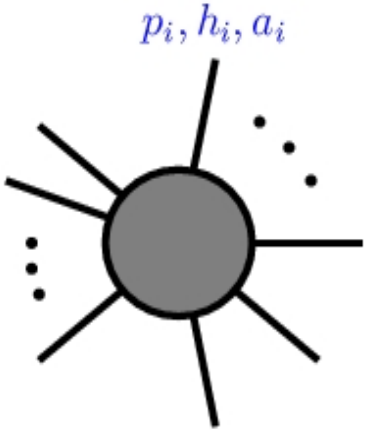
Spinors

[Xu,Zhang,Chang, Berends, Kleiss, De Causmaeker, Gastmans, Wu,Gunion, Kunzst]



- spinor identities

$$\langle 1|\gamma^\mu|2][3|\gamma_\mu|4\rangle = 2\langle 14\rangle[32] \qquad \text{Fierz}$$

$$\langle 12\rangle\langle 34\rangle = \langle 14\rangle\langle 32\rangle + \langle 13\rangle\langle 24\rangle \qquad \text{Shouten}$$

Exercise: proof the Fierz and Shouten identities

Hint: divide and multiply by $\langle 23\rangle$ and apply the Dirac identity

$$\gamma^\mu\gamma^\nu\gamma^\sigma\gamma_\mu = 4g^{\nu\sigma}$$

Exercises:

Calculate the scattering amplitudes and square amplitude for $e^+e^- \to q\bar{q}$ by using the helicity method, and compare with the traditional calculation

How many independent helicity amplitudes there are ?

$$M_{e^+e^-\to q\bar{q}} \sim \left[\bar{u}(p_1)\gamma^\mu v(p_2)\right]\left[\bar{v}(p_3)\gamma^\nu u(p_4)\right] d_{\mu\nu}(p_{12},n)$$

$$|M|^2 \sim \mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\sigma)\mathrm{Tr}(\not{p}_3\gamma^\nu\not{p}_4\gamma^\rho) d_{\mu\sigma}(p_{12},n) d_{\nu\rho}(p_{12},n)$$

Exercises:

Calculate by using BCFW the six-gluon amplitude

$$A_6(1^+,2^+,3^+,4^-,5^-,6^-) =$$
$$\frac{i}{\langle 2|1+6|5]}\left(\frac{\langle 6|1+2|3]^3}{\langle 61\rangle\langle 12\rangle[34][45]s_{126}}+\frac{\langle 4|5+6|1]^3}{\langle 23\rangle\langle 34\rangle[56][61]s_{561}}\right)$$

Exercise:

Calculate the splitting functions for the collinear processes $q \to qg$, $g \to q\bar{q}$ and $g \to gg$ by using the helicity method

Hint:

$$\mathbf{Sp}^{(0)}_{q\to q_1 g_2} = \mathbf{T}^a \frac{1}{s_{12}} \bar{u}(p_1) \not{\epsilon}(p_2) v(\widetilde{P})$$

$$P^{(0)}_{q\to q_1 g_2} = C_F \frac{1+z^2}{1-z} \qquad z = z_1 = \frac{n\cdot p_1}{n\cdot \widetilde{P}} \qquad z_2 = 1 - z$$