EXERCISES

Ex. 1 The τ-lepton is heavier than the muon, with

$$m_\tau \simeq 1777 \text{ MeV} \quad \text{vs} \quad m_\mu \simeq 106 \text{ MeV},$$

and can decay in several different ways, while muons decay $\simeq 100\%$ as $\mu \to e \bar{\nu}_e \nu_\mu$. One of the decay channels of the τ is exactly similar: $\tau \to e \bar{\nu}_e \nu_\tau$, and proceeds through the same Fermi coupling G_F. Knowing that the respective lifetimes are $T_\tau \simeq 2.9 \times 10^{-13}$ sec and $T_\mu \simeq 2.2 \times 10^{-6}$ sec. estimate the fraction of τ decays into $e \bar{\nu}_e \nu_\tau$ and compare with the observed $BR = (17.85 \pm 0.05)\%$.

What do you expect for $BR(\tau \to \mu \bar{\nu}_\mu \nu_\tau)$?

Ex. 2 Suppose the SM is the low-E EFT of a SUSY model with the mass of all superpartners $\Lambda_{SUSY} \simeq 1$ TeV. Assuming that SUSY fixes the higgs quartic $(\frac{1}{8} \lambda h^4)$ at the scale Λ_{SUSY} in terms of gauge couplings and the angle β as $\lambda(\Lambda_{SUSY}) = \frac{1}{4}(g^2 + g'^2)\cos^2 2\beta$.

Show how to calculate the Higgs boson mass using RG running from Λ_{SUSY} down to the EW scale and keeping only the leading effect to obtain:

$$M_h^2 = M_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2} \cdot \frac{M_t^4}{M_W^2} \log \frac{\Lambda_{SUSY}^2}{M_t^2}$$

where $M_t = \frac{1}{\sqrt{2}} h_t v$, $M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2$, $M_W^2 = \frac{1}{4} g^2 v^2$ $(v = 246 \text{ GeV})$

[Hint: $\beta_\lambda = \frac{d\lambda}{d \log \mu} = -\frac{3 h_t^4}{4\pi^2}$]