# Neutrino Physics

Arcadi Santamaria

IFIC/Univ. València

TAE 2014, Benasque, September 18, 2014

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## Outline

- Neutrino Physics I: Introduction and theory basics
- Neutrino Physics II: Neutrino phenomenology
- Neutrino Physics (Extra): Neutrino mass models

# References

**Links**

- C. Giunti and M. Laverder, “Neutrino Unbound”, **http://www.nu.to.infn.it/**

**Recent reviews**


- pdg 2014. K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014). Review on Neutrino mass, mixing, and oscillations.
- A. Romanino, “Neutrino Physics,” CERN Yellow Report CERN-2012-001, 153-182 [arXiv:1201.6158].
- P. Hernandez, “Neutrino physics,” CERN Yellow Report CERN-2010-001, 229-278 [arXiv:1010.4131].
- E. Ma, “Neutrino Mass: Mechanisms and Models,” arXiv:0905.0221.
- H. Nunokawa, S. J. Parke and J. W. F. Valle, “CP Violation and Neutrino Oscillations,” Prog. Part. Nucl. Phys. **60** (2008) 338 [arXiv:0710.0554].
- M. C. Gonzalez-Garcia and M. Maltoni,“Phenomenology with Massive Neutrinos,” Phys. Rept. **460** (2008) 1 [arXiv:0704.1800].


**Recent Fits**


- D.V. Forero, M. Tortola, J.W.F. Valle,“Neutrino oscillations refitted”. arXiv:1405.7540.
- F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, “Status of three-neutrino oscillation parameters, circa 2013” Phys. Rev. D **89** (2014) 093018 [arXiv:1312.2879]
- M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, “Global fit to three neutrino mixing: critical look at present precision” JHEP **1212** (2012) 123 [arXiv:1209.3023]


**Books**


- R. N. Mohapatra and P. B. Pal, “Massive neutrinos in physics and astrophysics. Second edition,” World Sci. Lect. Notes Phys. **60** (1998) 1 [World Sci. Lect. Notes Phys. **72** (2004) 1].
- G. G. Raffelt, “Stars as laboratories for fundamental physics: The astrophysics of neutrinos, axions, and other weakly interacting particles,” Chicago, USA: Univ. Pr. (1996) 664 p.

1. Introduction
   - Motivation and properties of $\nu$'s
   - $\nu$ masses in QFT (Dirac, Weyl and Majorana neutrinos)
   - $\nu$ masses in the SM and a slightly beyond
   - Description of neutrinos at low energies

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2. Neutrino oscillations in vaccum and in matter
   - Neutrino oscillacions in vacuum
   - Three neutrino oscillations
   - Neutrino oscillacions in matter (MSW)
   - The adiabatic approximation in the Sun

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# Why neutrino physics in 2014?

Very rich physics: from their invention by Pauli in 1930's to the last results on $\theta_{13}$ in 2012 many exciting discoveries:

# Why neutrino physics in 2014?

- Fermi theory for beta decay
- Majorana theory
- $\mu$ decay
- $\nu_{e,\mu,\tau}$ discoveries
- Neutrino oscillation proposal
- Solar $\nu$ problem
- Oscillations in matter (MSW)
- Atmospheric $\nu$ problem
- Supernova SN1987A
- Invisible $Z$-boson decay width
- Oscillations in solar $\nu$'s confirmed
- Oscilations in atmospheric $\nu$'s confirmed

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- **We do not have THE model of neutrino masses**

## There are experiments!

Planned experiments can answer many of them in a near future

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**Implications in cosmology**

- Contribution to the mass of the universe ($\Omega_\nu$)
- Effects in the cosmic microwave backgroud radiation (CMB)
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### Technological implications

- Communications in dense matter (underwater)
- Neutrino-graphies: earth core (search of oil, minerals $\cdots$)
- $\ldots$

# Intrinsic properties of neutrinos

## Before oscillation experiments

- Three types of neutrinos $\nu_e, \nu_\mu, \nu_\tau$
- Lepton numbers $L_e$, $L_\mu$, $L_\tau$ conserved separately
  - $\nu_e$ produces $e$'s and no $\mu$'s
  - No $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to e\gamma$, $\mu \to 3e$
- Total lepton number $L = L_e + L_\mu + L_\tau$ conserved (no $0\nu\beta\beta$)
- $\nu$ masses much smaller than charged lepton masses

$$m_{\nu_e} < 2\,\mathrm{eV}\,, \quad m_{\nu_\mu} < 170\,\mathrm{KeV}\,, \quad m_{\nu_\tau} < 18\,\mathrm{MeV} \quad \sum_a m_a \lesssim 14\,\mathrm{eV}$$

- $\nu$'s helicity $-1/2$ and $\bar{\nu}$'s helicity $+1/2$
- Magnetic moments very small: $\mu_\nu < 10^{-10}\mu_B\,, \quad \mu_\nu < 10^{-12}\mu_B)$

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## After oscillation experiments

- Neutrinos must be massive ($m_\nu \sim 1\,\mathrm{eV}$)
- They mix (with large mixings)
- LFV processes must exist (still not observed)

# Fermions in QFT

Dirac fermions reducible representation

$$\psi_L = P_L\psi = \begin{pmatrix} \xi \\ 0 \end{pmatrix}, \quad \psi_R = P_R\psi = \begin{pmatrix} 0 \\ \eta \end{pmatrix}$$

$$\xi \to \exp(-i\theta\vec{n}\cdot\vec{\sigma}\text{-}\vec{\beta}\cdot\vec{\sigma})\xi\,, \quad \eta \to \exp(-i\theta\vec{n}\cdot\vec{\sigma}+\vec{\beta}\cdot\vec{\sigma})\eta$$

In QFT the fundamental fields are two component spinors $\psi_L$ and $\psi_R$ and not the complete Dirac field $\psi$!

$$\mathscr{L} = i\overline{\psi_L}\partial\!\!\!/\,\psi_L + i\overline{\psi_R}\partial\!\!\!/\,\psi_R - m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L) =$$

$$= i\xi^\dagger\bar{\sigma}^\mu\partial_\mu\xi + i\eta^\dagger\sigma^\mu\partial_\mu\eta - m(\eta^\dagger\xi + \xi^\dagger\eta)$$

$$= i\xi_1^\dagger\bar{\sigma}^\mu\partial_\mu\xi_1 + i\xi_2^\dagger\bar{\sigma}^\mu\partial_\mu\xi_2 - m(\xi_2^T i\sigma_2\xi_1 - \xi_1^\dagger i\sigma_2\xi_2^*)$$

with $\xi_1 \equiv \xi$, $\xi_2 = i\sigma_2\eta^*$ ($\xi_2$ transforms like $\xi_1$)

# Fermions vs scalars

$$\mathcal{L} = i\xi_1^\dagger \bar{\sigma}^\mu \partial_\mu \xi_1 + i\xi_2^\dagger \bar{\sigma}^\mu \partial_\mu \xi_2$$

$$-\frac{i}{2}\left(m_1 \xi_1^T \sigma_2 \xi_1 + m_2 \xi_2^T \sigma_2 \xi_2 + 2m_{21} \xi_2^T \sigma_2 \xi_1 + \mathrm{h.c.}\right)$$

Kinetic terms invariant under $\xi_{1,2} \to e^{i\alpha_{1,2}}\xi_{1,2}$. $m_{1,2}$ break it

If $m_{1,2} = 0$, $\alpha_2 = -\alpha_1$ conserved $\to$ Dirac fields

$$\mathcal{L} = i\overline{\psi}\partial\!\!\!/\psi - m\overline{\psi}\psi\,, \quad \psi = \psi_L + \psi_R$$

Invariant under $\psi \to e^{i\alpha}\psi$

$$\mathcal{L} = \frac{1}{2}\partial\phi_1 \cdot \partial\phi_1 + \frac{1}{2}\partial\phi_2 \cdot \partial\phi_2 - \frac{1}{2}\left(m_1^2\phi_1^2 + m_2^2\phi_2^2 + 2m_{21}^2\phi_1\phi_2\right)$$

If $m_{21} = 0$ and $m_1 = m_2 \equiv m$. Invariant under rotations of $(\phi_1, \phi_2)$

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi\,, \quad \phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

Invariant under $\phi \to e^{i\alpha}\phi$

# Weyl and Majorana Fields

$\xi_2$ not necessary if there are no conserved charges: fermion fields can be massive with only two components (Majorana)

$$\mathscr{L}_{\mathrm{M}} = i\xi^\dagger \bar{\sigma}^\mu \partial_\mu \xi - \frac{i}{2}\left(m\xi^T \sigma_2 \xi + \mathrm{h.c.}\right)$$

$$i\bar{\sigma}^\mu \partial_\mu \xi - im\sigma_2 \xi^* = 0$$

If $m = 0$ (in momentum representation) $(E + \vec{p}\cdot\vec{\sigma})\xi(\vec{p}) = 0$, $E = \pm|\vec{p}|$

$$\frac{\vec{p}\cdot\vec{\sigma}}{|\vec{p}|}\xi(\vec{p}) = \begin{cases} -\xi(\vec{p}) & E > 0 \\ +\xi(\vec{p}) & E < 0 \end{cases}$$

**Weyl field:**

- Limit $m = 0$ of the Majorana field
- Particle helicity $-1/2$, antiparticle helicity $+1/2$.
- A U(1) charge conserved (invariance $\xi \to e^{i\alpha}\xi$)

# Quantization

$$\xi(x) = \sum_{\sigma=\pm} \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} \left( a_\sigma(\vec{p}) u_\sigma(\vec{p}) e^{-ip\cdot x} + a_\sigma^\dagger(\vec{p}) v_\sigma(\vec{p}) e^{ip\cdot x} \right)$$

Two helicities but particle and antiparticle are the same

In the limit $m \to 0$ $u_+(\vec{p}) = v_-(\vec{p}) = 0$

$$\xi(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} \left( a_-(\vec{p}) u_-(\vec{p}) e^{-ip\cdot x} + a_+^\dagger(\vec{p}) v_+(\vec{p}) e^{ip\cdot x} \right)$$

Particle has helicity $-1/2$ and antiparticle helicity $+1/2$

In four components define $\psi_L^c = (\psi_L)^c = C\overline{\psi_L}^T$ (is right-handed)

$$\mathscr{L}_M = i\overline{\psi_L}\partial\!\!\!/\psi_L - m\frac{1}{2}(\overline{\psi_L^c}\psi_L + \overline{\psi_L}\psi_L^c) = i\frac{1}{2}\overline{\psi_M}\partial\!\!\!/\psi_M - \frac{1}{2}m\overline{\psi_M}\psi_M$$

with $\psi_{\mathrm{M}} = \psi_L + \psi_L^c$ that satisfies $\left(i\partial\!\!\!/ - m\right)\psi_M = 0$

$$\psi_{\mathrm{M}}(x) = \sum_s \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p} \left( a(\vec{p},s) u(\vec{p},s) e^{-ip\cdot x} + a^\dagger(\vec{p},s) v(\vec{p},s) e^{ip\cdot x} \right)$$

Two helicities but particle and antiparticle equal

# Generalization to several fields

$$\overline{\psi_i^c}\psi_j = \overline{\psi_j^c}\psi_i \quad \rightarrow \quad \text{Symmetric mass matrices}$$

$$\overline{\psi_i^c}\gamma^\mu\psi_j = -\overline{\psi_j^c}\gamma^\mu\psi_i \quad \rightarrow \quad \text{Antisymmetric vector current}$$

$$\overline{\psi_i^c}\gamma^\mu\gamma_5\psi_j = \overline{\psi_j^c}\gamma^\mu\gamma_5\psi_i \quad \rightarrow \quad \text{Symmetric axial current}$$

$$\overline{\psi_i^c}\sigma^{\mu\nu}\psi_j = -\overline{\psi_j^c}\sigma^{\mu\nu}\psi_i \quad \rightarrow \quad \text{Antisymmetric magnetic moments}$$

$$\mathscr{L} = i\overline{\Psi_L}\partial\!\!\!/\Psi_L - \frac{1}{2}\left(\overline{\Psi_L^c}M\Psi_L + \mathrm{h.c.}\right)$$

with $\Psi_L = \mathrm{column}(\psi_{1L}, \psi_{1L}, \cdots \psi_{NL})$ and $M$ symmetric

$$M = V^T M_{\mathrm{diag}} V\,, \quad \Psi_M = V\Psi_L + V^*\Psi_L^c$$

$$\mathscr{L} = \frac{i}{2}\overline{\Psi_{\mathrm{M}}}\partial\!\!\!/\Psi_{\mathrm{M}} - \frac{1}{2}\overline{\Psi_{\mathrm{M}}}M_{\mathrm{diag}}\Psi_{\mathrm{M}}$$

# Masses of neutrinos in the SM

Simplest solution: add $\nu_R$ like in the quark sector

$$\mathscr{L}_{YL} = -\bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi} \nu_R + \text{h.c.}$$

But

- Why $m_\nu$ are so small?
- Why omit terms of the form $\overline{\nu_R^c} \nu_R$ in the Lagrangian?

Solution to the two questions: they are not omitted!

$$\mathscr{L}_{YL} \to \mathscr{L}_{YL} = -\bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi} \nu_R - \frac{1}{2} \overline{\nu_R^c} M \nu_R + \text{h.c.}$$

$$\mathscr{L}_{\nu M} = -\frac{1}{2} \left( \bar{\nu}_L, \overline{\nu_R^c} \right) \begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \text{h.c.}$$

if $M \gg M_D$ ("see-saw" mechanism):

- 3 **Heavy** Majorana neutrinos $\sim \nu_R$ with masses $\sim M$
- 3 **Light** Majorana neutrinos $\sim \nu_L$ with masses $\sim M_D^2 / M$

# Dirac and Majorana neutrinos

**Dirac:** if $M = 0$, $(M_\nu = M_D)$

$$\mathscr{L}_{\rm Dirac} = i\overline{\nu_L}\partial\!\!\!/\nu_L + \overline{\nu_R}\partial\!\!\!/\nu_R - (\overline{\nu_R}M_\nu\nu_L + {\rm h.c.})$$

- 4 degrees of freedom
- Conserve total lepton number (No $0\nu\beta\beta$ decay)
- Less natural (why $m_\nu$ are so small)

# Dirac and Majorana neutrinos

**Dirac:** if $M = 0$, $(M_\nu = M_D)$

$$\mathscr{L}_{\text{Dirac}} = i\overline{\nu_L}\partial\!\!\!/\nu_L + \overline{\nu_R}\partial\!\!\!/\nu_R - (\overline{\nu_R}M_\nu\nu_L + \text{h.c.})$$

- 4 degrees of freedom
- Conserve total lepton number (No $0\nu\beta\beta$ decay)
- Less natural (why $m_\nu$ are so small)

**Majorana:** if $M \gg M_D$, $(M_\nu = -M_D M^{-1} M_D^T)$

$$\mathscr{L}_{\text{Majorana}} = i\overline{\nu_L}\partial\!\!\!/\nu_L - \frac{1}{2}\left(\overline{\nu_L^c}M_\nu\nu_L + \text{h.c.}\right)$$

- 2 degrees of freedom
- Do not conserve total lepton number ($0\nu\beta\beta$ decay)
- More natural and more CP violating phases

# Neutrinos at low energies: Dirac

$$\mathscr{L}_{\text{Dirac}} = i\overline{\nu_L}\partial\!\!\!/\,\nu_L + \overline{\nu_R}\partial\!\!\!/\,\nu_R - (\overline{\nu_R}M_\nu\nu_L + \text{h.c.}) +$$
$$-\frac{G_F}{\sqrt{2}}J^\mu J_\mu^\dagger - \frac{G_F}{\sqrt{2}}J_Z^\mu J_{Z\mu} + \mathscr{L}_{\text{MM}} + \mathscr{L}_{\text{NSI}} + \cdots$$

$$J^\mu = 2\bar{\nu}_L\gamma^\mu e_L + \cdots\,, \qquad J_Z^\mu = \bar{\nu}_L\gamma^\mu\nu_L + \cdots$$

diagonalization

$$\nu_{\alpha L} = V_{\alpha i}\nu_{iL}\,, \quad \nu_{\alpha R} = U_{\alpha i}\nu_{iR}\,, \quad U^\dagger M_\nu V = M_{\text{diag}}\,, \quad \nu_i = \nu_{iL} + \nu_{iR}$$

$$J^\mu = 2\bar{\nu}\gamma^\mu V^\dagger P_L e + \cdots\,, \qquad J_Z^\mu = \bar{\nu}\gamma^\mu P_L\nu + \cdots$$

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}e^{i\delta} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Neutrinos at low energies: Majorana

$$\mathscr{L}_{\rm Dirac} = i\overline{\nu_L}\partial\!\!\!/ \nu_L - \frac{1}{2}\left(\overline{\nu_L^c} M_\nu \nu_L + \text{h.c.}\right) +$$

$$-\frac{G_F}{\sqrt{2}} J^\mu J_\mu^\dagger - \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu} + \mathscr{L}_{\rm MM} + \mathscr{L}_{\rm NSI} + \mathscr{L}_{0\nu\beta\beta} + \cdots$$

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$$J^\mu = 2\bar{\nu} V^\dagger P_L e + \cdots, \qquad J_Z^\mu = -\frac{1}{2}\bar{\nu}\gamma^\mu \gamma_5 \nu + \cdots$$

$$V_{\rm Majorana} = V_{\rm Dirac} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

1 Introduction

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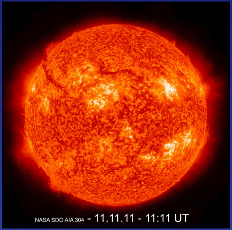
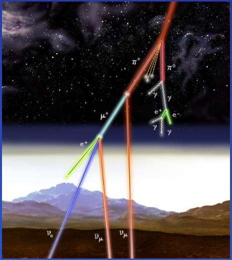
# Solar and atmospheric neutrino problems



## The Solar neutrino problem

- The Sun produces $\nu_e$'s, whose flux can be calculated using solar models
- The flux of $\nu_e$ measured in the earth in all experiments reduced by a factor 0.3–0.5
- Explained by oscillations $\nu_e \to \nu_{\mu,\tau}$

## The atmospheric neutrino problem

- $\pi$'s produced in the atmosphere should give a flux of $\nu_\mu$'s twice that of $\nu_e$'s
- The observed flux of $\nu_\mu$'s is largely reduced
- Explained in terms of oscillations $\nu_\mu \to \nu_\tau$

# Neutrino oscillacions in vacuum

Define $|\nu_e\rangle$ the state that produces $e^-$ and $|\nu_\mu\rangle$ the one that produces $\mu^-$. (Flavour eigenstates no energy eigenstates).

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

Where $\cos\theta = \langle\nu_1|\nu_e\rangle = \langle\nu_2|\nu_\mu\rangle$ and $\sin\theta = \langle\nu_2|\nu_e\rangle = -\langle\nu_1|\nu_\mu\rangle$

$$|\nu_e, t\rangle = e^{-iE_1 t}\cos\theta\,|\nu_1\rangle + e^{-iE_2 t}\sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu, t\rangle = -e^{-iE_1 t}\sin\theta\,|\nu_1\rangle + e^{-iE_2 t}\cos\theta\,|\nu_2\rangle$$

then

$$P(\nu_e \to \nu_\mu; t) = \left|\langle\nu_\mu|\nu_e, t\rangle\right|^2 = \sin^2(2\theta)\sin^2\left(\frac{(E_2 - E_1)t}{2}\right)$$

Definite momentum ultrarelativistic neutrinos ($p \gg m_i$),
$E_i = \sqrt{m_i^2 + p^2} \approx p + m_i^2/2p$, $L \approx t$ and $p \approx E$

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(2\pi\frac{L}{\lambda}\right)$$

with $\lambda$ oscillation length

$$\lambda = \frac{2\pi(E/\mathrm{GeV})}{1.27(\Delta m^2/\mathrm{eV}^2)}\mathrm{Km}\,, \quad \Delta m^2 = m_2^2 - m_1^2$$

## Not valid for

- $L \gg \lambda\frac{E}{\sigma}$ (decoherence, $\sigma$ wave packet width)
- $L \gg \lambda$ (Too fast oscillations: average)

$$\langle P(\nu_e \to \nu_\mu; t)\rangle = \frac{1}{2}\sin^2(2\theta)$$

Independent of $L, E$ and $\Delta m^2$

# Three neutrino oscillations

$$P(\nu_\alpha \to \nu_\beta; t) = \left|\langle \nu_\beta | \nu_\alpha, t\rangle\right|^2 = \left|\sum_i e^{-iE_i t} \langle \nu_\beta | \nu_i \rangle \langle \nu_i | \nu_\alpha \rangle\right|^2$$

if $\langle \nu_\beta | \nu_i \rangle = V_{\beta i}$ and $E_i \approx E + m_i^2/(2E)$

$$P(\nu_\alpha \to \nu_\beta; t) = \sum_{ij} e^{-i\Delta m_{ij}^2 t/2E} V_{\beta i} V_{\alpha i}^* V_{\alpha j} V_{\beta j}^* =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}\{V_{\beta i} V_{\alpha i}^* V_{\alpha j} V_{\beta j}^*\} \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) +$$

$$+2 \sum_{i>j} \mathrm{Im}\{V_{\beta i} V_{\alpha i}^* V_{\alpha j} V_{\beta j}^*\} \sin\left(\frac{\Delta m_{ij}^2 L}{2E}\right)$$

Effective hamiltonian $H = M_\nu^\dagger M_\nu/(2E) = V M_{diag}^2 V^\dagger/(2E)$
Phases of Majorana irrelevant (oscillations conserve LN)

# Neutrino oscillations in matter, MSW

$$\mathscr{L}_{\rm CC} = -\sqrt{2} G_F (\bar{e}\gamma_\mu P_L \nu_e)(\bar{\nu}_e \gamma^\mu P_L e) \to -\sqrt{2} G_F n_e (\bar{\nu}_e \gamma^0 P_L \nu_e)$$

$$H = C_{\rm univ} I + V \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m^2_{21}}{2E} & 0 \\ 0 & 0 & \frac{\Delta m^2_{31}}{2E} \end{pmatrix} V^\dagger + \begin{pmatrix} \tilde{V} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\tilde{V} = \pm b = \pm\sqrt{2} G_F n_e$ with $+$ for $\nu$'s and $-$ for $\bar{\nu}$'s

For two generations

$$H = \begin{pmatrix} \sin^2\theta + \frac{2E\tilde{V}}{\Delta m^2} & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{pmatrix} \frac{\Delta m^2}{2E} + \text{universal}$$

$$\sin 2\tilde{\theta} = \sin 2\theta \frac{\Delta m^2}{\Delta \tilde{m}^2}, \quad \Delta \tilde{m}^2 = \Delta m^2 \sqrt{1 + \left(\frac{2E\tilde{V}}{\Delta m^2}\right)^2 - 2\cos 2\theta \frac{2E\tilde{V}}{\Delta m^2}}$$

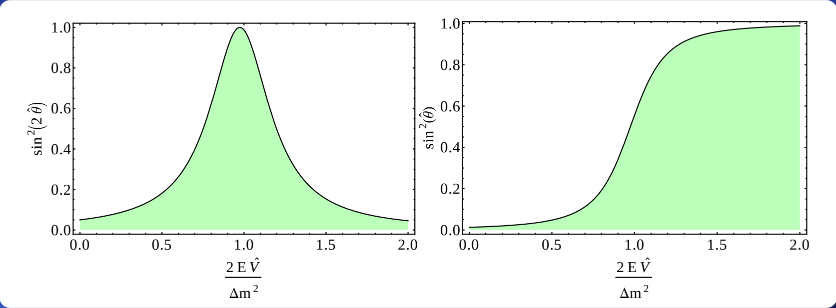
# The resonance

$$\Delta m^2 \cos 2\theta = \pm 2E\sqrt{2}G_F n_e \to \begin{cases} \Delta\tilde{m}^2 = \Delta m^2 \sin 2\theta \\ \sin^2 2\tilde{\theta} = 1 \end{cases}$$

$\Delta m^2 \cos 2\theta > 0$ for $\nu$'s and $\Delta m^2 \cos 2\theta < 0$ for $\bar{\nu}$'s
Ordering of $H$ eigenvalues such that $\Delta\tilde{m}^2 > 0$ implies

$$2E\tilde{V}/\Delta m^2 \ll 1 \,, \Delta\tilde{m}^2 \approx \Delta m^2,\ \tilde{\theta} \approx \theta \text{ and } |\tilde{\nu}_2\rangle \approx |\nu_2\rangle$$

$$2E\tilde{V}/\Delta m^2 \gg 1 \ \Delta\tilde{m}^2 \gg \Delta m^2,\ \tilde{\theta} = \frac{\pi}{2} \text{ and } |\tilde{\nu}_2\rangle \approx |\nu_e\rangle$$



# Adiabatic approximation in the Sun

If $n_e(x)$ changes slowly we can use the adiabatic theorem:
"If in $t=0$ the system is in one of the instantaneous eigenstates of $H(t=0)$, $H(t)|n,t\rangle = E_n(t)|n,t\rangle$ it will remain in the state $|n,t\rangle$ for $t>0$"

$$|\nu_e\rangle \overset{n_e\gg}{\approx} |\tilde{\nu}_2\rangle \overset{\text{Adiabat}}{\longrightarrow} |\tilde{\nu}_2, t\rangle \overset{\text{Adiabat}}{\longrightarrow} |\nu_2\rangle = \sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\mu\rangle$$

$$P(\nu_e \to \nu_e) = \sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$$

If $\theta \ll 1$ all the $\nu_e$ are transformed in $\nu_\mu$! (MSW)
General case

$$P(\nu_e \to \nu_e) = \frac{1}{2} + (\frac{1}{2} - P_{\rm LZ})\cos 2\hat{\theta}_0 \cos 2\theta$$

$$P_{\rm LZ} \approx e^{-\gamma}, \qquad \gamma \equiv \frac{\pi \Delta m^2}{4E|(n_e'/n_e)_{\rm res}|}\frac{\sin^2 2\theta}{\cos 2\theta}$$

# The solar neutrino triangles