Ex 5. The Lagrangian describing deviations from the SM in triple gauge-boson couplings is usually written as

$$\Delta\mathcal{L}_{TGC} = igc_w\Big[\delta g_1^z\, Z^\mu\big(W^{-\nu}W^+_{\mu\nu} - W^{+\nu}W^-_{\mu\nu}\big) + \delta\kappa_z\, Z^{\mu\nu}W^-_\mu W^+_\nu + \frac{\lambda_z}{M_W^2} Z^{\mu\nu}W^{-\rho}_\nu W^+_{\rho\mu}\Big] + igs_w\Big[\delta\kappa_\gamma\, A^{\mu\nu}W^-_\mu W^+_\nu + \frac{\lambda_\gamma}{M_W^2} A^{\mu\nu}W^{-\rho}_\nu W^+_{\rho\mu}\Big]$$

with 5 possible deviations parametrized by $\delta g_1^z, \delta\kappa_z, \lambda_z, \delta\kappa_\gamma$ and λ_γ. [Above: $s_w = \sin\theta_w$, $c_w = \cos\theta_w$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$].

In our $d=6$ Lagrangian only 4 operators contribute to such deviations in TGCs:

$$\Delta\mathcal{L}_{d=6} = \frac{1}{\Lambda^2}\Big[c_W O_W + \kappa_{HW} O_{HW} + \kappa_{HB} O_{HB} + \kappa_{3W} O_{3W}\Big]$$

where

$$O_W = \frac{ig}{2}\big(H^\dagger\sigma^a \overleftrightarrow{D}^\mu H\big) D^\nu W^a_{\mu\nu}$$

$$O_{HW} = ig\,(D^\mu H)^\dagger\sigma^a(D^\nu H)W^a_{\mu\nu}$$

$$O_{HB} = ig'\,(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu}$$

$$O_{3W} = \frac{1}{3!}\, g\,\epsilon_{abc} W^{a\,\nu}_\mu W^b_{\nu\rho} W^{c\,\rho\mu}$$

Show that the TGC deviations induced by $\Delta\mathcal{L}_{d=6}$ are:

$$\delta g_1^z = \frac{M_Z^2}{\Lambda^2}(c_W + \kappa_{HW}) \qquad \delta\kappa_\gamma = \frac{M_W^2}{\Lambda^2}(\kappa_{HW} + \kappa_{HB})$$

$$\delta\kappa_z = \delta g_1^z - \tan^2\theta_w\,\delta\kappa_\gamma \qquad \lambda_z = \lambda_\gamma = \frac{M_W^2}{\Lambda^2}\kappa_{3W}$$

This implies that only 3 parameters describe the leading ($1/\Lambda^2$) TGC deviations: a prediction of the EFT approach.

Ex 6. In SUSY models with heavy superpartners, R-parity forbids tree-level contributions to the irrelevant operators in the low-E EFT. The only exceptions to this rule are the ops. induced by the exchange of the (heavy) second Higgs doublet (R-even). Writing the relevant UV Lagrangian involving the heavy Higgs H' (taken to have $Y = 1/2$) as

$$\mathcal{L} = -\alpha_u y_u \bar{Q}_L \tilde{H}' u_R - \alpha_d y_b \bar{Q}_L H' d_R - \alpha_e y_e \bar{L}_L H' e_R$$
$$- \lambda' H'^{\dagger} H |H|^2 + h.c. + \ldots$$

(where $\alpha_u = -\cot\beta$, $\alpha_d = \alpha_e = \tan\beta$, $\lambda' = \frac{1}{8}(g^2 + g'^2)\sin 4\beta$ for the MSSM case) show that the EFT below $M_{H'}$ contains $d=6$ ops.

$$\Delta\mathcal{L}_{d=6} = \frac{g_*^2}{\Lambda^2}\Big[c_{y_t}\, y_t |H|^2 \bar{Q}_L \tilde{H} t_R + c_{y_b} y_b |H|^2 \bar{Q}_L H b_R + c_{y_\tau} y_\tau |H|^2 \bar{L}_L H \tau_R$$
$$+ h.c. + \lambda c_6 |H|^6 + c_{LR} (\bar{Q}_L \gamma^\mu Q_L)(\bar{t}_R \gamma_\mu t_R) +$$
$$c_{LR}^{(8)} (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{t}_R \gamma_\mu T^A t_R) \Big]$$
$$+ \frac{1}{\Lambda^2}\Big[c_{y_t y_b}\, y_t y_b (\bar{Q}_L^r t_R)\epsilon_{rs}(\bar{Q}_L^s b_R) +$$
$$+ c_{y_t y_\tau}\, y_t y_\tau (\bar{Q}_L^r t_R)\epsilon_{rs}(\bar{L}_L^s \tau_R) + h.c. \Big]$$

with coefficients

$$g_*^2 c_{y_t} = \alpha_t \lambda' \qquad g_*^2 c_{y_b} = \alpha_b \lambda' \qquad g_*^2 c_{y_\tau} = \alpha_\tau \lambda'$$
$$g_*^2 \lambda c_6 = \lambda'^2 \qquad c_{y_t y_b} = \alpha_t \alpha_b \qquad c_{y_t y_\tau} = \alpha_t \alpha_\tau$$
$$g_*^2 c_{LR}^{(8)} = 2N_c\, g_*^2 c_{LR} = -\alpha_t^2 y_t^2 \qquad \Lambda = M_{H'}$$

(Note: $\tilde{H}' = i\sigma_2 H'^*$. Quark fields carry color indices not shown, eg $\bar{Q}_L \gamma^\mu T^A Q_L = \bar{Q}_L^\alpha \gamma^\mu T^A_{\alpha\beta} Q_L^\beta$ where T^A are the $SU(3)_c$ generators. In $(\bar{Q}_L^r t_R)\epsilon_{rs}(\bar{L}_L^s \tau_R)$, r and s are $SU(2)_L$ indices.)