2. (B)SM EFT

D.o.f.s & Symmetries $\Rightarrow$ Most general $\mathcal{L} = \mathcal{L}_{d=4} + \mathcal{L}_{d=5} + \ldots$

$\mathcal{L}_{d\leq 4}$: Symmetries and naturalness problem

$\mathcal{L}_{d>4}$: What is Λ ? Neutrino masses and gravity

Λ_{max} in EFT

$\mathcal{L}_{d=6}$ ops. ($\not{B}$, $\not{L}_i$, FCNCs, Custodial, ...) Bounds

Hierarchy problem

D.o.f.s & Symmetries ⇨ Most general $\mathcal{L} = \mathcal{L}_{d=4} + \mathcal{L}_{d=5} + \dots$

The modern view on the SM is that it is a low-energy EFT with a limited range of applicability in energy. Its $\mathcal{L}$ is made of all possible local operators constructed with the light degrees of freedom:

quarks: $Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3,2,1/6)$ $\quad u_R \sim (3,1,2/3)$ $\quad d_R \sim (3,1,-1/3)$ (×3 families)

leptons: $L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1,2,-1/2)$ $\quad e_R \sim (1,1,-1)$ (×3 families)

Higgs: $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \sim (1,2,1/2)$

and respecting Poincaré and gauge invariance under

$$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

The gauge bosons associated to this group are gluons and EW gauge bosons: $\quad G_\mu^A, \; W_\mu^a, \; B_\mu$

We can decompose the general Lagrangian for the SM EFT as

$$\mathcal{L} = \Lambda^2 \mathcal{L}_{d=2} + \mathcal{L}_{d=4} + \frac{1}{\Lambda}\mathcal{L}_{d=5} + \frac{1}{\Lambda^2}\mathcal{L}_{d=6} + \dots$$

according to the scaling dimension d of the operators. Here Λ represents the mass scale where new physics BSM enters and the SM description fails. It's the UV cutoff of our EFT.

Relevant and marginal ops. ($\mathcal{L}_{d=2}$ and $\mathcal{L}_{d=4}$) would be determined by some unknown UV physics beyond Λ but we can do by measuring them experimentally.

The impact of the tower of irrelevant ops. ($\mathcal{L}_{d>4}$) will be suppressed by E_{EW}/Λ. Finding out what is the value of Λ is the single most important question the LHC tries to answer.

$\mathcal{L}_{d\leq 4}$: Symmetries and naturalness problem

$\mathcal{L}_{d\leq 4}$ describes low-energy physics with great precision. The $d=2$ part is simply the Higgs mass term

$$\Lambda^2 \mathcal{L}_{d=2} = -m^2 |H|^2$$

The natural value of this mass is $O(\Lambda)$ if Λ sets the scale of masses in the UV BSM theory. Having $m^2 \ll \Lambda^2$, so that H is in the low-E EFT to begin with, requires an explanation. This is the root of the SM hierarchy problem to be discussed later on. There are no mass terms for gauge bosons (forbidden by the gauge symmetry) or for fermions (forbidden by chiral symmetry): gauge bosons and fermions are natural dofs to have in a low-E EFT.

There's no $d=3$ term allowed by the symmetries and at $d=4$ we have, schematically:

$$\mathcal{L}_{d=4} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi + |D_\mu H|^2 - \lambda |H|^4$$
$$+ Y_u \bar{Q}_L \bar{H} u_R + Y_d \bar{Q}_L H d_R + Y_e \bar{L}_L H e_R + h.c.$$

This $\mathcal{L}_{d=4}$ has some symmetries beyond Poincaré, gauge and chiral symmetries that result from the restriction to $d=4$. They are

🚩 Baryon number. A $U(1)_B$ under which quarks have charge $1/3$. The proton is the lightest charged ($B=1$) state and is therefore stable (in the SM!). Experimentally

$p \to e^+ \pi^0$: $\quad \tau_p > 8.2 \times 10^{33}\,\mathrm{yr} \;\Rightarrow\; \Gamma_p = \frac{1}{\tau_p} < 2.5 \times 10^{-66}\,\mathrm{GeV}$ (!)

🚩 Lepton numbers. A $U(1)_L$ per family under which leptons have charge 1. Forbids e.g. $\mu \to e\gamma$. Experimentally

$$BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$$

We know that these are violated, though, in neutrino oscillations, which therefore imply $\mathcal{L}_{d=4}$ cannot be the whole story. Either there are right-handed neutrinos in the EFT or higher order ops induce such oscillations (see below).

🚩 (Approximate) flavor symmetry. This is a

$$U(3)^5 = U(3)_{Q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{L_L} \times U(3)_{e_R}$$

under which fermions rotate as $\psi \to U\psi$, $U \in U(3)_\psi$. It is only broken by the Yukawas (typically small except y_t). It explains a few cancellations in flavour physics (the SM flavor structure is far from "generic"). It is often extended to BSM theories where it goes under the name "Minimal Flavor Violation" with the Yukawas as only source of breaking.

🚩 (Approximate) Custodial Symmetry This is the $SO(4)$ symmetry of rotations of the 4 real components of the Higgs field (leaving invariant $|H|^2 = \frac{1}{2}\sum_{i=1}^{4} \varphi_i^2$ with $H = \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$)

It leads to

$$\rho \equiv \frac{M_W^2}{M_Z^2 c_W^2} \simeq 1$$

exact at tree-level but broken at loop-level by Yukawas and g'.

[In more detail, we can check custodial invariance by rewriting the Lagrangian in terms of a Higgs bidoublet, defined as

$$\mathcal{H} \equiv (\underbrace{\tilde{H}}_{i\sigma_2 H^*}, H)$$

which under the global custodial $SU(2)_L \times SU(2)_R \simeq SO(4)$

transforms as $\quad \mathcal{H} \to U_L \mathcal{H} U_R^\dagger \quad$ i.e. $\mathcal{H} \sim (2,2)$

while $\quad W_\mu \equiv \sigma^a W_\mu^a \to U_L W_\mu U_L^\dagger \quad$ i.e. $W_\mu \sim (3,1)$.

Then, for $g'=0$, $\quad D_\mu \mathcal{H} \to U_L D_\mu \mathcal{H} U_R^\dagger$

and the kinetic term $\mathcal{L}_{kin} = \frac{1}{2} \mathrm{Tr}\left[(D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H})\right]$ is custodially invariant (up to g' terms).

Similarly

$$\mathcal{L}_f = \sum_{ij} \underbrace{\bar{Q}_{L_i}}_{(2,1)} \mathcal{H} \begin{pmatrix} Y_u^{ij} & 0 \\ 0 & Y_d^{ij} \end{pmatrix} \underbrace{\begin{pmatrix} u_{R_j} \\ d_{R_j} \end{pmatrix}}_{(1,2)} + \text{h.c.} + \text{leptons}$$

would be custodial invariant up to $Y_u^{ij} \neq Y_d^{ij}$ effects, with $y_t \neq y_b$ giving the largest breaking effect.

The Higgs potential is, of course, custodial invariant as it is a function of $|H|^2 = \frac{1}{2} \mathrm{Tr}\left[\mathcal{H}^\dagger \mathcal{H}\right]$.

EWSB corresponds to $\langle \mathcal{H} \rangle = \frac{v}{\sqrt{2}} \mathbb{I}_2$ which realizes the breaking $SU(2)_L \times SU(2)_R \to SU(2)_{diag}$ and leads to $\rho = 1$. This holds because the W_μ^a form a custodial triplet and the

mass term for gauge bosons should be $\mathcal{L}_m \propto v^2 \sum_{a=1}^{3} W^a_\mu W^{a\mu} + O(g'B)$.
Using $Z_\mu = c_W W_{3\mu} - s_W B_\mu$ and $m_{A_\mu} = 0$, it must follow that $M_W^2 = M_Z^2 \cdot c_W^2$.]

These are accidental (or emergent) symmetries that hold in the IR but could be violated by higher order operators (if they are not true in the fundamental BSM theory).

$\mathcal{L}_{d>4}$: What is Λ? Neutrino masses and gravity

Beyond the well tested $\mathcal{L}_{d=4}$ we enter the unknown territory of higher order operators. The key point here is what is the suppression scale Λ, which is related to the presence of new physics beyond what we know. Of course, most likely there will be several such scales (with the lowest being the true cutoff of the SM). So far, we only have indirect hints about such scales, coming from neutrino oscillations and gravity.

At $d=5$ there is only one irrelevant operator (Weyl ferm. notation)

$$\frac{\kappa_{ij}}{\Lambda}(L_{l_i} H)(L_{l_j} H) + h.c. \qquad (1)$$

family index

It violates family (and global) Lepton-number ($\Delta L = 2$). After EWSB it generates Majorana neutrino masses, with

$$m_\nu \sim \frac{\kappa}{\Lambda} v^2$$

and the flavor structure of κ_{ij} can describe the observed ν oscillations. Assuming $\kappa \sim O(1)$ and $m_\nu \sim O(0.1\,eV) \Rightarrow \Lambda \sim 10^{15}\,GeV$.

It is tantalizing that this might be the first evidence of such remnants of BSM physics in irrelevant operators, perhaps to be expected as, at $d=5$ is the lowest irrel. op. However, notice that we cannot exclude light ν_R's as an explanation, through the Lagrangian terms

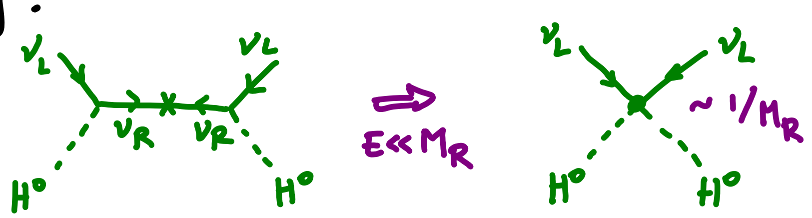
$$\mathcal{L}_{\nu_R} = \nu_R^\dagger i\bar{\sigma}\partial\nu_R + \left(Y_\nu \bar{L}_L \tilde{H}\nu_R + h.c.\right)$$

with $m_\nu \sim Y_\nu v$, even if $m_\nu \sim O(0.1)\,eV$ requires $Y_\nu \sim 10^{-13}$, hardly appealing as an explanation for the smallness of m_ν. However, as ν_R is a gauge singlet, we can add a Majorana mass term (allowed by gauge symmetry) to our Lagrangian

$$\delta\mathcal{L}_{\nu_R} = \frac{1}{2}M_R\,\nu_R^T \cdot \nu_R + h.c.$$

SU(2) product $i\sigma_2$

M_R has nothing to do with the EW scale and it's natural to expect it to be $M_R \gg M_{EW}$. Interestingly, if we remove it from the low-E EFT, it leaves behind the $d=5$ operator (1). This is the famous "see-saw" mechanism. Diagrammatically:

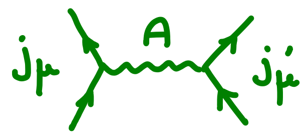


$\Rightarrow m_\nu \sim \dfrac{y_\nu^2 v^2}{M_R}$

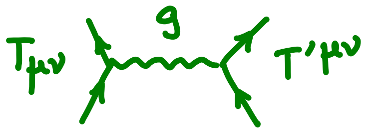
This is the simplest example of UV physics that could generate such operator explaining naturally the small value of m_ν, but it's not the only one, see Ex.3

The example also illustrates a generic difficulty in extracting from the effect of an irrelevant operator the mass of the heavy states that generate it. In this example, one gets $M_R \sim \frac{y_\nu^2 v^2}{m_\nu}$, which depends on the unknown y_ν's. For $y_\nu \sim O(y_t)$ we recover our previous estimate $M_R \sim 10^{15}\,\mathrm{GeV}$ but this could be much lower (eg $M_R \sim 1\,\mathrm{TeV}$ for $y_\nu \sim y_e$).

Gravity should also be considered as physics BSM. In the language of particle physics the gravitational interaction is due to the exchange of spin-2 massless particles, gravitons ($\gamma_{\mu\nu}$) which couple to the stress-energy momentum tensor $T_{\mu\nu}$ (basically to mass, at low energy). Such interaction is non-renormalizable, that is, it corresponds to an irrelevant operator. In analogy with the electromagnetic interaction



$\Rightarrow \quad j_\mu \frac{1}{k^2} j'^\mu \quad \Rightarrow \quad F_{em} = e^2 \frac{Q_1 Q_2}{r^2}$



$\Rightarrow \quad T^{\mu\nu} \underbrace{P_{\mu\nu,\rho\sigma}}_{\text{graviton propagator}} T'^{\rho\sigma} \quad \Rightarrow \quad F_g = G_N \frac{m_1 m_2}{r^2}$

Newton's constant $G_N = \frac{1}{8\pi m_P^2}$ with $m_P = 2.4\times 10^{18}\,\mathrm{GeV}$ sets the huge scale that suppresses this interaction.

In terms of Lagrangians, gravity is incorporated to the SM EFT by adding the pure gravity term $\Delta\mathcal{L} = \int d^4x \sqrt{-g}\, R$, where $g = \mathrm{Det}\, g_{\mu\nu}$ and $R \sim \partial^2 g$ is the Ricci scalar, and modifying

$$\int d^4x\, \mathcal{L}_{SM} \rightarrow \int d^4x \sqrt{-g}\, \mathcal{L}'_{SM}$$

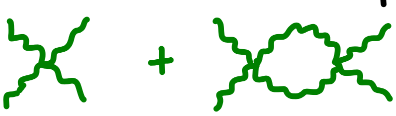
where $\mathcal{L}'_{SM}$ is just the $\mathcal{L}_{SM}$ made covariant so that $\partial_\mu\phi\,\partial^\mu\phi \rightarrow g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$, with $g^{\mu\nu}g_{\nu\rho} = \delta^\mu_\rho$ and so on. This is a nonrenormalizable theory and one can get the graviton interactions by rewriting the metric as

$$g_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{\text{flat background } \mathrm{diag}(1,-1,-1,-1)} + \frac{1}{m_P}\overbrace{\gamma_{\mu\nu}(x)}^{\text{graviton field}}$$

The kinetic term for $\gamma_{\mu\nu}$ comes from R (plus gauge fixing!) and the $1/m_P$ ensures the proper canonical normalization:

$$\mathcal{L}_{grav} = (\partial\gamma)^2 + \frac{1}{m_P^2}(\partial\gamma)^2 + \ldots$$

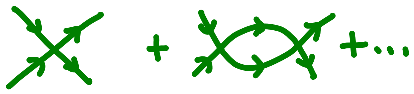
All interactions of the graviton with matter (coming from $\sqrt{-g}\,\mathcal{L}'_{SM}$) will carry this same scale suppression. This extended Lagrangian is the quantum theory of gravity at low-energy. This is an EFT that breaks down at a scale fixed by m_P beyond which we no longer have control over quantum gravity effects and we do not know the relevant theory that takes over. To be more precise, one can estimate the regime in which gravity becomes strongly coupled by checking when do loop effects become of the same order as tree-level ones. For example, looking at $gg \rightarrow gg$



$$+ \ldots = \left(\frac{E}{m_P}\right)^2 + \frac{1}{16\pi^2}\left(\frac{E}{m_P}\right)^4 + \ldots$$

one gets for that limiting scale $E \sim \Lambda_{grav} = 4\pi m_P \sim 10^{19}\,\mathrm{GeV}$.
This is the maximal scale below which a fully fledged quantum theory must enter and supersede the SM.

Λ_{max} in EFT The same exercise we did to estimate the upper bound on the cutoff for gravity can be done for any EFT. Eg, for Fermi theory:

 $= G_F E^2 + \frac{1}{16\pi^2}(G_F E^2)^2 + \ldots$

leads to $\Lambda_F \simeq 4\pi/\sqrt{G_F} = 4\pi v \sim$ few TeV. In this case, we know the UV theory that completes the Fermi EFT and Λ_F is not saturated: $W^\pm$ and Z^0 appear much below so that the UV completion is in fact weakly coupled.
The non-linear σ model describing pion interactions is an example in which the cutoff is saturated, with an strongly coupled UV completion: QCD.

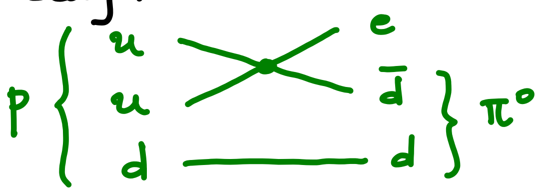
$\mathcal{L}_{d=6}$ ops. ($\not{B}$, $\not{L}_i$, FCNCs, Custodial,...) Bounds

At $d=6$ there are many irrel. operators. It is a laborious but simple exercise to build them out of the SM fields and respecting Poincaré and gauge symmetries. Among all these ops. some violate the accidental symmetries of $\mathcal{L}_{d\leq 4}$ we discussed before. Agreement with experiment requires that the suppression scale Λ for those ops. is much higher than the EW scale. Examples of such ops. are :

$\cancel{B}$ Ops. For instance, the 4-fermion op.

$$\frac{g_i^2}{\Lambda^2}(\bar{u}^c_R\gamma^\mu Q_L)(\bar{L}^c_L\gamma_\mu d_R)$$

violates baryon number by $\Delta B = 1$. It contributes directly to proton decay :



$\Rightarrow \Gamma_p = c \cdot \frac{g_i^4}{\Lambda^4} m_p^5$

As $p \to e^+\pi^0$ is extremely well constrained, with $\tau_p \gtrsim 8.2 \times 10^{33}\,\mathrm{yr}$, one gets the lower bound $\Lambda \gtrsim 10^{16}\,\mathrm{GeV} \times g_i$.

This is roughly compatible with the scale at which gauge couplings tend to unify. GUTs would typically produce such $\cancel{B}$ ops and observation of proton decay would give a very strong boost to such ideas.

Note also that the above op. has $\Delta(B-L) = 0$. All $d=6$ ops. satisfy that constraint. Ops that violate B only (like those leading to $n-\bar{n}$ oscillations) appear only at higher order and, therefore, are expected to be much more suppressed.

$\cancel{L_i}$ Ops. There are ops. that conserve L but violate individual L_i number. E.g.

$$\frac{g_i^2}{\Lambda^2}\bar{L}_L\sigma^{\mu\nu}\mu_R H^\dagger F_{\mu\nu}$$

which leads to $\mu \to e\gamma$ decay. Here $BR(\mu \to e\gamma) < 5.7 \times 10^{-13}$ translates into $\Lambda \gtrsim 10^6\,\mathrm{GeV} \times g_i$.

Notice that neutrino masses already violate L_i as we see in ν-oscillations. This violation propagates to other sectors of the theory via loops and in fact there is a contribution to $\mu \to e\gamma$ decay, which is nevertheless extremely small. Ex 4.

Flavor changing OPs for quarks.

Consider a neutral meson

same (light) type

$q_{i,j} \sim$ up or down

$$M^0 = q_i \bar{q}_j$$

diff. family

e.g. $u \overset{D^0}{\longleftrightarrow} c$

$d \overset{K^0}{\longleftrightarrow} s \overset{B_s}{\longleftrightarrow} b$, $\quad d \overset{B^0}{\longleftrightarrow} b$

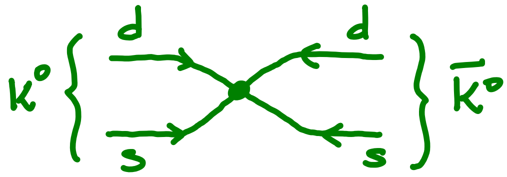
The anti-meson $\bar{M}^0$ is $\bar{q}_i q_j$.

If the amplitude $M^0 \to \bar{M}^0$ is nonzero, this acts as an off-diagonal entry in their mass matrix inducing a mass-splitting between the two mesons that can be probed experimentally. Eg. $\Delta m_K / m_K \sim 7 \times 10^{-15}$.

An op. like

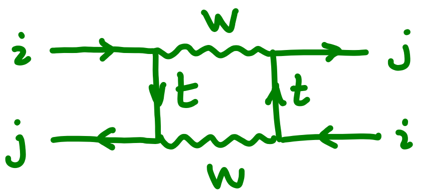
$$\frac{g_i^2}{\Lambda^2} (\bar{s}_R \gamma_\mu d_R)(\bar{s}_R \gamma^\mu d_R)$$

generates such mass splitting :



And one can extract the lower bound $\Lambda \gtrsim 10^6$ GeV. Similar bounds (a bit lower) come from other mesons.

Remember that the flavor symmetry is already violated by $\mathcal{L}_{d=4}$. This implies there is a contribution to Δm_K even in the absence of $d=6$ operators. At 1-loop we have



$$\rightarrow \underbrace{\frac{|V_{ti}V_{tj}|^2}{16\pi^2}}_{10^{-6},\ 10^{-10}} \frac{1}{M_W^2}$$

suppression factor (as $V_{CKM} \sim \mathbb{I}$)

The $d=6$ contribution $\sim g_i^2/\Lambda^2$ would be comparable for $\Lambda \sim 10^{5-7}\,\mathrm{GeV}$. We see that, if new physics BSM at $\Lambda \sim$ few TeV had a generic flavor structure we should have seen its effects already. This is the so-called "flavor problem". We should be careful though because there can be suppression effects in the coefficients of the $\mathcal{L}_{d=6}$ ops. E.g. they might be generated at loop-order only or flavor changing might be suppressed, as it is in the SM, by the symmetries of the UV theory (eg MFV).

Custodial breaking

Some $d=6$ ops can affect the good fit of EW data achieved by $\mathcal{L}_{d\leq 4}$. One typical example is the operator

$$\frac{c_T}{2}\frac{g_H^2}{\Lambda^2}|H^\dagger \overleftrightarrow{D}_\mu H|^2$$

which violates custodial symmetry and upsets the relation

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

as this operator (after EWSB) contributes only to M_Z as

$$M_Z^2 = \frac{1}{4}(g^2+g'^2)\left(1 - c_T g_H^2 \frac{v^2}{\Lambda^2}\right)$$

The ρ parameter deviates from 1 at the percent level due to radiative corrections from pure SM loops ($\mathcal{L}_{d\leq 4}$) and is known experimentally at the per mille level. This allows to bound the operator above as

$$g_H^2 c_T v^2/\Lambda^2 \lesssim O(10^{-3})$$

[It is convenient to write such bounds as a bound on the op. coefficient choosing $\Lambda = M_W$ (rather than, say, the arbitrary $\Lambda = 1\,\text{TeV}$) as this allows a more straightforward comparison with typical SM effects. That is, we will bound $c_i M_W^2/\Lambda^2$]

Such custodial breaking effects can be deadly for BSM theories at the TeV scale and it is customary to impose custodial symmetry on them to make them viable.

There are other ops. which are well constrained because they contribute to well measured quantities, even if no violation of $\mathcal{L}_{d=4}$ symmetries are involved. Examples are contributions to the muon g-2 anomalous magnetic moment or $e^+e^- \to l^+l^-$ LEP data, etc. We will discuss the hierarchy of constraints on $d=6$ ops. in the next lecture.

Hierarchy problem

We have many good reasons to think there must be new physics

BSM, like the evidence for dark matter, the need to explain inflation, the matter-antimatter asymmetry, and the very existence of gravity. The scale for such new physics Λ, where the SM description fails, is unknown and negative results at the LHC push it up to $O(\mathrm{TeV})$. The trouble with a very large Λ, $\Lambda \gg M_{EW}$ is that in such theory it is not natural to have a scalar like the Higgs with mass $m \sim M_{EW} \ll \Lambda$, as we discussed before. This problem is not an inconsistency of the SM but rather a finetuning problem. To be more precise, if we knew the full UV theory we could calculate the Higgs mass, including radiative corrections to it. We can split the final result in two pieces

$$M_h^2 = \Delta_{low} M_h^2 + \Delta_{high} M_h^2$$

where $\Delta_{low} M_h^2$ includes corrections from light modes below the SM cutoff Λ and $\Delta_{high} M_h^2$ from heavy modes. The first piece is calculable in the SM, coming from loops of SM particles, and reads

$$\Delta_{low} M_h^2 \simeq \frac{3}{64\pi^2}\left(3g^2 + g'^2 + 8\lambda - 8h_t^2\right)\Lambda^2$$

The UV piece is unknown but, assuming Λ represents the typical BSM scale we expect $\Delta_{high} M_h^2 = \kappa \Lambda^2$. (In general κ will depend on UV couplings not even present in the IR EFT.) The hierarchy problem is the problem of explaining the

cancellation between these two contributions to M_h^2 and it's harder to explain the larger Λ is, as then $\Delta_{\rm low} M_h^2$, $\Delta_{\rm high} M_h^2 \gg M_h^2$ and the cancellation seems more unlikely.
The degree of tuning involved is conventionally measured as

$$\Delta = \frac{\Delta_{\rm low} M_h^2}{M_h^2} \quad \text{or} \quad \frac{\Delta_{\rm high} M_h^2}{M_h^2}$$

and represents a tuning of 1 part in Δ. With $\Lambda \sim$ few TeV this number corresponds to more than 1% tuning.
The main two classes of models that address this naturalness problem are SUSY (that makes M_h natural by linking the Higgs to fermions, of mass protected by chiral symmetry) and composite Higgs models (in which the Higgs is a composite with a finite size, that cuts-off the dangerous Λ^2 corrections to M_h^2). In both, however, Λ should not be $\gg 1$ TeV. Natural explanations for the hierarchy problem require new physics not far from the TeV range, hopefully on the reach of the LHC. Such physics should "talk" to the Higgs (as it fixes the Higgs hierarchy problem) and for Λ not too far from the TeV should leave an imprint on the Higgs properties that we could measure as a deviation from SM predictions. This indirect probe of BSM could reach higher in E than direct searches and is an excellent place to look for $\mathcal{L}_{d=6}$ effects.