How big is the coupling ?

All the SM couplings (including $\overline{MS}$ mass/Yukawa) depend on the energy scale (obey Renormalization Group Equation RGE),
and the QCD coupling run fast

$$\frac{\partial a_S}{\partial \log \mu^2} = \beta(a_S) = -a_S^2 (b_0 + a_S\, b_1 + a_S^2\, b_2 + \ldots) \ , \qquad a_S = \frac{\alpha_S}{\pi}$$

$$\frac{\partial \log m_q}{\partial \log \mu^2} = \gamma_m(a_S) = -a_S (g_0 + a_S\, g_1 + a_S^2\, g_2 + \cdots) \ ,$$

$$b_0 = \frac{1}{12}(11 C_A - 2 N_F) \ , \qquad b_1 = \frac{1}{24}\left(17 C_A^2 - (5 C_A + 3 C_F) N_F\right)$$

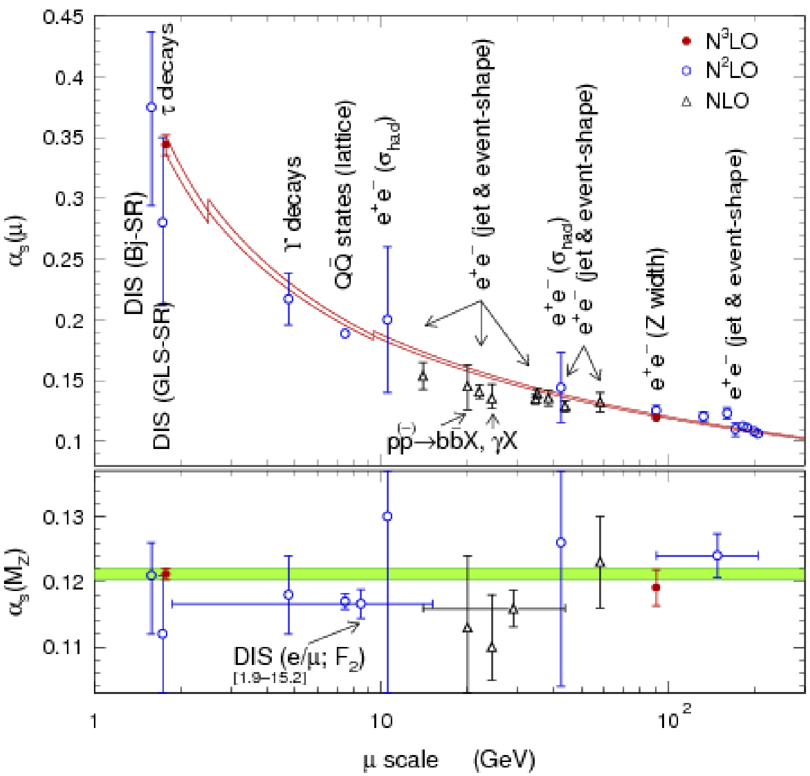
$$g_0 = 1 \qquad g_1 = \frac{1}{16}\left(\frac{202}{3} - \frac{20}{9} N_F\right)$$

- Sign $\beta(\alpha_S) < 0$: Asymptotic Freedom due to gluon self-interactions
  [Nobel Prize 2004, Gross, Politzer, Wilczek]
- **At high scales:** coupling becomes small, quarks and gluons are almost free, strong interactions are weak
- **At low scales:** coupling becomes large, quarks and gluons interact strongly, confined into hadrons, perturbation theory fails

Flavour thresholds

$$a_S^{(N_F)}(\mu_{\rm th}) = a_S^{(N_F-1)}(\mu_{\rm th}) \left[1 + \sum C_k(x)\,(a_S^{(N_F-1)}(\mu_{\rm th}))^k\right]$$

$$m_q^{(N_F)}(\mu_{\rm th}) = m_q^{(N_F-1)}(\mu_{\rm th}) \left[1 + \sum H_k(x)\,(a_S^{(N_F-1)}(\mu_{\rm th}))^k\right] \,, \qquad x = \log(\mu_{\rm th}^2/m_q^2)$$



$$C_1 = \frac{x}{6} \,, \qquad C_2 = -\frac{11}{72} + \frac{19}{24}x + \frac{x^2}{36}$$

$$H_1 = 0 \,, \qquad H_2 = -\frac{89}{432} + \frac{5}{36}x - \frac{x^2}{12}$$

- The $\beta(\alpha_S)$ and $\gamma_m(\alpha_S)$ functions depend on N_F
- Interpret it in the context of Effective Theories with different number of active flavours, and match the couplings at threshold
- Matching is independent of $\mu_{\rm th}$ (up to higher orders)

- α_S might become discontinuous, is that a problem ?
- Similar discussion for PDFs

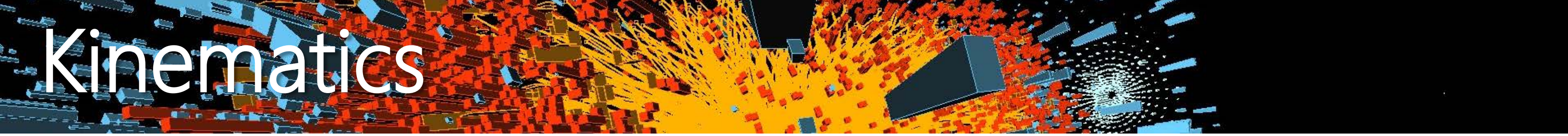
Exercises:

1. Integrate analytically the one-loop and two-loop RGE for the strong coupling, and one-loop for a quark mass
2. Calculate $\alpha_S(10\ \mathrm{GeV})$ and $\alpha_S(1\ \mathrm{TeV})$ from $\alpha_S(m_Z) = 0.1184\ \pm 0.0007$
3. If $m_b(m_b) = 4.2 \pm 0.1\ \mathrm{GeV}$, what is $m_b(m_Z)$
4. Hint

$$a_S(\mu) = \frac{a_S(\mu_0)}{1 + b_0\, a_S(\mu_0) \log \frac{\mu^2}{\mu_0^2}} \qquad \alpha_S(\mu) = \frac{\pi}{b_0 \log \frac{\mu^2}{\Lambda^2_{\mathrm{QCD}}}}$$

Then calculate Λ_{QCD}, the “fundamental” scale of QCD, at which coupling blows up (NB: it is not unambiguously defined at higher order)

Kinematics



Transverse plane

- Azimuthal angle ϕ
- Transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$
- Transverse mass $m_T = \sqrt{p_T^2 + m^2}$

Longitudinal variables

- Rapidity: $$y = \frac{1}{2} \log\left(\frac{E + p_z}{E - p_z}\right)$$
- Pseudo-rapidity: $$\eta = -\log\left(\tan(\theta/2)\right)$$

$$p^\mu = (m_T \cosh(y), p_T \cos(\phi), p_T \sin(\phi), m_T \sinh(y))$$

Exercises:

1. Show that $\eta = y$ for massless particles
2. Show that $\Delta y = y_i - y_j$ is invariant under boost