1.2. Lattice QCD: Discretization of gluon fields

Exercise: For a simple $U(1)$ gauge transformation, $e^{i\alpha(x)}$, show that

$$\begin{aligned} U_\mu^{(g)}(x) &= G(x)U_\mu(x)G^\dagger(x+a_\mu) \\ \psi^{(g)}(x) &= G(x)\psi(x) \\ \bar{\psi}^{(g)}(x) &= \bar{\psi}(x)G^\dagger(x) \end{aligned}$$

is equivalent to the QED-like gauge transformation in the continuum $A_\mu^{(g)} = A_\mu - \partial_\mu \alpha$.

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Exercise: Probe that the Wilson action

$$S_{g,QCD} = \beta \sum_p (1 - \frac{1}{N_c} \mathrm{ReTr} U_p); \quad \beta = \frac{2N_c}{g^2}$$

reduces to the continuum gluon action up to $\mathcal{O}(a^2)$ corrections.