Exercises

Q1) Explicitly show that both a strong and a weak phase are needed to generate CP asymmetry.

A1) Consider two decay amplitudes with strong phases $\delta_{1,2}$ and weak phases $\phi_{1,2}$

$$A_1 = |A_1| e^{i(\delta_1 - \phi_1)}$$
$$A_2 = |A_2| e^{i(\delta_2 - \phi_2)}$$

Similarly one has

$$\bar{A}_1 = |A_1| e^{i(\delta_1 + \phi_1)}$$
$$\bar{A}_2 = |A_2| e^{i(\delta_2 + \phi_2)}$$

Notice that only the CP-odd weak phase flips sign.

Now writing $\Delta\delta = \delta_1 - \delta_2$ and $\Delta\phi = \phi_1 - \phi_2$, we have

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2\mathrm{Re}(A_2^* A_1)$$
$$= |A_1|^2 + |A_2|^2 + 2|A_1 A_2| \cos(\Delta\phi + \Delta\delta)$$

Similarly

$$|\bar{A}_1 + \bar{A}_2|^2 = |A_1|^2 + |A_2|^2 + 2|A_1 A_2| \cos(\Delta\phi - \Delta\delta)$$

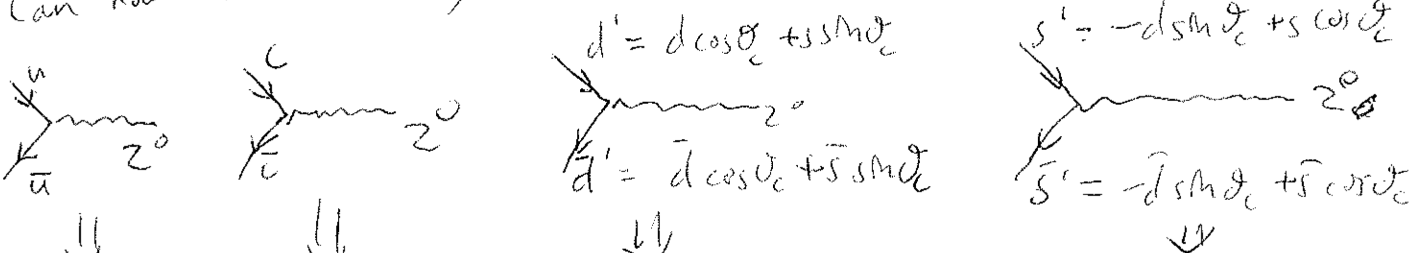
So because $\cos(-x) = \cos(x)$, both $\Delta\phi$ <u>and</u> $\Delta\delta$ must be non-zero for ~~this~~ the rates to differ and CP violation to occur.

Q2) Explicitly show the FCNC tree-level cancellation from the GIM mechanism.

A2) Consider two quark generations $\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$ with

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

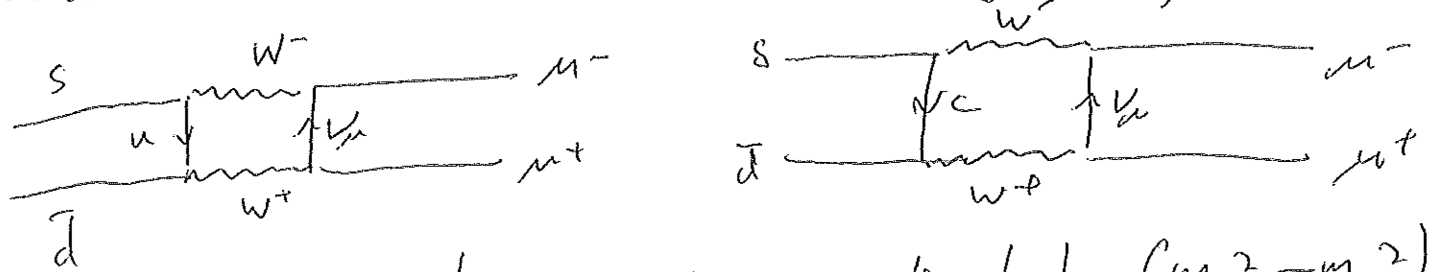
Can now write the diagrams



$$u\bar{u} + c\bar{c} + d\bar{d}\cos^2\theta_c + s\bar{s}\sin^2\theta_c + (\bar{d}s + d\bar{s})\cos\theta_c\sin\theta_c + d\bar{d}\sin^2\theta_c + s\bar{s}\cos^2\theta_c - (\bar{d}s + d\bar{s})\cos\theta_c\sin\theta_c$$

$$\Rightarrow u\bar{u} + c\bar{c} + d\bar{d} + s\bar{s}$$

otice that we can also draw non-tree diagrams e.g. for $K \to \mu\mu$



sum of diagrams is proportional to $(m_c^2 - m_u^2)$

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13) Why is $BR(\pi \to e\nu) \lll BR(\pi \to \mu\nu)$?

:3) The short answer is because of helicity suppression.
Consider decay in pion rest frame

$$\bar{\nu}_\mu (\bar{\nu}_e) \longleftarrow \pi \longrightarrow \mu^- \ (e^-)$$

⇒ pion has 0 spin so μ/ν_μ or e/ν_e spins are opposite

⇒ since weak interaction only couples to RH chiral anti-particles and ν is massless, ν is in RH helicity state

⇒ So μ/e are in an RH helicity state too

⇒ But only LH chiral particles couple to weak interaction. So need to project out LH chiral part of RH helicity state, prop. to

$$\left(1 - \frac{|p|}{E+m}\right)$$

as $m \to 0$, $E \to p$ and this goes to 0.

4) If $A^{meas}(t) = D_{tag} D_{res} A(t)$, how well do you need to know D_{tag}/D_{res} to have an overall systematic $< 1\%$?

14) From addition in quadrature, to better that 7‰ each.
For derivation see separate sheet.

The four undiluted decay rates are given apart from a common normalisation factor. The parameters η and $\overline{\eta}$ for the decay $B_d^0 \to D^*\pi$ are given by:

$$\eta = |\eta|\, e^{i(\Delta_{qcd}+(\phi_{mix}-\gamma))} \quad \overline{\eta} = |\eta|\, e^{i(\Delta_{qcd}-(\phi_{mix}-\gamma))}.$$

$$\begin{aligned}
&1)\, \Gamma(B_d^0 \to D^{*-}\pi^+) = R_{D^{*-}}(\tau) = e^{-\Gamma\tau}\Big\{ \left(1+|\eta|^2\right) + \left(1-|\eta|^2\right)\cos(\Delta m\tau) - 2\mathrm{Im}(\eta)\sin(\Delta m\tau) \Big\}\\
&2)\, \Gamma(\overline{B_d^0} \to D^{*-}\pi^+) = \overline{R}_{D^{*-}}(\tau) = e^{-\Gamma\tau}\Big\{ \left(1+|\eta|^2\right) - \left(1-|\eta|^2\right)\cos(\Delta m\tau) + 2\mathrm{Im}(\eta)\sin(\Delta m\tau) \Big\}\\
&3)\, \Gamma(\overline{B_d^0} \to D^{*+}\pi^-) = \overline{R}_{D^{*+}}(\tau) = e^{-\Gamma\tau}\Big\{ \left(1+|\overline{\eta}|^2\right) + \left(1-|\overline{\eta}|^2\right)\cos(\Delta m\tau) - 2\mathrm{Im}(\overline{\eta})\sin(\Delta m\tau) \Big\}\\
&4)\, \Gamma(B_d^0 \to D^{*+}\pi^-) = R_{D^{*+}}(\tau) = e^{-\Gamma\tau}\Big\{ \left(1+|\overline{\eta}|^2\right) - \left(1-|\overline{\eta}|^2\right)\cos(\Delta m\tau) + 2\mathrm{Im}(\overline{\eta})\sin(\Delta m\tau) \Big\}
\end{aligned} \quad (3.16)$$

study. From log-likelihood fits to the asymmetries, the error on γ is estimated.

The first step in this analysis is to derive the measured decay rates and asymmetries, including detector effects, from the theoretical ones given in sections 1.3.4 and 1.3.6. For ease of reference, and in order to define the notation used in the following sections, the undiluted decay rates are given again in equation 3.16 (page 95).

The asymmetries including the detector effects will be given in terms of $A(\tau)$, which stands for both $A_\eta(\tau)$ and $A_{\overline{\eta}}(\tau)$, before detector effects are taken into account.

3.5.1 Decay Rates, Including Detector Effects

Detector-effects are taken into account by assuming a Gaussian-distributed time resolution of 170 fs, a uniform mistag probability of 30 % and an acceptance function:

$$P_A(\tau) = \max\left(0, \frac{(a\tau)^3}{1+(a\tau)^3} - b\right) \quad (3.19)$$
$$\text{with } a = 0.96\,\mathrm{ps}^{-1},\quad b = 0.09$$

giving the probability that an event with decay-eigentime τ is recorded. The acceptance function is taken from [TP98]. It takes into account that the trigger, and also the final event selection, rejects events with small decay–lengths. A background to signal ratio of $B/S = 0.2$ is assumed.

Time Resolution

With a true decay rate

$$R_t(\tau) = e^{-\Gamma\tau}\left(a + b\cos(\omega\tau) + c\sin(\omega\tau)\right) \quad (3.20)$$

the measured decay rate is, taking into account the finite time–resolution and time–

dependent acceptance:

$$R_{A\sigma_\tau}(\tau_0) = \int_0^\infty P_A(\tau_0,\tau)\,R_t(\tau)\cdot g_{\tau_0}(\tau_0-\tau)\,\mathrm{d}\tau. \quad (3.21)$$

Here, τ is the decay eigentime of the $\mathrm{B^0_d}$, and τ_0 is the reconstructed decay eigentime; $P_A(\tau_0,\tau)$ is the acceptance function and $g_{\tau_0}(\tau_0-\tau)$ the resolution function. In general both might be quite complicated, and numerical methods will be necessary to perform the integral. Here, in order to be able to do the integral analytically, we will assume that the time resolution is described well by a Gaussian, and that the acceptance function is a function of the measured decay time, τ_0, only. Then the expression for the measured decay rate becomes:

$$R_{A\sigma_\tau}(\tau_0) = P_A(\tau_0)\int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_\tau} e^{-\frac{(\tau_0-\tau)^2}{2\sigma_\tau^2}} e^{-\Gamma\tau}\left(a + b\cos(\omega\tau) + c\sin(\omega\tau)\right)\mathrm{d}\tau. \quad (3.22)$$

All three parts of the above sum can be solved simultaneously by calculating:

$$F(\tau_0) = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_\tau} e^{-\frac{(\tau_0-\tau)^2}{2\sigma_\tau^2}} e^{-\Gamma\tau} e^{i\omega\tau}\mathrm{d}\tau. \quad (3.23)$$

Taking the part independent of τ outside the integral gives:

$$F(\tau_0) = \frac{1}{\sqrt{2\pi}\sigma_\tau} e^{-\frac{1}{2}\left(\frac{\tau_0}{\sigma_\tau}\right)^2} \int_0^\infty e^{-\frac{1}{2\sigma_\tau^2}\left(\tau^2 - 2\left(\tau_0 + (i\omega-\Gamma)\sigma_\tau^2\right)\tau\right)}\mathrm{d}\tau. \quad (3.24)$$

Defining

$$z \equiv \tau_0 + (i\omega - \Gamma)\,\sigma_\tau^2 \quad (3.25)$$

and completing the square in the exponent, this becomes:

$$F(\tau_0) = \frac{1}{\sqrt{2\pi}\sigma_\tau} e^{-\frac{1}{2}\left(\frac{\tau_0}{\sigma_\tau}\right)^2} e^{+\frac{1}{2}\left(\frac{z}{\sigma_\tau}\right)^2} \cdot \int_0^\infty e^{-\frac{(\tau-z)^2}{2\sigma_\tau^2}}\mathrm{d}\tau. \quad (3.26)$$

In practice, only events with long decay times pass the trigger, and therefore

$$\mathsf{Re}(z) = \tau_0 - \Gamma\sigma_\tau^2 > \text{a few } \sigma_\tau$$

for the relevant values of τ_0. Then the integral from 0 to ∞ can be replaced with an integral from $-\infty$ to ∞, and the solution is:

$$\begin{aligned} F(\tau_0) &= \frac{1}{\sqrt{2\pi}\sigma_\tau} e^{-\frac{1}{2}\left(\frac{\tau_0}{\sigma_\tau}\right)^2} e^{+\frac{1}{2}\left(\frac{z}{\sigma_\tau}\right)^2} \cdot \int_{-\infty}^\infty e^{-\frac{(\tau-z)^2}{2\sigma_\tau^2}}\mathrm{d}\tau \\ &= e^{-\frac{1}{2}\left(\frac{\tau_0}{\sigma_\tau}\right)^2} e^{+\frac{1}{2}\left(\frac{z}{\sigma_\tau}\right)^2} \\ &= e^{\frac{1}{2\sigma_\tau^2}\left((i\omega-\Gamma)^2\sigma_\tau^4 + 2\tau_0\left((i\omega-\Gamma)\sigma_\tau^2\right)\right)}. \end{aligned} \quad (3.27)$$

Re-ordering gives the final result:

$$F(\tau_0) = e^{-\frac{1}{2}\sigma_\tau^2\cdot(\Gamma^2+\omega^2)} e^{-\Gamma(\tau_0-\Gamma\sigma_\tau^2)} e^{i\omega(\tau_0-\Gamma\sigma_\tau^2)} \quad (3.28)$$

So the effect of the finite time-resolution on the function

$$R(\tau) = e^{-\Gamma\tau}\left(a + b\cos(\omega\tau) + c\sin(\omega\tau)\right)$$

can be described by simultaneously scaling the amplitudes a, b, and c, and shifting the parameter τ according to:

$$\begin{aligned} a &\longrightarrow ae^{-\frac{1}{2}\sigma_\tau^2\cdot\Gamma^2} \\ b &\longrightarrow be^{-\frac{1}{2}\sigma_\tau^2\cdot(\Gamma^2+\omega^2)} \\ c &\longrightarrow ce^{-\frac{1}{2}\sigma_\tau^2\cdot(\Gamma^2+\omega^2)} \\ \tau &\longrightarrow \tau - \Gamma\sigma_\tau^2. \end{aligned} \quad (3.29)$$

This transforms the asymmetry to:

$$A_{\sigma_\tau}(\tau) = e^{-\frac{1}{2}\sigma_\tau^2\cdot(\Delta m)^2} A(\tau - \Gamma\sigma_\tau^2) \quad (3.30)$$

The measured decay rates, taking into account the finite time resolution, but not the acceptance function, are given in equation 3.31 (page 98).

Background

In this study it is assumed that the background is independent of the decay considered; this is to say that at any given time, the number of background events interpreted as $B_d^0 \to D^{*-}\pi^+$ decays is the same as the number interpreted as $\overline{B_d^0} \to D^{*-}\pi^+$ decays. As will be shown in section 3.5.6, this results in a conservative estimate on the statistical precision in γ. For a background to signal ratio of B/S, the measured decay rate for $B_d^0 \to D^{*-}\pi^+$, $R_{\frac{B}{S}\,D^{*-}}(\tau)$, becomes, in terms of the decay rates without background:

$$R_{\frac{B}{S}\,D^{*-}}(\tau) = R_{D^{*-}}(\tau) + \frac{B}{S}\cdot\frac{1}{2}\left(R_{D^{*-}}(\tau) + \overline{R}_{D^{*-}}(\tau)\right) \quad (3.32)$$

and similarly for the other decay rates. The measured asymmetry, $A_{\frac{B}{S}}(\tau)$, is given by:

$$A_{\frac{B}{S}}(\tau) = \frac{1}{1+B/S}A(\tau), \quad (3.33)$$

where $A(\tau)$ is the asymmetry without background.

Mistag

Including a mistag fraction of ω_{tag}, the measured decay rate for $B_d^0 \to D^{*-}\pi^+$, $R_{\omega_{\text{tag}}\,D^{*-}}(\tau)$, becomes, in terms of the decay rate without mistag:

$$R_{\omega_{\text{tag}}\,D^{*-}}(\tau) = (1-\omega_{\text{tag}})\,R_{D^{*-}}(\tau) + \omega_{\text{tag}}\overline{R}_{D^{*-}}(\tau) \quad (3.34)$$

and similarly for the other decay rates. The measured asymmetry, $A_{\omega_{\text{tag}}}(\tau)$, is given by

$$A_{\omega_{\text{tag}}}(\tau) = (1-2\omega_{\text{tag}})\,A(\tau) \quad (3.35)$$

where $A(\tau)$ is the asymmetry for perfect tagging.