EXERCISES

Ex. 1 The τ-lepton is heavier than the muon, with

$$m_\tau \simeq 1777 \text{ MeV} \quad \text{vs} \quad m_\mu \simeq 106 \text{ MeV},$$

and can decay in several different ways, while muons decay $\simeq 100\%$ as $\mu \to e \bar{\nu}_e \nu_\mu$. One of the decay channels of the τ is exactly similar: $\tau \to e \bar{\nu}_e \nu_\tau$, and proceeds through the same Fermi coupling G_F. Knowing that the respective lifetimes are $T_\tau \simeq 2.9 \times 10^{-13}$ sec and $T_\mu \simeq 2.2 \times 10^{-6}$ sec. estimate the fraction of τ decays into $e \bar{\nu}_e \nu_\tau$ and compare with the observed $BR = (17.85 \pm 0.05)\%$.

What do you expect for $BR(\tau \to \mu \bar{\nu}_\mu \nu_\tau)$?

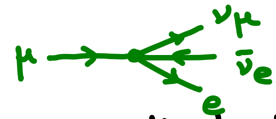
Ex. 2 Suppose the SM is the low-E EFT of a SUSY model with the mass of all superpartners $\Lambda_{SUSY} \simeq 1$ TeV. Assuming that SUSY fixes the Higgs quartic $\left(\frac{1}{8}\lambda h^4\right)$ at the scale Λ_{SUSY} in terms of gauge couplings and the angle β as $\lambda(\Lambda_{SUSY}) = \frac{1}{4}(g^2 + g'^2)\cos^2 2\beta$.

Show how to calculate the Higgs boson mass using RG running from Λ_{SUSY} down to the EW scale and keeping only the leading effect to obtain:

$$M_h^2 = M_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2} \cdot \frac{M_t^4}{M_W^2} \log \frac{\Lambda_{SUSY}^2}{M_t^2}$$

where $M_t = \frac{1}{\sqrt{2}} h_t v$, $M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2$, $M_W^2 = \frac{1}{4} g^2 v^2$ $(v = 246 \text{ GeV})$

[Hint: $\beta_\lambda = \frac{d\lambda}{d\log\mu} = -\frac{3h_t^4}{4\pi^2}$]

Sol. 1 The amplitude for the muon decay 

is proportional to the Fermi constant $a \sim G_F$, so that the decay probability goes like $|a|^2 \sim G_F^2 \sim \text{mass}^{-4}$. Neglecting the electron mass ($m_e \ll m_\mu$), the only relevant mass scale is m_μ, and therefore, the muon width $\Gamma_\mu = 1/T_\mu = C G_F^2 m_\mu^5$, where C is some numerical coefficient that takes care of the kinematics. The partial width of the τ into the $\nu_\tau \bar{\nu}_e e$ channel is also controlled by G_F and, within the same approximation of neglecting $m_e \ll m_\tau$, will go as $\Delta\Gamma_\tau = C G_F^2 m_\tau^5$ with the same constant C. Knowing the total widths $1/T_\mu$, $1/T_\tau = \Gamma_\tau$ we get

$$BR(\tau \to \nu_\tau e \bar{\nu}_e) = \frac{\Delta\Gamma_\tau}{\Gamma_\tau} = \frac{\Gamma_\mu}{\Gamma_\tau} \cdot \frac{\Delta\Gamma_\tau}{\Gamma_\mu} = \frac{T_\tau}{T_\mu} \cdot \left(\frac{m_\tau}{m_\mu}\right)^5 = 0.18$$

which is a very good estimate of the measured BR.

The decay $\tau \to \nu_\tau \mu \bar{\nu}_\mu$ will proceed along a similar G_F coupling and the corresponding BR will be of the same order, except a little bit lower due to phase space factors (m_μ is not as small as m_e). Indeed the PDG quotes $BR(\tau \to \nu_\tau \mu \bar{\nu}_\mu) \simeq 17.36\%$

Sol. 2 The problem requires finding λ at the EW scale by integrating its RGE $d\lambda/d\log\mu \simeq -\frac{3h_t^2}{4\pi^2}$. This gives

$$\int_{M_{EW}}^{\Lambda_{SUSY}} d\lambda = \lambda(\Lambda_{SUSY}) - \lambda(M_{EW}) = -\int_{M_{EW}}^{\Lambda_{SUSY}} \frac{3h_t^2}{4\pi^2}\, d\log\mu \simeq -\frac{3h_t^2}{4\pi^2} \log\frac{\Lambda_{SUSY}}{M_{EW}}$$

Once $\lambda(M_{EW})$ is known, the Higgs mass is approximated as the second derivative of the Higgs potential at its minimum $\langle h \rangle = v$ giving $M_h^2 = \lambda(M_{EW}) v^2$. Plugging the $\lambda(M_{EW})$ obtained from integrating the RG and rewriting it in terms of M_Z, M_t and M_W reproduces the expression given. (The argument M_{EW} in the logarithm can be taken as $M_{EW} = M_t$ because the dominant term $\beta_\lambda \sim -h_t^4$ comes from a top loop, so that it makes sense to stop the running of λ when the top threshold is reached.)