Ex 5. The Lagrangian describing deviations from the SM in triple gauge-boson couplings is usually written as

$$\Delta\mathcal{L}_{TGC} = igc_w\Big[\delta g_1^z\, Z^\mu\left(W^{-\nu}W^+_{\mu\nu} - W^{+\nu}W^-_{\mu\nu}\right)$$
$$+\,\delta\kappa_z\, Z^{\mu\nu}W^-_\mu W^+_\nu$$
$$+\,\frac{\lambda_z}{M_W^2} Z^{\mu\nu}W^{-\rho}_\nu W^+_{\rho\mu}\Big]$$
$$+\,igs_w\Big[\delta\kappa_\gamma\, A^{\mu\nu}W^-_\mu W^+_\nu$$
$$+\,\frac{\lambda_\gamma}{M_W^2} A^{\mu\nu}W^{-\rho}_\nu W^+_{\rho\mu}\Big]$$

with 5 possible deviations parametrized by δg_1^z, $\delta\kappa_z$, λ_z, $\delta\kappa_\gamma$ and λ_γ. [Above: $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$].

In our $d=6$ Lagrangian only 4 operators contribute to such deviations in TGCs:

$$\Delta\mathcal{L}_{d=6} = \frac{1}{\Lambda^2}\Big[c_W\, O_W + \kappa_{HW}\, O_{HW} + \kappa_{HB}\, O_{HB} + \kappa_{3W}\, O_{3W}\Big]$$

where

$$O_W = \frac{ig}{2}\left(H^\dagger\sigma^a \overleftrightarrow{D}^\mu H\right)D^\nu W^a_{\mu\nu}$$
$$O_{HW} = ig\,(D^\mu H)^\dagger\sigma^a(D^\nu H)\,W^a_{\mu\nu}$$
$$O_{HB} = ig'\,(D^\mu H)^\dagger(D^\nu H)\,B_{\mu\nu}$$
$$O_{3W} = \frac{1}{3!}\, g\,\epsilon_{abc}\, W^{a\,\nu}_\mu W^b_{\nu\rho} W^{c\,\rho\mu}$$

Show that the TGC deviations induced by $\Delta\mathcal{L}_{d=6}$ are:

$$\delta g_1^z = \frac{M_Z^2}{\Lambda^2}\left(c_W + \kappa_{HW}\right) \qquad \delta\kappa_\gamma = \frac{M_W^2}{\Lambda^2}\left(\kappa_{HW} + \kappa_{HB}\right)$$

$$\delta\kappa_z = \delta g_1^z - \tan^2\theta_W\,\delta\kappa_\gamma \qquad \lambda_z = \lambda_\gamma = \frac{M_W^2}{\Lambda^2}\kappa_{3W}$$

This implies that only 3 parameters describe the leading ($1/\Lambda^2$) TGC deviations: a prediction of the EFT approach.

Ex 6. In SUSY models with heavy superpartners, R-parity forbids tree-level contributions to the irrelevant operators in the low-E EFT. The only exceptions to this rule are the ops. induced by the exchange of the (heavy) second Higgs doublet (R-even). Writing the relevant UV Lagrangian involving the heavy Higgs H' (taken to have $Y = 1/2$) as

$$\mathcal{L} = -\alpha_u y_u \bar{Q}_L \tilde{H}' u_R - \alpha_d y_b \bar{Q}_L H' d_R - \alpha_e y_e \bar{L}_L H' e_R$$
$$-\lambda' H'^{\dagger} H |H|^2 + h.c. + \ldots$$

(where $\alpha_u = -\cot\beta$, $\alpha_d = \alpha_e = \tan\beta$, $\lambda' = \frac{1}{8}(g^2 + g'^2)\sin 4\beta$ for the MSSM case) show that the EFT below $M_{H'}$ contains $d=6$ ops.

$$\Delta\mathcal{L}_{d=6} = \frac{g_*^2}{\Lambda^2}\Big[c_{y_t}\, y_t |H|^2 \bar{Q}_L \tilde{H} t_R + c_{y_b} y_b |H|^2 \bar{Q}_L H b_R + c_{y_\tau} y_\tau |H|^2 \bar{L}_L H \tau_R$$
$$+ h.c. + \lambda c_6 |H|^6 + c_{LR} (\bar{Q}_L \gamma^\mu Q_L)(\bar{t}_R \gamma_\mu t_R) +$$
$$c_{LR}^{(8)} (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{t}_R \gamma_\mu T^A t_R)\Big]$$
$$+ \frac{1}{\Lambda^2}\Big[c_{y_t y_b}\, y_t y_b (\bar{Q}_L^r t_R)\epsilon_{rs}(\bar{Q}_L^s b_R) +$$
$$+ c_{y_t y_\tau}\, y_t y_\tau (\bar{Q}_L^r t_R)\epsilon_{rs}(\bar{L}_L^s \tau_R) + h.c.\Big]$$

with coefficients

$$g_*^2 c_{y_t} = \alpha_t \lambda' \qquad g_*^2 c_{y_b} = \alpha_b \lambda' \qquad g_*^2 c_{y_\tau} = \alpha_\tau \lambda'$$
$$g_*^2 \lambda c_6 = \lambda'^2 \qquad c_{y_t y_b} = \alpha_t \alpha_b \qquad c_{y_t y_\tau} = \alpha_t \alpha_\tau$$
$$g_*^2 c_{LR}^{(8)} = 2N_c\, g_*^2 c_{LR} = -\alpha_t^2 y_t^2 \qquad \Lambda = M_{H'}$$

(Note: $\tilde{H}' = i\sigma_2 H'^*$. Quark fields carry color indices not shown, eg $\bar{Q}_L \gamma^\mu T^A Q_L = \bar{Q}_L^\alpha \gamma^\mu T^A_{\alpha\beta} Q_L^\beta$ where T^A are the $SU(3)_c$ generators. In $(\bar{Q}_L^r t_R)\epsilon_{rs}(\bar{L}_L^s \tau_R)$, r and s are $SU(2)_L$ indices.)

Sol. 5 Using $H=\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ and $\langle H\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} v/\sqrt{2}$ in the covariant derivative $D_\mu H = \partial_\mu H - ig\frac{\sigma^a}{2}W^a_\mu H - ig'\frac{1}{2}B_\mu H$ one gets $\langle D_\mu H\rangle = -\frac{iv}{2\sqrt{2}}\begin{pmatrix} g\sqrt{2}\,W^+_\mu \\ -G Z_\mu \end{pmatrix}$.

Remembering further that

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

and

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu$$

$$D^\nu W_{\mu\nu} = \partial^\nu W_{\mu\nu} - ig\,[W^\nu, W_{\mu\nu}]$$

it's a matter of simple algebra to get the final results.

As an example, take $\Delta\mathcal{L} = \frac{\kappa_{HW}}{\Lambda^2}O_{HW} = \frac{\kappa_{HW}}{\Lambda^2}\, ig(D^\mu H)^\dagger\sigma^a(D^\nu H)W^a_{\mu\nu}$

Each (Higgs covariant derivative) gives one gauge boson, so as we are interested in contributions to TGCs only we only need the linear part of $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \ldots$

Plugging $\langle D^\mu H\rangle^\dagger$ and $\langle D^\nu H\rangle$ one gets

$$ig\frac{v^2}{8}\left(g\sqrt{2}\,W^{\mu-}, -GZ^\mu\right)\begin{pmatrix} W^3_{\mu\nu} & W^1_{\mu\nu} - iW^2_{\mu\nu} \\ W^1_{\mu\nu} + iW^2_{\mu\nu} & -W^3_{\mu\nu} \end{pmatrix}\begin{pmatrix} g\sqrt{2}\,W^{\nu+} \\ -GZ^\nu \end{pmatrix}$$

and, using

$$W^3_{\mu\nu} = s_W A_{\mu\nu} + c_W Z_{\mu\nu}$$

$$W^1_{\mu\nu} \pm iW^2_{\mu\nu} = \sqrt{2}\,W^\mp_{\mu\nu}$$

we get $\delta g_1^Z = \frac{M_Z^2}{\Lambda^2}\kappa_{HW}$ $\quad \delta\kappa_\gamma = \frac{M_W^2}{\Lambda^2}\kappa_{HW} = \delta\kappa_Z$.

The rest of contributions can be obtained in a similar way.

Sol. 6 Writing the part of the Lagrangian that involves H' (including the mass and kinetic terms) in the form

$$\mathcal{L} = -H'^{\dagger}(\partial^2 + M^2_{H'})\, H' + H'^{\dagger}\Delta + \Delta^{\dagger} H'$$

the EoM for H' reads

$$\frac{\delta \mathcal{L}}{\partial H'^{\dagger}} = -(M^2_{H'} + \partial^2) H' + \Delta = 0$$

and, to orden $1/M^2_{H'}$ this gives $H' = \Delta / M^2_{H'}$. Plugging this back in $\mathcal{L}$ we get the effective Lagrangian, with $\Lambda = M^2_{H'}$,

$$\mathcal{L}_{eff} = +\frac{1}{\Lambda^2}|\Delta|^2 + O\left(\frac{1}{\Lambda^4}\right)$$

The explicit expression for Δ can be obtained immediately as (remember that $\tilde{H} \equiv \epsilon H^*$, with $\epsilon = i\sigma_2$, so that

$$\bar{Q}_L \tilde{H}' t_R = \bar{Q}^s_L \tilde{H}'_s t_R = \bar{Q}^s_L \epsilon_{sr} H'^*_r t_R = -H'^{\dagger} \epsilon\, \bar{Q}^T_L t_R\ )$$

$$\Delta = -\lambda' H|H|^2 + \alpha_t y_t \epsilon \bar{Q}^T_L t_R - \alpha_b y_b \bar{b}_R Q_L - \alpha_\tau y_\tau \bar{\tau}_R L_L$$

Plugging this in $\mathcal{L}_{eff}$ one gets the following contributions:

$O(\lambda'^2)$: $$\delta\mathcal{L}_{eff} = \frac{1}{M^2_{H'}}(\lambda')^2 |H|^6 \Rightarrow g_*^2 \lambda c_6 = (\lambda')^2$$

$O(\lambda' y_f)$: $$\delta\mathcal{L}_{eff} = \frac{1}{M^2_{H'}}(-\lambda'|H|^2)\Big[-\alpha_t y_t \bar{Q}_L \tilde{H} t_R - \alpha_b y_b \bar{b}_R H^{\dagger} Q_L - \alpha_\tau y_\tau \bar{\tau}_R H^{\dagger} L_L + h.c.\Big]$$

$$\Rightarrow g_*^2 c_{y_f} = \alpha_f \lambda' y_f$$

$O(y_t y_{f\neq t})$: $$\delta\mathcal{L}_{eff} = -\frac{\alpha_t y_t}{M^2_{H'}}\left[\alpha_f y_f (\bar{F}_L f_R)\, \epsilon\, \bar{Q}_L t_R\right] \Rightarrow c_{y_t y_f} = \alpha_t \alpha_f$$

$O(y_t^2)$: $$\delta\mathcal{L}_{eff} = \frac{-\alpha_t^2 y_t^2}{M^2_{H'}}\Big[(\bar{t}_R Q^T_L \underbrace{\epsilon)(\epsilon}_{-I} \bar{Q}^T_L t_R)\Big] = \frac{\alpha_t^2 y_t^2}{M^2_{H'}}(\bar{t}_R Q_L)(\bar{Q}_L t_R)$$

To rewrite the operator $(\bar{t}_R Q_L)(\bar{Q}_L t_R)$ in terms of the operators
$$c_{LR}(\bar{Q}_L\gamma^\mu Q_L)(\bar{t}_R\gamma_\mu t_R)$$
$$+\, c_{LR}^{(8)}(\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{t}_R\gamma_\mu T^A t_R)$$
we use the Fierz rearrangement formula (paying attention to color indices):
$$(\bar{t}_R^\alpha Q_L^\alpha)(\bar{Q}_L^\beta t_R^\beta) = -\frac{1}{2}(\bar{t}_R^\alpha t_R^\beta)(\bar{Q}_L^\beta Q_L^\alpha)$$
and for the $SU(3)_c$ generators $T^A_{\alpha\beta}$:
$$T^A_{\alpha\beta}T^A_{\rho\sigma} = \frac{1}{2}\left(\delta_{\alpha\sigma}\delta_{\rho\beta} - \frac{1}{N_c}\delta_{\alpha\beta}\delta_{\rho\sigma}\right)$$
getting:
$$(\bar{Q}_L t_R)(\bar{t}_R Q_L) = -(\bar{Q}_L T^A\gamma_\mu Q_L)(\bar{t}_R T^A\gamma^\mu t_R)$$
$$-(\bar{Q}_L\gamma^\mu Q_L)(\bar{t}_R\gamma^\mu t_R)/(2N_c)$$
And finally:
$$g_*^2 c_{LR}^{(8)} = 2N_c g_*^2 c_{LR} = -\alpha_t^2 y_t^2$$