Coherent superposition of current flows in an Atomtronic Quantum Interference Device

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Outlook

Atomtronics: a new stage for quantum simulation ?

– optical circuits with lithographic accuracy
– neutrality of atoms
– bosons / fermions
– flexibility on interactions

Quantum information ?

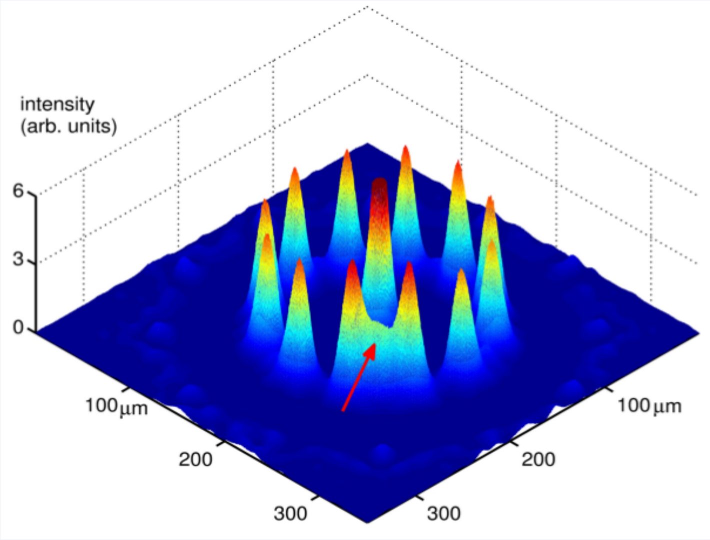
What could be a **qubit** ?

Particle current flowing in a ring-shaped potential
a barrier creates an interference state (SQUID)

the cold-atom analog of a *flux qubit*

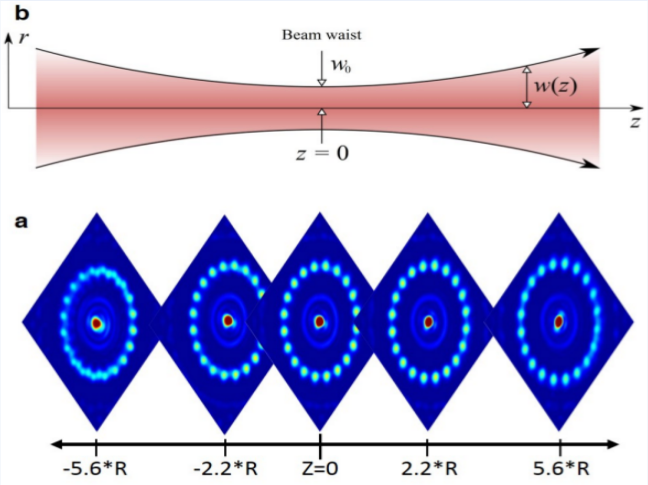
D. Solenov, D. Mozyrsky (2011)
L. Amico, D. Aghamalyan, F. Auksztol, H. Crepaz, R. Dumke, L.C. Kwek (2014)

Our model



- Interacting bosons on a ***1D lattice***
- Localized potential on one lattice site
- Magnetic flux piercing the ring

✔ No vortex formation
✔ Easier to localize a barrier
✔ Ring-ring interactions



L. Amico et al. (2014)

→ Rainer's talk
→ Davit's talk

Previous studies in limiting cases:
Hallwood, Ernst, Brand (2010)
Nunnenkamp, Rey, Burnett (2011)

Our model

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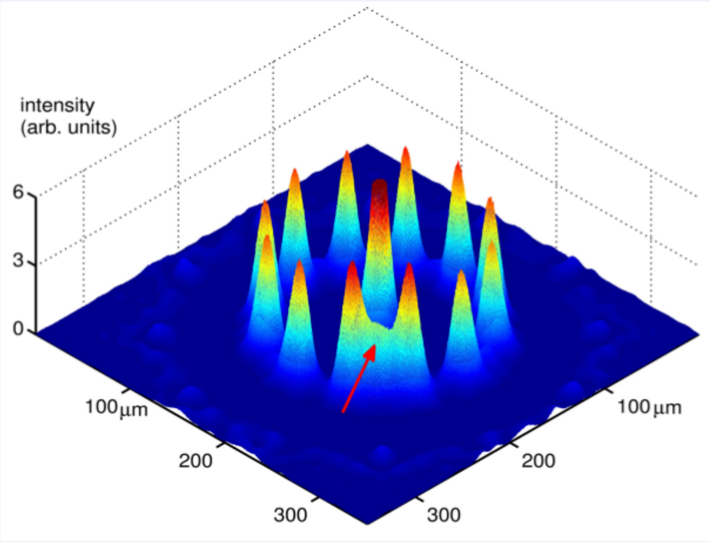


tight-binding

$$H = \sum_{j=1}^{M} \left[-t\left(e^{-i\Omega/M} b_j^\dagger b_{j+1} + \mathrm{H.c.}\right) + \boxed{\frac{U}{2} n_j(n_j - 1)} + \boxed{\Lambda_j n_j} \right]$$

artificial magnetic flux

on-site interactions

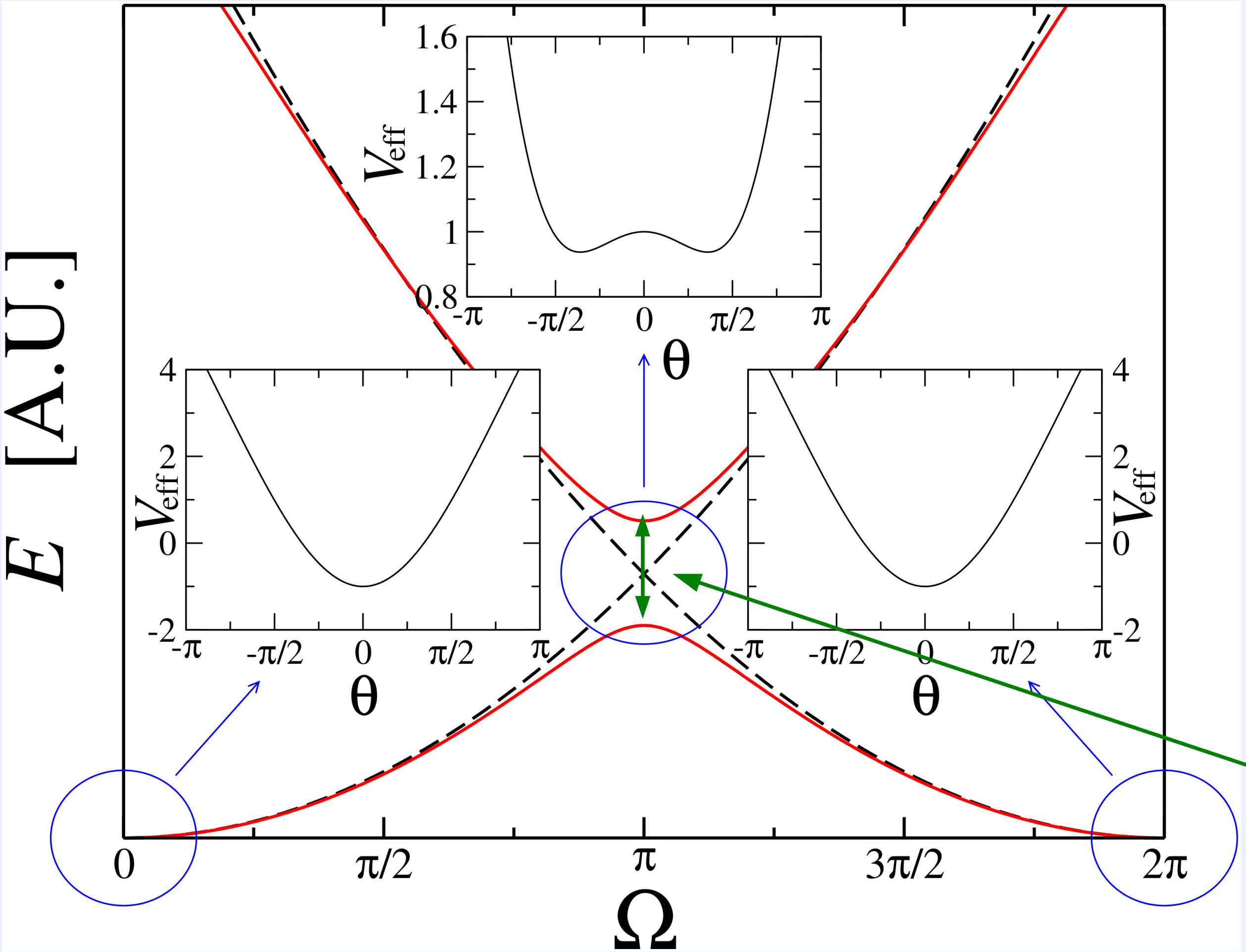
local potential

An effective ***Bose-Hubbard model***

→ hopping renormalized by the magnetic flux $t/U \to (t/U)\cos(\Omega/M)$

Niemeyer, Freericks, Monien (1999)

Effective two-level system



WITHOUT barrier:
rotational invariance
→ set of parabolas with defined angular momentum

WITH barrier:
symmetry breaking
→ avoided crossing

gap separating first two bands

@ large fillings: *quantum phase model* → Davit's talk

@ normal fillings, n ≈ 1: **Bose-Hubbard** → **this talk**

Effective two-level system

An effective "**qubit**" (two-level system) may be identified

→ energy splitting of the two levels should be sufficiently large

→ higher excitations should be energetically far enough from the two competing ground states

Bose-Hubbard model:
the low-lying spectrum

Check the dependence of $\boxed{\Delta E_1 = E_1 - E_0}$ & $\boxed{\Delta E_2 = E_2 - E_0}$ on

→ interactions *U*
→ barrier strength Λ
→ system size *M*
→ filling factor N / M

$\boxed{\Delta E_1 / \Delta E_2}$ our quality factor for the qubit

Mostly ***numerical*** study:

exact diagonalization (ED)
density-matrix renortmalization group (DMRG)
Tonks-Girardeau (TG) mapping
Gross-Pitaevskii (GP) approximation

Bose-Hubbard model:
<u>the low-lying spectrum</u>

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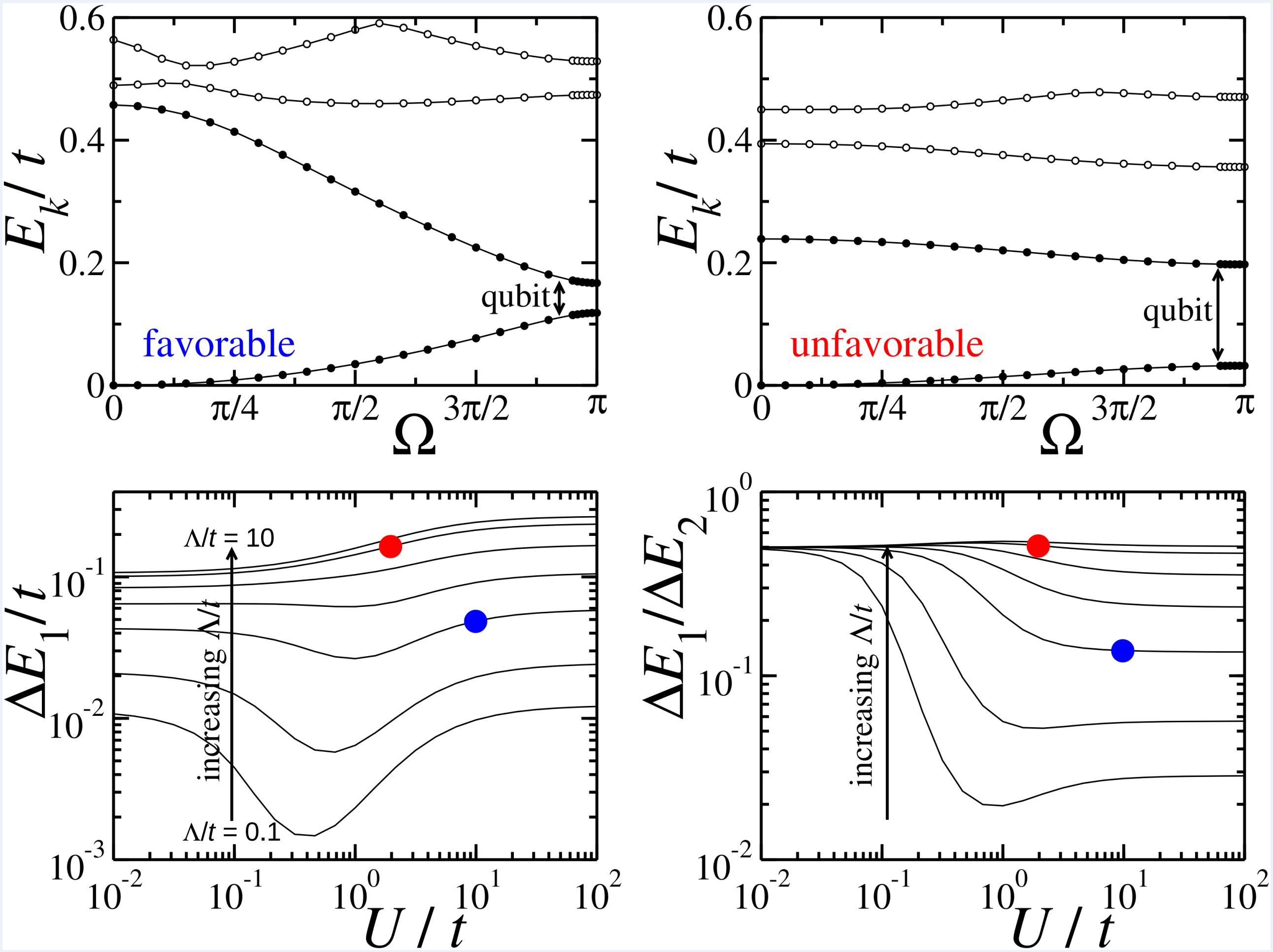
Dependence on interactions & barrier strength



✔ $U / t = 10$, $\Lambda / t = 0.5$

→ large interactions
→ moderate barrier

✗ $U / t = 2$, $\Lambda / t = 5$

→ weaker interactions
→ larger barrier

ED

$N = 4$ bosons
$M = 16$ sites

Too weak interactions **cannot** suffice to isolate the qubit !

Bose-Hubbard model:

the low-lying spectrum

Check the dependence of $\boxed{\Delta E_1 = E_1 - E_0}$ & $\boxed{\Delta E_2 = E_2 - E_0}$ on

→ interactions U

→ barrier strength Λ

→ $\boxed{\text{system size} \quad M}$

→ filling factor N / M

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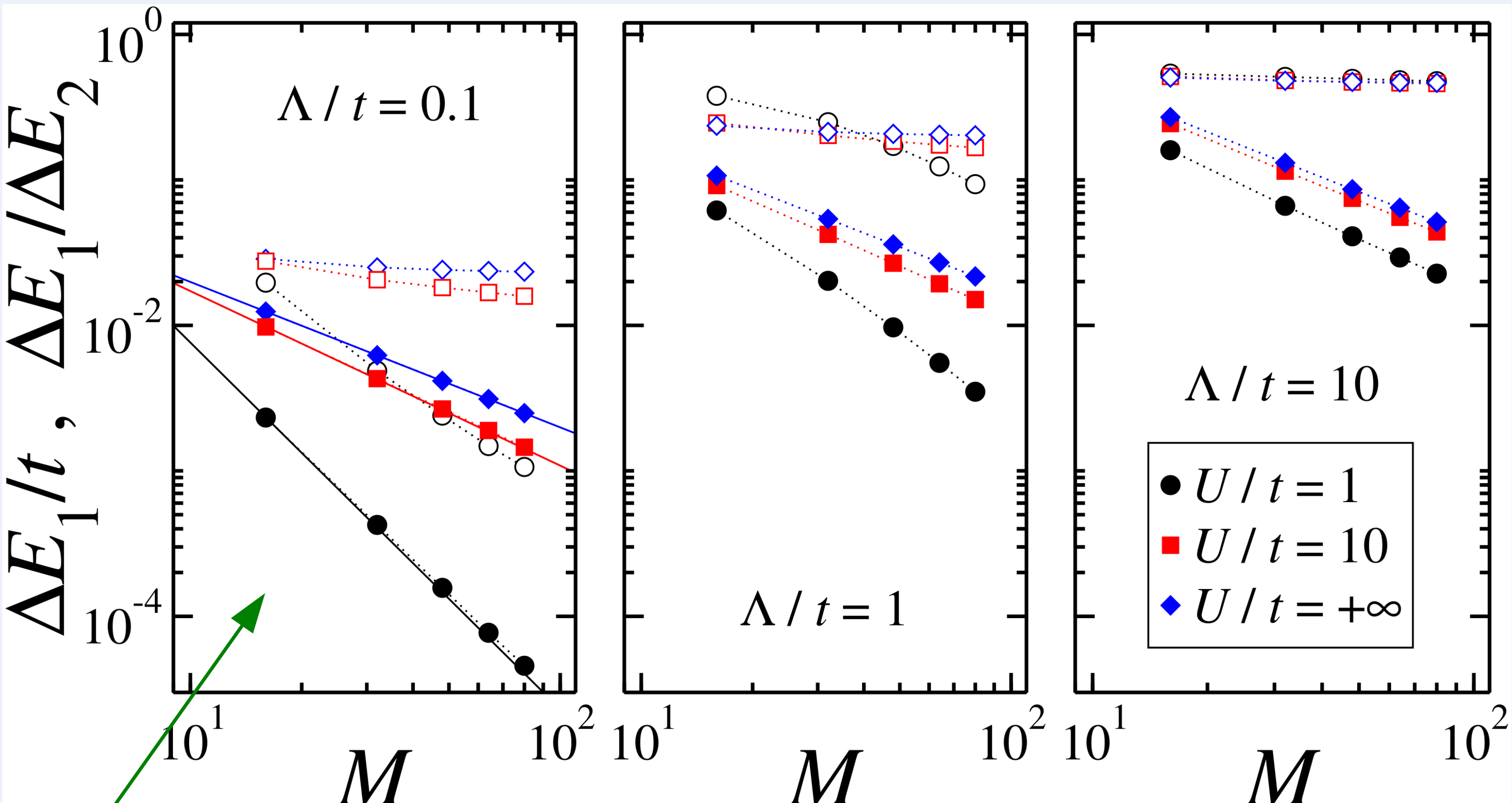
Dependence on the system size

DMRG, filling: $N\ /\ M = 1/4$



small barrier intermediate barrier large barrier

best regime: *small barrier*
mesoscopic *size*

Similar to the scaling of *persistent currents*:
Cominotti, DR, Rizzi, Hekking, Minguzzi (2014)

Bose-Hubbard model:
<u>**the low-lying spectrum**</u>

Check the dependence of $\boxed{\Delta E_1 = E_1 - E_0}$ & $\boxed{\Delta E_2 = E_2 - E_0}$ on

→ interactions U

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→ system size M

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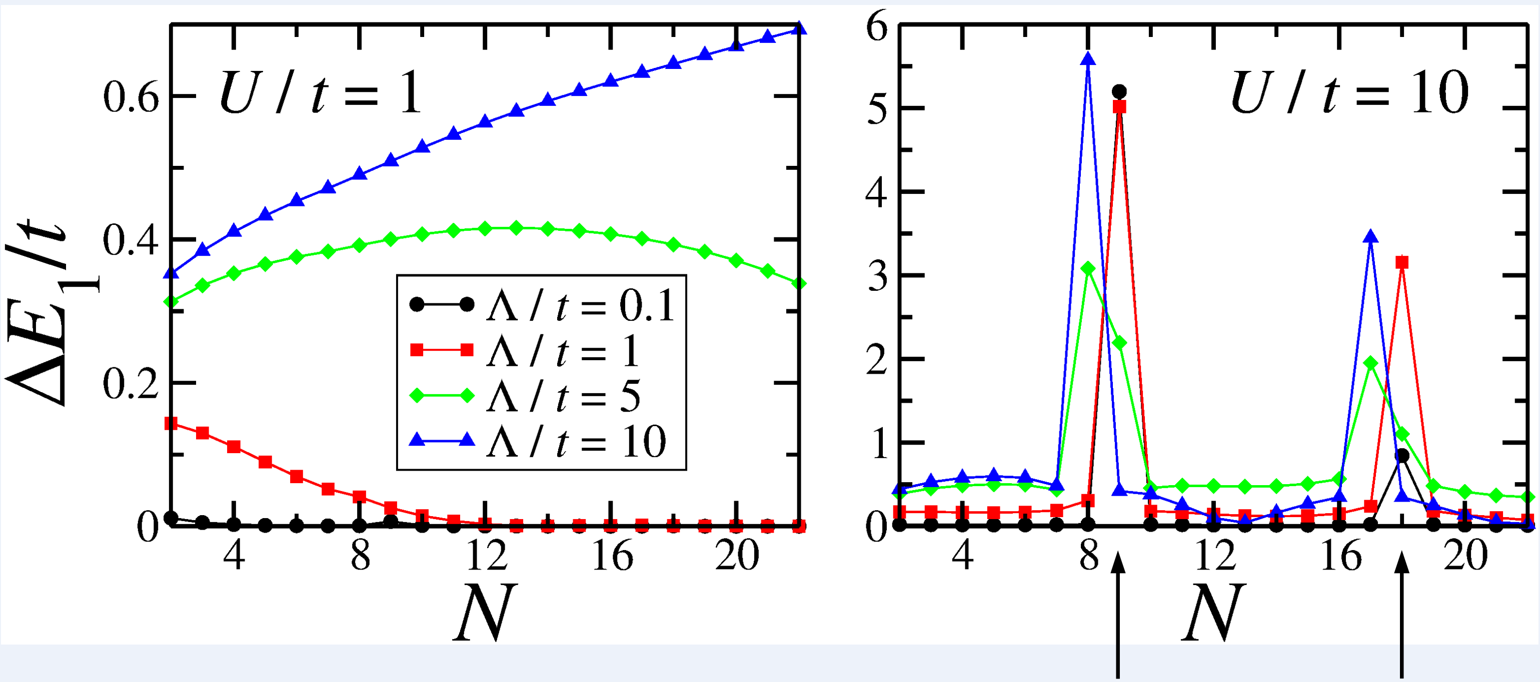
$\boxed{\Delta E_1 / \Delta E_2}$ our quality factor for the qubit

Dependence on the filling

ED, M = 9 (mesoscopic ring)



Superfluid regime:
smooth dependence on *N*

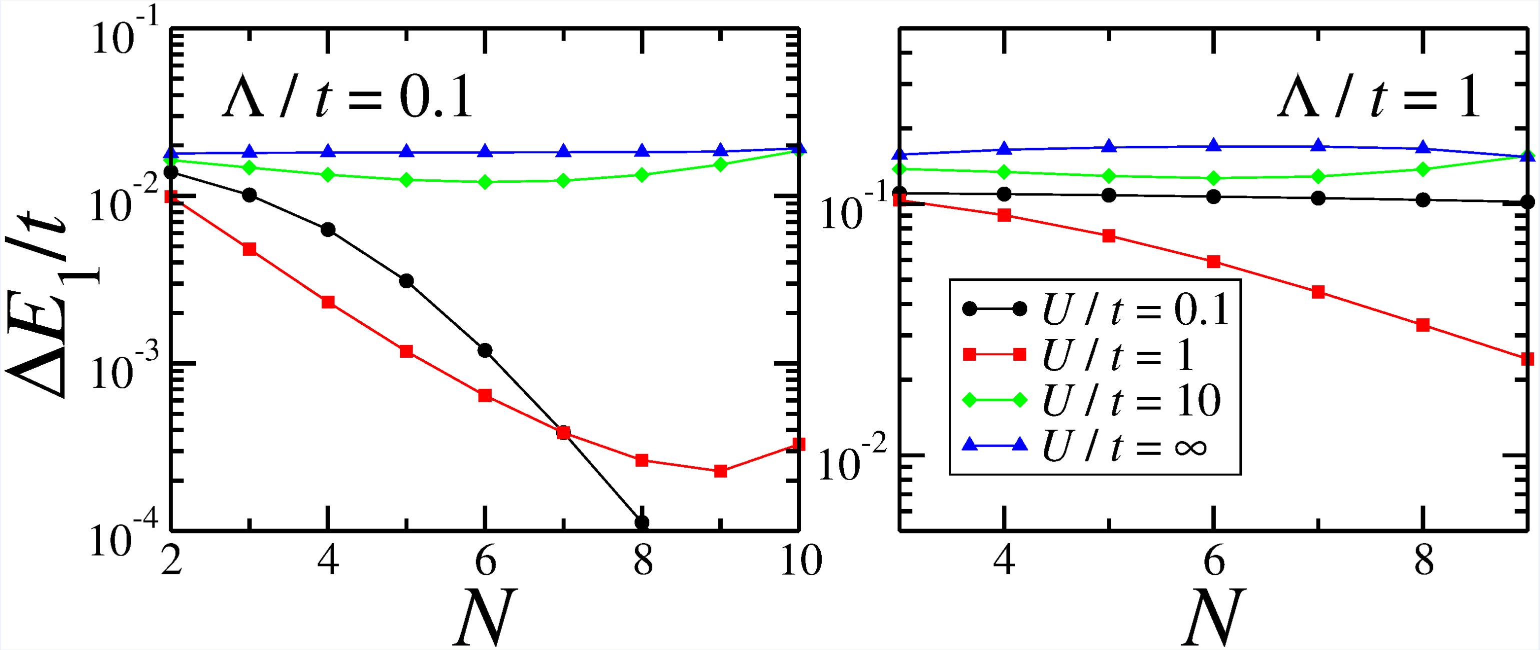
small barrier → screening limit
large barrier → tunnel limit

Mott regime:
reminiscent of gapped phases

(finite-size effects @ large Λ)

Dependence on the filling

ED & TG, M = 11 (mesoscopic ring)



Non-monotonic dependence on U

small interactions → level mixing of single-particle energies increases with N
large interactions → TG limit: single-particle gaps are identical for all level crossings

What are the most advantageous “**working points**” ?

✔ Moderate-to-strong interactions
✔ Small-barrier limit

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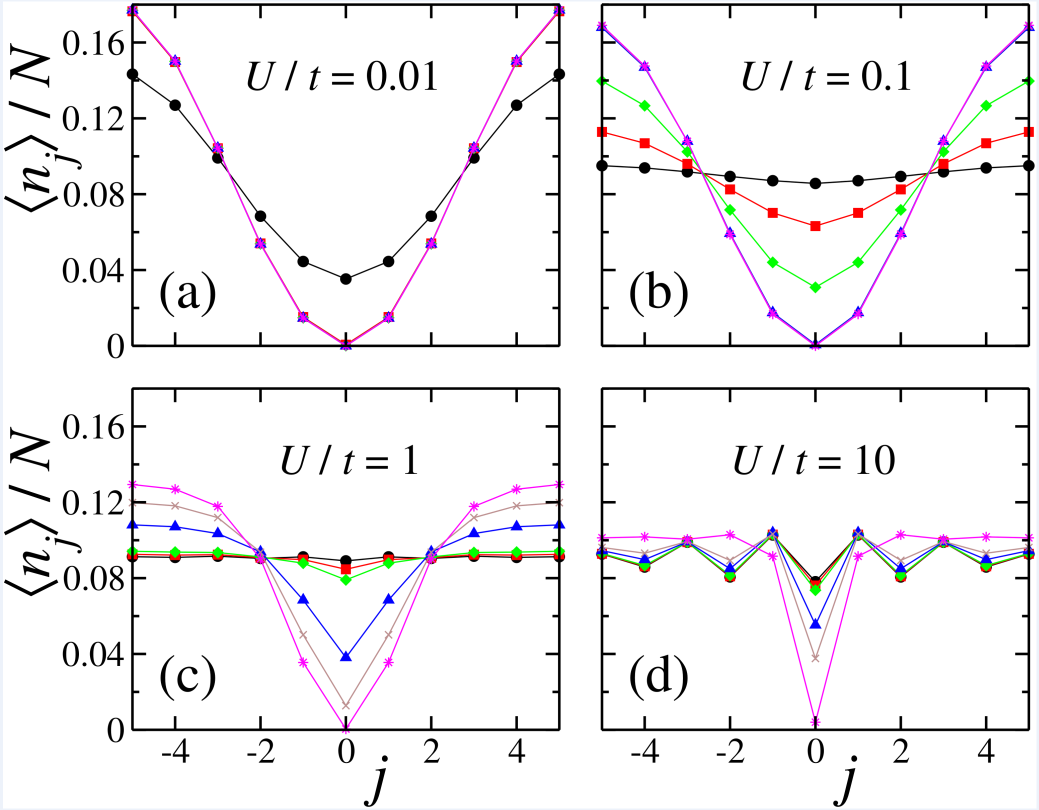
The ratio Λ / U as a useful *benchmark parameter* to define the qubit quality



density profiles along the ring…
→ Matteo's talk

@ different interaction regimes U / t
@ different barriers
(Λ / t = 0.01, 0.05, 0.1, 0.5, 1, 5)

ED
N = 5 bosons
M = 11 sites

What are the most advantageous "**working points**" ?

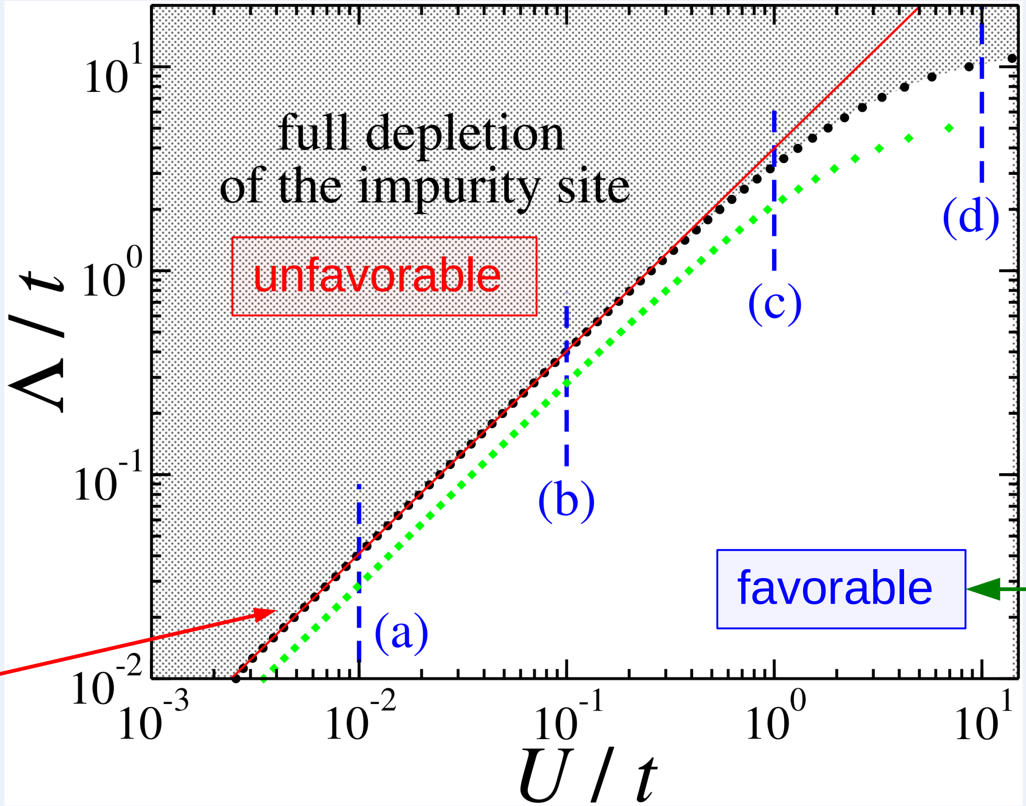
✔ Moderate-to-strong interactions
✔ Small-barrier limit

The ratio Λ / *U* as a useful *benchmark parameter* to define the qubit quality

… & barrier strength required to **disconnect** the ring

(up to a given threshold)

$(\Lambda/t)_c \propto U/t$



Momentum distribution

Focus on the ground state: detectability of macroscopic superposition of circulating states

$$\boxed{n(\mathbf{k}) = \int \mathrm{d}\mathbf{x} \int \mathrm{d}\mathbf{x}' \langle \psi^\dagger(\mathbf{x}) \psi(\mathbf{x}') \rangle e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}}$$

time-of-flight expansion

on a lattice… $\psi(\mathbf{x}) = \sum_{j=1}^{M} w(\mathbf{x}-\mathbf{x}_j) b_j$

$$n(\mathbf{k}) = |\tilde{w}(\mathbf{k})|^2 \sum_{l,j=1}^{M} \langle b_l^\dagger b_j \rangle e^{i\mathbf{k}\cdot(\mathbf{x}_l - \mathbf{x}_j)}$$

Momentum distribution

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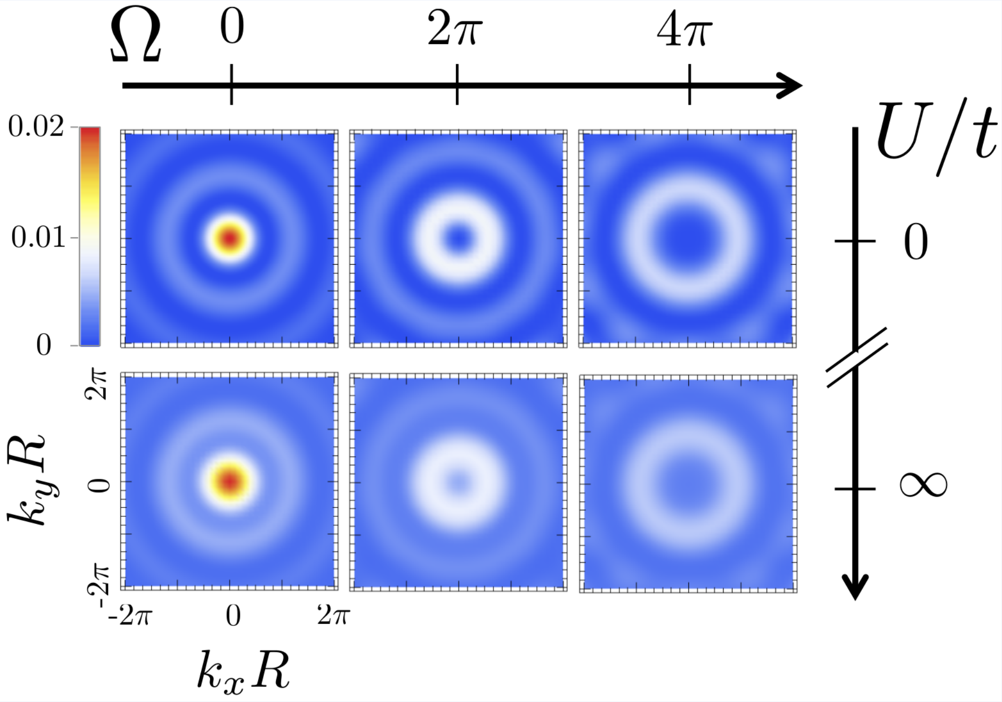
time-of-flight expansion

In absence of barrier ($\Lambda = 0$)

– rotational invariant system
– currents unaffected by interactions
– smeared signal at large *U*

$0 < \Omega < \pi$ no circulation
$\pi < \Omega < 2\pi$ one quantum of circulation
$\Omega = \pi$ *interference* of them



Without interactions (a single-particle problem)

Absence of barrier ($\Lambda = 0$)

$$\psi(\theta) = \frac{1}{\sqrt{2\pi}} e^{in\theta} \qquad n \in \mathbb{Z}$$

$$n(\mathbf{k}) = |J_n(|\mathbf{k}|R)|^2$$

n = 0 peaked at **k** = 0
n > 0 ring shaped, radius growing with *n*

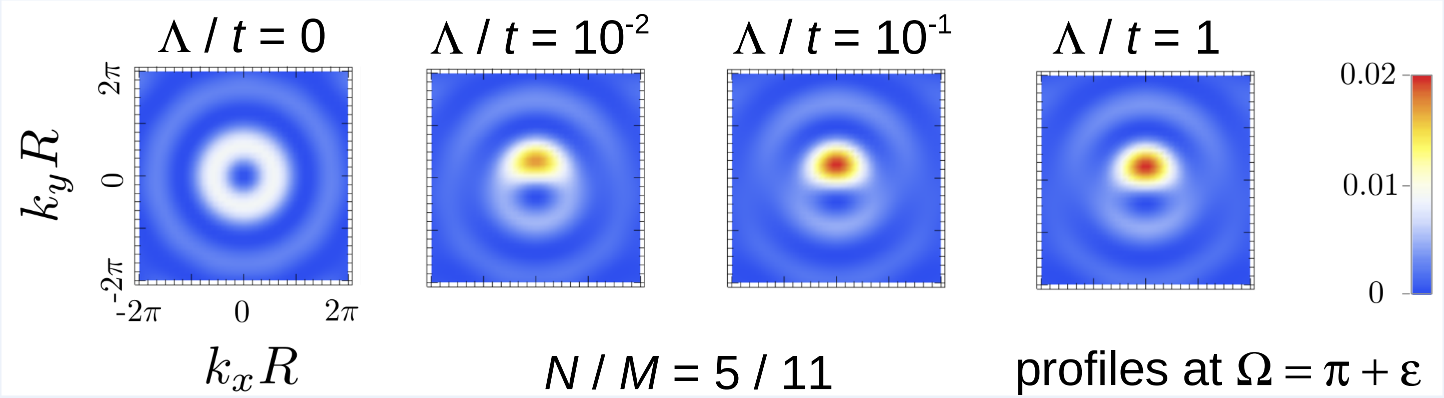
Presence of barrier ($\Lambda \neq 0$)

$$\psi(\theta) \approx \frac{1}{\sqrt{2\pi}} \left[\sin\left(\frac{\varphi}{2}\right) e^{in\theta} + \cos\left(\frac{\varphi}{2}\right) e^{i(n+1)\theta} \right]$$

mix states with different angular momentum

$$\begin{aligned} n(\mathbf{k}) \approx\ & \sin^2\left(\frac{\varphi}{2}\right) J_n^2(|\mathbf{k}|R) \\ & + \cos^2\left(\frac{\varphi}{2}\right) J_{n+1}^2(|\mathbf{k}|R) \\ & + \boxed{\sin(\varphi)\cos(\gamma_{\mathbf{k}}) J_n(|\mathbf{k}|R) J_{n+1}(|\mathbf{k}|R)} \end{aligned}$$

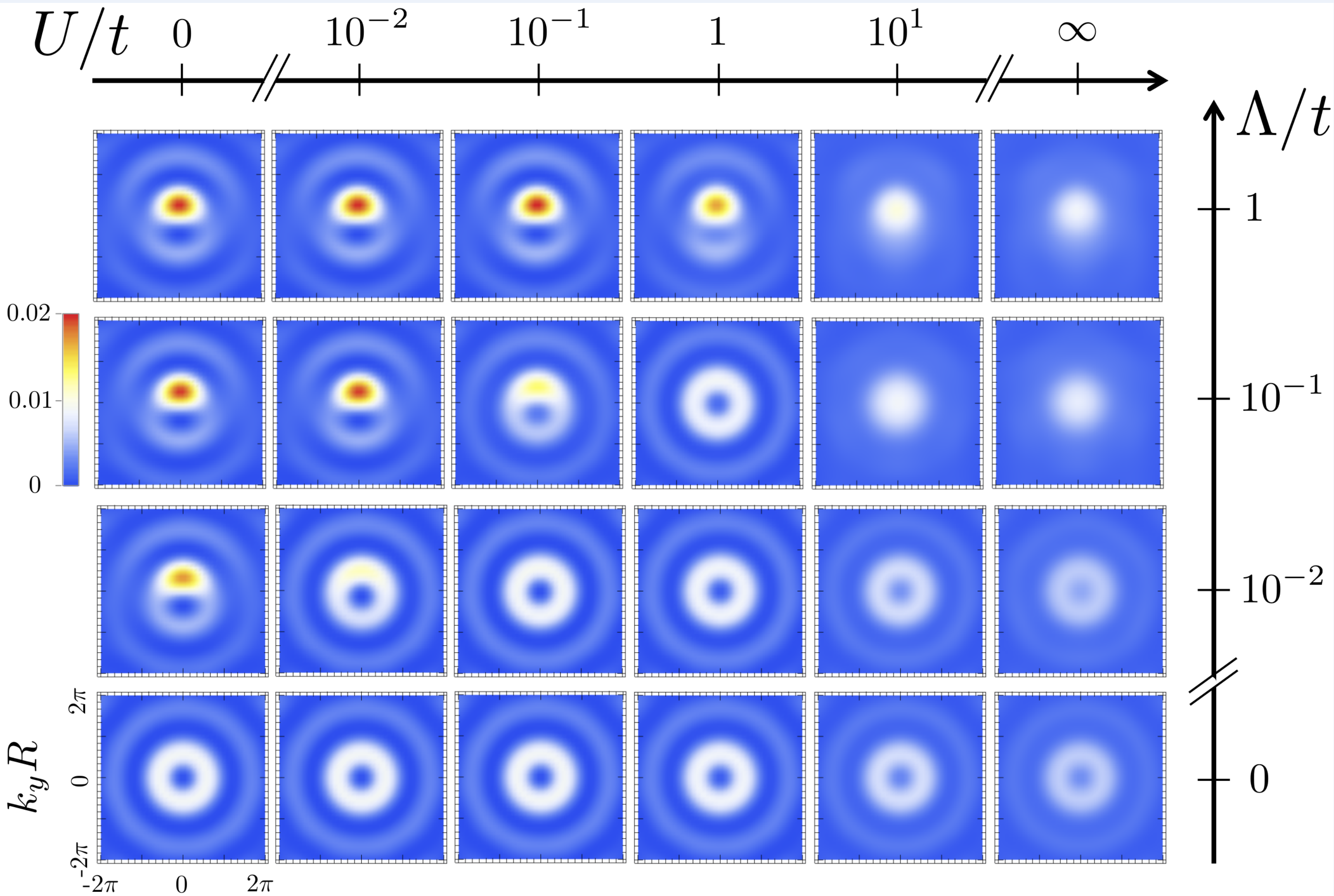
interference term



slight offset from frustration point ($\Omega = \pi$)

nontrivial dependence on Λ and *U*

With interactions (a many-body problem)



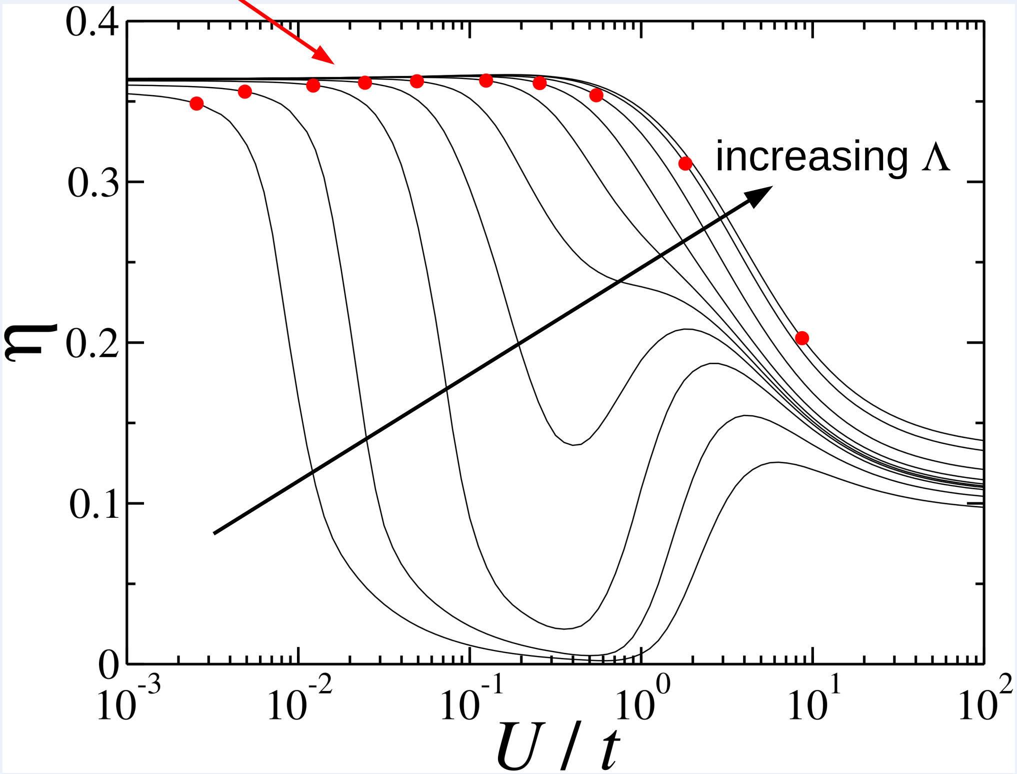
N / *M* = 5 / 11

profiles at $\Omega = \pi + \varepsilon$

With interactions (a many-body problem)

TOF images are *independent of the barrier* above a given critical value Λ_c

η constant @ U_c defined as the interaction strength required to disconnect the ring



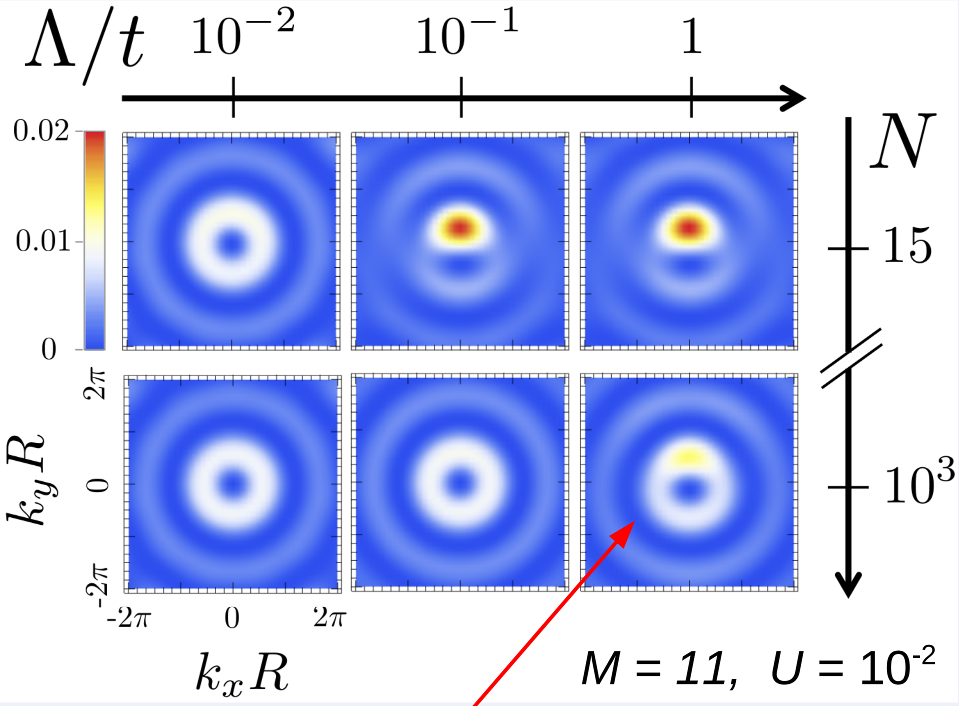
contrast figure of merit:

$$\eta = \frac{\int d\mathbf{k}\ |n_{\Lambda\neq0}(\mathbf{k}) - n_{\Lambda=0}(\mathbf{k})|}{\int d\mathbf{k}\ n_{\Lambda\neq0}(\mathbf{k}) + n_{\Lambda=0}(\mathbf{k})}$$

non-monotonic screening of the barrier vs. *U*

→ Matteo talk

With interactions (a many-body problem)



@ large fillings

(GPE analysis)

A larger barrier strength is required to observe superposition features

Large *N* enhances the screening of the barrier!

Summary

Interacting bosons on a ring-shaped 1D lattice with a localized barrier:

an effective **qubit** [@ low-energies]

interference between forward/backward scattered bosons

✔ Scaling of the energy gap for the qubit

→ *appreciable* for small / mesoscopic systems
→ *suppressed* in the thermodynamic limit

✔ Superposition of circulation states: momentum distribution

The ratio U / Λ locates the ***optimal working point*** for gap resolution & TOF detectability

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