Quantum Repeaters for Long Distance Quantum Communication

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Benasque Workshop

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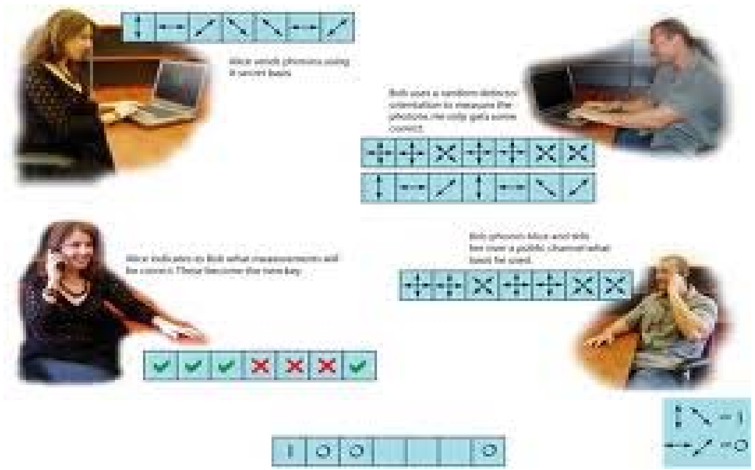
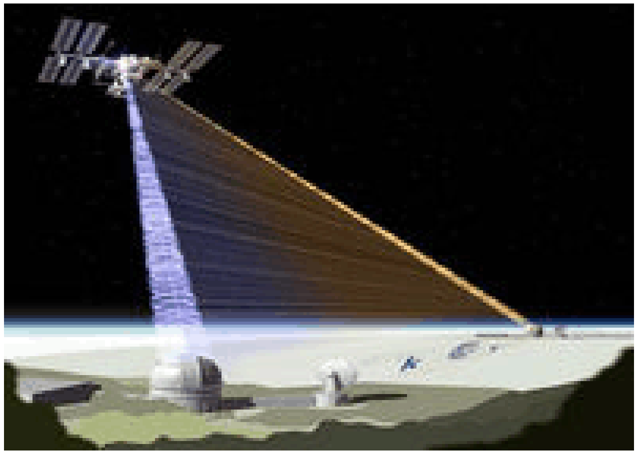
Quantum Communication

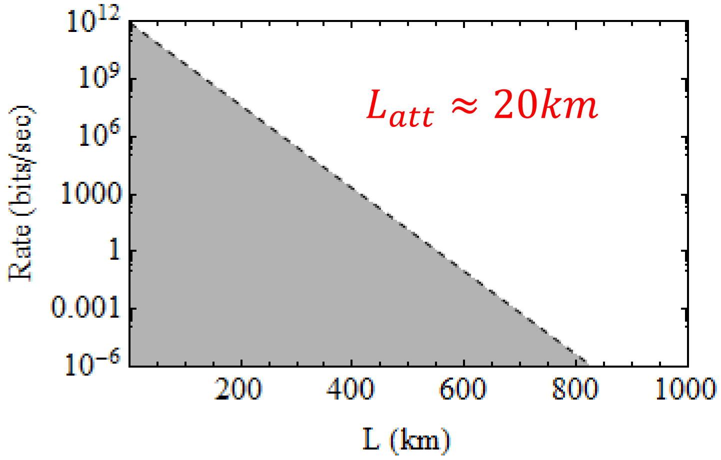
Quantum Key Distribution (e.g., BB84, Ekert91)



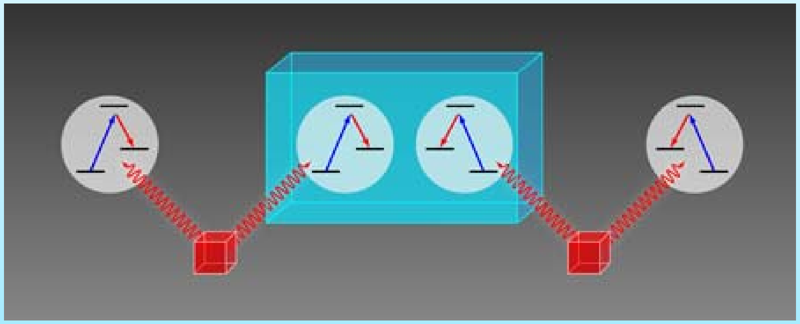
Solution 1: Satellite based QKD



Major Challenge – Fiber Attenuation



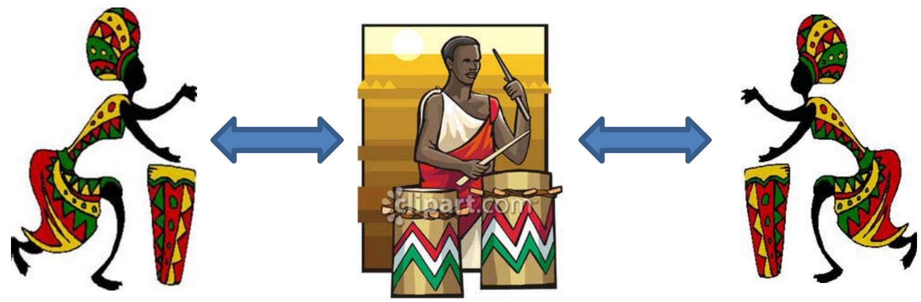
Solution 2: Quantum Repeaters



Classical Repeaters

Repeaters based on Smoke Signal
(Great Wall, 900BC)

Repeaters based on Sound
(Africa Drum Communication)



Repeaters based on Optical Signal
(Undersea Cables)

Our Quantum World

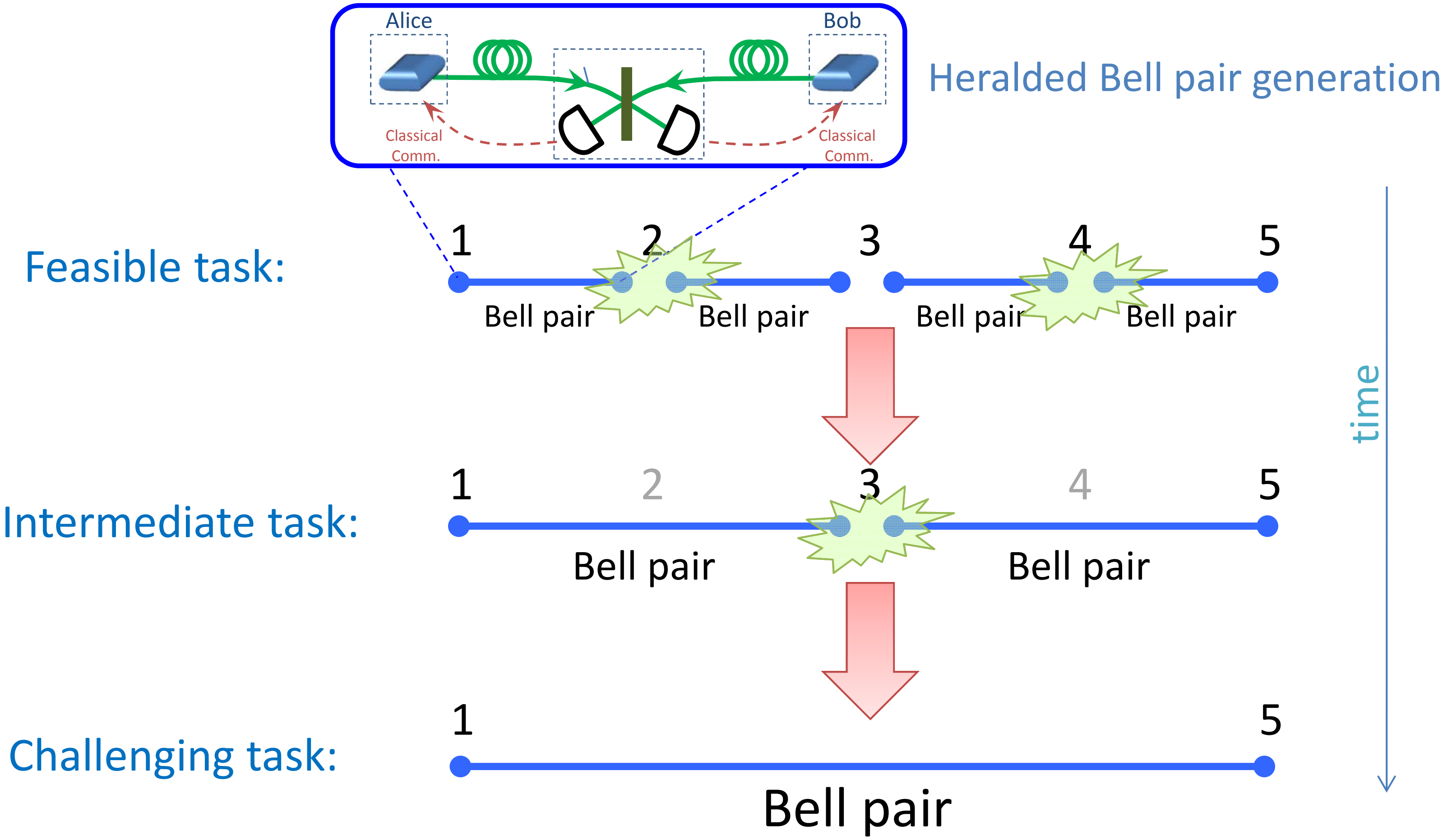
- **Challenges**: Quantum No-Cloning Theorem
  - Unknown quantum states cannot be perfectly cloned
- **Opportunities**: Quantum Entanglement
  - Quantum state teleportation
  - Entanglement swapping
  - Non-local coupling gate
- **Imperfections**:
  - *Loss Errors* (Fiber loss L_{att}≈20km, coupling & detector inefficiency)
  - *Operation Errors* (Channel decoherence, memory errors, local gate/measurement errors)

Quantum Repeaters

- Key Ideas for Quantum Repeaters
- Three Generations of Quantum Repeaters
- Compare Various Repeater Protocols
- Further Improvement

Key Ideas for Quantum Repeaters

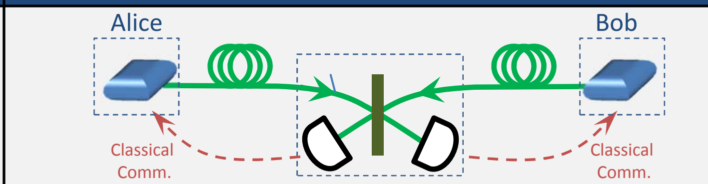
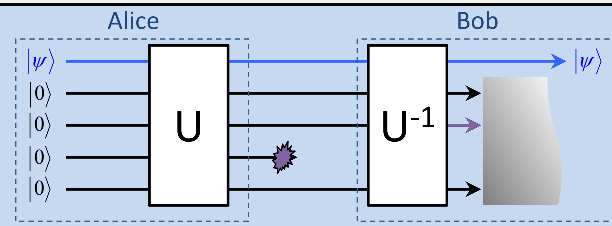
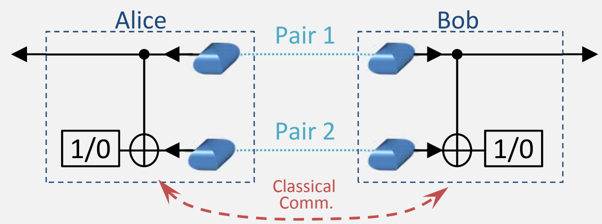
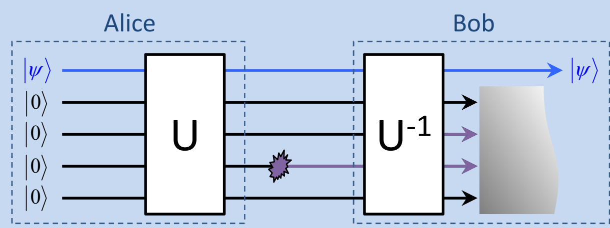
- To overcome ***loss errors***



- For ***operation error*** ε, final infidelity $\sim\varepsilon N \sim \varepsilon \frac{L_{tot}}{L_{att}} \approx 0.1.$

$$L_{tot} \sim 0.1 L_{att}/\varepsilon$$

How to Overcome Both Loss & Operation Errors?

	Approaches	Example	Requirement
Loss Errors	Heralded Generation (*)		Prob. & Heralded, ***Two-Way*** Comm.
	Quantum Error Correction		Deterministic, ***One-Way*** Comm. Suppress $\varepsilon \to \varepsilon^{2t+1}$.
Operation Errors	Heralded Purification (**)		Prob. & Heralded, ***Two-Way*** Comm.
	Quantum Error Correction		Deterministic, ***One-Way*** Comm. Suppress $\varepsilon \to \varepsilon^{t+1}$.

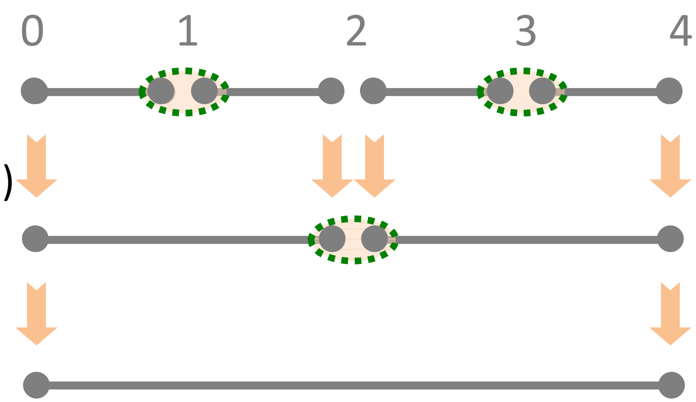
(*) Experiments with ions, atoms, NVs, QDs, … (**) Experiments with photons, ions, …

Three Generations of QRs

	Approaches	1st Generation	2nd Generation	3rd Generation
Loss Errors	Heralded Generation [*Two-Way* Comm.]	✓	✓	
Loss Errors	Quantum Error Correction [*One-Way* Comm.]			✓
Operation Errors	Heralded Purification [*Two-Way* Comm.]	✓		
Operation Errors	Quantum Error Correction [*One-Way* Comm]		✓	✓

1^{St} Generation QRs

Key Idea: Nested self-similar architecture of heralded ent. generation & purification.



- Procedure
  - ***Heralded*** Entanglement Generation (loss errors)
  - Connection
  - ***Heralded*** Entanglement Purification (operation errors)
- Time Scaling
  - ***Poly(L)***
- Implementations (memory)
  - Single Emitters (e.g., ions, atoms, NVs, QDs),
  - Ensemble Approach (e.g., atoms, rare-earth-ion doped crystal)
- Challenges
  - ***Low key rates*** for long distances
  - ***Significant*** memory errors *[Hartmann, Kraus, Dür, Briegel. PRA 75, 032310 (2007)]*
- Limitation
  - ***Heralded*** Entanglement Purification

    -> ***Two-way*** communication,

    -> slows for long distances!

Briegel, Dür, Cirac, Zoller. PRL 81, 5932 (1998);
Duan, Lukin, Cirac, Zoller, Nature, 414, 413 (2001); Sangouard, Simon, de Riedmatten, Gisin, RMP 83, 33 (2011)

Three Generations of QRs

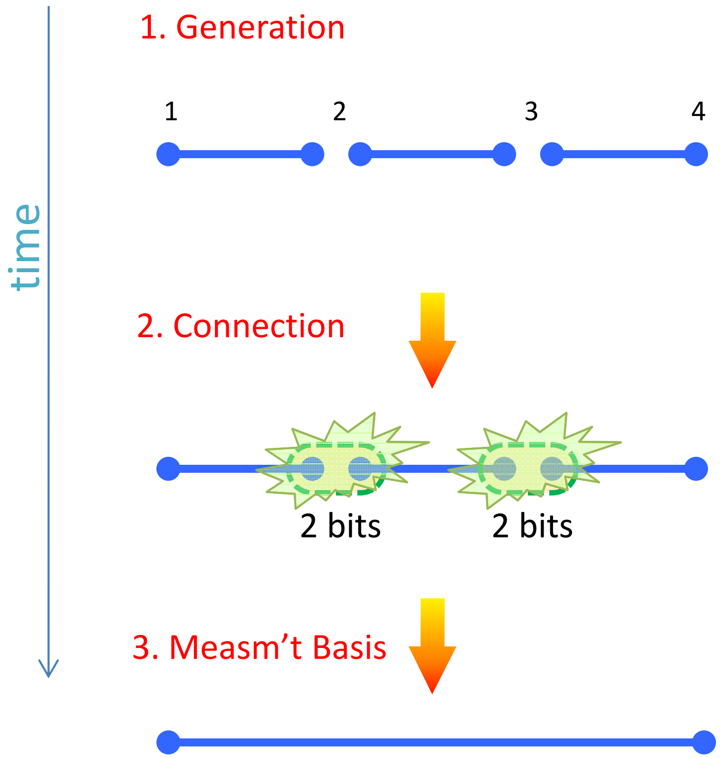
	Approaches	1st Generation	2nd Generation	3rd Generation
Loss Errors	Heralded Generation [*Two-Way* Comm.]	✓	✓	
	Quantum Error Correction [*One-Way* Comm.]			✓
Operation Errors	Heralded Purification [*Two-Way* Comm.]	✓		
	Quantum Error Correction [*One-Way* Comm]		✓	✓

2^{nd} Generation QRs

Key Idea: Create almost perfect Bell pairs at the encoded level

Without Encoding

(Operation Error: ε)

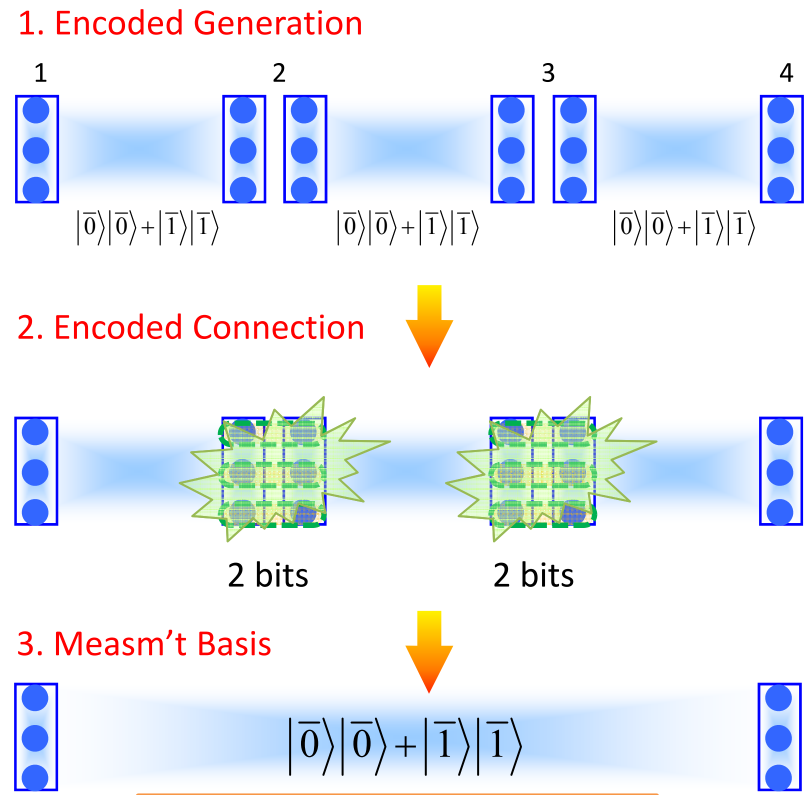


Final Infidelity:
$1-(1-\varepsilon)^N \approx N\varepsilon$

$L_{tot} \sim L_{att}/\varepsilon$

With Quantum Encoding [[n,1,2t+1]]

(Suppressed Opr. Errors: ε-> ε^{t+1})

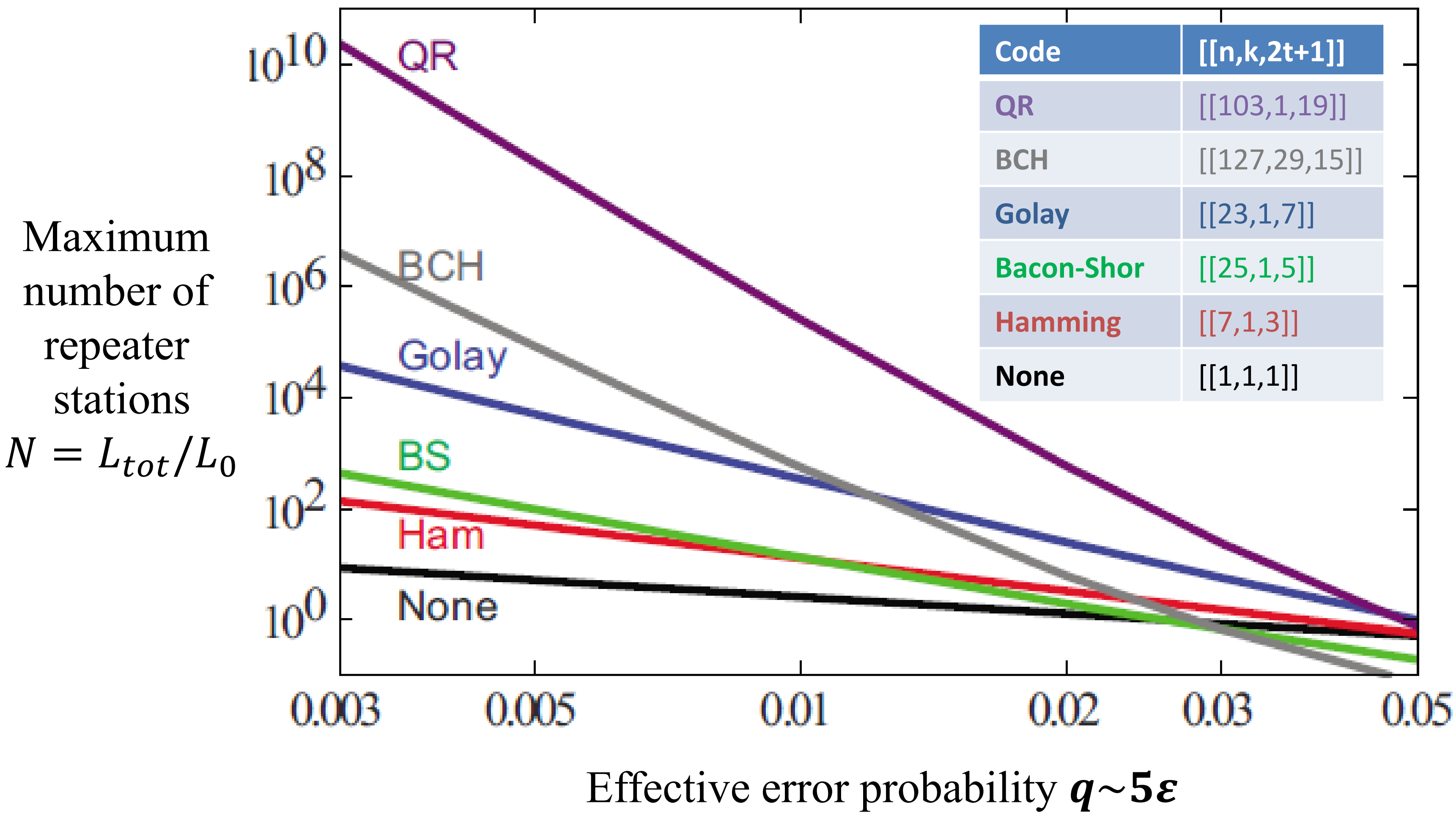


Final Infidelity:
$1-(1-\varepsilon^{t+1})^N \approx N\varepsilon^{t+1}$

$L_{tot} \sim L_{att}/\varepsilon^{t+1}$

[L.J., Taylor, Nemoto, Munro, Van Meter, Lukin, PRA 79, 032325 (2009)]

2nd Generation QRs

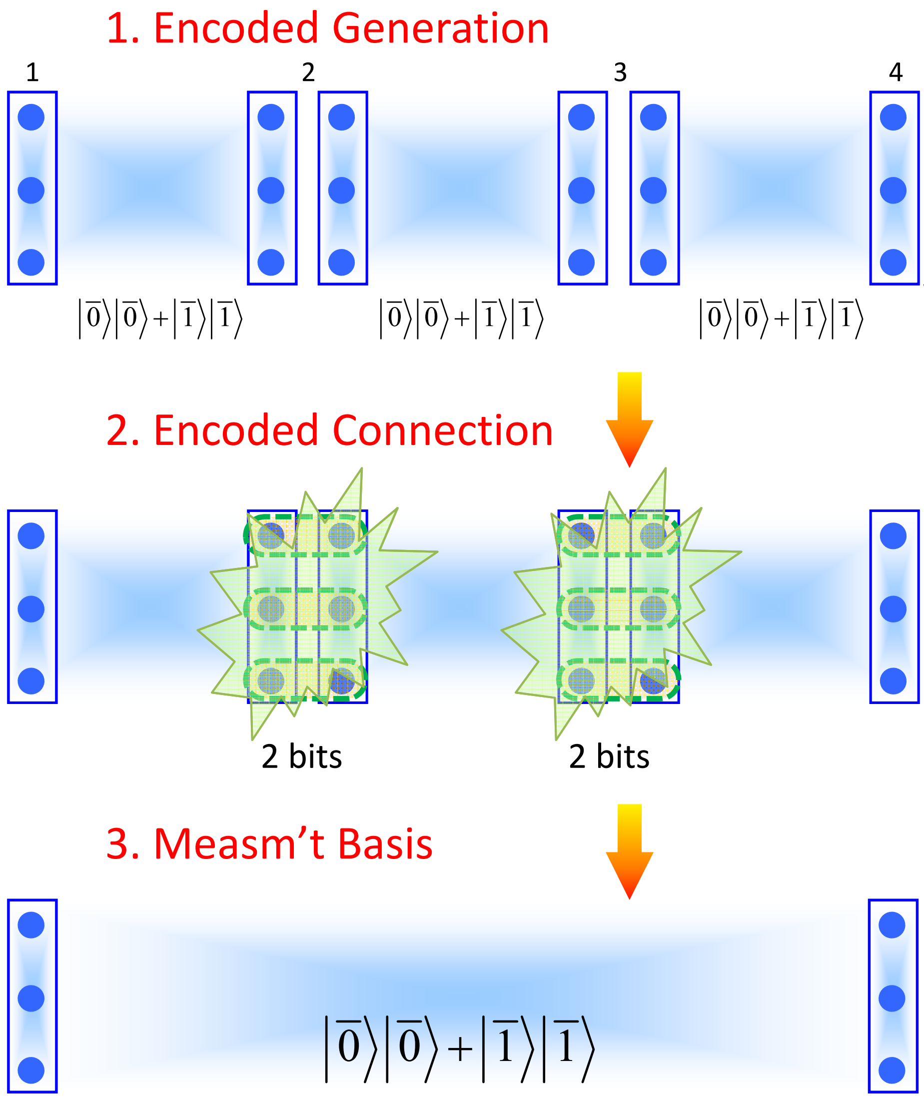


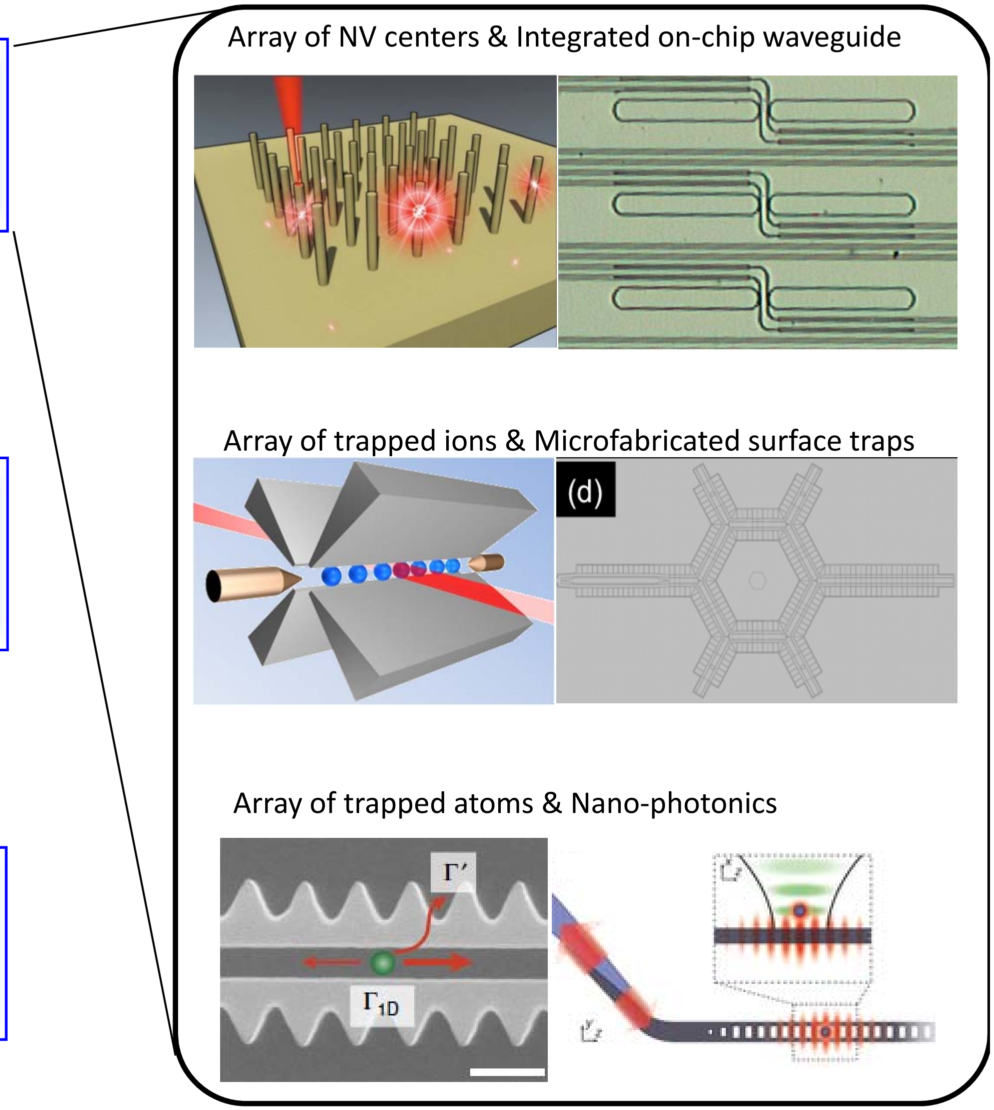
Code	[[n,k,2t+1]]
QR	[[103,1,19]]
BCH	[[127,29,15]]
Golay	[[23,1,7]]
Bacon-Shor	[[25,1,5]]
Hamming	[[7,1,3]]
None	[[1,1,1]]

[L.J., Taylor, Nemoto, Munro, Van Meter, Lukin, PRA 79, 032325 (2009)]

2nd Generation QRs

-- Potential Implementations with Ion/Atom/NV/QD





2^{nd} Generation QRs

- Key Idea:
  - Create almost perfect Bell pairs at the encoded level (CSS code)
    [L.J., Taylor, Nemoto, Munro, Van Meter, Lukin, PRA 79, 032325 (2009)]
- Procedure
  - ***Heralded*** Entanglement Generation/Purification [loss errors]
  - ***Deterministic*** Entanglement Connection & Error Correction [operation errors]
- Time Scaling
  - $Poly\big(Log(L)\big) * \tau_0$, with pair generation time

$$\tau_0 \propto \frac{l_0}{c}\frac{\exp\left(l_0/l_{att}\right)}{\eta^2} \approx \frac{0.1ms}{\eta^2} \quad \text{for repeater spacing } l_0 = 10km.$$

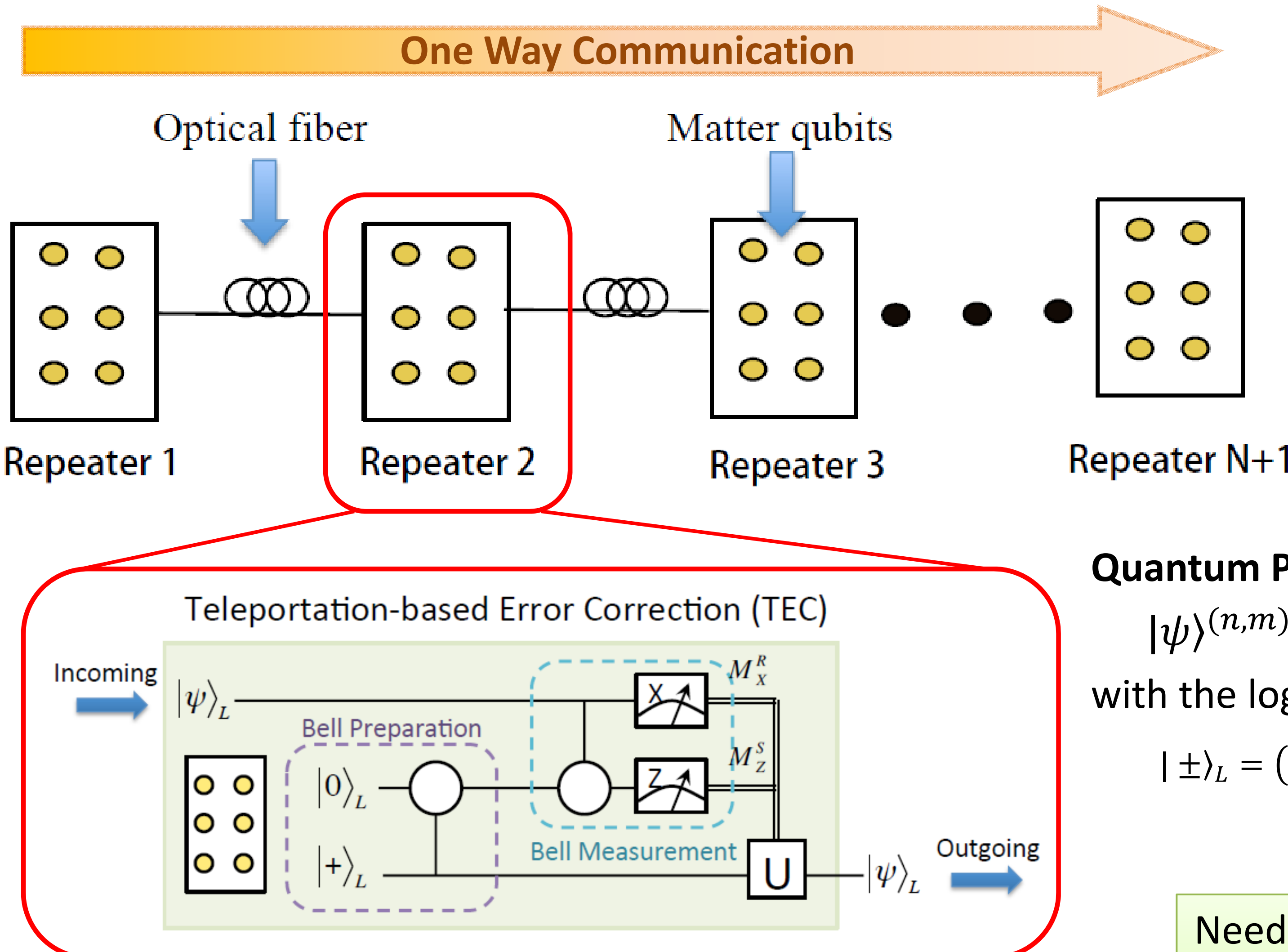
- Advantages
  - **Fast** for **long** distances (>Kbits/sec)
  - **Suppressed** memory errors *[Munro, et al., Nature Photonics 4, 792 (2010)]*
- Limitation
  - ***Heralded*** Entanglement Generation still requires ***two-way*** communication between neighboring stations to suppress loss errors, which limits τ_0.
  - Can we design efficient QR protocols to overcome loss errors?

Three Generations of QRs

	Approaches	1st Generation	2nd Generation	3rd Generation
Loss Errors	**Heralded Generation [*Two-Way* Comm.]**	✓	✓	
	Quantum Error Correction [*One-Way* Comm.]			✓
Operation Errors	**Heralded Purification [*Two-Way* Comm.]**	✓		
	Quantum Error Correction [*One-Way* Comm]		✓	✓

3rd Generation QRs

-- Architecture based on Quantum Error Correction



Quantum Parity Code:

$$|\psi\rangle^{(n,m)} = \alpha|+\rangle_L + \beta|-\rangle_L$$

with the logical basis

$$|\pm\rangle_L = \left(|0\ldots0\rangle_{1\ldots m} \pm |1\ldots1\rangle_{1\ldots m}\right)^{\otimes n}$$

Need correct outcomes:
$\widetilde{M}_X$ & $\widetilde{M}_Z$
to complete state transfer.

Munro, Stephens, Devitt, Harrison, Nemoto, Nature Photonics 6, 777 (2012)

Muralidharan, Kim, Lütkenhaus, Lukin, L.J., PRL 112, 250501 (2014)

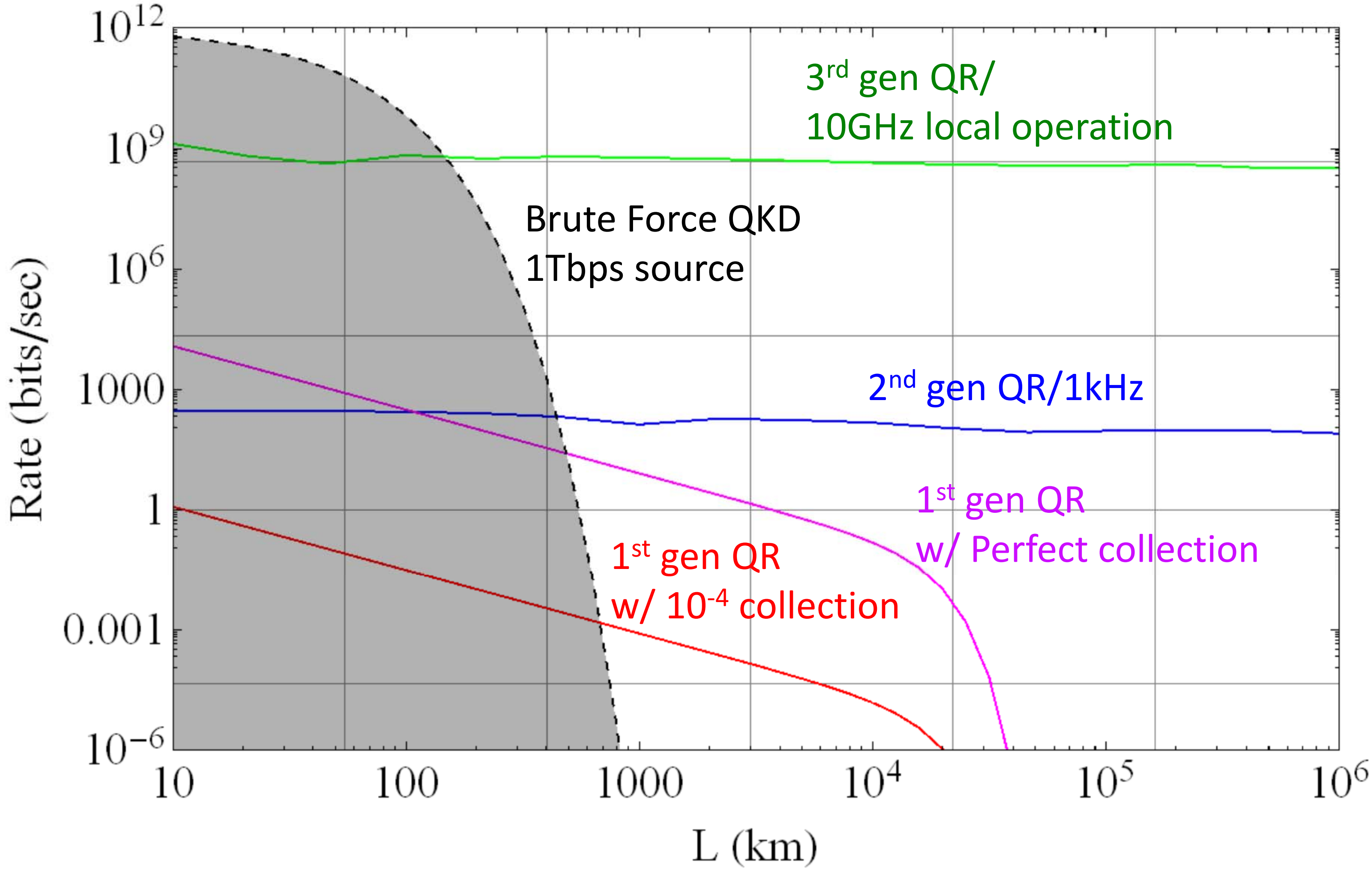
3rd Generation QRs

- Key Idea:
  - Use Quantum Error Correction for **BOTH** loss and operation errors
    [Fowler, Wang, Hill Ladd, Van Meter, Hollenberg, PRL 104, 180503 (2010)]
    [Munro, Stephens, Devitt, Harrison, Nemoto, Nature Photonics 6, 777 (2012)]
    [Muralidharan, Kim, Lütkenhaus, Lukin, L.J., PRL 112, 250501 (2014)]
- Procedure
  - Quantum Encoding & Send/Propagate Quantum States over Optical Channel
  - Apply Quantum Error Correction (loss errors & operation errors)
- Time Scaling
  - $Poly\big(Log(L)\big) * \boldsymbol{\tau_{opr}}$, with local gate operational time τ_{opr}.
- Advantages
  - **Ultra Fast** for long distances (>Mbits/sec)
  - **Suppressed** memory errors *[Munro, et al., Nature Photonics 6, 777 (2012)]*
- Challenges
  - Local operation takes time, which limits τ_{opr}.
  - Large encoding block (hundreds or more qubits)
  - No more than 50% loss (no-cloning)

Three Generations of QRs

	Approaches	1st Generation	2nd Generation	3rd Generation
Loss Errors	Heralded Generation [*Two-Way* Comm.]	✓	✓	
	Quantum Error Correction [*One-Way* Comm.]			✓
Operation Errors	Heralded Purification [*Two-Way* Comm.]	✓		
	Quantum Error Correction [*One-Way* Comm]		✓	✓
Key Rate		$\ll c/L_{tot}$	$\sim \frac{c}{\eta^2 L_0}$	$\sim \frac{1}{\tau_{opr}}$

Rate Estimates



Cost Function & Cost Coefficient

Resources to generate a secret key:

- **Time**: $1/R$
- **Qubits**: $n \times L_{tot}/L_0$

Cost = Time × Qubits: $C(L_{tot}) = \min_{n,L_0} \frac{n}{R}\frac{L_{tot}}{L_0}$ [qubit*time/sbit]

Cost coefficient: $C'(L_{tot}) = C(L_{tot})/L_{tot}$ [qubit*time/(sbit*km)]

Muralidharan, Kim, Lütkenhaus, Lukin, L.J., PRL 112, 250501 (2014)

Comparison Among Various QR Protocols

- Preliminary results of comparison among four QR protocols:

Protocol Name	Properties & Refs
1st Gen	BDCZ scheme [*PRL 81, 5932 (1998)*]
2nd Gen	QR with [[n,k,d]] code *[PRA 79, 032325 (2009]*
3rd Gen	Optimized quantum parity code *[arXiv: 1310.5291]*

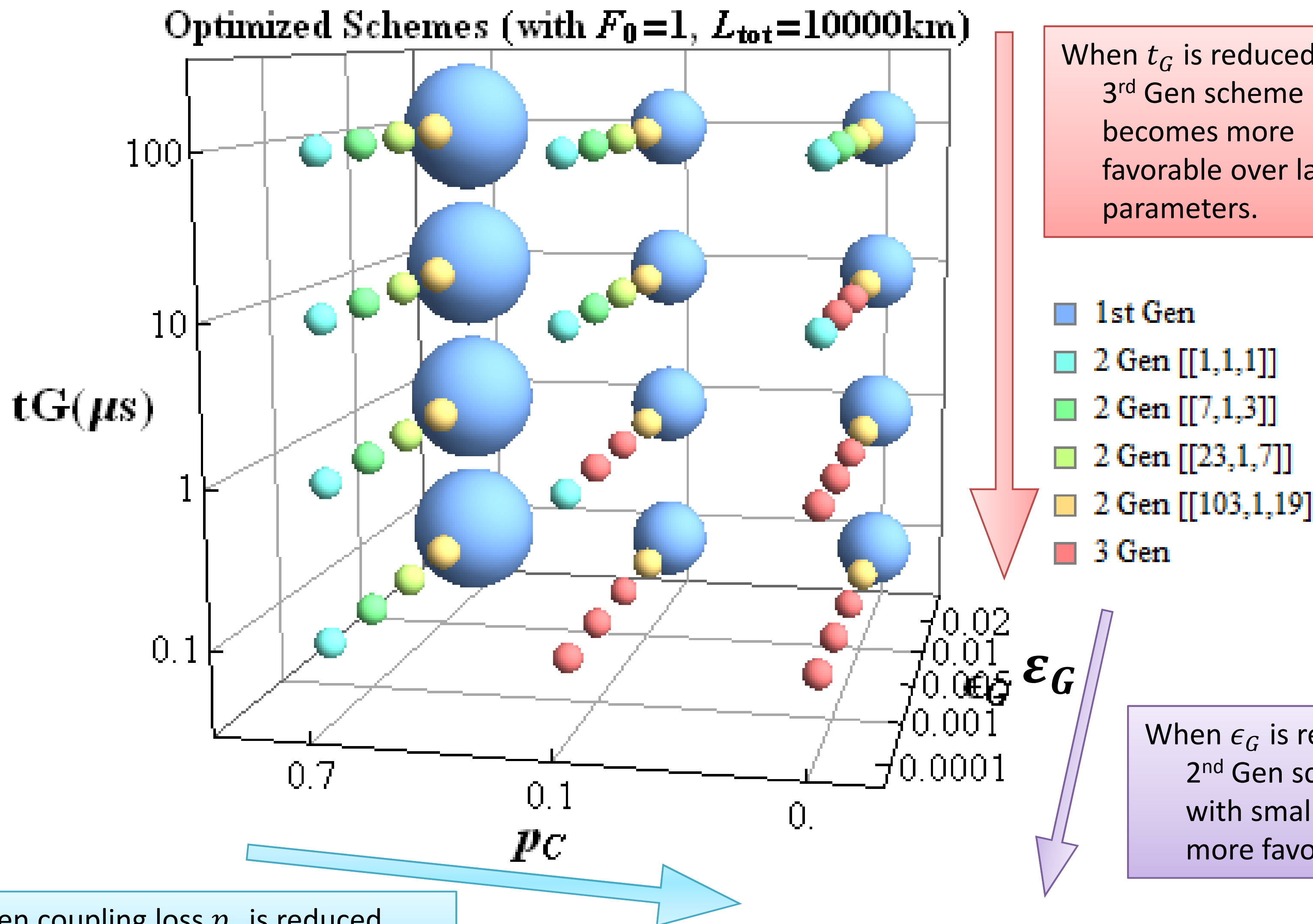
- The control parameters:

Parameter	Description
L_{tot}	Total distance
F_0	Fidelity of unpurified Bell pair
ε_G	Probability of gate error
p_c	Coupling loss (between qubit and fiber)
t_G	Time of local operations

- Optimization criterion: Cost coefficient

$$C' \equiv \frac{qubit \times time}{sbit \times distance}$$

Comparison Among Various QR Protocols



When t_G is reduced, 3rd Gen scheme becomes more favorable over larger parameters.

When ϵ_G is reduced, 2nd Gen scheme with small code is more favorable.

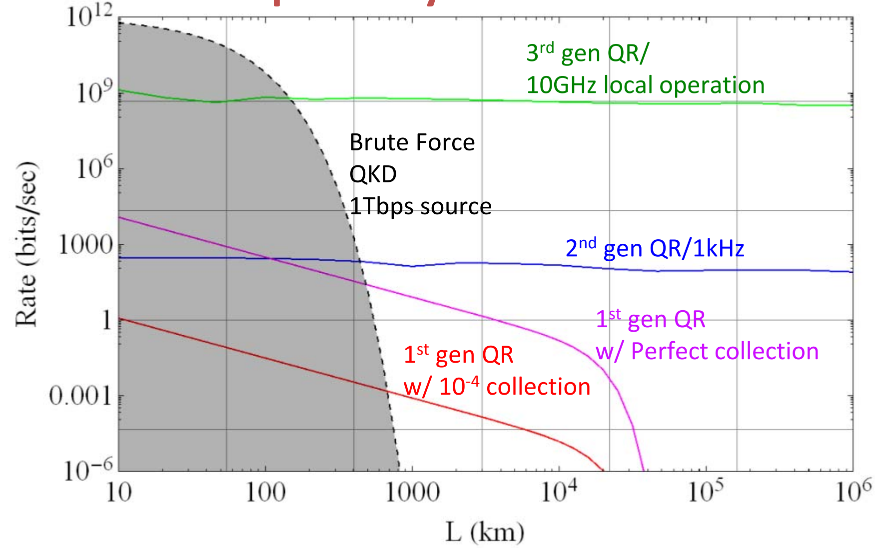
When coupling loss p_c is reduced, cost coefficient is more favorable.

Summary

Three Generations of QRs

	Approaches	1st Generation	2nd Generation	3rd Generation
Loss Errors	Heralded Generation [*Two-Way* Comm.]	✓	✓	
	Quantum Error Correction [*One-Way* Comm.]			✓
Operation Errors	Heralded Purification [*Two-Way* Comm.]	✓		
	Quantum Error Correction [*One-Way* Comm]		✓	✓

Compare Key Generation Rate



Compare Cost Coefficients