

Massive spin-1 particles

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- equations of motion:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} - \frac{\partial \mathcal{L}}{\partial A_\nu} = 0$$

show: $[g^{\mu\nu}(\Box + m^2) - \partial^\mu \partial^\nu] A_\nu = 0$
and $\partial_\mu A^\mu = 0$

- plane-wave solutions:

$$A^\nu = \varepsilon^\nu e^{\pm i p x}$$

a) what is the condition on polarization vector ε^ν ?

b) given: $(p^\mu) = (E, 0, 0, p)$, $p = |\vec{p}|$
construct 3 orthogonal ε^ν with $\varepsilon^2 = -1$

c) show that $\varepsilon_3^\nu \equiv \varepsilon_L^\nu \simeq \frac{p^\nu}{m}$ for $p \gg m$

- propagator:

verify that $D_{\rho\nu}(k) = \left(-g_{\rho\nu} + \frac{k_\rho k_\nu}{m^2} \right) \frac{1}{k^2 - m^2 + i\varepsilon}$

is the solution of

$$[(-k^2 + m^2) g^{\mu\rho} + k^\mu k^\rho] D_{\rho\nu}(k) = g^\mu_\nu$$



Scalar QED

- a) Construct Lagrangian $\mathcal{L}$ for scalar QED, with a scalar field $\phi(x)$, $\phi(x) \neq \phi^\dagger(x)$ and a photon field $A_\mu(x)$.
- b) Identify interaction terms and derive the Feynman rules

