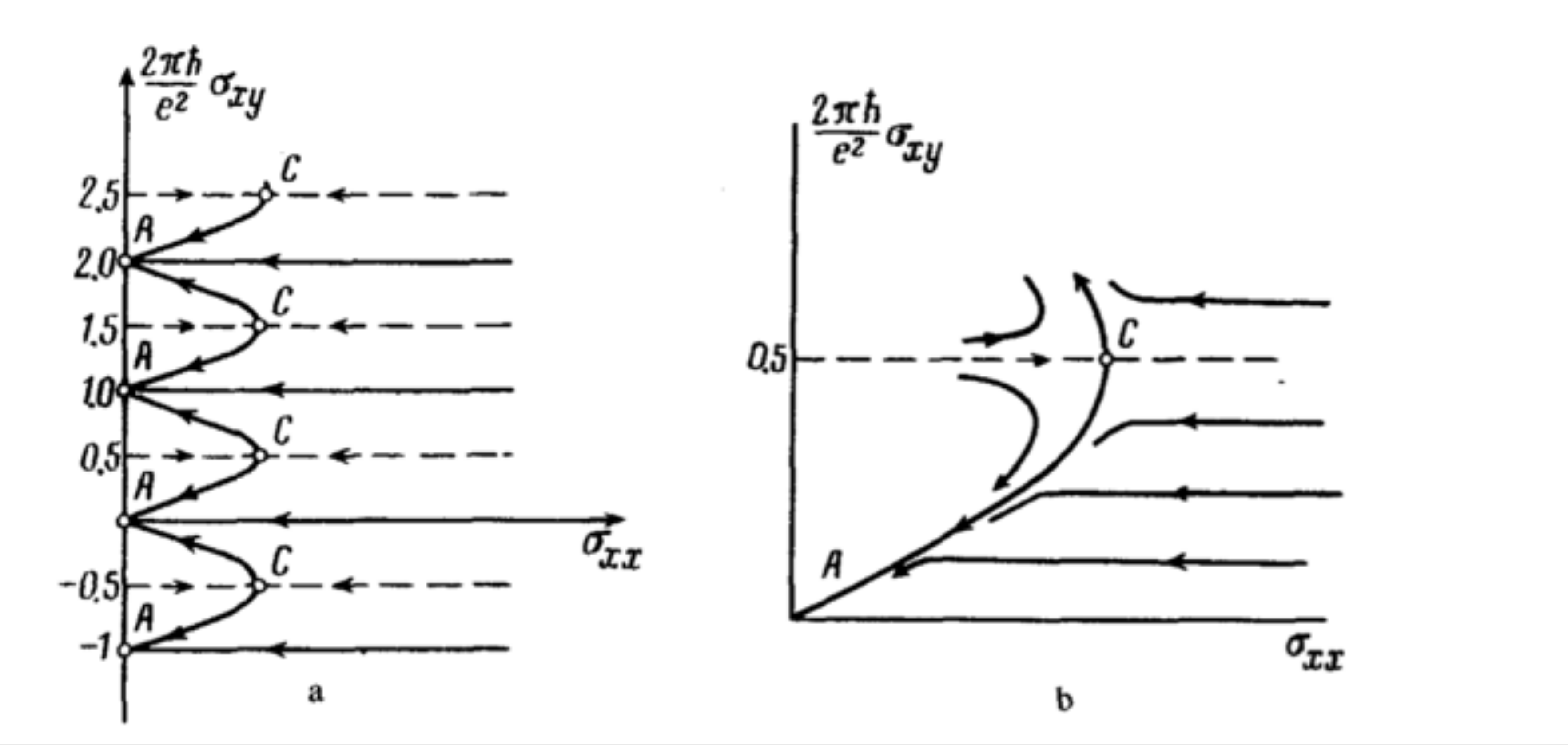
quantum Hall continued

- universal interplay disorder/topology described by two parameters

σ_{xx} : average longitudinal transport coefficient

σ_{xy} : average topological index

- two parameter criticality



Khmelnitskii, 1983

quantum Hall continued

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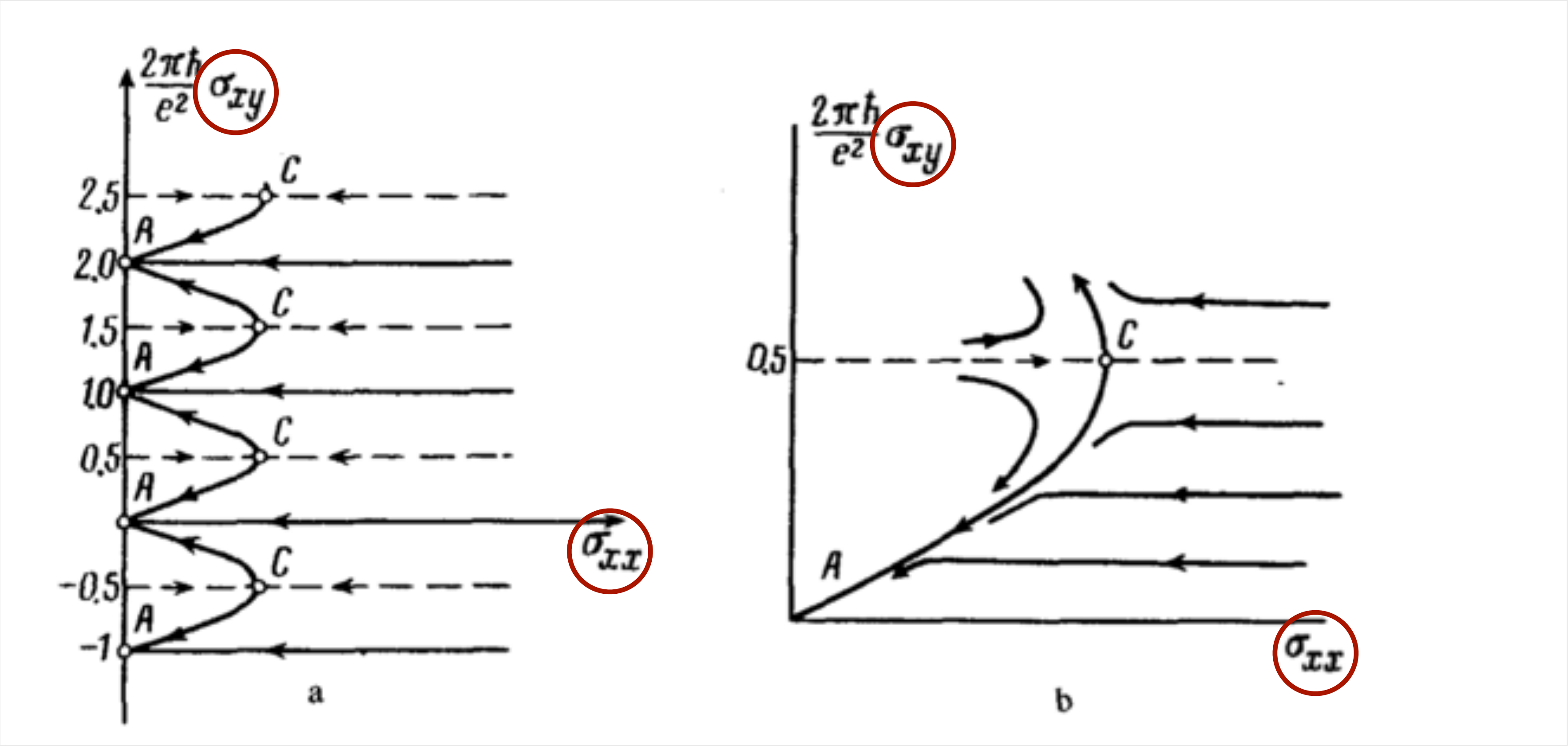

Khmelnitskii, 1983

disordered topological matter — time line

80s

quantum Hall

disordered topological matter — time line

80s

quantum Hall

90s

symmetry classes

disordered topological matter — time line

80s

quantum Hall

90s

symmetry classes

- ▷ free fermion systems classified by unitary and anti-unitary symmetries
- ▷ pre-nineties: 3 Wigner-Dyson classes
- ▷ there exist 10=(3+7) symmetry classes, (Zirnbauer, AA, 96)
- ▷ labeled A, AI, AII, AIII, BDI, C, CI, CII, D, DIII (courtesy Cartan)

disordered topological matter — time line

80s

quantum Hall

90s

symmetry classes

disordered topological matter — time line

80s

quantum Hall

90s

1d delocalization

symmetry classes

novel quantum Hall effects

00s

2d delocalization

1d delocalization phenomena

- delocalization in quasi-one dimensional geometries
  - 1998 AIII quantum wire (Brouwer, Mudry, Simons, AA)
  - 1999 D quantum wire (Brouwer, Mudry, Furusaki)
  - 2004 AIII, D, BDI, DIII, CII (Read, Gruzberg, Vishveshwara)

1d delocalization phenomena

- delocalization in quasi-one dimensional geometries
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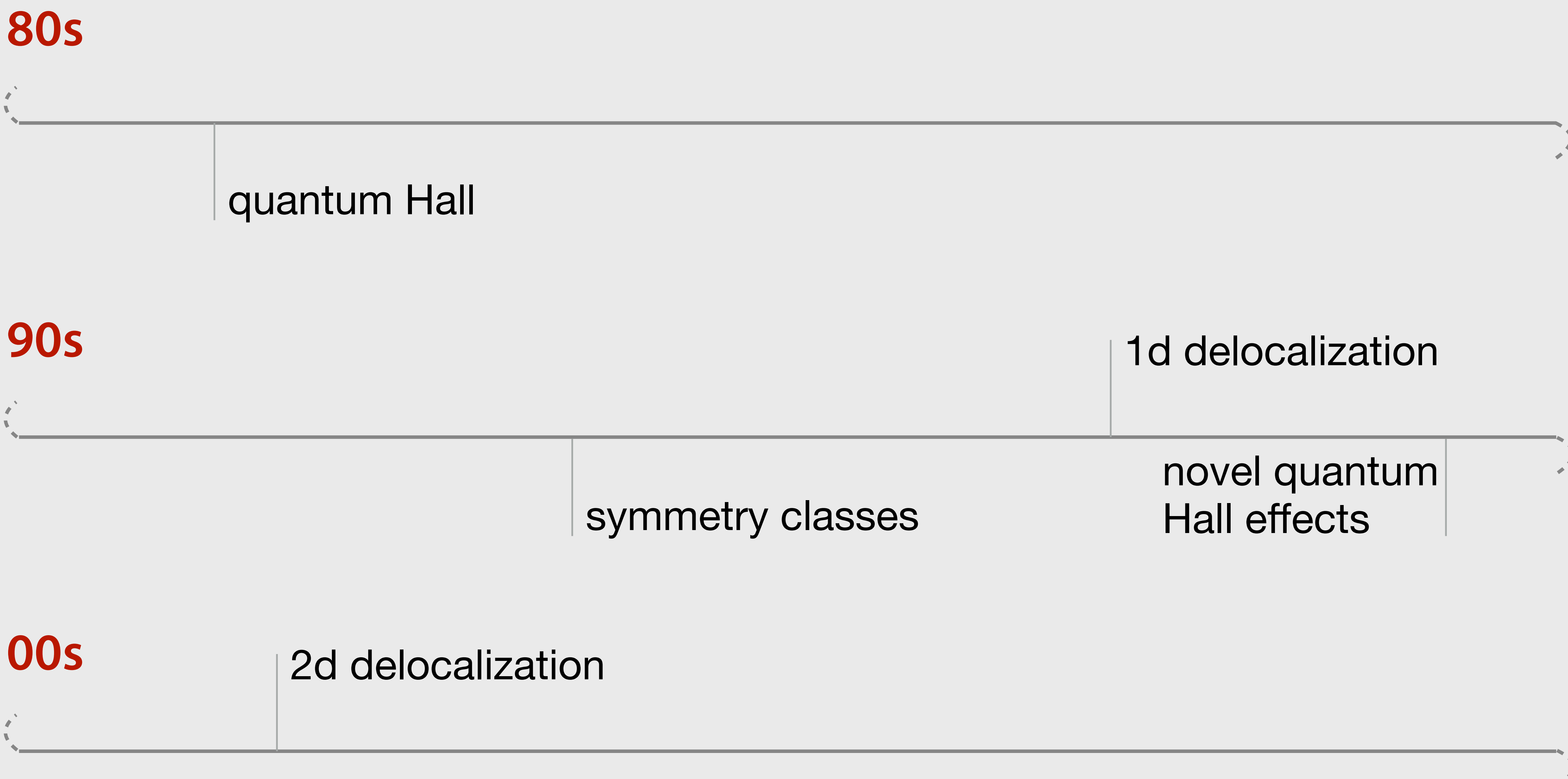
  - 2004 AIII, D, BDI, DIII, CII (Read, Gruzberg, Vishveshwara)

Unconventional criticality in 2d

  - 2001 Class C spin quantum Hall effect (Chalker et al.)
  - 2001 Class D quantum criticality (Fisher et al., Read et al.)

- universal interpretation: topological insulators at quantum critical point

disordered topological matter — time line



disordered topological matter — time line

80s

quantum Hall

90s

1d delocalization

symmetry classes

novel quantum
Hall effects

00s

2d delocalization

topological matter

disordered topological matter — time line

80s

quantum Hall

90s

1d delocalization

symmetry classes

novel quantum
Hall effects

00s

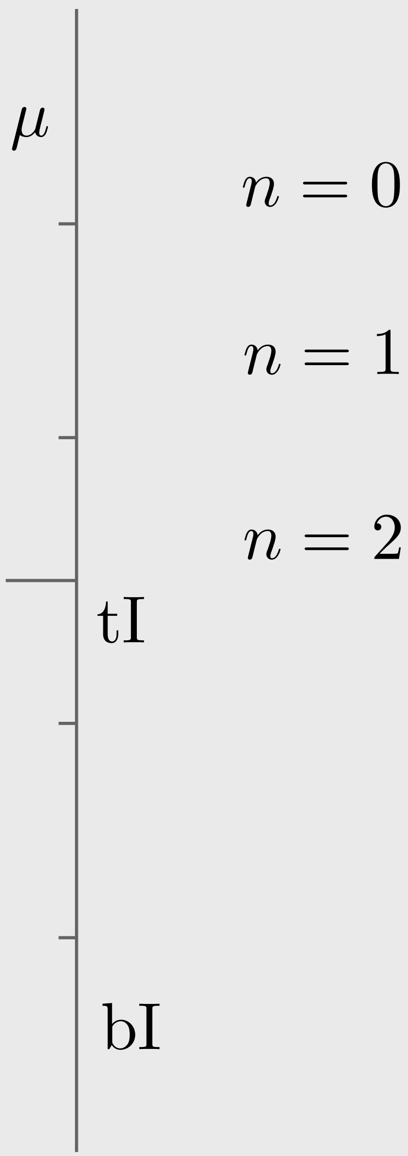
2d delocalization

topological matter

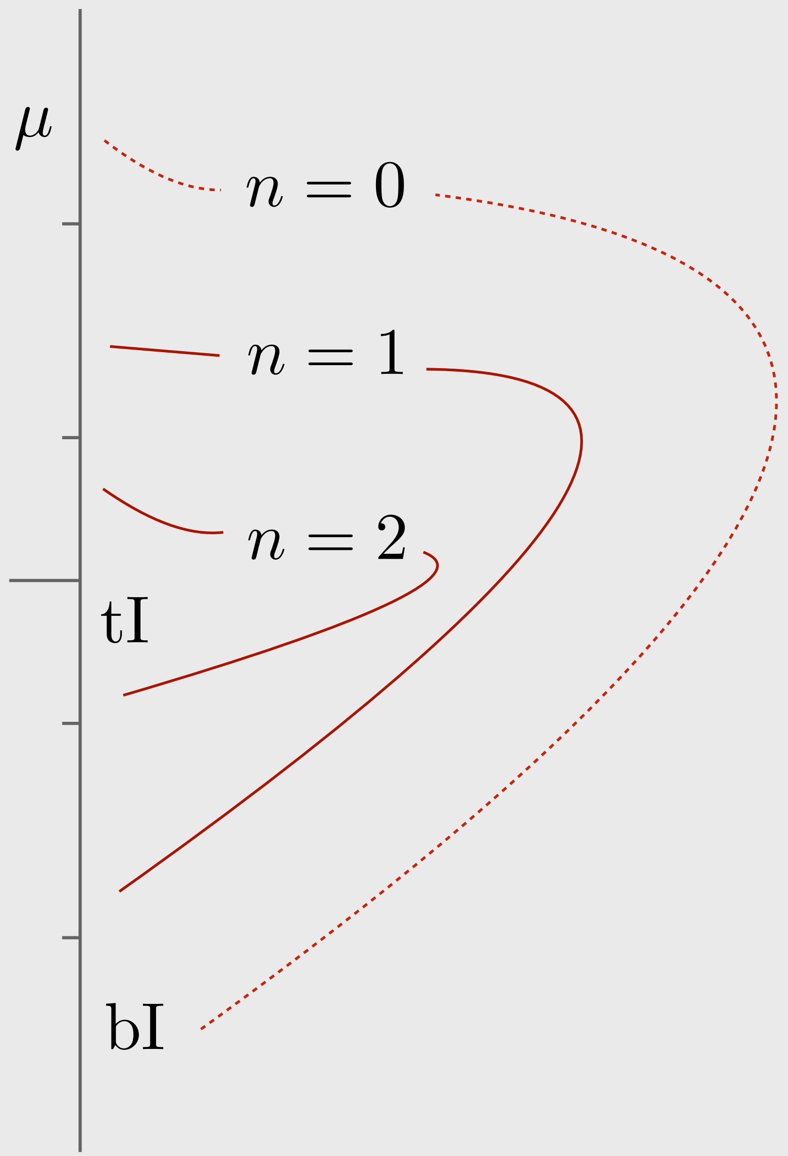
10s

“topological Anderson insulator”

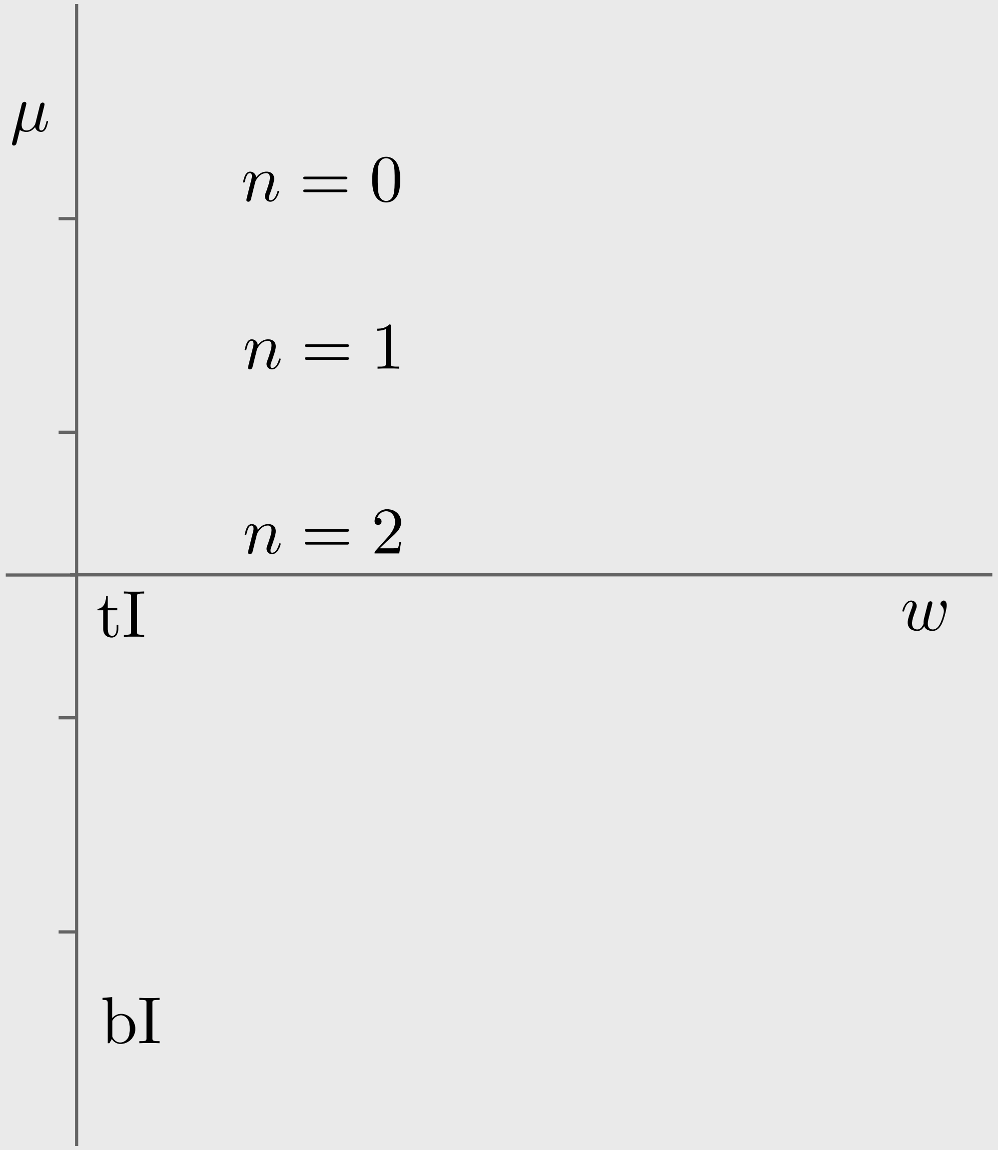
topological Anderson insulator



topological Anderson insulator



topological Anderson insulator

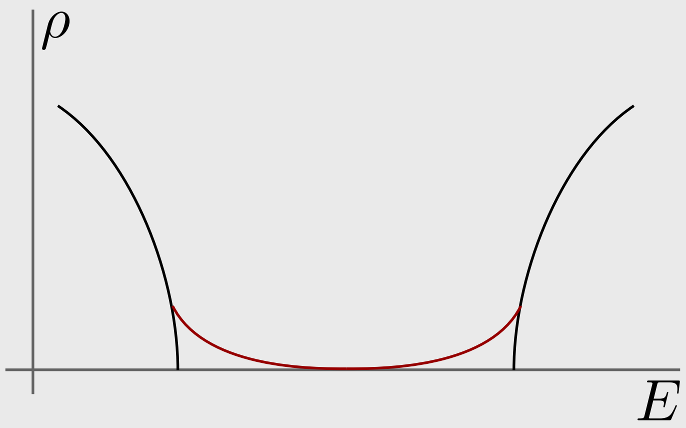


Addition of disorder to a clean band insulator …

- invalidate k-theory of band gaps/topological indices

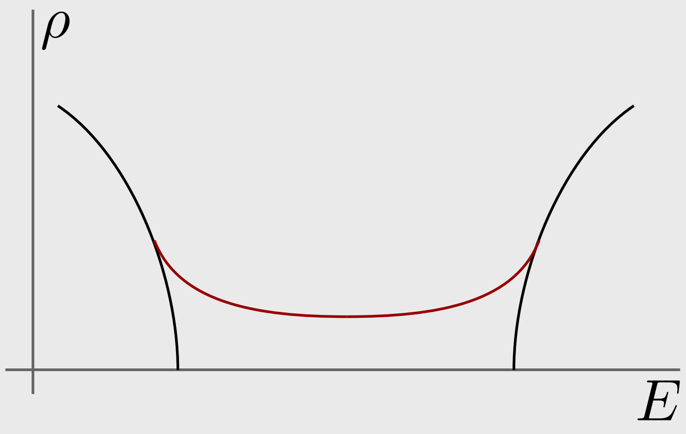
Addition of disorder to a clean band insulator …

- invalidate k-theory of band gaps/topological indices
- destroys band gaps and



Addition of disorder to a clean band insulator …

- invalidate k-theory of band gaps/topological indices
- destroys band gaps and
- turns insulator into nominal metal
- renders topological indices statistically distributed



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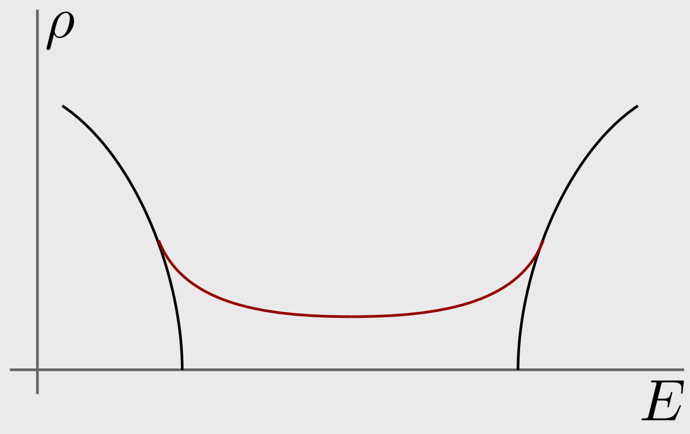
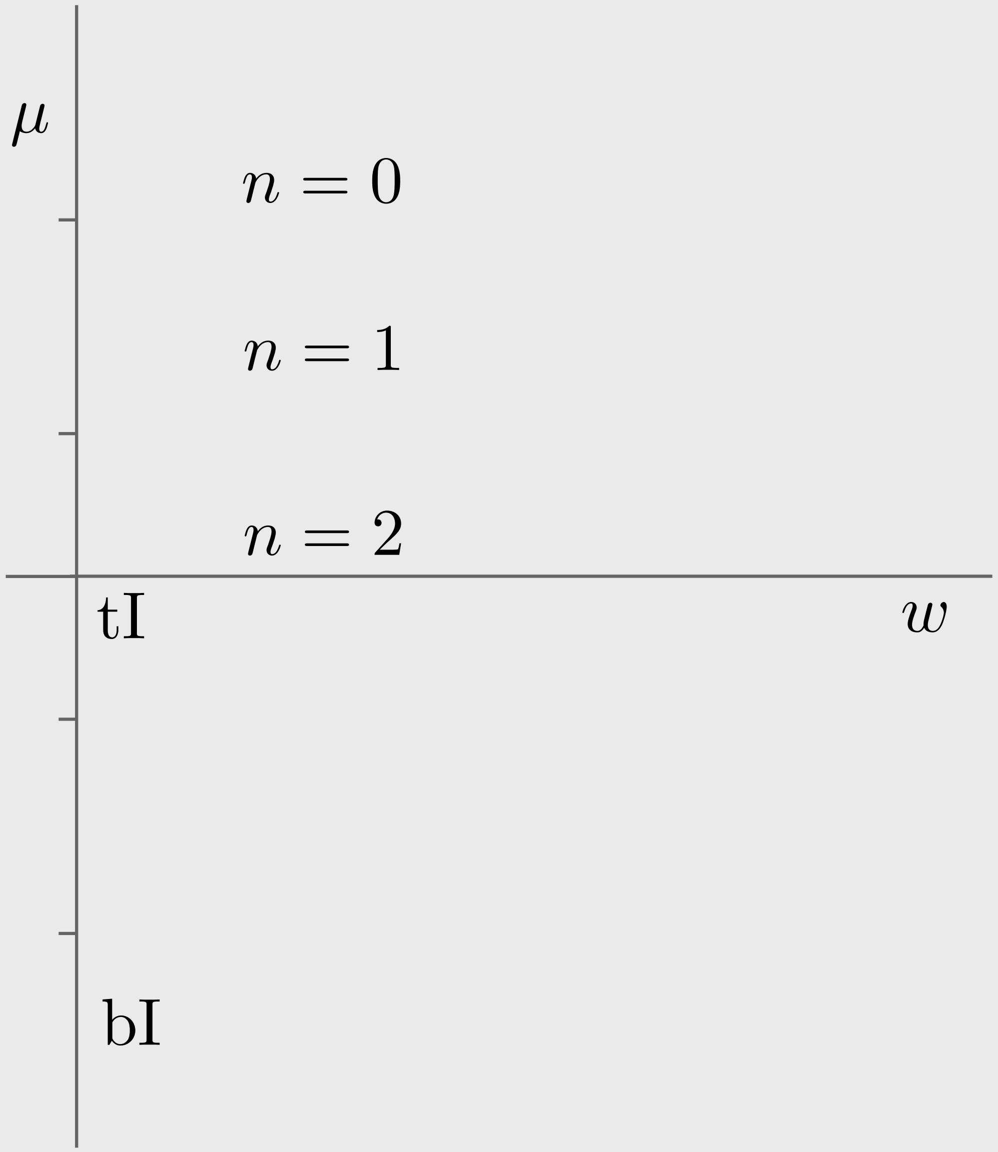


	clean	disordered
k-theory	+	-
band gap	+	-
index	+	(+)

topological Anderson insulator

cf. Motrunich, Damle and Huse 2001, Groth et al. 2010



topological Anderson insulator

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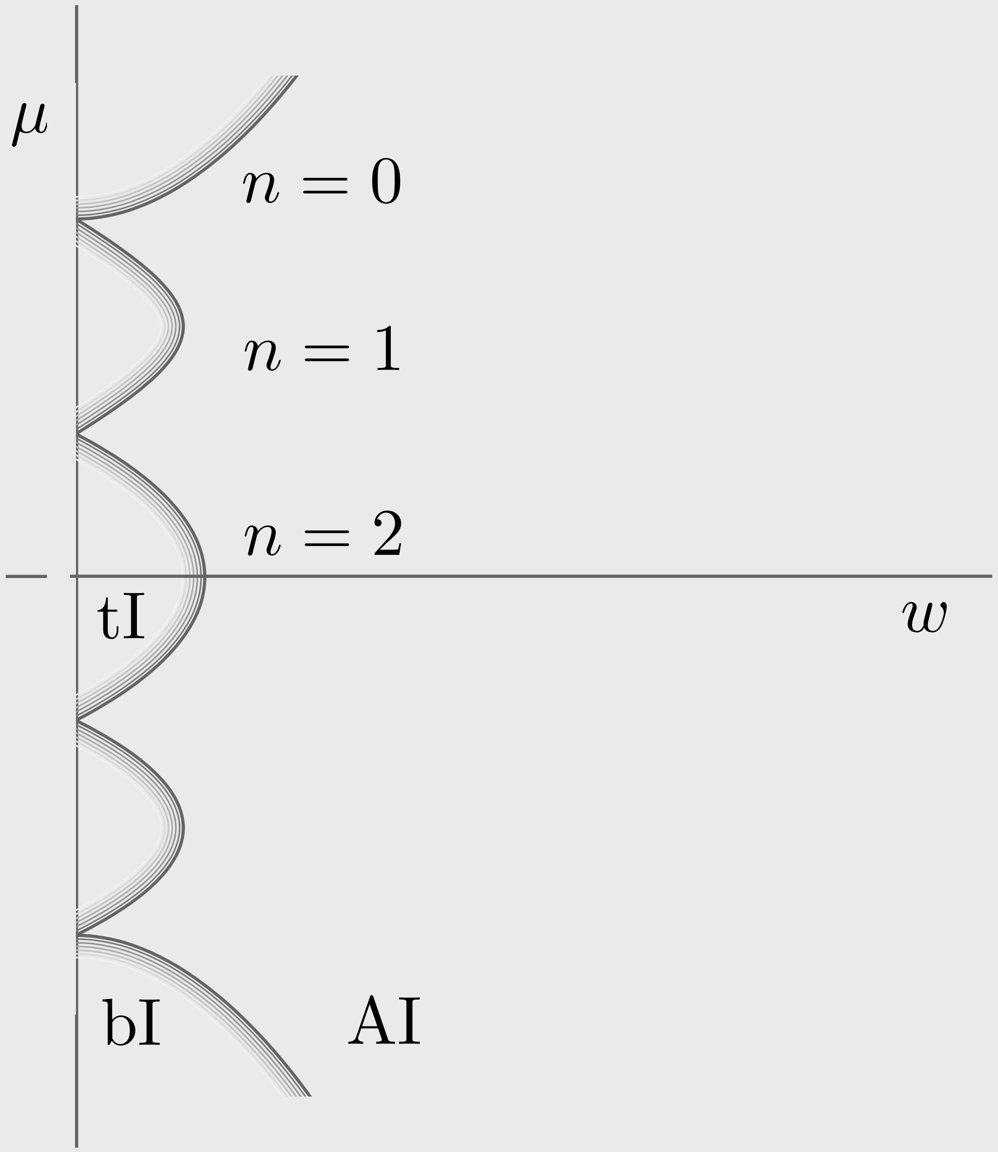

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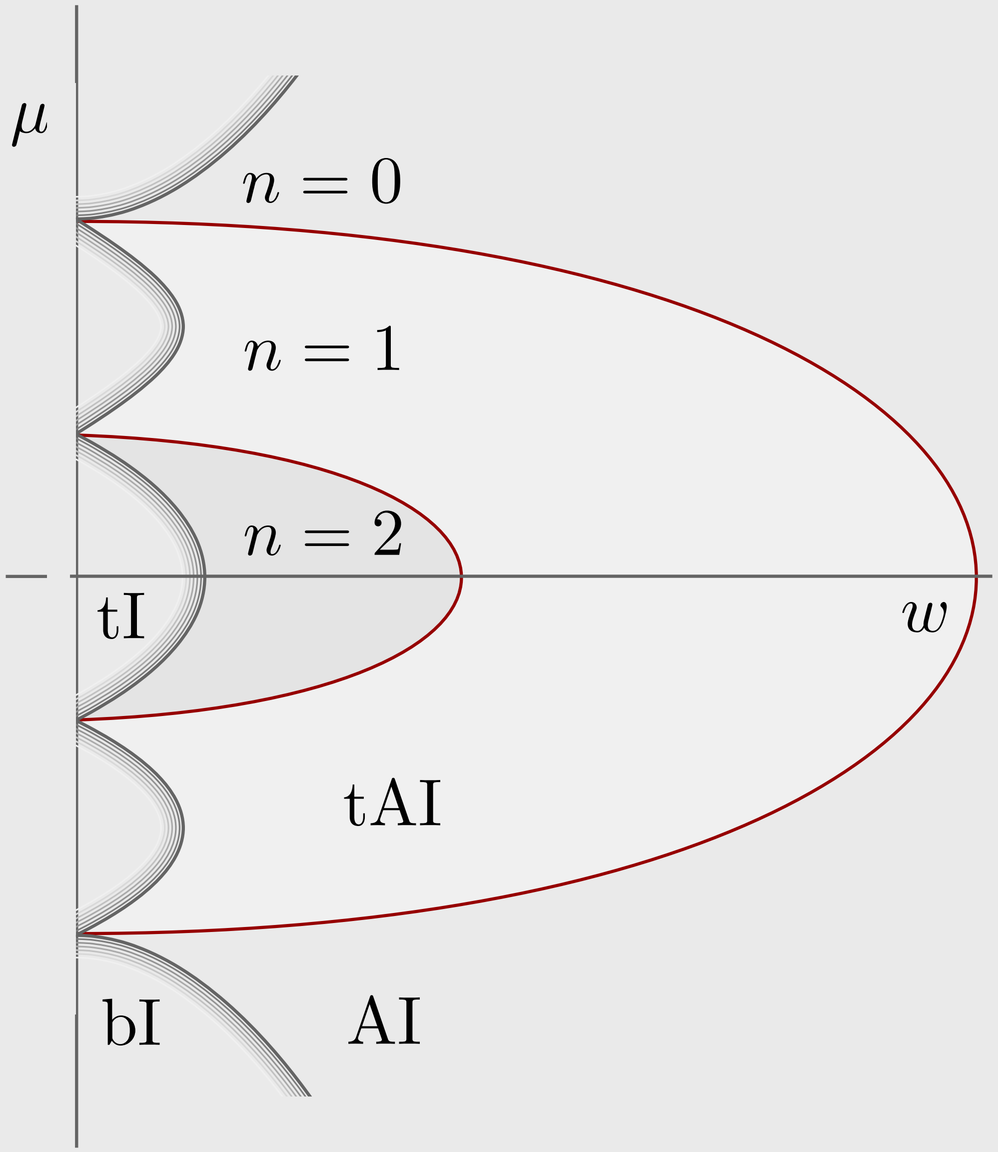

topological Anderson insulator

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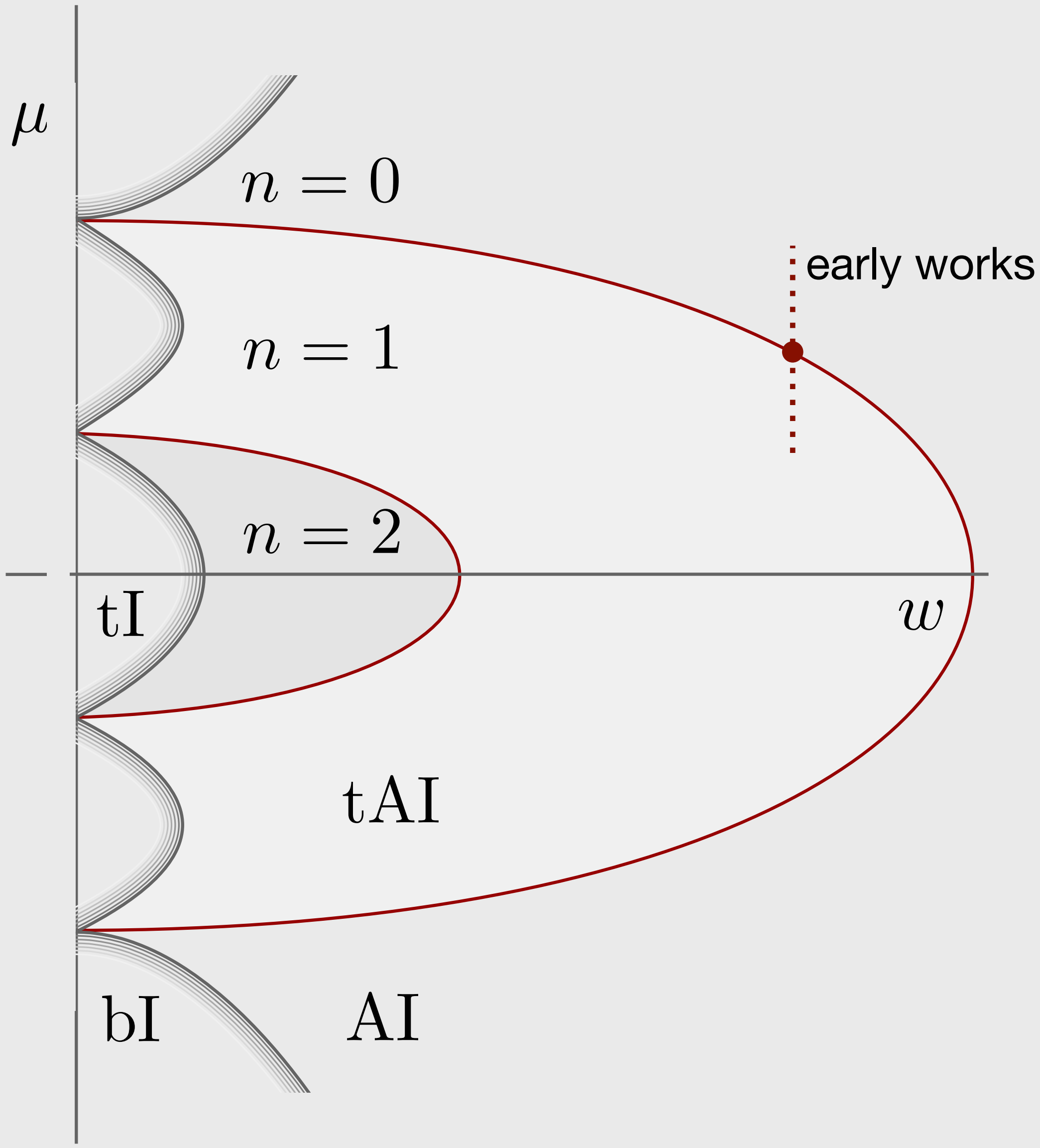

topological Anderson insulator

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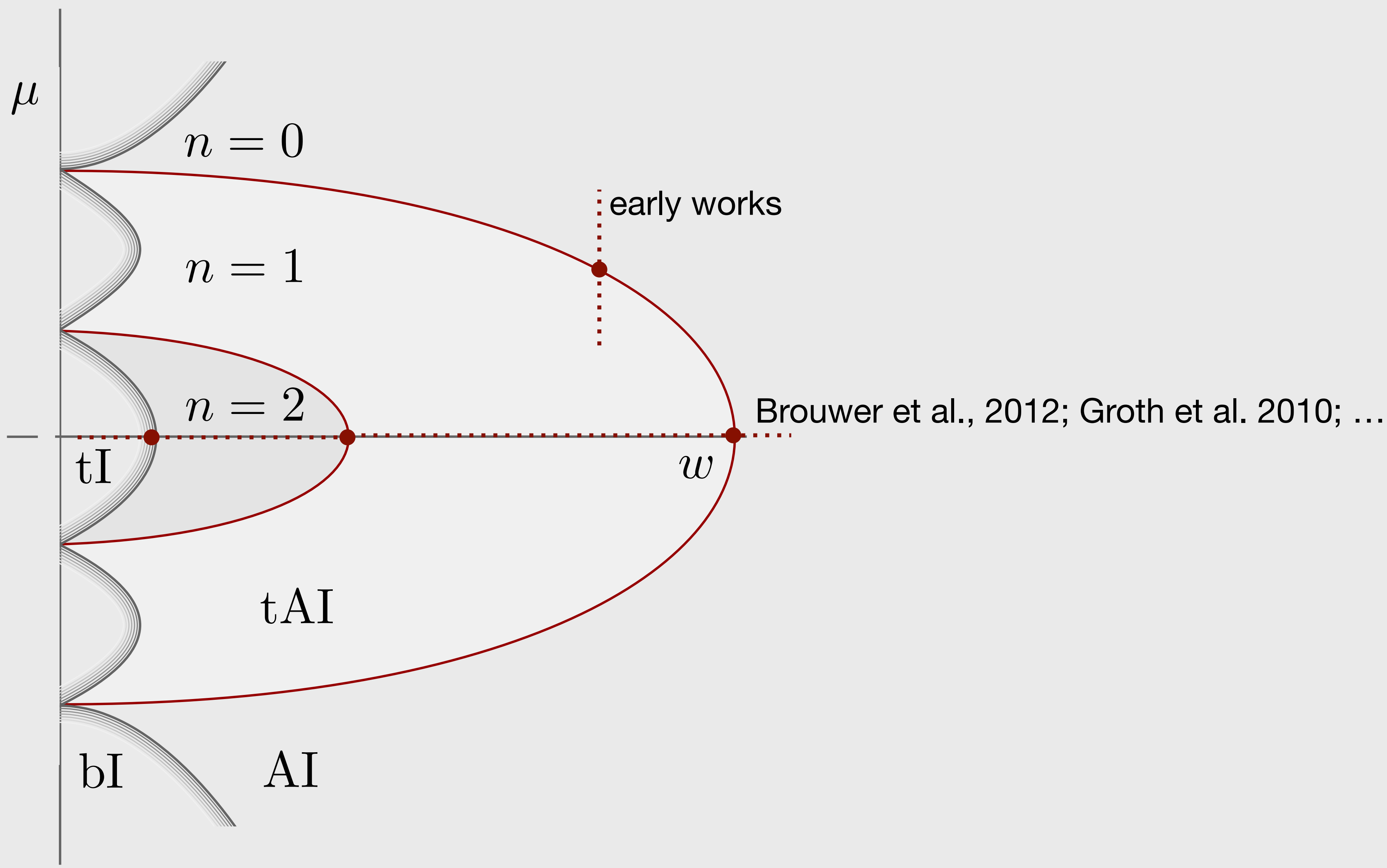

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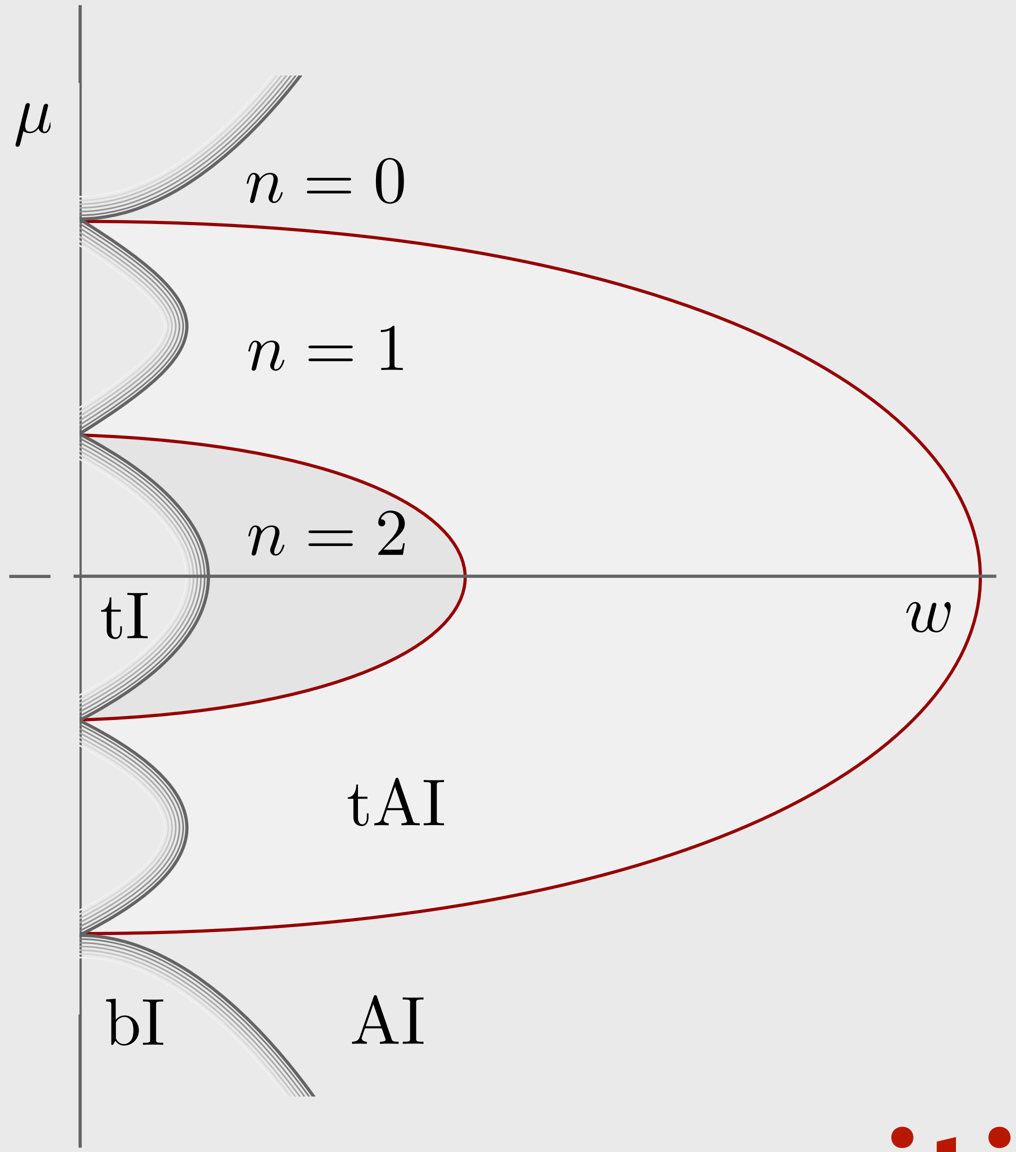

critical theory ?

Universal theory of disordered TI — checklist

▷ describe topological invariants without reference to k-space

▷ describe criticality in terms of two observables:

g : transport coefficient

χ : configurational average of index

▷ state bare values $g = g(\mu, w, \ldots),\ \chi = \chi(\mu, w, \ldots)$

generically: $(g, \chi) \xrightarrow{L\to\infty} (0, n)$

critical: $(g, \chi) \xrightarrow{L\to\infty} (g_{\mathrm{crit.}}, n + 1/2)$

▷ describe edge state formation

▷ similar architecture in all symmetry classes, d=1,2

Universal theory of disordered TI — checklist

complex case:

Cartan\\d	0	1	2	3	4	5	6	7	8	9	10	11	$\cdots$
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$

real case:

Cartan\\d	0	1	2	3	4	5	6	7	8	9	10	11	$\cdots$
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\cdots$
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$\cdots$

Universal theory of disordered TI — checklist

complex case:

Cartan\d	0	1	2	3	4	5	6	7	8	9	10	11	⋯
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	⋯
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	⋯

IQH

SSH

real case:

Cartan\d	0	1	2	3	4	5	6	7	8	9	10	11	⋯
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	⋯
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	⋯
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	⋯
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	$\mathbb{Z}_2$	$\mathbb{Z}$	⋯
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	⋯
CII	0	$2\mathbb{Z}$	0	[illegible]	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	⋯
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	⋯
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	⋯

‘Kitaev chain’

‘Majorana quantum wire’

‘topolgical insulator (HgTe)’

spin QH

Universal theory of disordered TI — checklist

complex case:

Cartan\\d	0	1	2	3	4	5	6	7	8	9	10	11	$\cdots$
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Cartan\\d	0	1	2	3	4	5	6	7	8	9	10	11	$\cdots$
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D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\cdots$
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$\cdots$

quantum criticality of the 1d topological Anderson insulator

Benasque, 26.6.14

Alexander Altland, Dmitry Bagrets (Cologne)

Alex Kamenev (Minnesota)

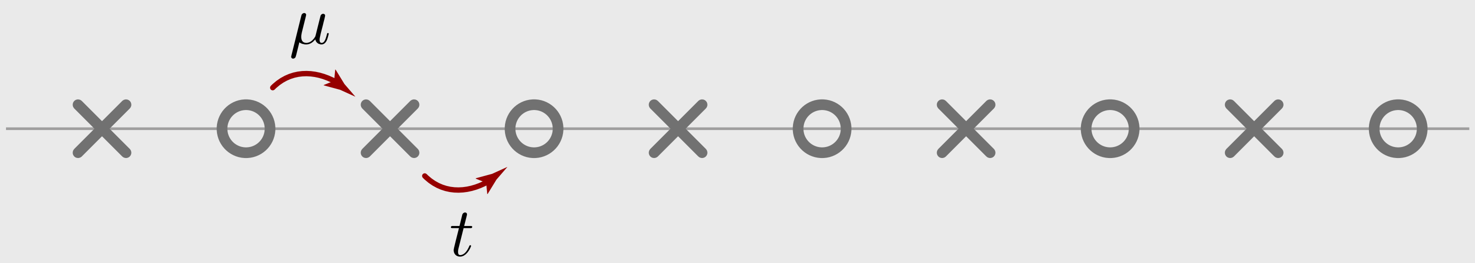
Lars Fritz (Utrecht)

- ▷ the 1d AIII insulator
- ▷ topological quantum criticality
- ▷ generalization to 1d BDI, CII
- ▷ comparison to 2d A, C, D
- ▷ generalization to Z2 classes, 1d D,DIII, 2d AII, DIII

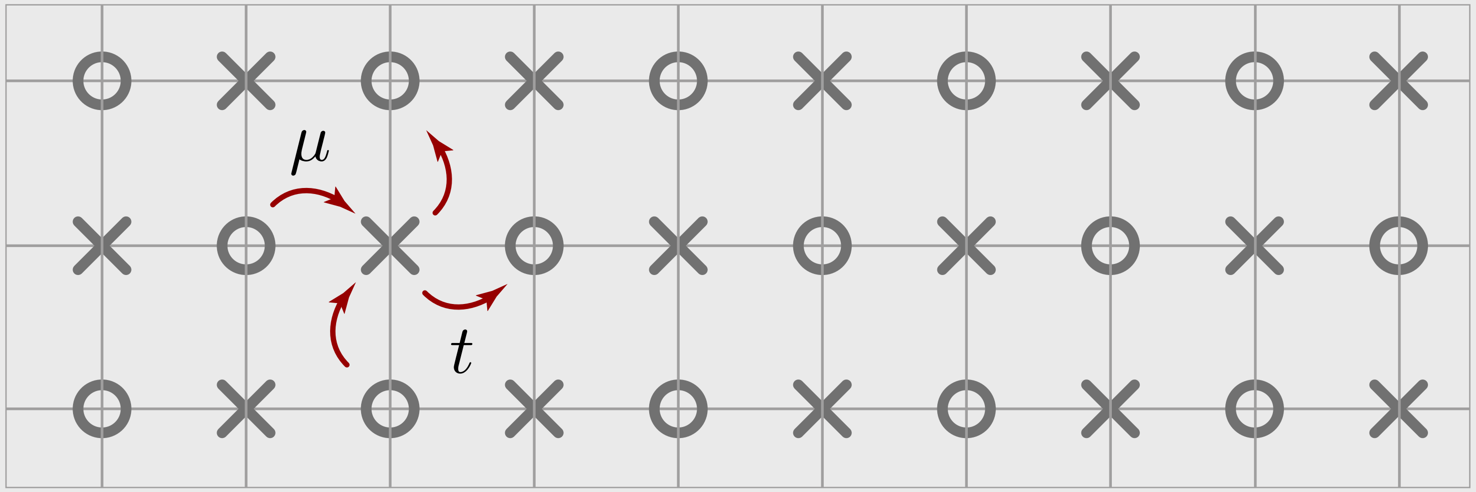
arXiv 1402.1738; PRL **112**, 206602 (2014)

the 1d AIII system (aka SSH)

AIII quantum wire

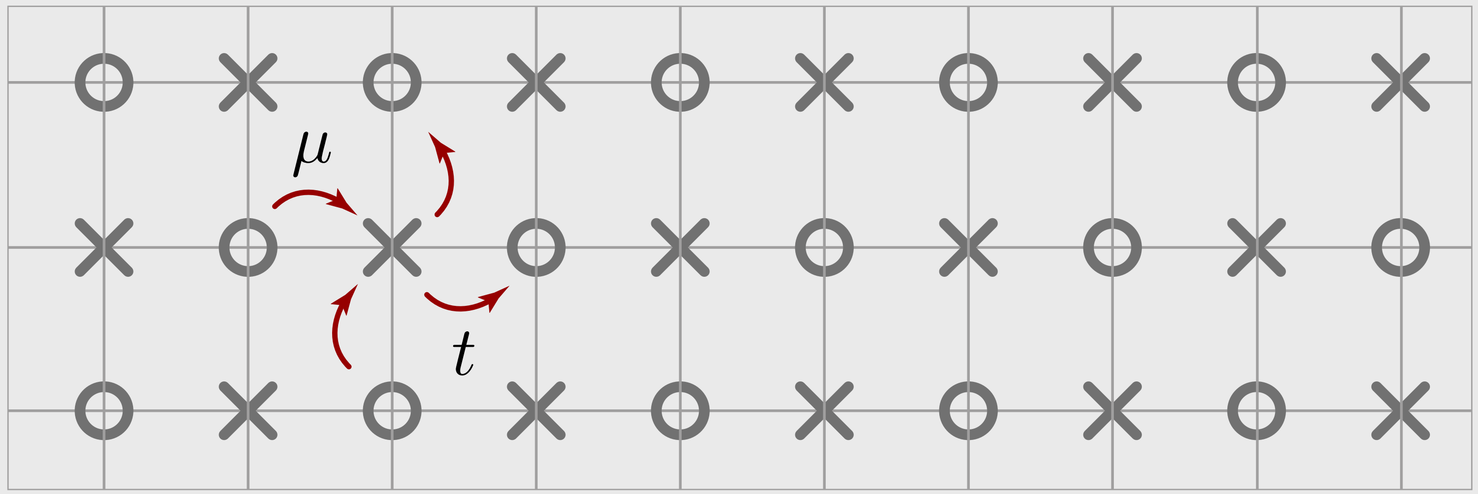


AIII quantum wire

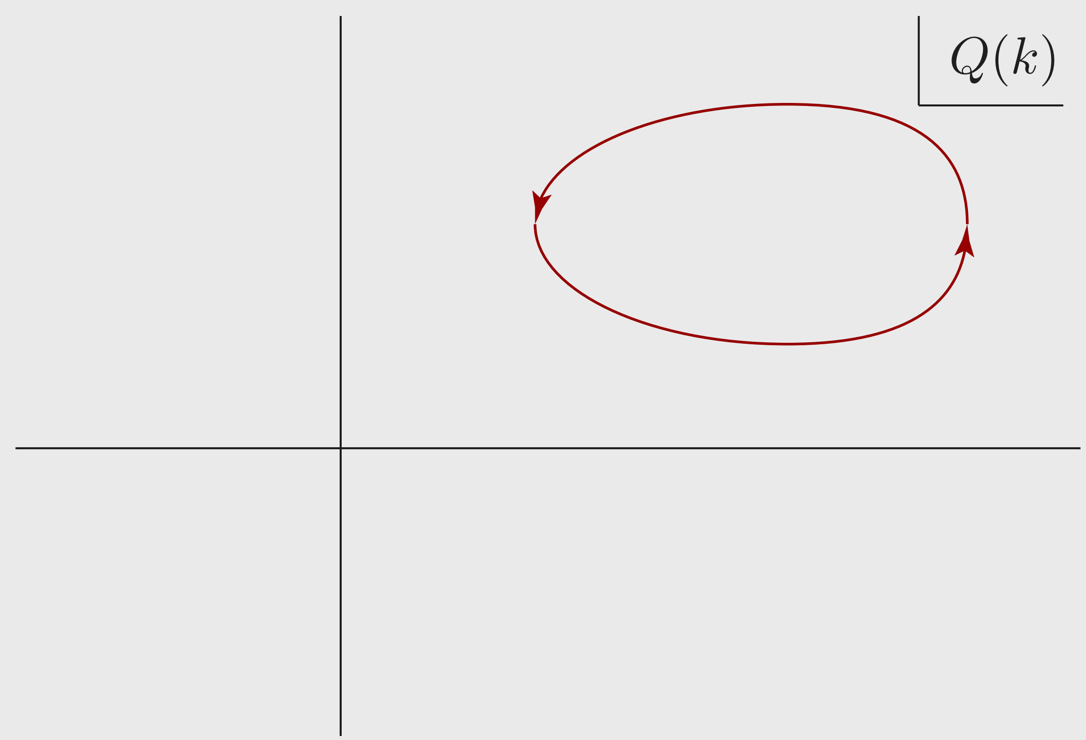


$$\begin{array}{cc} \hat{H} = \begin{pmatrix} & \hat{Q} \\ \hat{Q}^\dagger & \end{pmatrix} & \begin{array}{c} \times \\ \circ \end{array} \end{array}$$

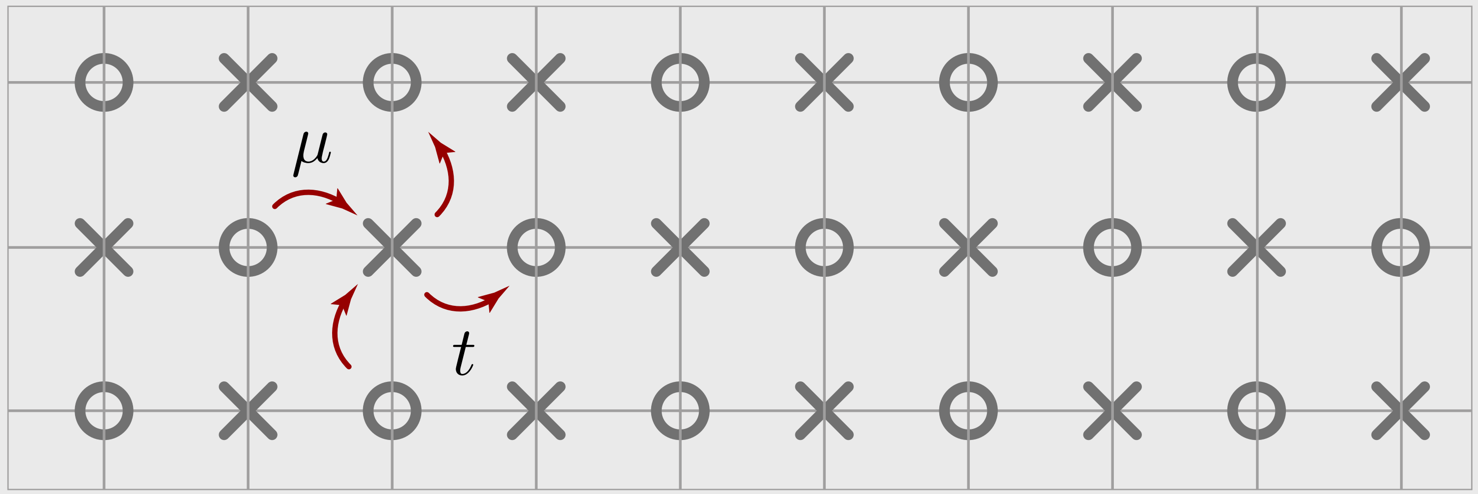
AIII quantum wire



$$\boxed{\hat{H} = \begin{pmatrix} & \hat{Q} \\ \hat{Q}^\dagger & \end{pmatrix} \begin{array}{c} \times \\ \circ \end{array}}$$



AIII quantum wire



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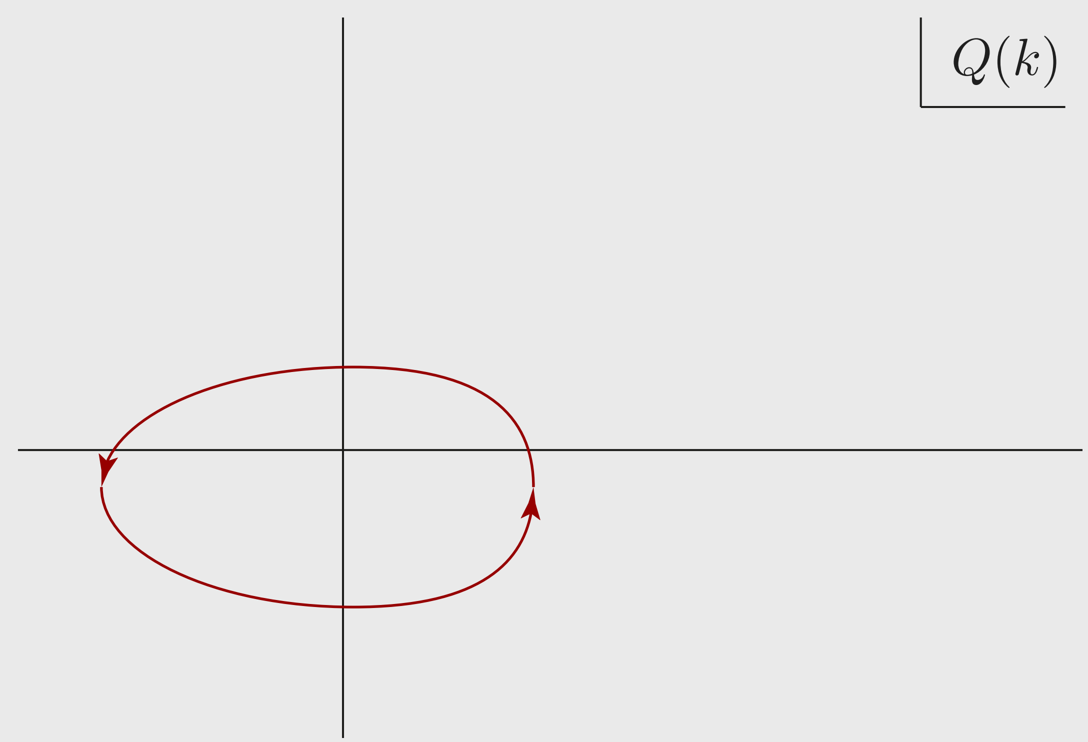
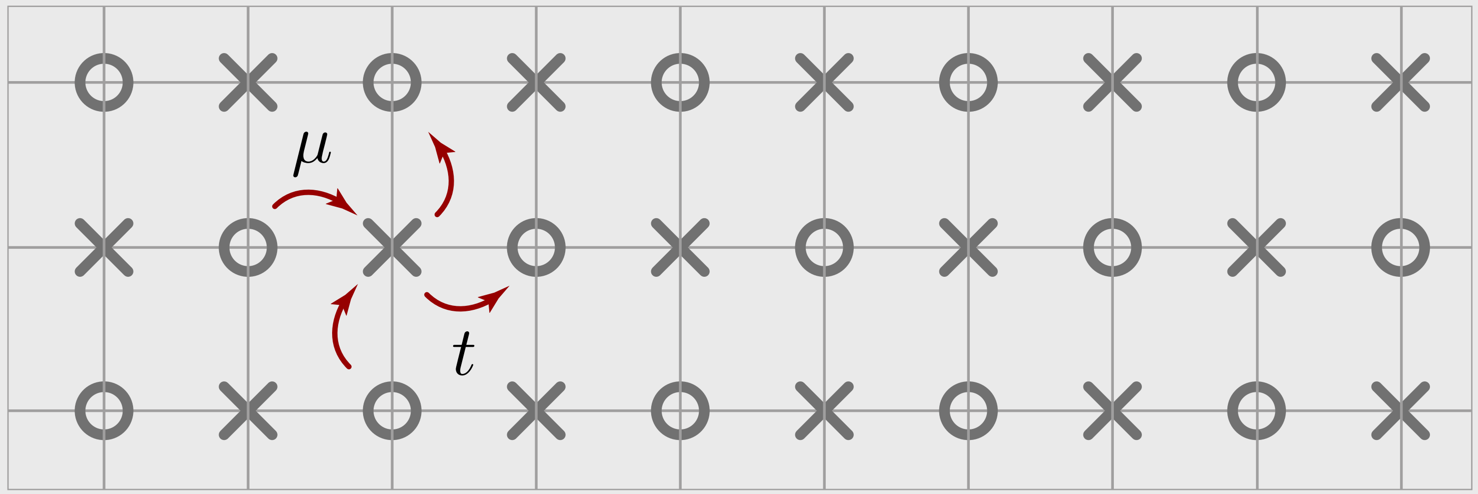


AIII quantum wire



$$\boxed{\hat{H} = \begin{array}{cc} \begin{pmatrix} & \hat{Q} \\ \hat{Q}^\dagger & \end{pmatrix} & \begin{array}{c} \times \\ \circ \end{array} \end{array}}$$

▷ winding number (clean)

$$n = -\frac{i}{2\pi}\int_0^{2\pi} dk\,\mathrm{tr}\,(\hat{Q}^{-1}\partial_k\hat{Q})$$

single channel: $n = \Theta(t-\mu)$

N channels: $n \in [0, N]$

generalized definition of winding number

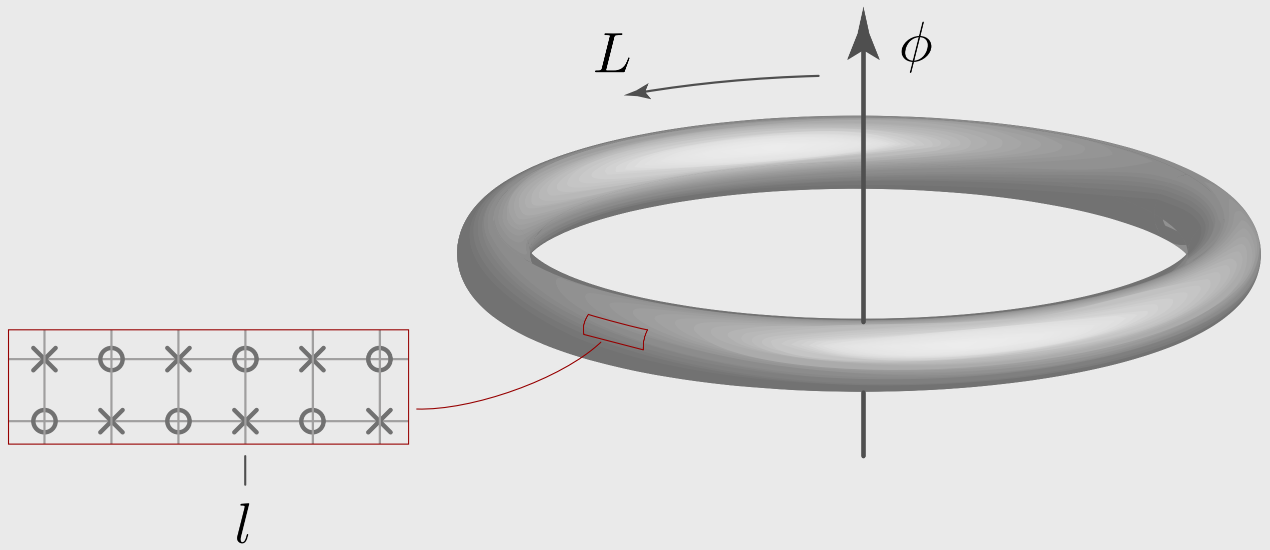
$$n = -\frac{i}{2\pi}\int_0^{2\pi} dk\,\mathrm{tr}\,(\hat{Q}^{-1}\partial_k \hat{Q})$$

generalized definition of winding number

$$n = -\frac{i}{2\pi}\int_0^{2\pi} dk \,\mathrm{tr}\,(\hat{Q}^{-1}\partial_k \hat{Q})$$

- generalized definition

$$|\circ_l\rangle \rightarrow e^{+i\frac{\phi l}{L}}\,|\circ_l\rangle$$
$$|\times_l\rangle \rightarrow e^{-i\frac{\phi l}{L}}\,|\times_l\rangle$$



generalized definition of winding number

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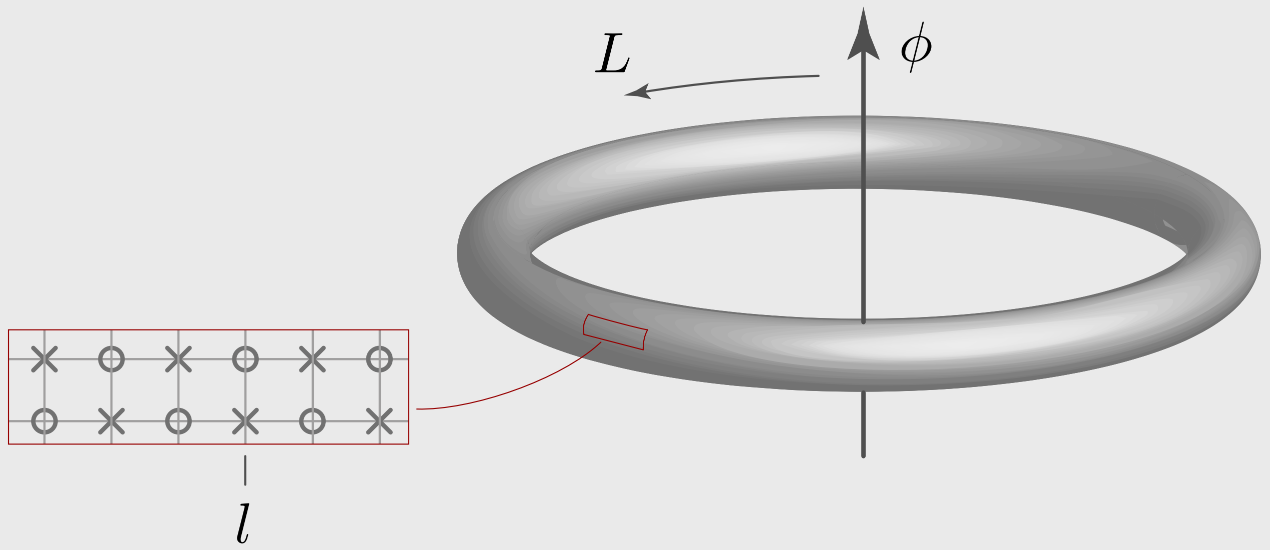
▷ generalized definition

$$\boxed{\begin{aligned} |\circ_l\rangle &\to e^{+i\frac{\phi l}{L}}\,|\circ_l\rangle \\ |\times_l\rangle &\to e^{-i\frac{\phi l}{L}}\,|\times_l\rangle \end{aligned}}$$



$$Z(\phi) \equiv \frac{\det \hat{G}_0(\phi_0)}{\det \hat{G}_0(i\phi_1)}, \qquad G_0(\theta) \equiv (i0 - \hat{H}(\theta))^{-1}, \qquad \phi = (\phi_0, \phi_1)^T,$$

cf. Nazarov, 94

$$n \equiv \chi = -\frac{1}{2\pi}\int_0^{2\pi} d\phi_0\, Z(\phi)\big|_{\phi_1=0},$$

$$g = (\partial^2_{\phi_0} + \partial^2_{\phi_1})\big|_{\phi=0} Z(\phi)$$

topological quantum criticality

field integral

$$Z(\phi) = \int \mathcal{D}T \, \exp(-S[T])$$

$$S[T] = \int_0^L dx \left[\frac{\tilde{\xi}}{4} \mathrm{str}(\partial_x T \partial_x T^{-1}) + \tilde{\chi} \, \mathrm{str}(T^{-1} \partial_x T) \right]$$

AA & Merkt, 2001

- matrix fields $T = U \begin{pmatrix} e^{y_1} & \\ & e^{iy_0} \end{pmatrix} U^{-1}$
- boundary conditions $T(L) = T(0) \begin{pmatrix} e^{\phi_1} & \\ & e^{i\phi_0} \end{pmatrix}$
- $(\tilde{\xi}, \tilde{g})$ bare (SCBA) values of loc. length and topological parameter, resp.

$$\tilde{\xi} = Nl, \qquad \tilde{\chi} = -\frac{i}{2} \mathrm{tr}(\hat{G}^+ \hat{v})$$

- good picture: path integral of quantum point particle in time L.

diffusive systems

$$Z(\phi) = \int \mathcal{D}T \, \exp(-S[T])$$

$$S[T] = \int_0^L dx \left[\frac{\tilde{\xi}}{4} \mathrm{str}(\partial_x T \partial_x T^{-1}) + \tilde{\chi} \, \mathrm{str}(T^{-1} \partial_x T) \right]$$

▷ for $L < \tilde{\xi}$:

$$T(x) = \mathrm{diag}(e^{\phi_1 x/L}, e^{i\phi_0 x/L}),$$

$$S[T] = \frac{\tilde{\xi}}{L}(\phi_1^2 + \phi_2^2) + \tilde{\xi}(\phi_1 + i\phi_0)$$

▷ diffusive conductance $g = \frac{\tilde{\xi}}{L}$

▷ non-integer average invariant $\chi = \tilde{\chi}$

Understanding the path integral

$$Z(\phi) = \int \mathcal{D}T \, \exp(-S[T])$$

$$S[T] = \int_0^L dx \left[\frac{\tilde{\xi}}{4} \mathrm{str}(\partial_x T \partial_x T^{-1}) + \tilde{\chi} \, \mathrm{str}(T^{-1} \partial_x T) \right]$$

$$T = U \begin{pmatrix} e^{y_1} & \\ & e^{i y_0} \end{pmatrix} U^{-1}$$

Understanding the path integral

$$\boxed{\begin{aligned} Z(\phi) &= \int \mathcal{D}T \, \exp(-S[T]) \\ S[T] &= \int_0^L dx \left[\frac{\tilde{\xi}}{4} \operatorname{str}(\partial_x T \partial_x T^{-1}) + \tilde{\chi} \operatorname{str}(T^{-1}\partial_x T) \right] \end{aligned}}$$

$$T = U \begin{pmatrix} e^{y_1} & \\ & e^{iy_0} \end{pmatrix} U^{-1}$$

Understanding the path integral

- focus on compact sector: $y_0 = \theta$

- theory reduces to QM on a ring with twisted boundary conditions and subject to magnetic flux $\tilde{\chi}$

$$Z(\phi) = \int \mathcal{D}\theta \, \exp(-S[\theta])$$

$$S[\theta] = \int_0^L dx \left[\frac{\tilde{\xi}}{4} \, \partial_x\theta\partial_x\theta + i\tilde{\chi} \, \partial_x\theta \right]$$

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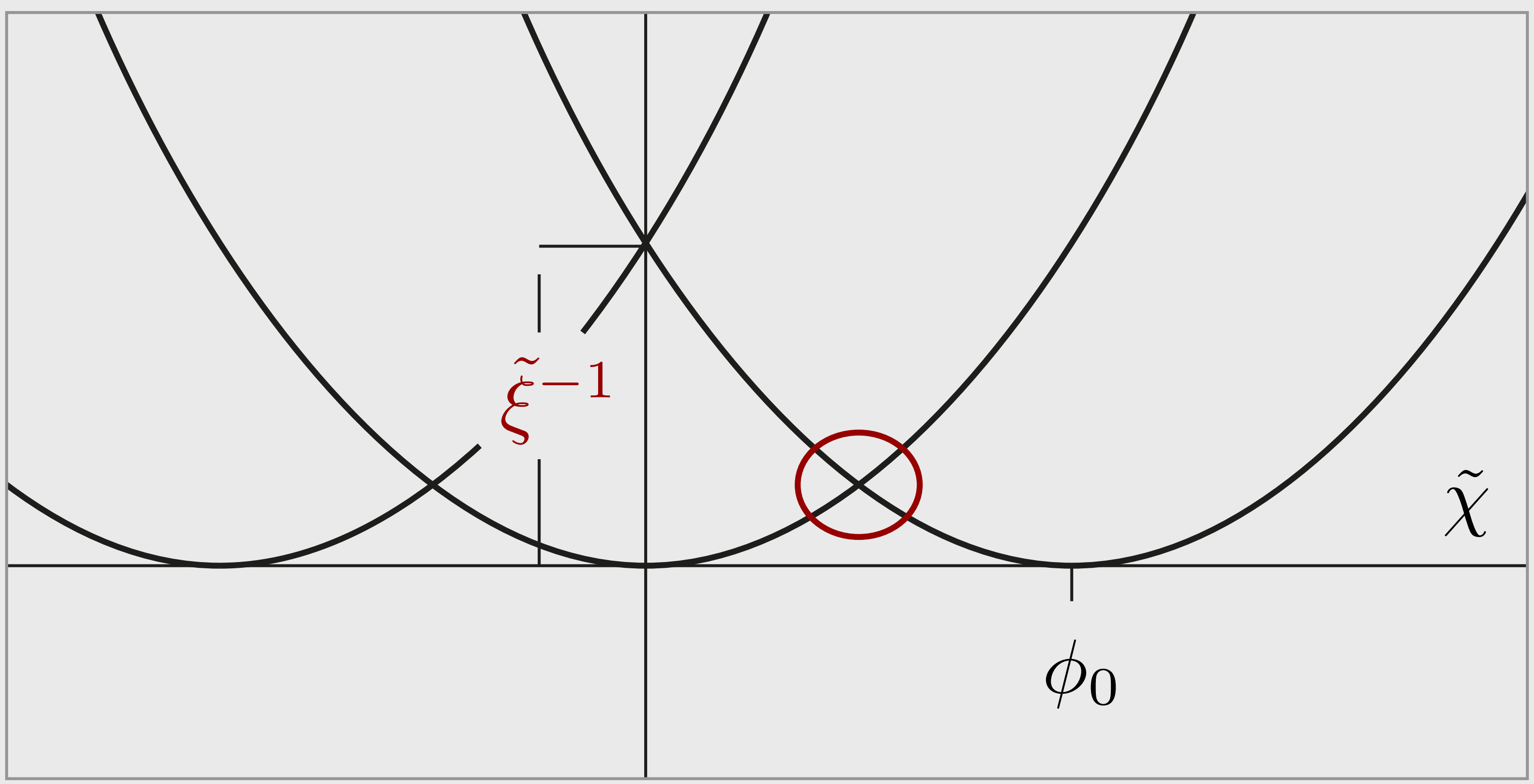

Understanding the path integral

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$$S[\theta] = \int_0^L dx \left[\frac{\tilde{\xi}}{4} \partial_x\theta\partial_x\theta + i\tilde{\chi}\,\partial_x\theta \right]$$

$$\Big\downarrow$$

$$\tilde{\xi}\partial_x\Psi(\theta, x) = (\partial_\theta - iA)^2\Psi(x,\theta), \qquad A = \tilde{\chi}$$

transfer matrix solution of full problem

$$Z(\phi) = \int \mathcal{D}T \, \exp(-S[T])$$

$$S[T] = \int_0^L dx \left[\frac{\tilde{\xi}}{4} \operatorname{str}(\partial_x T \partial_x T^{-1}) + \tilde{\chi} \operatorname{str}(T^{-1} \partial_x T) \right]$$

transfer matrix solution of full problem

$$\tilde{\xi}\,\partial_x \Psi(y,x) = \frac{1}{J(y)}(\partial_\alpha - iA_\alpha)J(y)(\partial_\alpha - iA_\alpha)\Psi(y,x),$$

$$J(y) = \sinh^{-2}\left(\frac{1}{2}(y_1 - iy_0)\right) \qquad A = \tilde{\chi}(1,i)^T$$

- solvable problem:
  - eigenfunctions: $\Psi_l(y) = \sinh\left(\frac{1}{2}(y_1 - iy_0)\right)e^{il_\alpha y_\alpha} \qquad (l_0, l_1) \in (\mathbb{Z} + \frac{1}{2}, \mathbb{R})$
  - eigenvalues: $\epsilon_l = (l_0 - \tilde{\chi})^2 + (l_1 - i\tilde{\chi})^2$
  - solution by spectral sum:

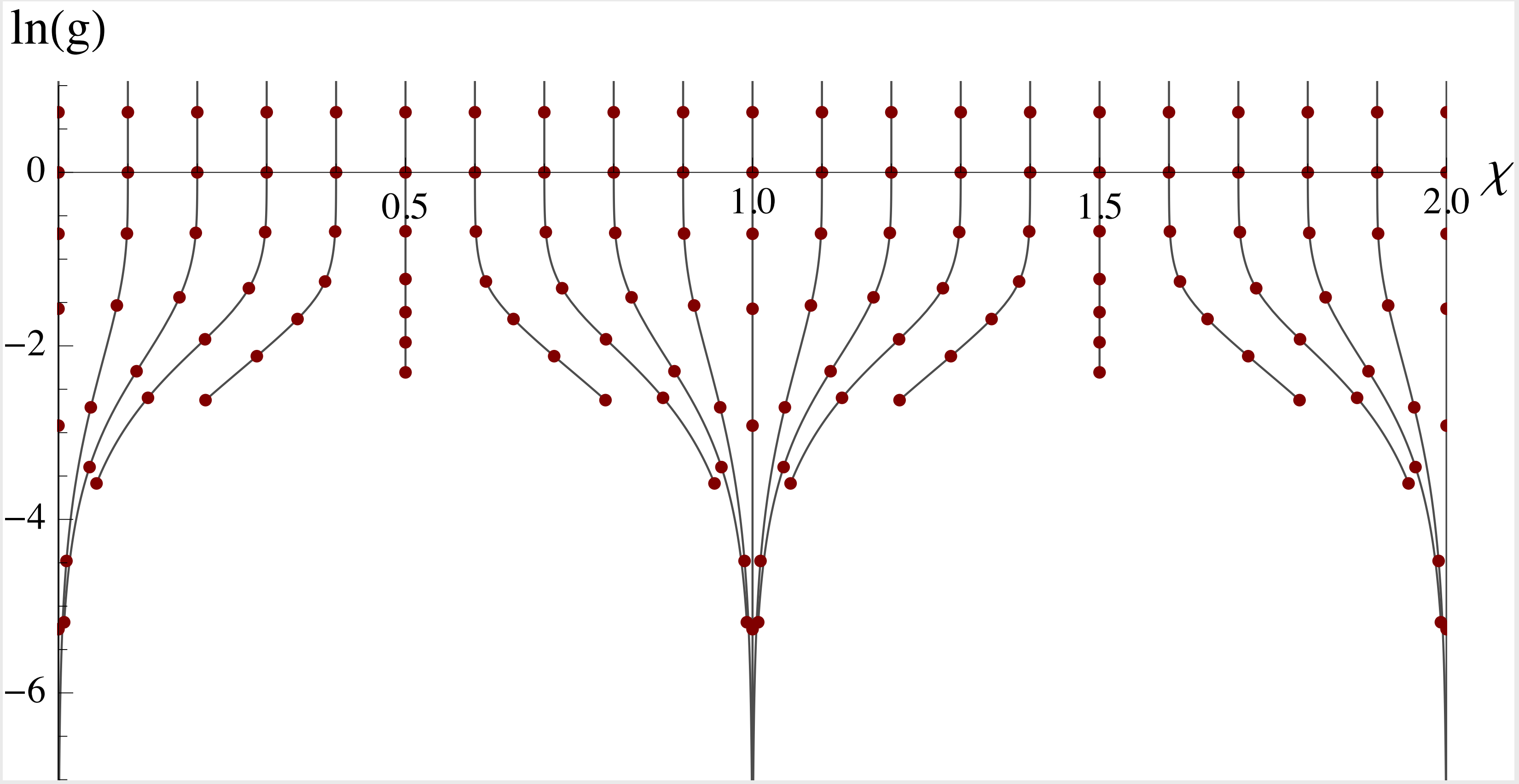
$$\Psi(\phi, L) = 1 + \frac{1}{\pi}\sum_{l_0 \in \mathbb{Z}+\frac{1}{2}} \int dl_1 \, \frac{\Psi_l(\phi)}{l_0 + il_1}\, e^{-\epsilon_l L/\tilde{\xi}}$$

results

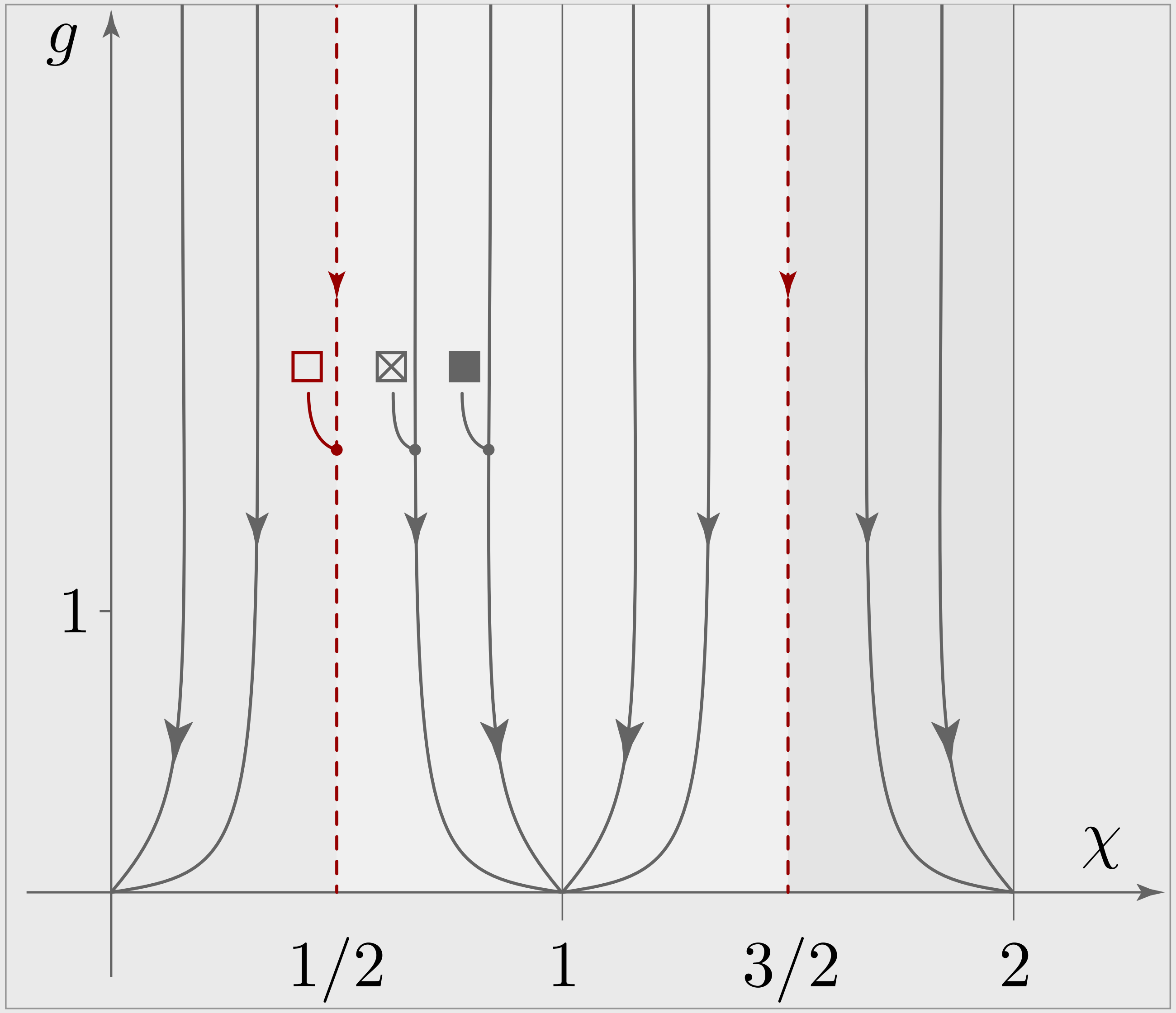
$$g = \sqrt{\frac{\tilde{\xi}}{\pi L}} \sum_{l_0 \in \mathbb{Z}+1/2} e^{-(l_0-\tilde{\chi})^2 L/\tilde{\xi}},$$

$$\chi = n - \frac{1}{4} \sum_{l_0 \in \mathbb{Z}+1/2} \left[\mathrm{erf}\left(\sqrt{\frac{L}{\tilde{\xi}}} (l_0 - \delta\tilde{\chi}) \right) - (\delta\tilde{\chi} \leftrightarrow -\delta\tilde{\chi}) \right],$$

where $\chi = n + \tilde{\chi}$



phase diagram



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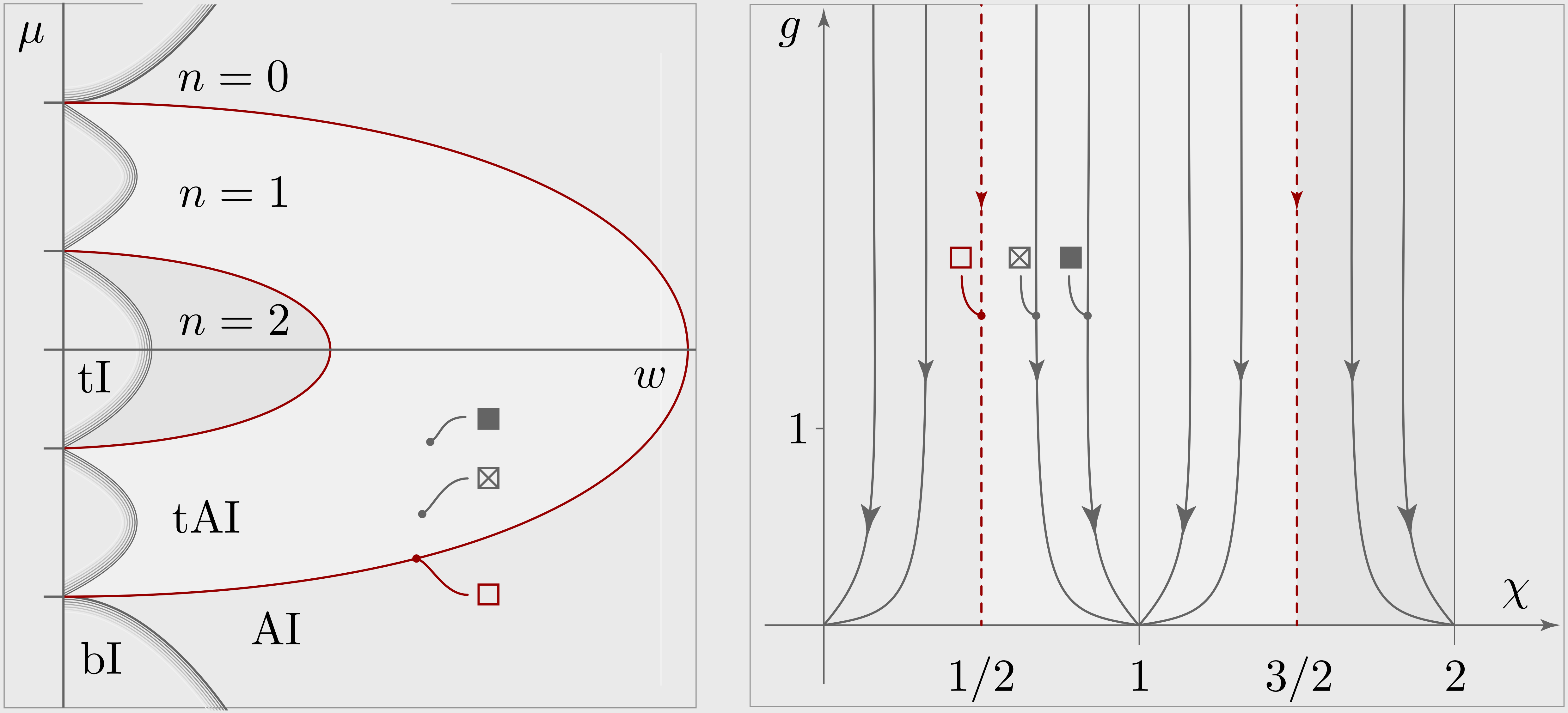
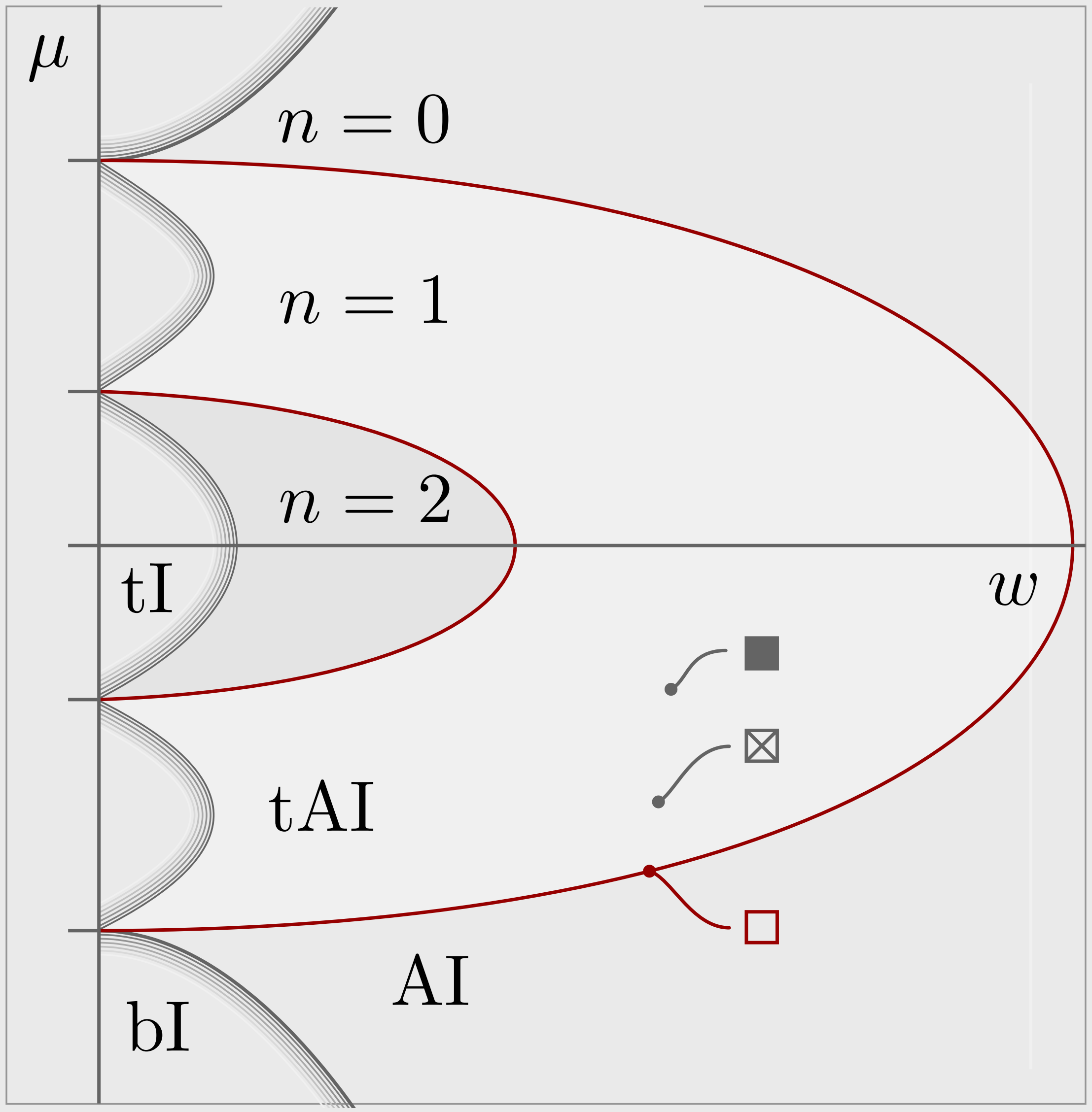
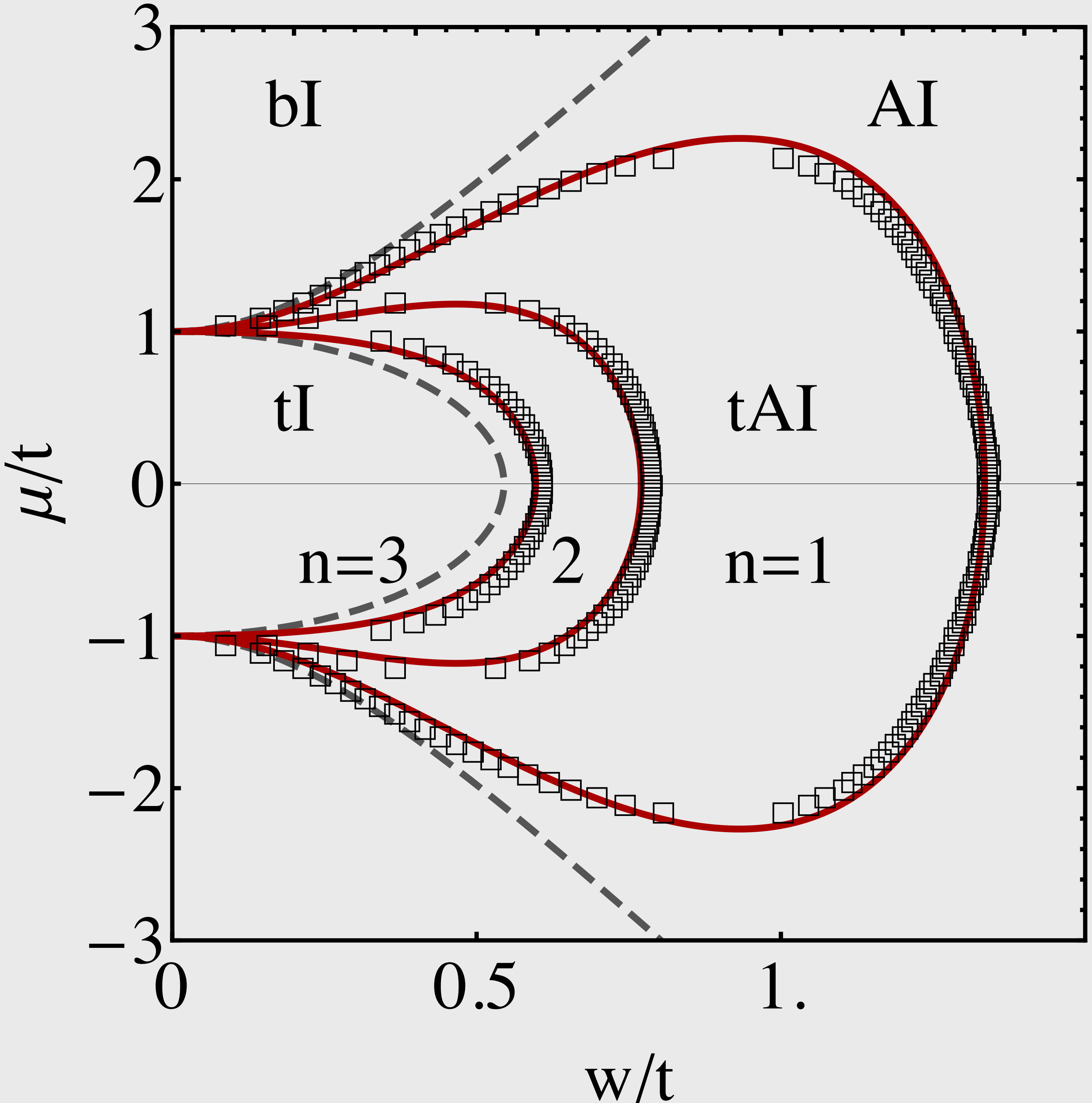


- ▷ phase boundaries: lines of half integer bare topological index
- ▷ flow describes stabilization of self-averaging topological phase/boundary state generation

phase diagram





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physics at boundary

- deep localization regime

$$S[T] = \int_0^L dx \left[\frac{\tilde{\xi}}{4} \operatorname{str}(\partial_x T \partial_x T^{-1}) + \tilde{\chi} \operatorname{str}(T^{-1} \partial_x T) \right].$$

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$$\Big\downarrow \; L \gg \tilde{\xi}$$

$$S[T] = n \int dx\, \mathrm{str}\,(T^{-1}\partial_x T) = n \int dx\, \partial_x \mathrm{str}\; \ln(T) = n(\mathrm{str}\; \ln(T(L)) - \mathrm{str}\; \ln(T(0)))$$

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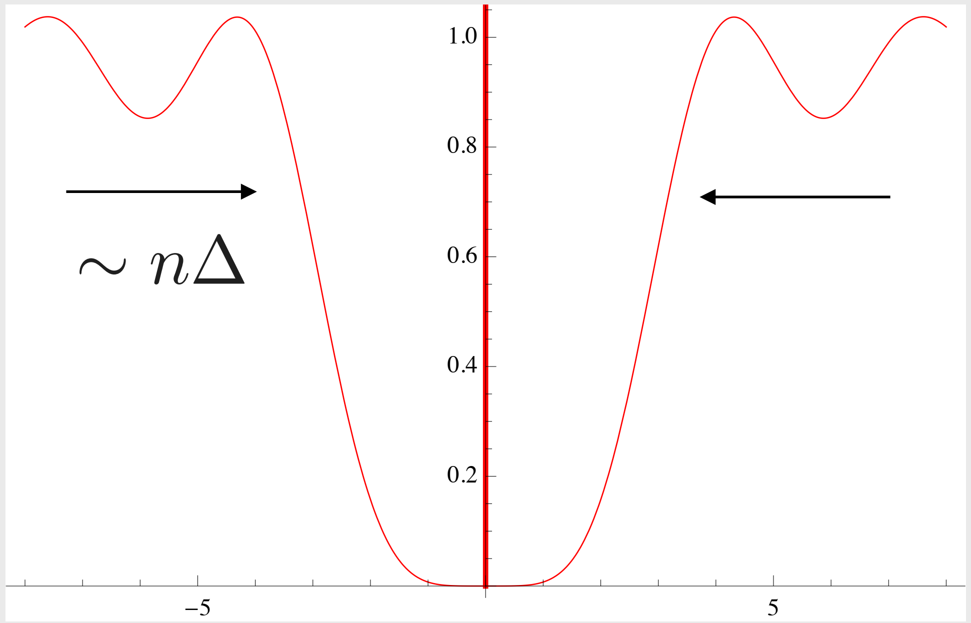
- boundary action (generalized for finite excitation energies)

$$S[T] = \frac{s}{4} \operatorname{str}(T + T^{-1}) + n \operatorname{str}\, \ln(T),$$

$$s = \frac{\pi\epsilon}{\Delta}$$



$$\rho(s) = \frac{\pi|s|}{\Delta} \left[J_n(s)^2 - J_{n-1}(s) J_{n+1}(s) \right]$$

physics at boundary

▷ deep localization regime

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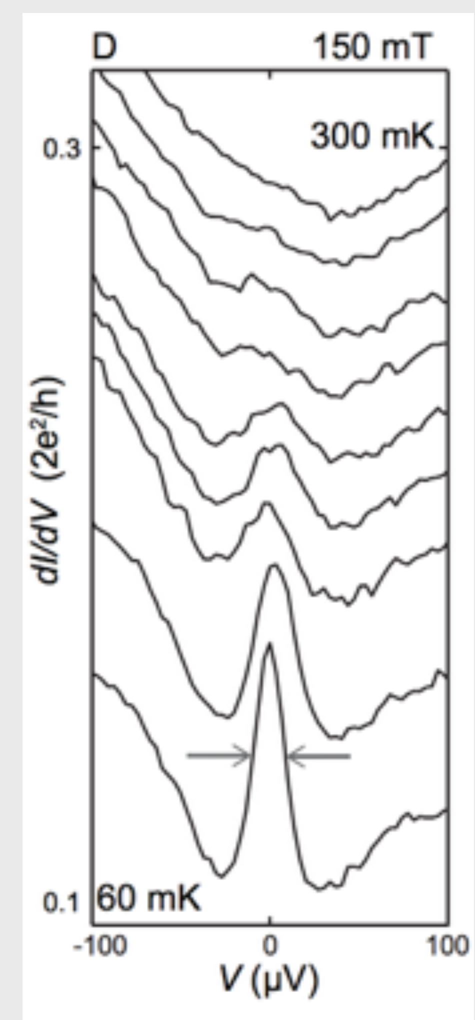


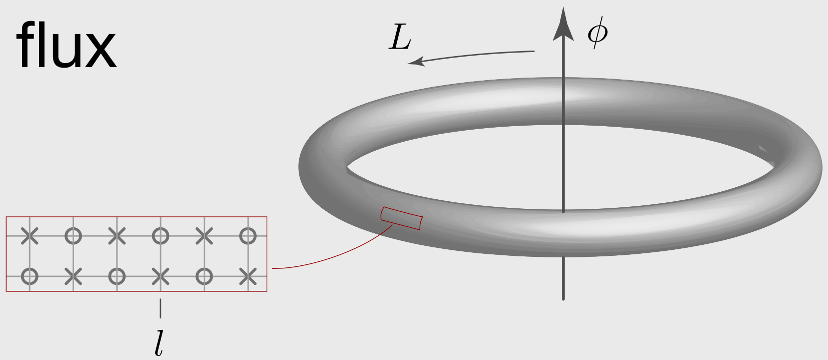
Mourik et al, 12

universality

1d universality

▷ classes AIII, BDI, CII described by unified approach:

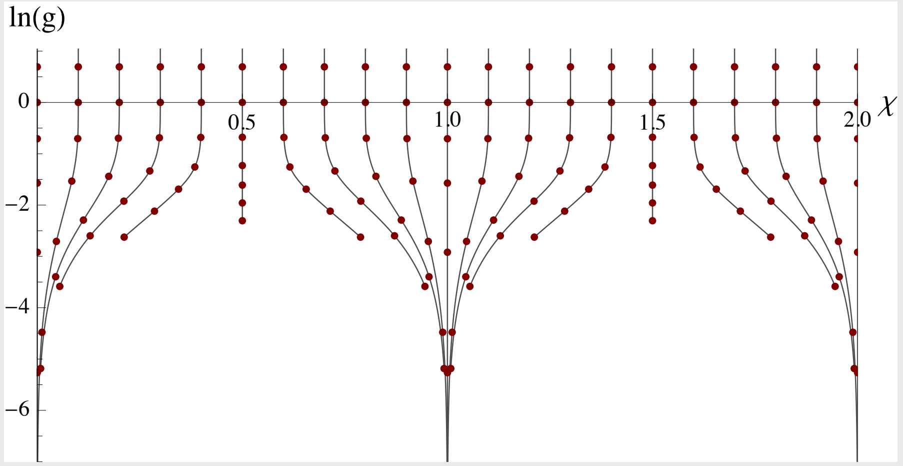
▷ physical response probed by insertion of gauge flux



▷ structurally identical low energy action

$$S[T] = \int_0^L dx \left[\frac{\tilde{\xi}}{4} \mathrm{str}(\partial_x T \partial_x T^{-1}) + \tilde{\chi}\, \mathrm{str}(T^{-1} \partial_x T) \right].$$

▷ and flow of system parameters.



ON LOCALIZATION IN THE THEORY OF THE QUANTIZED HALL EFFECT: A TWO-DIMENSIONAL REALIZATION OF THE θ-VACUUM

A M M PRUISKEN

Schlumberger-Doll Research, PO Box 307, Ridgefield, CT 06877, USA

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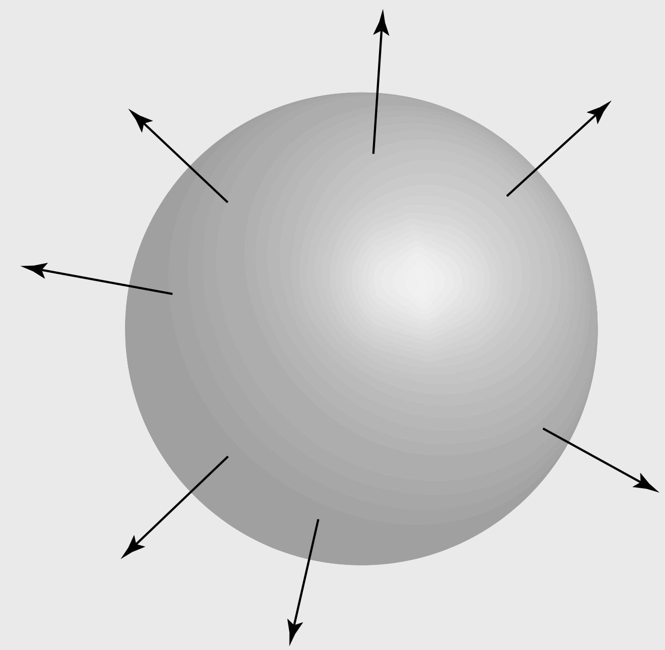
It is shown that the localization problem in the theory of the quantized Hall effect is governed by the zero-component grassmannian U($2m$) non-linear σ-model with a θ-term, a two-dimensional analogue of the θ-vacuum in Yang-Mills theory In this case, θ is to be interpreted as the "bare" value for the Hall conductivity, determined by an underlying non-critical theory A detailed derivation is presented starting from the replica method and a delta function distribution for the impurities

2d universality

▷classes A, C, D described by unified approach:

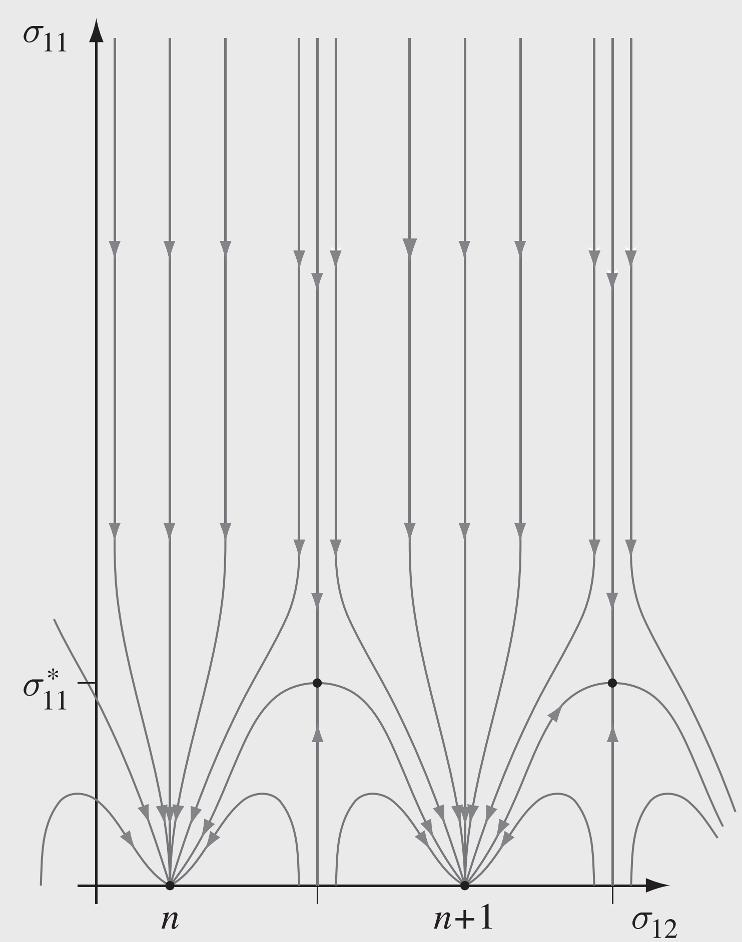


▷ probed by insertion of gauge flux (Pruisken's background field, cf. Laughlin gauge argument)

▷ structurally identical low energy action

$$S[Q] = \frac{\tilde{g}}{2} \int d^2x \, \mathrm{str}(\partial_\mu Q \partial_\mu Q) + \frac{\tilde{\chi}}{2} \int d^2x \, \epsilon_{\mu\nu} \, \mathrm{str}(Q \partial_\mu Q \partial_\nu Q)$$

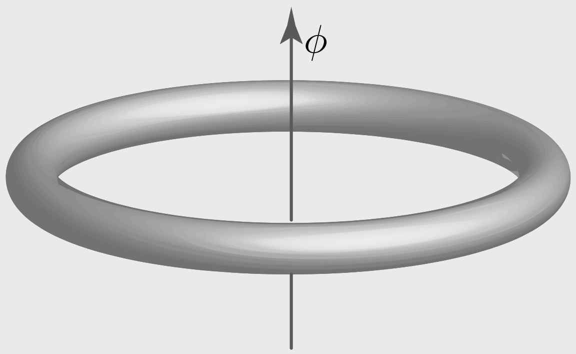
▷ and flow of system parameters.

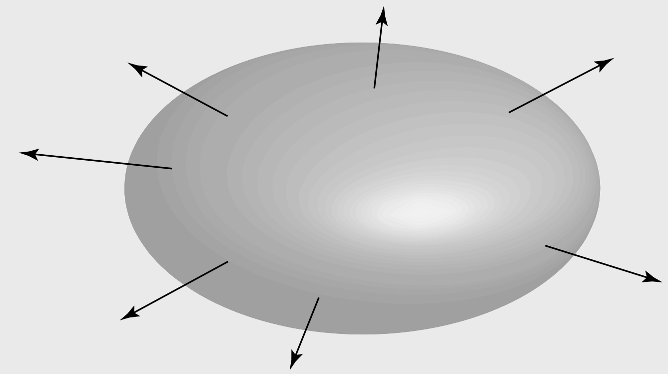


Z-universality

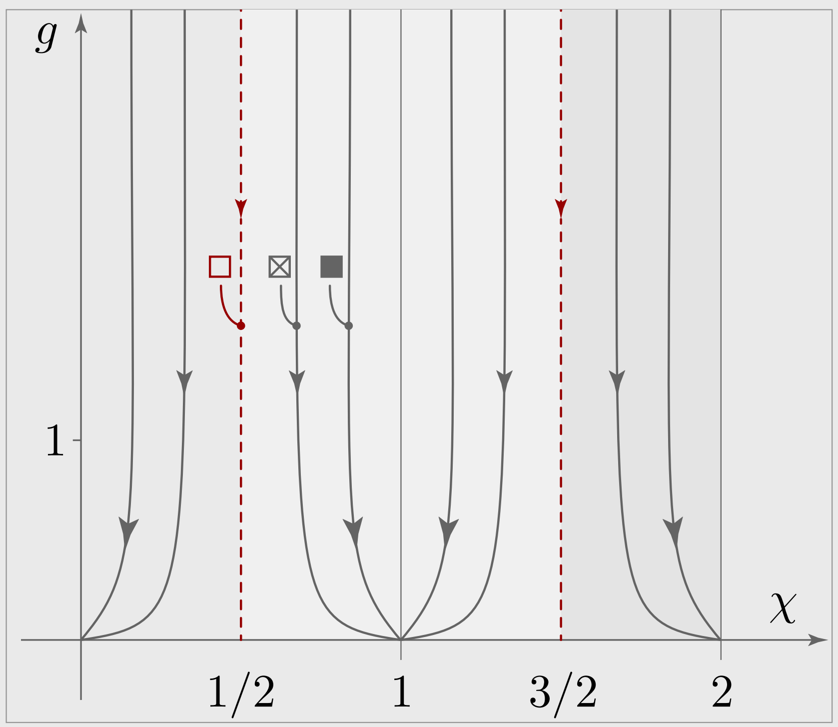
1d AIII, BDI, CII

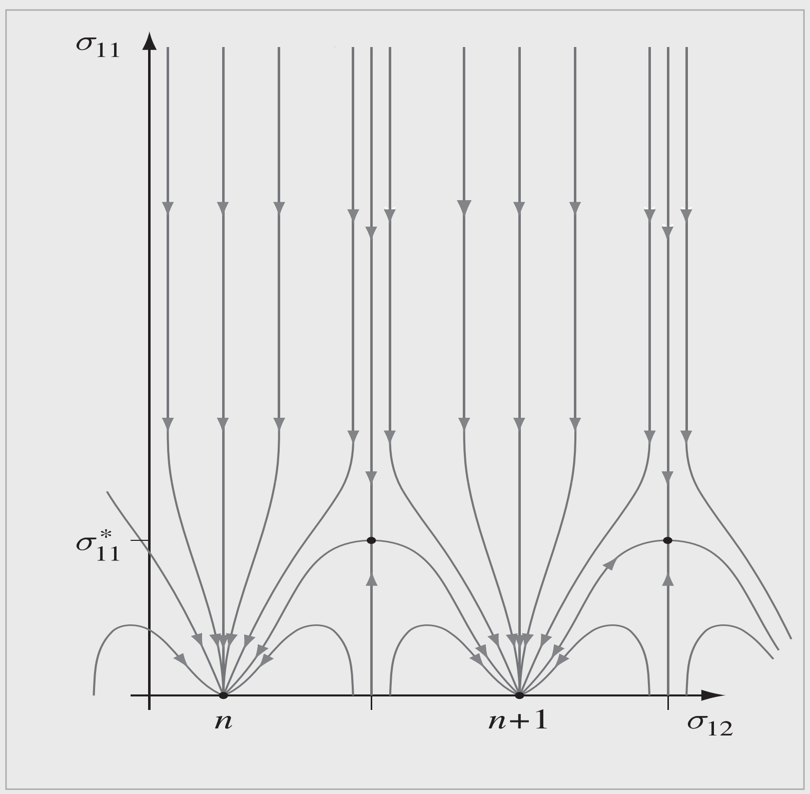
2d A, C, (D)





$$S[M] = \tilde{g}\, S_{\mathrm{diff}}[M] + \tilde{\chi}\, S_{\mathrm{top}}[M]$$



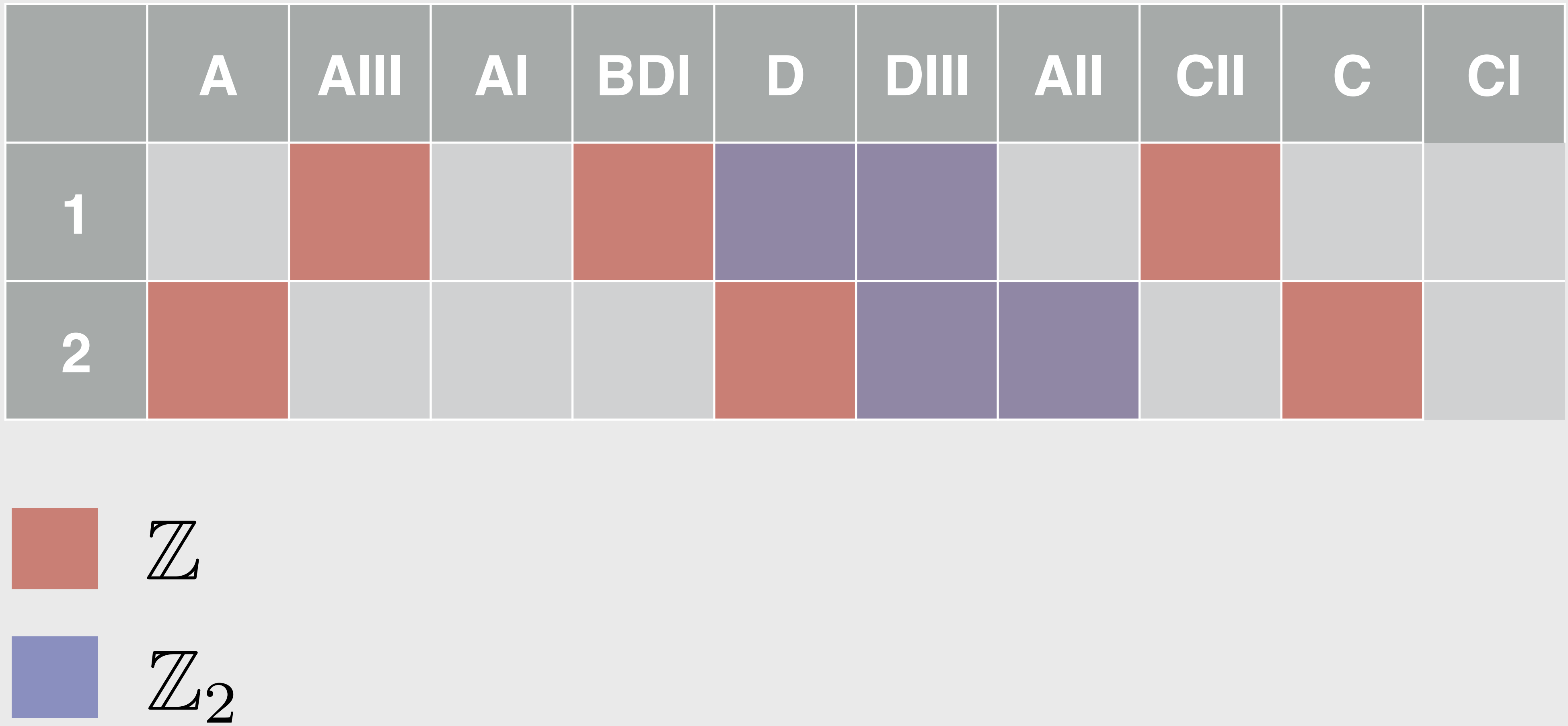


generic flow $(g, \chi) \xrightarrow{L\to\infty} (0, n)$

$$n S_{\mathrm{top}}[M] \longrightarrow n\, S_{\mathrm{boundary}}[T]$$

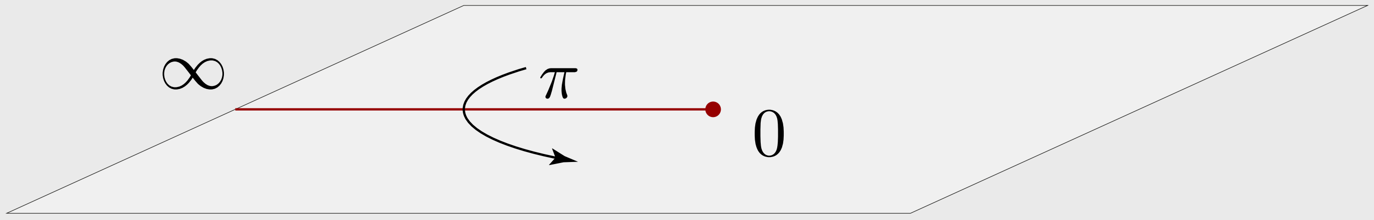
Z2

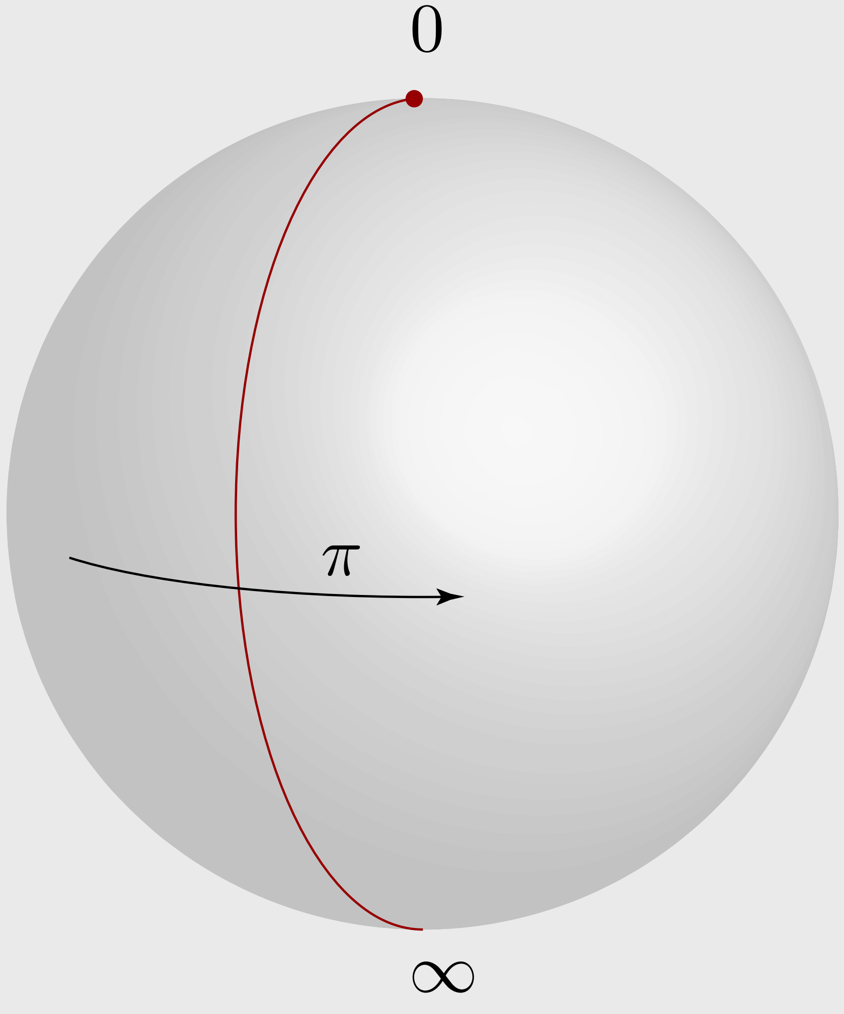
generalization to Z2



generalization to Z2

- case study: AII, d=2 (Kane & Fu, 2012)
  - system probed by topological point defects
  - field theory admits point-like excitations
  - theta-term —> fugactiy term
  - 2 parameter criticality





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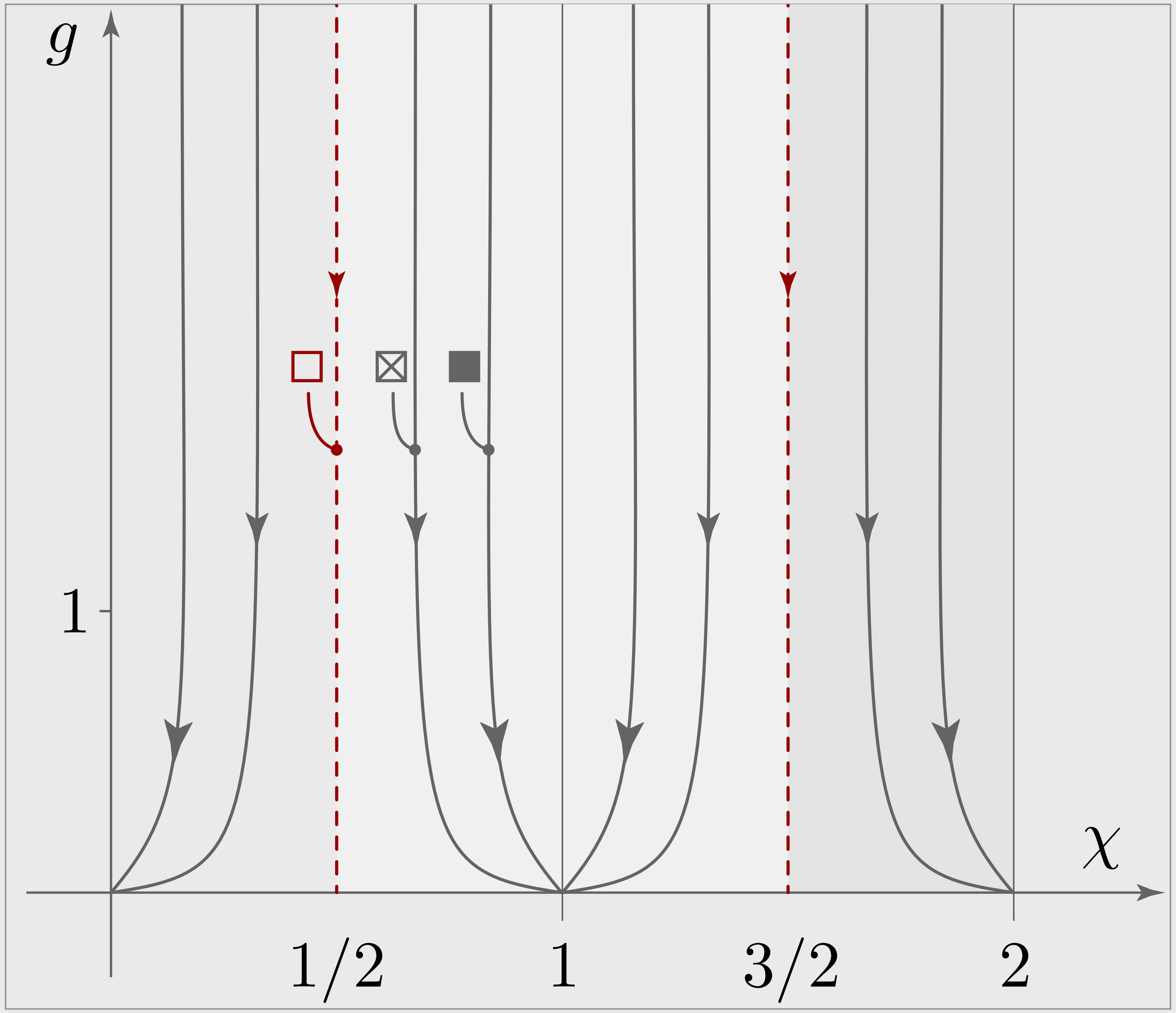

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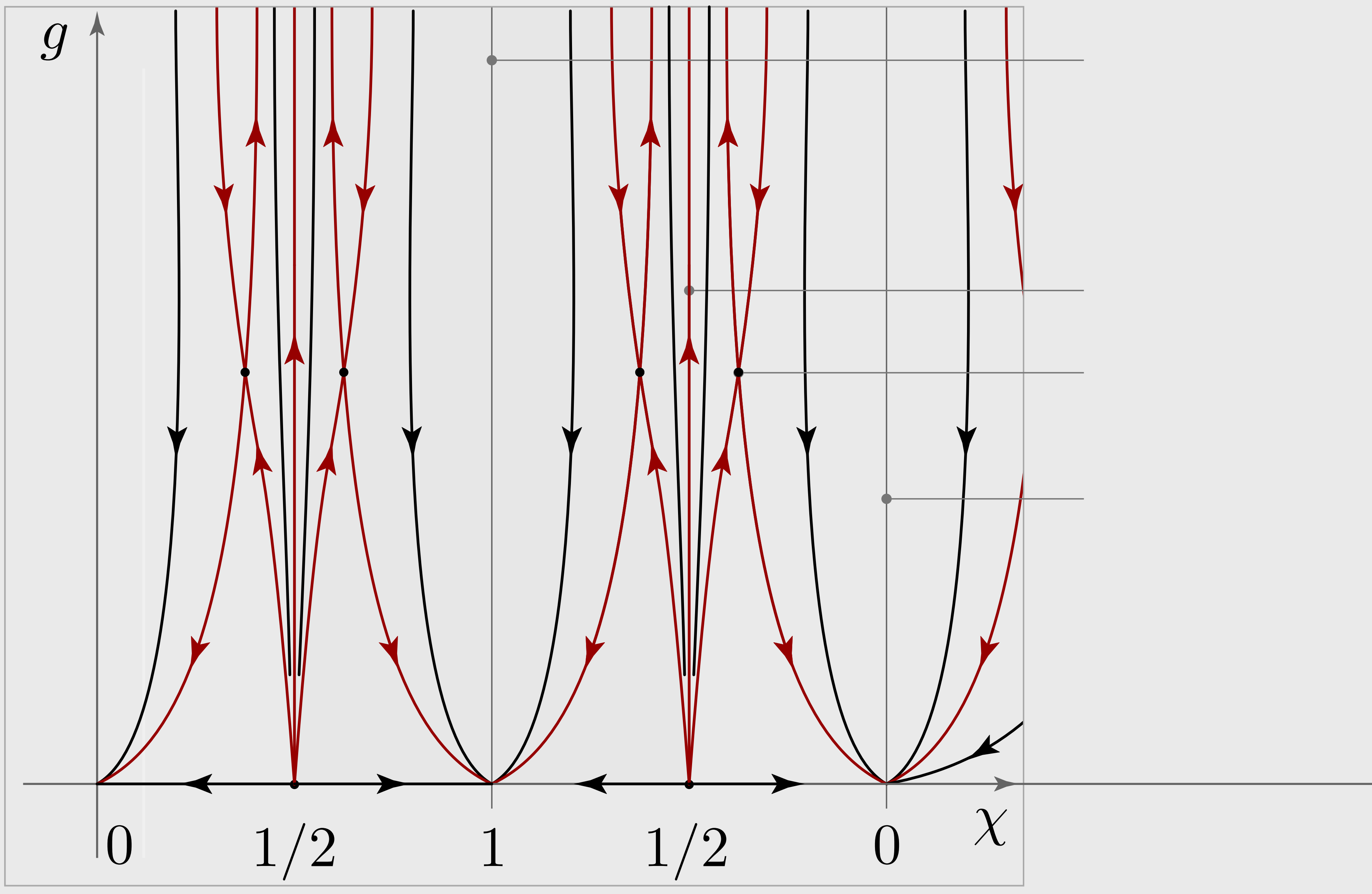
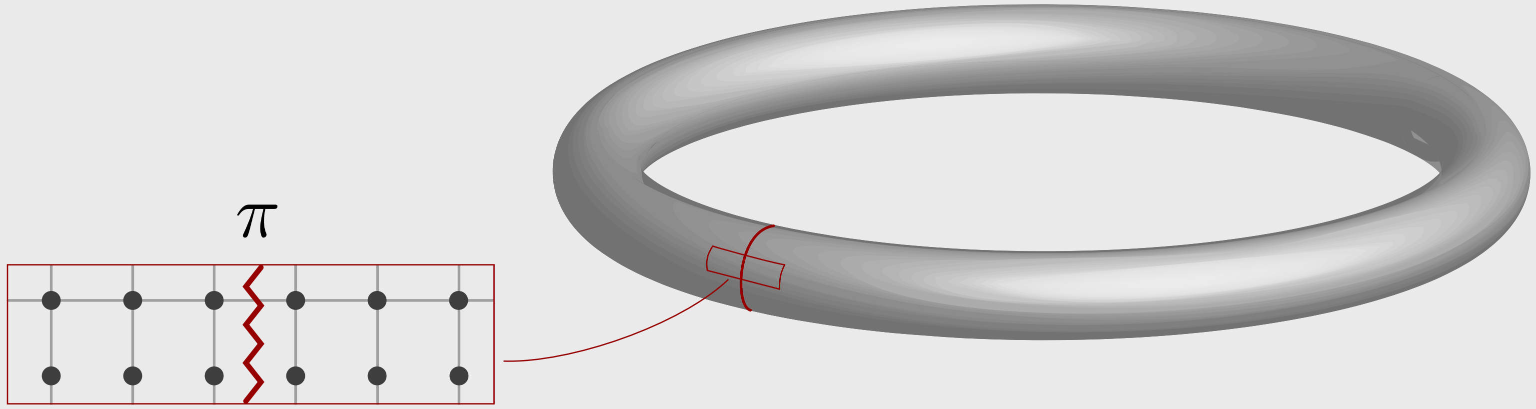


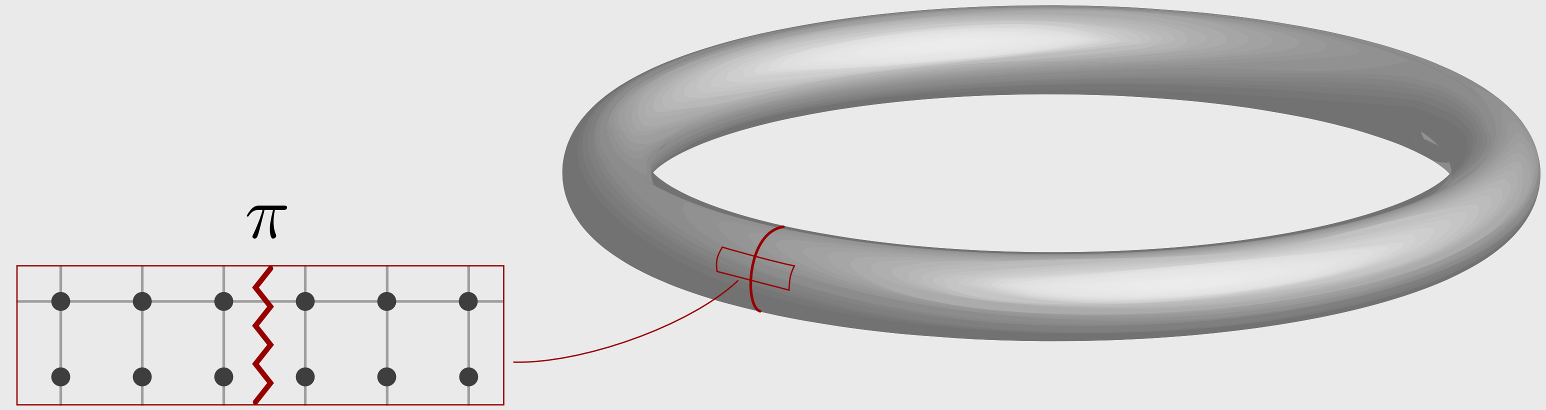
1d Z2 — D and DIII



▷ field theory

$$S[T] = \frac{\tilde{\xi}}{4}\int_0^L dx\, \mathrm{tr}\left(\partial_x T \partial_x T^{-1}\right) + \ln(\tilde{\chi}) \times (\#\text{ of kinks})$$

1d Z2 — D and DIII



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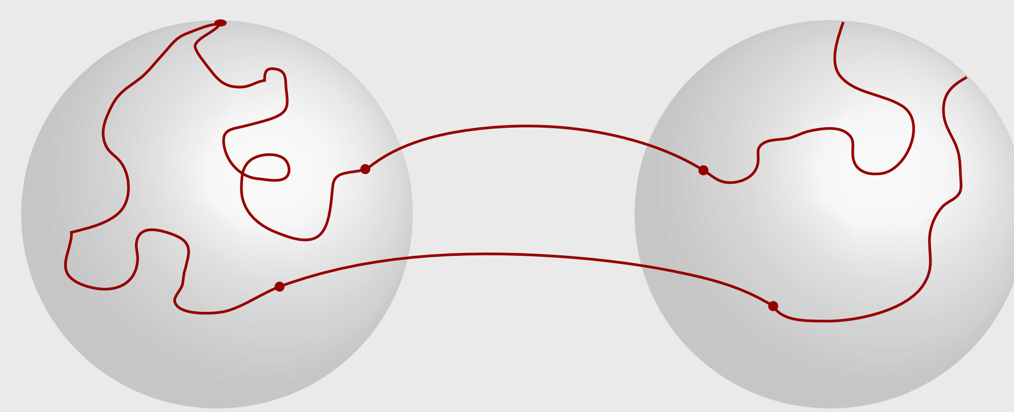

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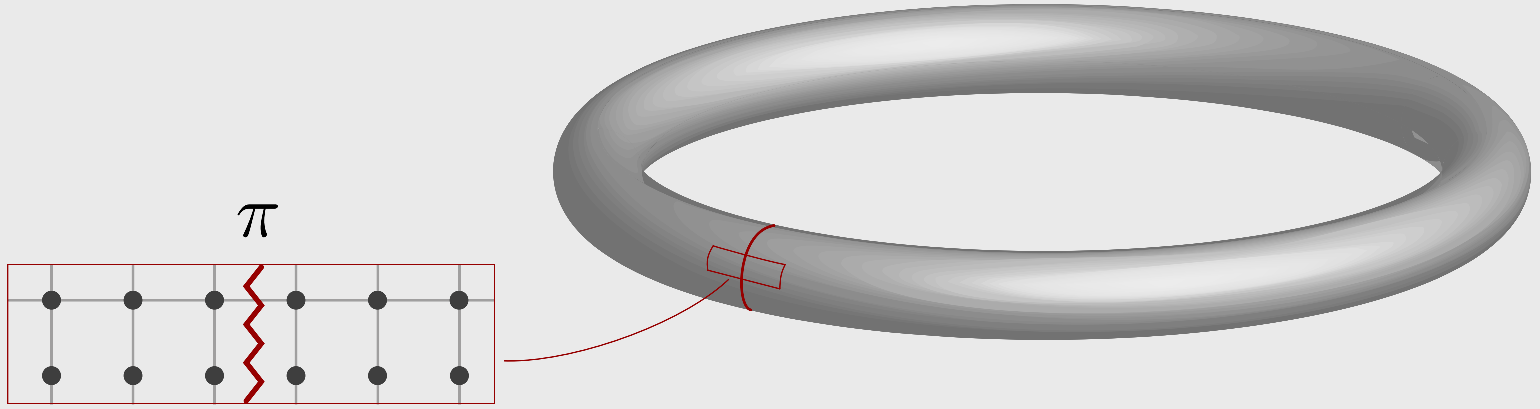


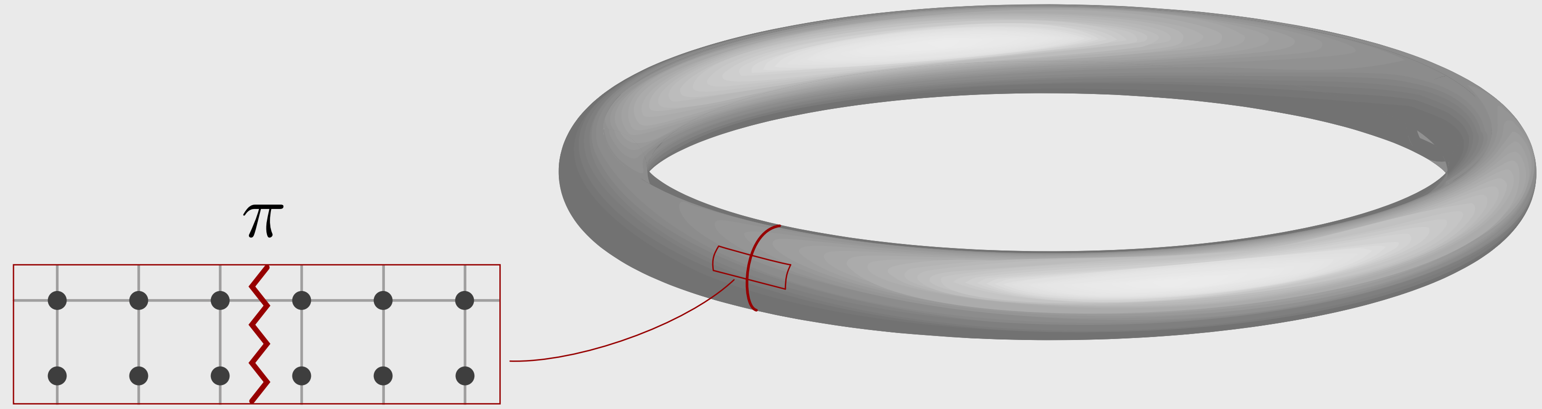
▷ field theory

$$S[T] = \frac{\tilde{\xi}}{4}\int_0^L dx\,\mathrm{tr}\,(\partial_x T \partial_x T^{-1}) + \ln(\tilde{\chi}) \times (\#\text{ of kinks})$$

$$\partial_x \begin{pmatrix} \Psi^{(+)} \\ \Psi^{(-)} \end{pmatrix} = \begin{pmatrix} g^{-1}J^{-1}\partial_i(J\partial_i) & -\frac{vg^{-1/2}}{\sqrt{J(\phi,\theta)}}\,\partial_\phi \int d\theta \\ -vg^{-1/2}\,\partial_\phi \int d\theta \sqrt{J(\phi,\theta)} & g^{-1}\partial_i\partial_i \end{pmatrix} \begin{pmatrix} \Psi^{(+)} \\ \Psi^{(-)} \end{pmatrix},$$

$$J = \frac{\sinh^2\phi}{(\cosh\phi - \cos\theta)^2}$$

1d Z2 — D and DIII



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$$S[T] = \frac{\tilde{\xi}}{4}\int_0^L dx\,\mathrm{tr}\,(\partial_x T \partial_x T^{-1}) + \ln(\tilde{\chi}) \times (\#\text{ of kinks})$$

▷ generic two parameter flow $(g, \chi) \to (0, \pm 1)$

Summary

- real space approach to translationally non-invariant topological insulators
  - 2-parameter field theory
  - probed by continuous (Z) or point-like (Z2) topological sources
  - universal scaling
  - stabilization of topology by localization

Thouless topology 2d class A

Khmelnitskii/Pruisken criticality 2d class A

Chalker et al. 2d class C

Ludwig et al. exact solution 2d class C

Fisher et al., Read et al., Zirnbauer/Serban 2d class D

Zirnbauer 1d super-Fourier analysis

Mirlin, Mudry, Gruzberg et al. disordered topological matter

Beenakker et al., Brouwer et al. scattering theory of topological matter

Read, Gruzberg, Vishveshwara 1d topological quantum criticality

Ludwig et al. disorder vs. bulk-boundary correspondence

Kane/Fu, 2d class AII

previous work

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