Supersymmetry

– Lecture Benasque, Sept. 2014, 3 hours –

Dominik Stöckinger, TU Dresden

Plan:

- Aim: understand structure of SUSY theories: SUSY-QED, ..., MSSM, ...
- fundamental motivation
- "guess" SUSY-QED $\rightsquigarrow$ important lessons
- derivation:
    - SUSY algebra
    - superspace, superfields
    - SUSY Lagrangian
- applications

Special Relativity [Einstein, 1905]

- "Nature is Lorentz invariant"

QFT:

Lorentz transf. (rotation etc)

$$\begin{array}{llllllll}
\text{boson:} & 0 & \longleftrightarrow & 1 & \longleftrightarrow & 2 & \longleftrightarrow & \ldots \\
\text{fermion:} & \frac{1}{2} & \longleftrightarrow & \frac{3}{2} & \longleftrightarrow & \frac{5}{2} & \longleftrightarrow & \ldots
\end{array}$$

Wouldn't it be more natural to have a symmetry with $\Delta J = \frac{1}{2}$?

$$0 \leftrightarrow \frac{1}{2} \leftrightarrow 1 \leftrightarrow \frac{3}{2} \leftrightarrow 2 \leftrightarrow \frac{5}{2} \leftrightarrow \ldots$$

Yes, indeed

$\longrightarrow$ this is supersymmetry (SUSY) — a unique F–B symmetry

1. Introduction

1.1 Motivation

SUSY = fundamental new symmetry of relativistic QFT

- changes spin : $\Delta j = \pm \frac{1}{2}$
- Fermions $\Longleftrightarrow$ Bosons
- $Q \mathrel{\hat{=}} \sqrt{\text{Poincaré}}$

$\rightsquigarrow$ improves QFT, maybe Quantum Gravity
$\rightsquigarrow$ relation to cosmological constant

1. Introduction

1.1 Motivation

SUSY = fundamental new symmetry of relativistic QFT

- changes spin: $\Delta j = \pm \frac{1}{2}$
- Fermions $\Leftrightarrow$ Bosons
- $Q \mathrel{\hat{=}} \sqrt{\text{Poincaré}}$

$\leadsto$ improves QFT, maybe Quantum Gravity
$\leadsto$ relation to cosmological constant

Phenomenological motivation for SUSY at TeV-scale:

- fine tuning problem of M_{Higgs}
- radiative electroweak symmetry breaking

$\Big\}$ "understand scalars"!

- unification of gauge couplings
- dark matter candidate

1.2 "Guess" SUSY-QED – Important aspects of SUSY

$$\mathcal{L}_{QED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}\,(i D^{\mu} \gamma_{\mu} - m)\,\psi \quad , D^{\mu} = \partial^{\mu} + ie A^{\mu}$$

1st guess: need selectron (charged scalar), same mass / interaction as electron

$\longrightarrow$ scalar field ϕ and

$$\mathcal{L}_{\phi} = |D^{\mu} \phi|^2 - m^2 |\phi|^2$$

$\longrightarrow$ not enough!

ψ: 4 deg. of freedom $(e^{\pm}_{L,R})$

ϕ: 2 deg. of freedom

1.2 "Guess" SUSY-QED – Important aspects of SUSY

$$\mathcal{L}_{QED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\Psi}(i D^\mu \gamma_\mu - m)\Psi \quad , D^\mu = \partial^\mu + ie A^\mu$$

1st guess: need selectron (charged scalar), same mass / interaction as electron

$\longrightarrow$ scalar field ϕ and

$$\mathcal{L}_\phi = |D^\mu \phi|^2 - m^2 |\phi|^2$$

$\longrightarrow$ not enough! $\quad \Psi$: 4 deg. of freedom $(e^\pm_{L,R})$

ϕ: 2 deg. of freedom

2nd guess: need same # deg. of freedom for selectrons

$\longrightarrow$ two scalar fields $A_L, A_R \quad \hat{=} \quad \tilde{e}^\pm_L, \tilde{e}^\pm_R$

$$\mathcal{L}_{A_L} + \mathcal{L}_{A_R}$$

$\longrightarrow$ not enough !

3rd guess: Need photino ! $\quad \tilde{\gamma} = \tilde{\gamma}^c$ (Majorana spinor), massless

$$\mathcal{L}_{\tilde{\gamma}} = \overline{\tilde{\gamma}}\, i\cancel{\partial}\, \tilde{\gamma}$$

$\longrightarrow$ not enough ! $\qquad$ Interactions ?

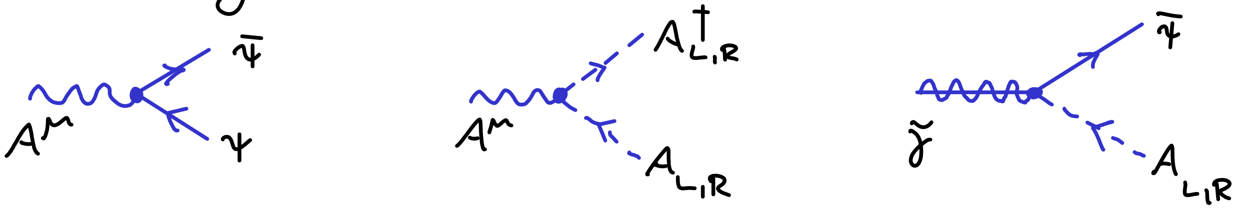
$\longrightarrow$ not enough!

3rd guess: Need photino! $\tilde{\gamma} = \tilde{\gamma}^c$ (Majorana spinor), massless

$$\mathcal{L}_{\tilde{\gamma}} = \bar{\tilde{\gamma}}\, i\partial\!\!\!/\, \tilde{\gamma}$$

$\longrightarrow$ not enough! Interactions?

4th guess:

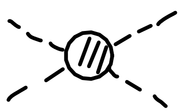
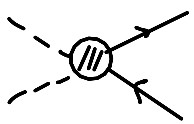


all "equal"

$$\mathcal{L}_{\tilde{\gamma}\text{-int}} = -\sqrt{2}\, e \left(\bar{\tilde{\gamma}} P_L \psi A_L^\dagger - \bar{\tilde{\gamma}} P_R \psi A_R^\dagger + h.c. \right)$$

$\longrightarrow$ still not enough! Need also

$$\mathcal{L}_{\phi^4} = -\frac{e^2}{2} \left(|A_L|^2 - |A_R|^2 \right)^2$$

(Then: "  =  ")

Result : $\mathcal{L}_{SUSY-QED} = \mathcal{L}_{QED} + \mathcal{L}_{A_L} + \mathcal{L}_{A_R} + \mathcal{L}_{\tilde{\gamma}} + \mathcal{L}_{\tilde{\gamma}\text{-int}} + \mathcal{L}_{A^4}$

Important lessons :

- superpartners for all particles, same # deg. of freedom
- same mass, interactions
- $(\text{scalar})^4$ - terms predicted ($\rightsquigarrow$ Higgs mass/quartic fixed!)
- leads to rigid, but complex, structure of $\mathcal{L}$

$\longrightarrow$ Now: prove everything

introduce formalism of superfields

2. SUSY-Algebra

Symmetries in quantum theory $\longrightarrow$ operators on Hilbert space of states

e.g. rotational invariance $\longrightarrow$ angular momentum $\vec{J}$

$$[J_x, J_y] = i J_z \quad \text{etc.}$$

rotation e.g. $|\Psi\rangle \longrightarrow e^{i\alpha J_z} |\Psi\rangle$

translational $(\vec{x}, t)$ invariance $\longrightarrow$ momentum operator P^μ

translation e.g. $|\Psi\rangle \rightarrow e^{i a^\mu P_\mu} |\Psi\rangle$

Relativistic QFT : need P^μ (translations)

$J^{\mu\nu}$ (rotations & boosts)

SUSY: need operator Q

- Fermions $\leftrightarrow$ Bosons $\Longrightarrow$ Q is fermionic (anticommuting)
- $\Delta J = \pm\frac{1}{2}$ $\Longrightarrow$ Q must transform as a relativistic spinor
  $\rightsquigarrow$ simplest: Majorana-spinor

$$\Longrightarrow \quad [Q, P^\mu] = 0,$$
$$[Q, J^{\mu\nu}] = \text{fixed}$$

- algebra $\{Q_\alpha, \bar{Q}_\beta\}$ must be
  - symmetry
  - transforms like $\Psi_\alpha \bar{\Psi}_\beta$
  - cannot vanish since positive definite

$$\Rightarrow \qquad = 2\,\gamma^\mu_{\alpha\beta}\, P_\mu \quad \text{(up to normalization)}$$

SUSY - algebra result:

$$[Q, P^\mu] = 0$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\gamma^\mu_{\alpha\beta} P_\mu$$

$$\tilde{Q} = Q^c \quad = \text{Majoranaspinor}$$

proof that this is most general possibility: Haag, Lopuszanski, Sohnius.

SUSY - algebra result :

$$[Q, P^\mu] = 0$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\gamma^\mu_{\alpha\beta} P_\mu$$

$$Q = Q^c = \text{Majoranaspinor}$$

proof that this is most general possibility : Haag, Lopuszanski, Sohnius.

Direct consequences :

- $[Q, P^2] = 0$ → equal rest mass of particle/superpartner
- multiply algebra by $\gamma^0_{\beta\alpha}$ ⇒ $\{Q_\alpha, Q^\dagger_\alpha\} = 2\,\mathrm{Tr}(\gamma^\mu\gamma^0) P_\mu$

$$Q_1 Q_1^\dagger + \ldots + Q_4^\dagger Q_4 = 8 \text{ Hamiltonian}$$

- " $Q \hat{=} \sqrt{H}$ " → $H \geq 0$, relation to cosm. const.
- on states with $E \neq 0$: "Q invertible" ⟹ # F = # B

3. Superspace and superfields

3.1 Construction

Aim: SUSY QFT $\longrightarrow$ made much simpler by beautiful formalism

3. Superspace and superfields

3.1 Construction

Aim: SUSY QFT $\longrightarrow$ made much simpler by beautiful formalism

So far: abstract operators : $\vec{J}$: rotations

$P^\mu, J^{\mu\nu}$: translations, Lorentz

Q : susy

Idea: translations, rotations etc described much simpler by

$$\left.\begin{aligned} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &\longrightarrow R(\alpha,\beta,\gamma)\begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ x^\mu &\longrightarrow x^\mu + a^\mu \end{aligned}\right\}$$ transformations on suitable coordinate space

3. Superspace and superfields

3.1 Construction

Aim: SUSY QFT $\longrightarrow$ made much simpler by beautiful formalism

So far: abstract operators : $\vec{J}$: rotations

$P^\mu, J^{\mu\nu}$: translations, Lorentz

Q : susy

Idea: translations, rotations etc described much simpler by

$$\left.\begin{aligned} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &\longrightarrow R(\alpha,\beta,\gamma)\begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ x^\mu &\longrightarrow x^\mu + a^\mu \end{aligned}\right\} \text{transformations on suitable coordinate space}$$

$\Rightarrow$ Hope: susy :

$$\left\{\begin{aligned} x^\mu &\longrightarrow x^\mu + \dots \\ \theta &\longrightarrow \theta + \xi \end{aligned}\right\} \longleftarrow \text{new coordinate \& transf. parameter}$$

$\Longrightarrow$ Need to know relation between coordinate / operator representation

Operator $\Leftrightarrow$ coordinate for translations:

(1) coordinate translation by a^μ: $\quad x^\mu \longrightarrow x'^\mu := x^\mu + a^\mu$

(2) define fields on coordinate space: $\quad \phi(x)$

$$\phi \longrightarrow \phi' \text{ with } \phi'(x') = \phi(x)$$

[math.: linear representation of translation group]

(3) define translation operators on space of functions: $\quad e^{i a^\mu P_\mu}$

$$\phi' = e^{i a^\mu P_\mu} \phi$$

$$\Rightarrow \quad \phi(x-a) = e^{i a^\mu P_\mu} \phi(x) \quad \forall \phi$$

$$\Rightarrow \quad P_\mu = i \partial_\mu \qquad \text{(Taylor operator)}$$

$\Rightarrow$ the operators constructed in this way define the commutation rules of the abstract operators.

For SUSY: go backwards!

Operator $\leftrightarrow$ coordinate for SUSY:

(3) abstract algebra $\rightarrow$ differential operators on some space

(2) how do they act on the arguments of fields

(1) coordinate transformations

Operator $\Leftrightarrow$ coordinate for SUSY:

(3) abstract algebra $\rightarrow$ differential operators on some space
(2) how do they act on the arguments of fields
(1) coordinate transformations

(3): which space?

Q anticommuting, spinor $\Longrightarrow$ fermionic spinor coordinate

$\{Q, \bar{Q}\}$ = translation $\Rightarrow$ describe susy + translations

→ superspace coordinates

$\left\{ \begin{matrix} x^\mu \\ \theta \end{matrix} \right\}$ ← 4 bosonic dimensions
← fermionic Majorana spinor 4 fermionic dimensions

Diff. operators:

$$Q := i\left(\frac{\partial}{\partial\bar{\theta}} + i\gamma^\mu\theta\partial_\mu\right), \quad \bar{Q} := i\left(-\frac{\partial}{\partial\theta} - i\bar{\theta}\gamma^\mu\partial_\mu\right)$$

$$P^\mu := i\partial^\mu$$

⟶ satisfy the susy algebra (check!)

(2) field transformations :

$$\boxed{\text{superfields} \quad F(x,\vartheta) \;=\; \text{fields on superspace}}$$

SUSY: $\quad F \longrightarrow F' := e^{i\bar{Q}\xi} F \quad , \; \xi = $ fermionic Majorana spinor

evaluate:

$$F'(x,\vartheta) = e^{-(-\frac{\partial}{\partial\vartheta}\xi \; - i\bar{\vartheta}\gamma^\mu\xi\,\partial_\mu)} F(x,\vartheta)$$

$$= F(x^\mu + i\bar{\vartheta}\gamma^\mu\xi \;,\; \vartheta - \xi)$$

(1) identify with $\quad F(x^\mu - \Delta x^\mu \;,\; \vartheta - \xi)$

↝ SUSY coordinate transformation on superspace:

$$\boxed{\begin{Bmatrix} x^\mu \\ \vartheta \end{Bmatrix} \longrightarrow \begin{Bmatrix} x'^\mu = x^\mu + \Delta x^\mu \\ \vartheta' = \vartheta + \xi \end{Bmatrix}}$$

units : $\quad [x] = \frac{1}{\text{Mass}} = [\vartheta^2] \quad , \quad [\vartheta] = \frac{1}{\sqrt{\text{Mass}}}$

What is this good for?

- Nice and easier than $\{Q, \bar{Q}\} = \dots$
- construct susy theories : e.g.

define many superfields $F_1, \dots, F_n$
$\mathcal{L} = \mathcal{L}(F_1 \dots F_n)$

$\Rightarrow \int d^4x \, d^4\theta \, \mathcal{L}$ is susy invariant
(provided $\int d^4\theta$ is transl. invariant)

$\int d^4\theta \, \mathcal{L}$ only depends on x
$\rightsquigarrow$ ordinary Lagrangian

3.2 Types of superfields

Fields with special properties $\longrightarrow$ special theories

3.2 Types of superfields

Fields with special properties $\longrightarrow$ special theories

First: properties of ϑ and functions of ϑ:

a) $$\vartheta = P_L \vartheta + P_R \vartheta =: \vartheta_L + \vartheta_R$$ $\longrightarrow \vartheta_{L,R}$ don't mix under Lorentz/SUSY

$$\vartheta = \vartheta^c = i\gamma^0\gamma^2 \bar{\vartheta}^T$$

$$\overline{\vartheta_R} = \bar{\vartheta} P_L = \overline{\vartheta^c} P_L = \vartheta^T \overline{i\gamma^0\gamma^2} P_L = (P_L\vartheta)^T \overline{i\gamma^0\gamma^2} =: \vartheta_L^T \epsilon$$

b) $\vartheta_L \sim \overline{\vartheta_R}^T$ contains two indep. entries; each $(\text{entry})^2 = 0$

$$\Longrightarrow \quad \vartheta_L \ldots \vartheta_L \ldots \vartheta_L = \vartheta_L \ldots \overline{\vartheta_R} \ldots \vartheta_L = 0$$

any $$f(\vartheta_L) = a + \overline{\vartheta_R}\, \Psi_L + \underbrace{\overline{\vartheta_R}\vartheta_L}_{=\vartheta_L^T \epsilon \vartheta_L}\, b$$

c) ϑ contains four indep. entries: any $f(\vartheta) = a + \ldots + (\bar{\vartheta}\vartheta)(\bar{\vartheta}\vartheta) b$

Interlude: Weyl spinors

SUSY $\Longrightarrow$ spinors important

Weyl = minimal Lorentz covariant spinors

γ-matrices: $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}$, $P_L = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}$, $P_R = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix}$

4-spinor $\Psi = \begin{pmatrix} \psi_{L\alpha} \\ \bar\psi_R^{\dot\alpha} \end{pmatrix}$ $\quad \Psi_L = \begin{pmatrix} \psi_{L\alpha} \\ 0 \end{pmatrix}$ $\quad \Psi_R = \begin{pmatrix} 0 \\ \bar\psi_R^{\dot\alpha} \end{pmatrix}$

$\Psi^c = \begin{pmatrix} \psi_{R\alpha} \\ \bar\psi_L^{\dot\alpha} \end{pmatrix}$ $\quad \theta = \begin{pmatrix} \theta_\alpha \\ \bar\theta^{\dot\alpha} \end{pmatrix}$

$\bar\Psi = \begin{pmatrix} \psi_R^\alpha & \bar\psi_{L\dot\alpha} \end{pmatrix}$, $\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta$, $\varepsilon^{\alpha\beta}$ = antisymm.

mass term $m\bar\Psi\Psi = m(\bar\Psi_R\Psi_L + \bar\Psi_L\Psi_R)$

$= m(\psi_R^\alpha\psi_{L\alpha} + \bar\psi_{L\dot\alpha}\bar\psi_R^{\dot\alpha})$

kinetic term $\bar\Psi\gamma^\mu\Psi = \bar\Psi_L\gamma^\mu\Psi_L + \bar\Psi_R\gamma^\mu\Psi_R$

$\bar\Psi_L\gamma^\mu\Psi_L = \bar\psi_L\bar\sigma^\mu\psi_L = \bar\psi_L(-\sigma^2\sigma^{\mu T}\sigma^2)\psi_L = -\psi_L\sigma^\mu\bar\psi_L$

general superfields: any function $F(x, \theta)$

"vector" superfields: real function : $V(x,\theta) = V^\dagger(x,\theta)$

general superfields: any function $F(x,\theta)$

"vector" superfields: real function: $V(x,\theta) = V^\dagger(x,\theta)$

expansion in θ: $V(x,\theta) = a(x) + \ldots + \overline{\theta_L}\,\gamma^\mu \theta_L\, V_\mu(x) + \ldots$

$\longrightarrow$ contains vector field $V_\mu(x)$

unit: $[\overline{\theta}_L \gamma^\mu \theta_L V_\mu] = (\text{Mass})^0 \rightarrow$ dimensionless

search for simpler superfields: try $\mathbb{F}_{try} := \{ f(x,\theta_L) \}$

$f(x,\theta_L) \xrightarrow{\text{susy}} f(x + i\bar{\theta}\gamma^\mu\xi,\ \theta_L - \xi_L)$

$\hookrightarrow$ depends also on θ_R

$\notin \mathbb{F}_{try}$

chiral superfields:

$$\mathbb{F}_{ch,L} := \left\{ f\left(x^\mu - i\,\overline{\vartheta}_R \gamma^\mu \vartheta_R \,,\; \vartheta_L\right) \right\}$$

$$f \xrightarrow{\text{susy}} f\left(x^\mu + i\,\overline{\vartheta}\gamma^\mu \xi - i\,\overline{\vartheta_R - \xi_R}\,\gamma^\mu(\vartheta_R - \xi_R) \,,\; \vartheta_L - \xi_L\right)$$

$$= f\left(x^\mu - i\,\overline{\vartheta}_R \gamma^\mu \vartheta_R + 2i\,\underbrace{\overline{\vartheta}_R}_{\sim\vartheta_L} \gamma^\mu \xi_R \,,\; \vartheta_L - \xi_L\right)$$

$$= g\left(x^\mu - i\,\overline{\vartheta}_R \gamma^\mu \vartheta_R \,,\; \vartheta_L\right)$$

$$\in \mathbb{F}_{ch,L}$$

chiral superfields: $\quad \mathbb{F}_{ch,L} := \left\{ f\left(x^\mu - i\,\overline{\vartheta}_R \gamma^\mu \vartheta_R \,,\ \vartheta_L \right) \right\}$

$$f \xrightarrow{\text{susy}} f\left(x^\mu + i\,\overline{\vartheta}\gamma^\mu \xi - i\,\overline{\vartheta_R - \xi_R}\,\gamma^\mu(\vartheta_R - \xi_R) \,,\ \vartheta_L - \xi_L \right)$$

$$= f\left(x^\mu - i\,\overline{\vartheta}_R \gamma^\mu \vartheta_R + 2i\,\overline{\vartheta}_R \gamma^\mu \xi_R \,,\ \vartheta_L - \xi_L \right)$$

$\sim \vartheta_L$

$$= g\left(x^\mu - i\,\overline{\vartheta}_R \gamma^\mu \vartheta_R \,,\ \vartheta_L \right)$$

$$\in \mathbb{F}_{ch,L}$$

$\longrightarrow$ susy invariant set of simpler superfields

$\longrightarrow$ sums/products are again chiral, but $f^\dagger$ is not!

expansion in ϑ: $\quad \Phi \in \mathbb{F}_{ch,L}$: $\quad \Phi(x,\vartheta) = \phi\big(\underbrace{x^\mu - i\,\overline{\vartheta}_R \gamma^\mu \vartheta_R}_{=: y^\mu}, \vartheta_L\big)$

where

$$\phi(y, \vartheta_L) = A(y) + \sqrt{2}\,\overline{\vartheta}_R\, \psi_L(y) + \overline{\vartheta}_R \vartheta_L\, F(y)$$

Lorentz scalar $\qquad$ L-spinor $\qquad$ scalar

Exercise 3a, 4a!

3.3 Lagrangians

Suppose we define integral $\int d^4\theta = \int d^2\theta_L \int d^2\theta_R$ such that

$$\int d^4\theta \; f(\theta) = \int d^4\theta \; f(\theta+\xi) \quad \text{etc.}$$

$\Longrightarrow$ easy to construct susy Lagrangians out of vector and chiral superfields Φ_i

general: $\int d^4x \, \mathcal{L} = \int d^4x \int d^4\theta \; \underbrace{K \quad [\text{arbitrary combination}]}_{= \text{superfield}}$

chiral: $\int d^4x \, \mathcal{L} = \int d^4x \int d^2\theta_L \; \underbrace{W \quad [\text{arbitrary polynomial in } \Phi_i \text{ (not } \Phi_i^\dagger)]}_{= \text{"superpotential"} \; = \text{chiral}}$

Summary and lessons:

- Susy transformations become simple on superspace $\left\{ \begin{matrix} x^\mu \\ \theta \end{matrix} \right\}$

- Superfields have simple transformations

  chiral s.f. only depend on θ_L explicitly

- $\int d^4\theta$, $\int d^2\theta_L$ integrals yield susy Lagrangians

Summary and lessons:

- Susy transformations become simple on superspace $\left\{ \begin{matrix} x^\mu \\ \theta \end{matrix} \right\}$

- Superfields have simple transformations

  chiral s.f. only depend on θ_L explicitly

- $\int d^4\theta$, $\int d^2\theta_L$ integrals yield susy Lagrangians

Tasks now:

- define integrals (translationally invariant!)

- evaluate integrals to obtain explicit form of $\mathcal{L}$

4. Detailed Lagrangians

4.1 Detailed evaluations (no vector superfields)

Integral definitions: [first only one ϑ-component]

we want $\int d\vartheta \; f(\vartheta + \xi) = f(\vartheta)$

but we know $f(\vartheta) = a + b\,\vartheta$ since $\vartheta^2 = 0$

therefore, $f(\vartheta + \xi) = a + b\,\xi + b\,\vartheta$

$\Longrightarrow$ only possibility: $\int d\vartheta \; f(\vartheta) := b$ (times normalization)

$\int d\vartheta \sim \frac{\partial}{\partial \vartheta}$ for fermionic variables

Similarly:

$$\int d^2\theta_L \; f(\theta_L) := f\Big|_{\substack{\text{coefficient of } \bar{\theta}_R\theta_L = \theta^\alpha\theta_\alpha \\ \text{with } \theta_R = 0}}$$

$$\int d^4\theta \; f(\theta) := f\Big|_{\text{coefficient of } \frac{1}{2}(\bar{\theta}\theta)(\bar{\theta}\theta) = \theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}}$$

Similarly:

$$\int d^2\theta_L \; f(\theta_L) := f\Big|_{\substack{\text{coefficient of } \bar{\theta}_R \theta_L = \theta^\alpha \theta_\alpha \\ \text{with } \theta_R = 0}}$$

$$\int d^4\theta \; f(\theta) := f\Big|_{\text{coefficient of } \frac{1}{2}(\bar{\theta}\theta)(\bar{\theta}\theta) = \theta^\alpha \theta_\alpha \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}}$$

Units and renormalizability: (no couplings $\sim \frac{1}{\text{Mass}^n}$)

$$\mathcal{L} = \underbrace{\int d^4\theta}_{\sim \text{Mass}^2} \; \underbrace{K}_{\sim \text{Mass}^2} \implies \boxed{K \sim \underbrace{\Phi_i^\dagger \; \Phi_i}_{\sim \text{Mass}^2} \; * f(V)}$$

$$\mathcal{L} = \underbrace{\int d^2\theta_L}_{\sim \text{Mass}} \; \underbrace{W}_{\sim \text{Mass}^3} \implies \boxed{W = c_i \, \Phi_i + \frac{m_{ij}}{2} \Phi_i \Phi_j + \frac{Y_{ijk}}{6} \Phi_i \Phi_j \Phi_k}$$

Computations of $\Phi^\dagger \Phi$, $\Phi\Phi$: (use Weyl spinors)

$$\Phi(x,\theta) = \phi(\overbrace{x^\mu - i\,\bar\theta_R \gamma^\mu \theta_R}^{=y^\mu}, \theta_L)$$

$$= A(y) + \sqrt{2}\,\overline{\theta_R}\,\Psi_L(y) + \bar\theta_R \theta_L\, F(y)$$

$\Phi_1 \Phi_2$:

Computations of $\Phi^\dagger \Phi$, $\Phi \Phi$: (use Weyl spinors)

$$\Phi(x,\vartheta) = \phi(\underbrace{x^\mu - i\,\bar\vartheta_R \gamma^\mu \vartheta_R}_{=y^\mu}, \vartheta_L)$$
$$= A(y) + \sqrt{2}\,\bar\vartheta_R \Psi_L(y) + \bar\vartheta_R \vartheta_L F(y)$$

$\Phi_1 \Phi_2$: $(y^\mu = x^\mu)$

$$\left(A_1 + \sqrt{2}\,\vartheta^\alpha \psi_{1L\alpha} + \vartheta^\alpha \vartheta_\alpha F_1\right)\left(A_2 + \sqrt{2}\,\vartheta^\alpha \psi_{2L\alpha} + \vartheta^\alpha \vartheta_\alpha F_2\right)$$
$$= A_1 A_2 + \sqrt{2}\,\vartheta^\alpha\left(\psi_{1L\alpha} A_2 + \psi_{2L\alpha} A_1\right)$$
$$+ \vartheta^\alpha \vartheta_\alpha \left(F_1 A_2 + A_1 F_2\right) + 2 \underbrace{\vartheta^\alpha \psi_{1L\alpha}\, \vartheta^\beta \psi_{2L\beta}}_{= -\frac{1}{2} \vartheta^\alpha \vartheta_\alpha \psi_{1L}^\beta \psi_{2L\beta}}$$

$$\Rightarrow \int d^2\vartheta_L\, \Phi_1 \Phi_2 = F_1 A_2 + A_1 F_2 - \psi_{1L}^\alpha \psi_{2L\alpha}$$

Spinor rearrangements:

$$\vartheta^1 = \varepsilon^{12}\vartheta_2 \qquad \vartheta^1\vartheta_1 = \varepsilon^{12}\vartheta_2\vartheta_1 = -\varepsilon^{12}\vartheta_1\vartheta_2 = +\vartheta^2\vartheta_2$$

$$\vartheta^\alpha\vartheta_\beta = \delta^\alpha{}_\beta \quad \tfrac{1}{2}\vartheta^\gamma\vartheta_\gamma$$

$$\psi^\alpha\vartheta_\alpha = \varepsilon^{\alpha\beta}\psi_\beta\vartheta_\alpha = -\varepsilon^{\alpha\beta}\vartheta_\alpha\psi_\beta = +\vartheta^\beta\psi_\beta$$

Spinor rearrangements:

$$\vartheta^1 = \varepsilon^{12}\vartheta_2 \qquad \vartheta^1\vartheta_1 = \varepsilon^{12}\vartheta_2\vartheta_1 = -\varepsilon^{12}\vartheta_1\vartheta_2 = +\vartheta^2\vartheta_2$$

$$\vartheta^\alpha\vartheta_\beta = \delta^\alpha{}_\beta \quad \tfrac{1}{2}\vartheta^\gamma\vartheta_\gamma$$

$$\psi^\alpha\vartheta_\alpha = \varepsilon^{\alpha\beta}\psi_\beta\,\vartheta_\alpha = -\varepsilon^{\alpha\beta}\vartheta_\alpha\psi_\beta = +\vartheta^\beta\psi_\beta$$

$$\vartheta^\alpha\psi_{1L\alpha}\quad\vartheta^\beta\psi_{2L\beta} = \psi_{1L}{}^\alpha\,\vartheta_\alpha\,\vartheta^\beta\,\psi_{2L\beta} = \psi_{1L}{}^\alpha\left(-\tfrac{1}{2}\delta^\beta{}_\alpha\,\vartheta^\gamma\vartheta_\gamma\right)\psi_{2L\beta}$$

$$= -\tfrac{1}{2}\vartheta^\gamma\vartheta_\gamma\;\psi_{1L}{}^\alpha\psi_{2L\alpha}$$

$$\vartheta^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\vartheta^{\dot\alpha}\quad\vartheta^\beta\sigma^\nu_{\beta\dot\beta}\bar\vartheta^{\dot\beta} = -\varepsilon^{\beta\gamma}\,\tfrac{1}{2}(\vartheta\vartheta)\quad\sigma^\mu_{\gamma\dot\alpha}\bar\vartheta^{\dot\alpha}\quad\sigma^\nu_{\beta\dot\beta}\bar\vartheta^{\dot\beta}$$

$$= -\varepsilon^{\beta\gamma}\,\tfrac{1}{2}(\vartheta\vartheta)\quad\varepsilon^{\dot\alpha\dot\beta}\,\tfrac{1}{2}(\bar\vartheta\bar\vartheta)\;\sigma^\mu_{\gamma\dot\alpha}\quad\sigma^\nu_{\beta\dot\beta}$$

$$= \tfrac{1}{4}\;\vartheta\vartheta\,\bar\vartheta\bar\vartheta\;\mathrm{Tr}\left(\sigma^2\,\sigma^\mu\,\sigma^2\,\sigma^{\nu T}\right)$$

$$= \tfrac{1}{2}\;\vartheta\vartheta\,\bar\vartheta\bar\vartheta\quad g^{\mu\nu}$$

$$\underset{\sim\sim\sim}{\Phi^\dagger \Phi}\Big|_{\theta^4} :$$

$$\Phi = \phi\left(x - i\bar{\theta}_R \gamma^\mu \theta_R \;, \theta_L\right)$$

$$= e^{-i\bar{\theta}_R \gamma^\mu \theta_R \partial_\mu} \phi(x, \theta_L)$$

explictly, Weyl notation:

$$\Phi = \left[1 - i\theta\sigma^\mu\bar{\theta}\partial_\mu \underbrace{- \tfrac{1}{2}(\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta})\partial_\mu\partial_\nu}_{= -\frac{1}{4}\theta\theta\,\bar{\theta}\bar{\theta}\,\Box}\right]\left[A + \sqrt{2}\,\theta\psi_L + \theta\theta F\right]$$

$$\underbrace{\Phi^\dagger \Phi}\Big|_{\theta^4} : \qquad \Phi = \phi(x - i\bar{\theta}_R \gamma^\mu \theta_R \,, \theta_L)$$

$$= e^{-i\bar{\theta}_R \gamma^\mu \theta_R \partial_\mu} \phi(x, \theta_L)$$

explictly, Weyl notation:

$$\Phi = \Big[1 - i\theta\sigma^\mu\bar{\theta}\partial_\mu \underbrace{- \tfrac{1}{2}(\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta})\partial_\mu\partial_\nu}_{= -\frac{1}{4}\theta\theta\,\bar{\theta}\bar{\theta}\,\Box}\Big]\Big[A + \sqrt{2}\,\theta\psi_L + \theta\theta F\Big]$$

$$\Phi^\dagger \Phi\Big|_{\theta^4} = A^\dagger\left(-\tfrac{1}{4}\theta^4 \Box A\right) + (-i\theta\sigma^\mu\bar{\theta}\partial_\mu A)^\dagger(-i\theta\sigma^\nu\bar{\theta}\partial_\nu A) - \tfrac{1}{4}\theta^4(\Box A^\dagger)A$$

$$+ \sqrt{2}\,\bar{\theta}\bar{\psi}_L\left(-i\theta\sigma^\mu\bar{\theta}\partial_\mu \sqrt{2}\,\theta\psi_L\right) + \text{h.c.}$$

$$+ \bar{\theta}\bar{\theta} F^\dagger \;\theta\theta F$$

$$= \theta^4 \Big\{ (\partial^\mu A^\dagger)(\partial_\mu A) + \bar{\psi}_L \bar{\sigma}^\mu i\partial_\mu \psi_L + F^\dagger F \Big\}$$

Result: general $\mathcal{L}$ for chiral superfields

$$(1) = \int d^4\theta \; \Phi_i^\dagger \Phi_i = (\partial^\mu A_i)^\dagger (\partial_\mu A_i) + \bar{\psi}_{Li} \bar{\sigma}^\mu i \partial_\mu \psi_{Li} + F_i^\dagger F_i$$

$$\int d^2\theta_L \; \tfrac{1}{2} m_{ij} \Phi_i \Phi_j = \tfrac{1}{2} m_{ij} \left(A_i F_j + A_j F_i - \psi_{Li} \psi_{Lj} \right)$$

$$(2) = \int d^2\theta_L \; W(\Phi) = \sum_i F_i \frac{\partial W(A)}{\partial A_i} - \frac{1}{2} \sum_{ij} \psi_{Li} \psi_{Lj} \frac{\partial^2 W(A)}{\partial A_i \partial A_j}$$

Result: general $\mathcal{L}$ for chiral superfields

$$(1) = \int d^4\theta \; \Phi_i^\dagger \Phi_i = (\partial^\mu A_i)^\dagger (\partial_\mu A_i) + \bar{\psi}_{Li} \bar{\sigma}^\mu i \partial_\mu \psi_{Li} + F_i^\dagger F_i$$

$$\int d^2\theta_L \; \tfrac{1}{2} m_{ij} \Phi_i \Phi_j = \tfrac{1}{2} m_{ij} \left(A_i F_j + A_j F_i - \psi_{Li} \psi_{Lj} \right)$$

$$(2) = \int d^2\theta_L \; W(\Phi) = \sum_i F_i \frac{\partial W(A)}{\partial A_i} - \frac{1}{2} \sum_{ij} \psi_{Li} \psi_{Lj} \frac{\partial^2 W(A)}{\partial A_i \partial A_j}$$

$\Longrightarrow$ no kinetic term for F $\Longrightarrow$ no deg. of freedom $\Longrightarrow$ eliminate

eq.o.motion: $\quad 0 = \dfrac{\partial \mathcal{L}}{\partial F_i} = F_i^\dagger + \dfrac{\partial W(A)}{\partial A_i} \Longrightarrow F_i^\dagger = \dfrac{\partial W(A)}{\partial A_i}$

$$\Longrightarrow \boxed{(1) + (2) = \mathcal{L} = \text{kinetic terms for } A_i, \psi_{Li} - \sum_i \left| \frac{\partial W(A)}{\partial A_i} \right|^2 - \frac{1}{2} \sum_{ij} \psi_{Li} \psi_{Lj} \frac{\partial^2 W(A)}{\partial A_i \partial A_j}}$$

4.2 Example: SUSY-QED matter

$$\Phi = A(y) + \sqrt{2}\,\bar\vartheta_R\,\Psi_L(y) + \bar\vartheta_R\,\vartheta_L\,F(y)$$

$A(y)$: complex scalar; $\Psi_L(y)$: left-handed spinor; $F(y)$: no deg. of freedom

Describe electron: $\Psi = \Psi_L + \Psi_R = \begin{pmatrix} \psi_{L\,\alpha} \\ \bar\psi_R^{\,\dot\alpha} \end{pmatrix}$

$\Psi_L \longrightarrow$ chiral superfield Φ_L, $Q_L = -1$

$\Psi_R = \begin{pmatrix} 0 \\ \bar\psi_R^{\,\dot\alpha} \end{pmatrix} \longrightarrow\ ?$

$\bar\Psi_R = (\psi_R^{\,\alpha} \quad 0) \longrightarrow$ chiral superfield Φ_R, $Q_R = +1$

(physics: anti-electron/selectron-R)

Electron mass: $m\,\bar\Psi\,\Psi = m\left(\bar\Psi_R\Psi_L + \bar\Psi_L\Psi_R\right) = m\left(\psi_R^{\,\alpha}\psi_{L\,\alpha} + \bar\psi_{L\,\dot\alpha}\bar\psi_R^{\,\dot\alpha}\right)$

$\longrightarrow$ superpotential term $\quad m\,\Phi_L\,\Phi_R$

Hence, choose

$$\Phi_L = A_L + \sqrt{2}\,\vartheta^\alpha \psi_{L\alpha} + \vartheta^\alpha\vartheta_\alpha\, F_L$$

$\uparrow$

$\tilde{e}_L$, charge -1

$$\Phi_R = A_R + \sqrt{2}\,\vartheta^\alpha \psi_{R\alpha} + \vartheta^\alpha\vartheta_\alpha\, F_R$$

$\uparrow$

$\tilde{e}_R^\dagger$, charge $+1$

$$\mathcal{L} = \int d^4\vartheta \left(\Phi_L^\dagger \Phi_L + \Phi_R^\dagger \Phi_R \right) + \int d^2\vartheta \; W + h.c. \quad , \quad W = m\,\Phi_L \Phi_R$$

$$= \mathcal{L}_{\text{kinetic}, A_L, A_R, \psi_L, \psi_R} - m\left(\psi_L^\alpha \psi_{R\alpha} + \bar{\psi}_{L\dot\alpha}\bar{\psi}_R^{\dot\alpha}\right) - m^2\left(|A_L|^2 + |A_R|^2\right)$$

- scalar superpartners $\tilde{e}_L$, $\tilde{e}_R$
- kinetic terms for all fields ; mass terms with equal masses

4.3 Vector superfields

contain V^μ $\rightsquigarrow$ only consistent in gauge theory !

4.3 Vector superfields

contain V^μ $\rightsquigarrow$ only consistent in gauge theory!

non-susy : (scalar) matter $\phi := \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} \xrightarrow{\text{gauge transf.}} e^{i T^a \alpha_a} \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$

$\alpha_a = \text{real}$

covariant deriv. $\quad D^\mu = \partial^\mu + ig\, T^a V_a^\mu$

such that $\quad (D^\mu \phi)^\dagger (D_\mu \phi) = \text{gauge invariant}$

4.3 Vector superfields

contain V^μ $\rightsquigarrow$ only consistent in gauge theory !

non-susy : (scalar) matter $\phi := \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} \xrightarrow{\text{gauge transf.}} e^{iT^a \alpha_a} \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$

$\alpha_a = \text{real}$

covariant deriv. $\mathcal{D}^\mu = \partial^\mu + ig\, T^a\, V_a^{\ \mu}$

such that $(\mathcal{D}^\mu \phi)^\dagger\, (\mathcal{D}_\mu\, \phi) = \text{gauge invariant}$

susy : chiral matter $\Phi \xrightarrow{\text{gauge transf.}} e^{iT^a \Lambda_a}\, \Phi$

$\Lambda_a = \text{chiral}$

postulate term $e^{2g\, T^a V_a}$

such that $\Phi^\dagger\, e^{2g\, T^a V_a}\, \Phi$ is gauge-invariant

requires : $e^{2g\, T^a V_a} \xrightarrow{\text{gauge transf.}} e^{iT^a \Lambda^\dagger_a}\, e^{2g\, T^a V_a}\, e^{-iT^a \Lambda_a}$

Result: consistent theory with vector fields

$$\mathcal{L} = \int d^4\theta \quad \Phi^\dagger \; e^{2g\, T^a V_a} \; \Phi$$

susy and gauge invariant

- multiplet of chiral matter fields Φ
- multiplet of vector superfields V_a (adjoint rep.)

Result: consistent theory with vector fields

$$\mathcal{L} = \int d^4\vartheta \quad \Phi^\dagger \quad e^{2g\, T^a V_a} \quad \Phi$$

susy and gauge invariant

- multiplet of chiral matter fields Φ
- multiplet of vector superfields V_a (adjoint rep.)

Immediate simplification: Wess-Zumino gauge:

use gauge transformation $\Lambda \longrightarrow$ achieve $V_a\big|_{1,\vartheta,\vartheta^\alpha\vartheta_\alpha} = 0$

$$V(x,\vartheta) = \bar{\vartheta}_L \gamma^\mu \vartheta_L \; V_\mu(x) + (\bar{\vartheta}\vartheta)\,\bar{\vartheta}\lambda(x) + \tfrac{1}{4}(\bar{\vartheta}\vartheta)(\bar{\vartheta}\vartheta)D(x)$$

$$= \vartheta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\vartheta}^{\dot{\alpha}} \; V_\mu(x) + \vartheta^\alpha\vartheta_\alpha\, \bar{\vartheta}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}(x) + \tfrac{1}{2}\, \vartheta^\alpha\vartheta_\alpha\, \bar{\vartheta}_{\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}} D(x)$$
$$+ \bar{\vartheta}_{\dot{\alpha}} \bar{\vartheta}^{\dot{\alpha}}\, \vartheta^\alpha \lambda_\alpha(x)$$

Final ingredient: field strength superfield

- one more possible susy, gauge invariant term:
- take certain derivative of vector superfield
- result = chiral, gauge covariant superfield:

$$W_\alpha = \lambda_\alpha - \frac{i}{2}\left(\sigma^\mu \bar{\sigma}^\nu \theta\right)_\alpha F_{\mu\nu} + \theta_\alpha D + \ldots$$

$$\rightsquigarrow \frac{1}{4}\int d^2\theta_L \; W_a^\alpha W_{a\alpha} + \text{h.c.} = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{\lambda}_a \bar{\sigma}^\mu i D_\mu \lambda_a + \frac{1}{2} D_a^2$$

$$= \text{kinetic terms for } V_a^\mu,\ \lambda_a \quad + \frac{1}{2} D_a^2$$

4.4 General renormalizable $\mathcal{L}_{susy}$

multiplet of chiral superfields + suitable vectorsuperfields

$$\mathcal{L} = \int d^4\vartheta \left(\Phi^\dagger \, e^{2gT^aV_a} \, \Phi \right)$$

$$+ \int d^2\vartheta_L \left(\frac{1}{4} W^\alpha_a W_{a\alpha} + W(\Phi) \right) + h.c.$$

4.4 General renormalizable $\mathcal{L}_{susy}$

multiplet of chiral superfields + suitable vectorsuperfields

$$\mathcal{L} = \int d^4\vartheta \left(\Phi^\dagger \, e^{2gT^aV_a} \, \Phi \right)$$

$$+ \int d^2\vartheta_L \left(\tfrac{1}{4} W_a^\alpha W_{a\alpha} + W(\Phi) \right) + h.c.$$

evaluate $\rightarrow$ F_i , D_a have no kinetic terms $\rightarrow$ eliminate

$$F_i^\dagger = - \frac{\partial W(A)}{\partial A_i} \quad , \quad D_a = - g A^\dagger T^a A$$

4.4 General renormalizable $\mathcal{L}_{susy}$

multiplet of chiral superfields + suitable vectorsuperfields

$$\mathcal{L} = \int d^4\vartheta \left(\Phi^\dagger \, e^{2gT^aV_a} \, \Phi \right)$$
$$+ \int d^2\vartheta_L \left(\frac{1}{4} W_a^\alpha W_{a\alpha} + W(\Phi) \right) + h.c.$$

evaluate $\rightarrow$ F_i, D_a have no kinetic terms $\rightarrow$ eliminate

$$F_i^\dagger = - \frac{\partial W(A)}{\partial A_i} \,, \qquad D_a = -g A^\dagger T^a A$$

Result:

$$\mathcal{L} = \mathcal{L}_{kinetic,\, A,\, \psi,\, V^\mu,\, \lambda}\Big|_{\partial^\mu \to D^\mu} + \mathcal{L}_{gaugino} + \mathcal{L}_{superpot.} + \mathcal{L}_F + \mathcal{L}_D$$

$$\mathcal{L}_{gaugino} = -\sqrt{2}\, g \left(\bar{\psi}_{\dot\alpha} T^a \bar{\lambda}_a^{\dot\alpha} A + A^\dagger T^a \lambda_a^\alpha \psi_\alpha \right)$$

$$\mathcal{L}_{superpot} = -\frac{1}{2} \psi_i^\alpha \psi_{j\alpha} \frac{\partial^2 W(A)}{\partial A_i \partial A_j} + h.c.$$

$$\mathcal{L}_F = - \left| \frac{\partial W(A)}{\partial A} \right|^2 \qquad \mathcal{L}_D = -\frac{1}{2} g^2 (A^\dagger T^a A)(A^\dagger T^a A)$$

4.5 Full SUSY-QED

two chiral superfields

$\Phi_L : A_L, \psi_{L\alpha} \qquad Q_L = -1$

$\Phi_R : A_R, \psi_{R\alpha} \qquad Q_R = +1$

electron $\Psi = \begin{pmatrix} \psi_{L\alpha} \\ \bar{\psi}_R^{\dot\alpha} \end{pmatrix}$ 4 fermionic

selectrons $\tilde{e}_L = A_L$, $\tilde{e}_R = A_R^\dagger$ 4 bosonic

$\left.\begin{matrix} \text{4 fermionic} \\ \text{4 bosonic} \end{matrix}\right\}$ deg.o.f.

one vector superfield

$V : A^\mu, \lambda_\alpha, \bar{\lambda}_{\dot\alpha}$

photon A^μ

photino $\tilde{\gamma} = \begin{pmatrix} \lambda_\alpha \\ \bar{\lambda}^{\dot\alpha} \end{pmatrix} = \tilde{\gamma}^c$ (Majorana)

$\left.\begin{matrix} \text{2 bos.} \\ \text{2 ferm.} \end{matrix}\right\}$ deg.o.f.

Lagrangian:

$$\mathcal{L} = \int d^4\theta \left(\Phi_L^\dagger e^{2eQ_L V} \Phi_L + L \to R \right)$$
$$+ \int d^2\theta_L \left(\frac{1}{4} W^\alpha W_\alpha + m \Phi_L \Phi_R \right) + h.c.$$

Lagrangian: $\mathcal{L} = \int d^4\vartheta \left(\Phi_L^\dagger e^{2eQ_L V} \Phi_L + L \to R \right)$
$+ \int d^2\vartheta_L \left(\frac{1}{4} W^\alpha W_\alpha + m \Phi_L \Phi_R \right) + h.c.$

evaluate: kinetic terms + gaugino + superpot. + F + D -terms

$$\mathcal{L}_{kinetic} = |D^\mu A_L|^2 + |D^\mu A_R|^2 + \bar{\Psi} i\gamma^\mu D_\mu \Psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \bar{\tilde{\gamma}} i\gamma^\mu \partial_\mu \tilde{\gamma}$$

$$\mathcal{L}_{gaugino} = -\sqrt{2}\, e\, Q_L \left(\bar{\psi}_{L\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} A_L - \bar{\psi}_{R\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} A_R + A_L^\dagger \lambda^\alpha \psi_{L\alpha} - A_R^\dagger \lambda^\alpha \psi_{R\alpha} \right)$$

$$\mathcal{L}_{superpot} = -m\, \psi_L^\alpha \psi_{R\alpha} + h.c. = -m \bar{\Psi} \Psi$$

$$\mathcal{L}_F = -m^2 |A_L|^2 - m^2 |A_R|^2$$

$$\mathcal{L}_D = -\frac{1}{2} e^2 Q_L^2 \left(|A_L|^2 - |A_R|^2 \right)^2$$

compare with "guess":
- equal # deg.o.f. : implemented in superfields
- equal masses $\Leftrightarrow$ superpot. & F-terms
- equal interactions with γ and $\tilde{\gamma}$ <u>and</u> (scalar)^4 -terms

Summary:

- SUSY algebra $\leftarrow$ $\Delta J = \pm \frac{1}{2}$, $F \leftrightarrow B$
- implement on superspace / fields
- $\int d^4\theta$, $\int d^2\theta_L$ give susy Lagrangians
- evaluation needs lots of spinor algebra
- chiral fields appear as $\Phi^\dagger \Phi$ $\rightarrow$ kinetic terms
  and in superpotential
- vector superfields appear in $\Phi^\dagger e^{2gV} \Phi$ $\rightarrow$ gauge/gaugino interactions
  and in $W^\alpha W_\alpha$ $\rightarrow$ kinetic terms
- implements a lot of physics:
  - equal # d.o.f.
  - equal interactions, masses
  - $(\text{scalar})^4$ interactions always fixed by F-, D-terms

5. Supersymmetric Standard Model

Strategy:

gauge field $\longrightarrow$ vector s.f. $\longrightarrow$ V^μ, λ_α, $\bar{\lambda}_{\dot{\alpha}}$

scalar field $\longrightarrow$ chiral s.f. $\longrightarrow$ scalar, ψ_α

fermion $\Psi = \begin{pmatrix} \psi_{L\alpha} \\ \bar{\psi}_R^{\dot{\alpha}} \end{pmatrix}$ (charge Q)

$\psi_{L\alpha} \longrightarrow$ chiral s.f. $\longrightarrow$ $\tilde{f}_L$, $\psi_{L\alpha}$ (Q)

$\psi_{R\alpha} \longrightarrow$ chiral s.f. $\longrightarrow$ $\tilde{f}_R^\dagger$, $\psi_{R\alpha}$ $(-Q)$

(anti-(s)fermion-R)

5.1 MSSM fields

			$Q_{e.m} = T_3 + Y/2$	$SU(3)_c$
$\hat{Q}$	$\tilde{q}_L = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$q_{L\alpha} = \begin{pmatrix} u_{L\alpha} \\ d_{L\alpha} \end{pmatrix}$	$+2/3$ $-1/3$	3
$\hat{U}$	$\tilde{u}_R^\dagger$	$u_{R\alpha}$	$-2/3$	3^*
$\hat{D}$	$\tilde{d}_R^\dagger$	$d_{R\alpha}$	$+1/3$	3^*
$\hat{L}$	$\tilde{\ell}_L = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$	$\ell_{L\alpha} = \begin{pmatrix} \nu_{L\alpha} \\ e_{L\alpha} \end{pmatrix}$	0 -1	1
$\hat{E}$	$\tilde{e}_R^\dagger$	$e_{R\alpha}$	$+1$	1
$\hat{H}_u$	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_{u\alpha}^+ \\ H_{u\alpha}^0 \end{pmatrix}$	$+1$ 0	1
$\hat{H}_d$	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} H_{d\alpha}^0 \\ H_{d\alpha}^- \end{pmatrix}$	0 -1	1
$\hat{V}_s^a$	$G^{\mu a}$	$\lambda_{s\alpha}^a, \bar{\lambda}_{s\dot{\alpha}}^a$	0	8
$\hat{V}_w^a$	$W^{\mu a}$	$\lambda_\alpha^a, \bar{\lambda}_{\dot{\alpha}}^a$	$0, \pm 1$	1
$\hat{V}'$	B^μ	$\lambda'_\alpha, \bar{\lambda}'_{\dot{\alpha}}$	0	1

5.2 MSSM Lagrangian

$$\mathcal{L} = \mathcal{L}_{susy} + \mathcal{L}_{susy-breaking} \rightsquigarrow \text{later}$$

$$\mathcal{L}_{susy} = \int d^4\theta \left[\hat{Q}^\dagger e^{2g_s\hat{V}_s + 2g\hat{V} + 2g' Y_Q \hat{V}'} \hat{Q} + \ldots + \hat{H}_d^\dagger e^{2g\hat{V} + 2g' Y_{Hd}\hat{V}'} \hat{H}_d \right]$$

$$+ \int d^2\theta_L \left[\frac{1}{4} W_s^{a\alpha} W^a_{s\alpha} + \frac{1}{4} W^{a\alpha} W^a_\alpha + \frac{1}{4} W'^\alpha W'_\alpha + W \right] + h.c.$$

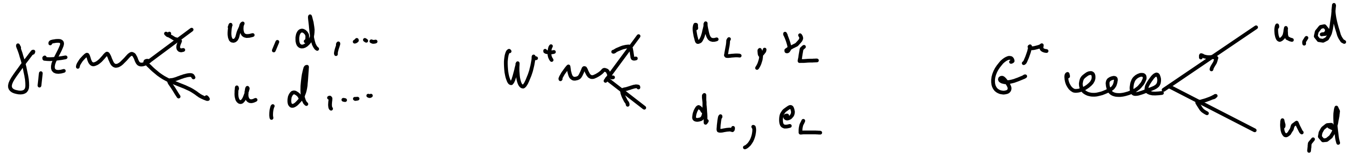
- kinetic terms, gauge interactions, gaugino interactions
- superpotential, F-, D-terms

$$W = -y_u \underbrace{\hat{H}_u \hat{Q}}_{} \hat{U} + y_d \hat{H}_d \hat{Q} \hat{D} + y_e \hat{H}_d \hat{L} \hat{E} + \mu \hat{H}_u \hat{H}_d$$

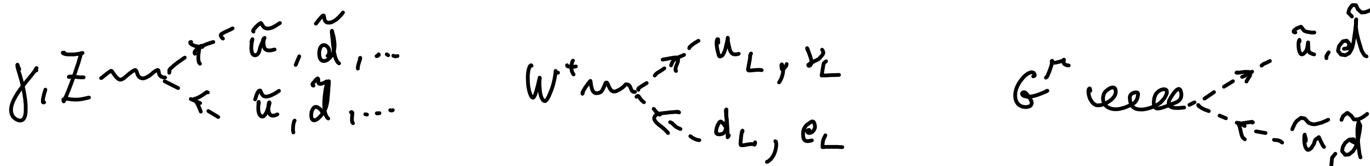
$$= \hat{H}_u^1 \hat{Q}^2 - \hat{H}_u^2 \hat{Q}^1 \qquad (SU(2)\text{-invariant, without } \hat{H}_u^\dagger)$$

Obvious Feynman rules:

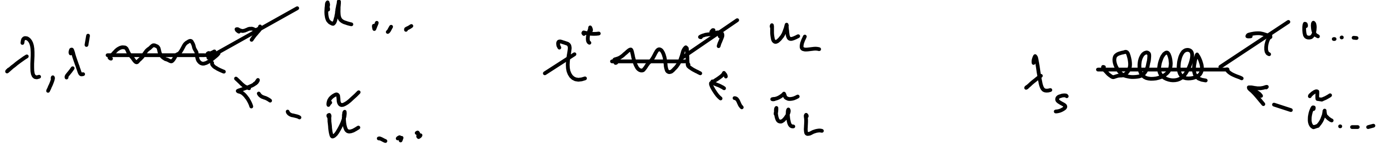
SM gauge interactions :



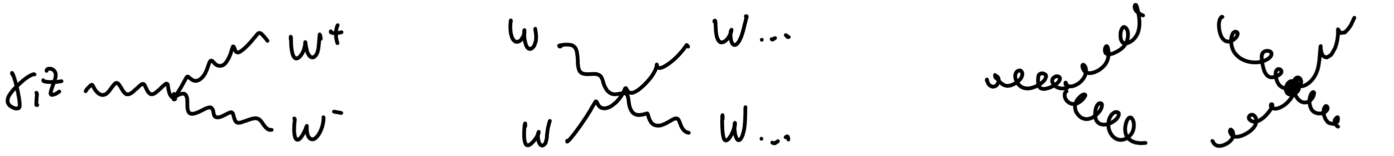
sfermion gauge " :



gaugino



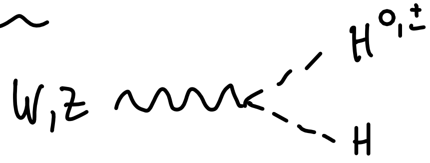
SM $F^{\mu\nu}F_{\mu\nu}$ - int. :

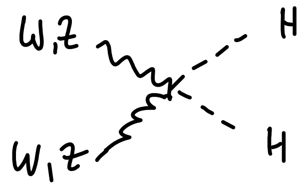


gaugino :

Feynman rules for $H, \tilde{H}$:

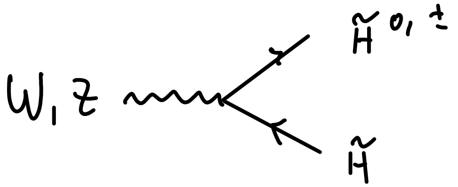
SM Higgs:



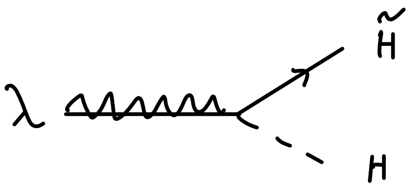


$\longrightarrow$ $WW\ v^2$
mass term ...

gauge - Higgsino:

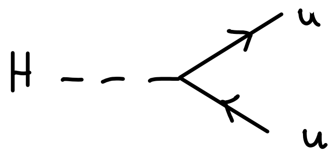


gaugino - Higgsino-Higgs:



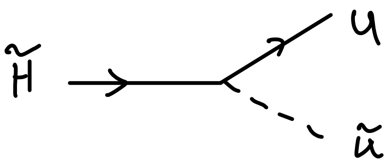
$\longrightarrow$ $\lambda\ \tilde{H}\ v$
mass/mixing term

SM Yukawa:



$\longrightarrow$ $\bar{u}\ u\ v$
mass term

Higgsino/sfermions:



Sample: gluino-squark interactions:

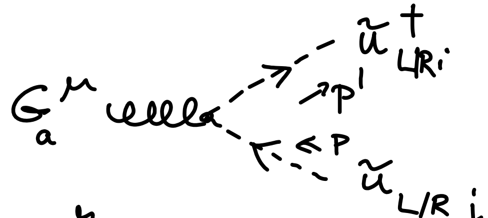
$y_u = y_d = 0$, massless (s)quarks
neglect $SU(2) \times U(1)$

$$D^\mu \tilde{u}_L = (\partial^\mu + i g_s G_a^\mu T^a)\, \tilde{u}_L$$

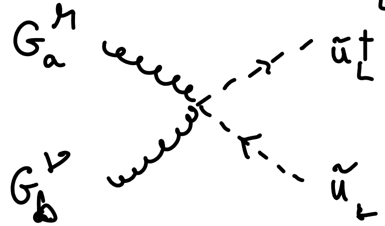
$$D^\mu \tilde{u}_R^\dagger = (\partial^\mu + i g_s G_a^\mu (-T^{a*}))\, \tilde{u}_R^\dagger$$

$$D^\mu \tilde{u}_R = (\partial^\mu + i g_s G_a^\mu T^a)\, \tilde{u}_R$$

$$\mathcal{L} = |D^\mu \tilde{u}_L|^2 + |D^\mu \tilde{u}_R|^2 \longrightarrow$$



$$= -i g_s (p+p')^\mu T^a_{ij}$$



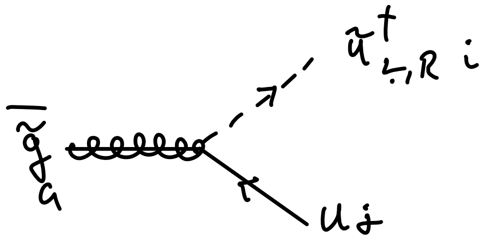
$$= i g_s^2 g^{\mu\nu} \{T^a, T^b\}$$

$$\mathcal{L}_{\text{gaugino}} = -\sqrt{2}\, g_s \Big(\bar{u}_{L\dot{\alpha}} T^a \bar{\lambda}^{\dot{\alpha}}_{sa} \tilde{u}_L + \tilde{u}_L^\dagger T^a \lambda^\alpha_{sa} u_{L\alpha} + \bar{u}_{R\dot{\alpha}} (-T^{a*}) \bar{\lambda}^{\dot{\alpha}}_{sa} \tilde{u}_R^\dagger + \tilde{u}_R (-T^{a*}) \lambda^\alpha_{sa} u_{R\alpha} \Big)$$

Combine to 4-spinors:

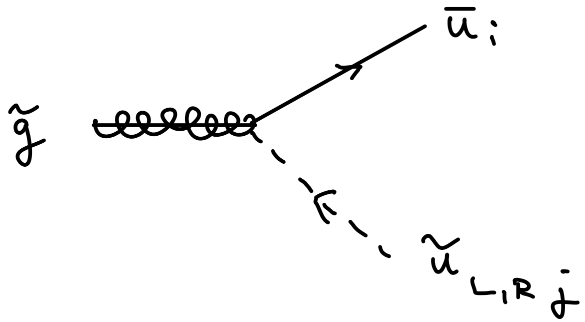
$$u = \begin{pmatrix} u_{L\,\alpha} \\ \bar{u}_R^{\dot\alpha} \end{pmatrix} \quad , \quad \tilde{g}_a = \begin{pmatrix} \lambda_{sa\,\alpha} \\ \bar{\lambda}_{sa}^{\dot\alpha} \end{pmatrix}$$

$$(2)+(3) = \tilde{u}_L^\dagger \, \overline{\tilde{g}_a} \, T^a P_L u - \tilde{u}^\dagger_{R\,j} \, \overline{\tilde{g}_a} \, \underbrace{T^{a*}_{ij}}_{= T^a_{ji}} P_R u_i$$



$$= -\sqrt{2}\, i g_s \, T^a_{ij} \begin{cases} + P_L \\ - P_R \end{cases}$$

$$(1)+(4) = \bar{u} P_R T^a \tilde{g}_a \tilde{u}_L - \bar{u}_j P_L \underbrace{T^{a*}_{ij}}_{= T^a_{ji}} \tilde{g}_a \tilde{u}_{R\,i}$$



$$= -\sqrt{2}\, i g_s \, T^a_{ij} \begin{cases} + P_L \\ - P_R \end{cases}$$