EXERCISES

Ex. 3 Determine what types of heavy particle could generate (à la Fermi) the $d=5$ operator

$$\frac{\kappa_{ij}}{\Lambda}(L_{L_i}H)(L_{L_j}H) + h.c.$$

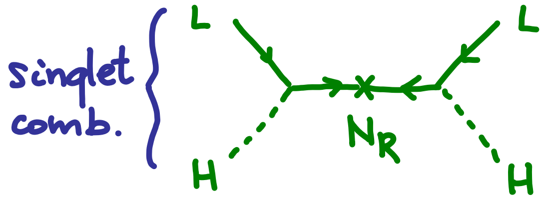
that leads to Majorana masses for ν_L's. Find all different possibilities, giving for each the quantum numbers of the heavy particle being exchanged.

Ex. 4 Given that neutrinos are massive and their flavors mix, they induce a contribution to $\mu\to e\gamma$, which gives

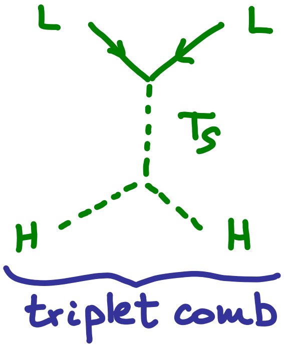
$$BR(\mu\to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i}U_{ei}\frac{m^2_{\nu_i}}{M^2_W}\right|^2 \sim 10^{-52} \quad (!)$$

where $U_{\alpha i}$ is the flavor mixing matrix for neutrinos, that relates flavor and mass eigenstates as $\nu_{L\alpha} = \sum_i U_{\alpha i}\nu_i$. Draw the diagram that contributes to this decay and explain, using symmetry arguments, why the BR above is proportional to the fourth power of neutrino masses.

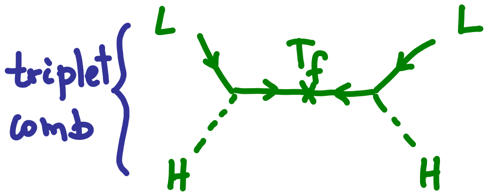
Sol. 3



N_R heavy fermion
singlet under $SU(2)_L \times U(1)_Y$
(Type I in the lit.)

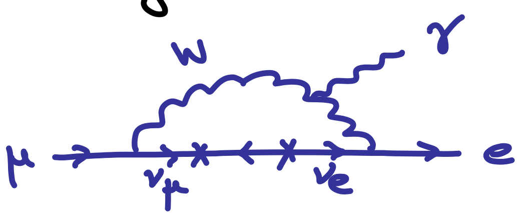


T_S heavy scalar
triplet of $SU(2)_L$, $Y = -1$
(Type II in the lit.)



T_f heavy fermion
triplet of $SU(2)_L$, $Y = 0$
(Type III in the lit.)

Sol. 4 The relevant diagram is



with two neutrino mass insertions as indicated. To see this, notice that the decay has $\Delta L_\mu = -1$, $\Delta L_e = +1$ while a neutrino mass term $\nu_\alpha M_{\alpha\beta} \nu_\beta + h.c$ has $\Delta L_\alpha + \Delta L_\beta = \pm 2$. Therefore, the amplitude for the decay above has to depend on neutrino masses as $a \sim \sum_\alpha M^*_{\mu\alpha} M_{\alpha e}$ which can be rewritten in terms of neutrino mass eigenvalues as

$$a \sim \sum_i U^*_{\mu i}\, m^2_{\nu_i}\, U_{ie}$$

The decay rate going as $|a|^2$ implies $BR \sim m_\nu^4$.