Driving a mobile agent in a guidance by repulsion model

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Partial differential equations, optimal design and numerics – BENASQUE

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EXCELENCIA SEVERO OCHOA

"Guidance by repulsion" model

$$\dot{\vec{u}}_p(t) = \vec{v}_p(t), \tag{1}$$

$$\dot{\vec{u}}_e(t) = \vec{v}_e(t), \tag{2}$$

$$\dot{\vec{v}}_p(t) = -\frac{C_P^E}{m_p} \frac{\vec{u}_p - \vec{u}_e}{\|\vec{u}_e(t) - \vec{u}_p(t)\|^2} \left[1 + \frac{1}{\|\vec{u}_e(t) - \vec{u}_p(t)\|^2} \left(\frac{C_{\mathrm{R}}}{C_P^E} d_1^4 - d_c^2 \right) \right] \tag{3}$$

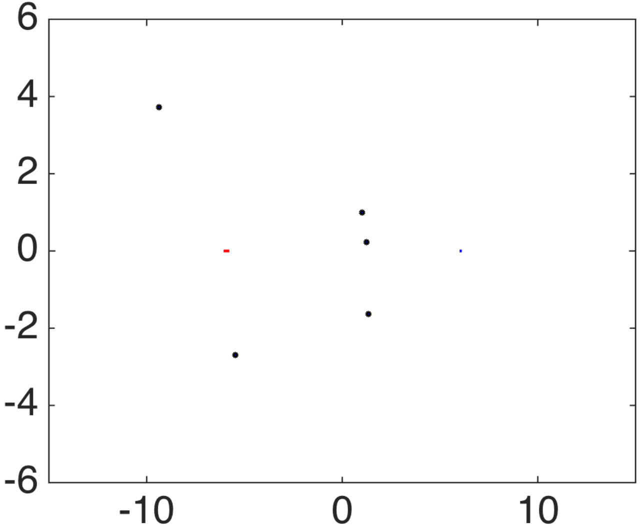
$$+ \kappa d_1^4 d_2 \frac{C_{\mathrm{R}}}{m_p} \frac{(\vec{u}_p - \vec{u}_e)^{\perp}}{\|\vec{u}_e(t) - \vec{u}_p(t)\|^5} - \frac{\nu_p}{m_p} \vec{v}_p, \tag{4}$$

$$\dot{\vec{v}}_e(t) = \frac{1}{m_e} \left[C_E^P \frac{\vec{u}_e(t) - \vec{u}_p(t)}{\|\vec{u}_e(t) - \vec{u}_p(t)\|^2} - \nu_e \vec{v}_e(t) \right], \tag{5}$$

$$\vec{u}_p(t_0) = \vec{u}_p^0, \quad \vec{u}_e(t_0) = \vec{u}_e^0, \quad \vec{v}_p(t_0) = 0 \quad \text{and} \quad \vec{v}_e(t_0) = 0. \tag{6}$$

Variables and parameters of the model

- The variables:
  - Driver position vector: $\vec{u}_p \in \mathbb{R}^2$
  - Driver velocity vector: $\vec{v}_p \in \mathbb{R}^2$
  - Evader position vector: $\vec{u}_e \in \mathbb{R}^2$
  - Evader velocity vector: $\vec{v}_e \in \mathbb{R}^2$
- The control: $\kappa(t) \in \{-1, 0, 1\}$
- $\vec{u}_T \in \mathbb{R}^2$
- Parameters (with the values used in this work):
  - $d_1 = 2, d_2 = 2, d_c = 2$ are distances,
  - $C_R = 0.5$, $C_P^E = 3$, $C_E^P = 2$ are coefficients of the atraction-repulsion force and circumvection force.
  - $m_e = 1, m_p = 0.4$ are the masses of the evader and the driver.
  - $\nu_e = 2, \nu_p = 1$ are the frictions of the evader and the driver.
- We want fast driver, faster than the evader at least, so:
  - $C_R d_1^4 < C_P^E d_c^2$, $m_p/\nu_p < m_e/\nu_e < 1$, $C_E^P < C_P^E$


6
4
2
0
-2
-4
-6
-10
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10

Pursuit and Circumvection modes

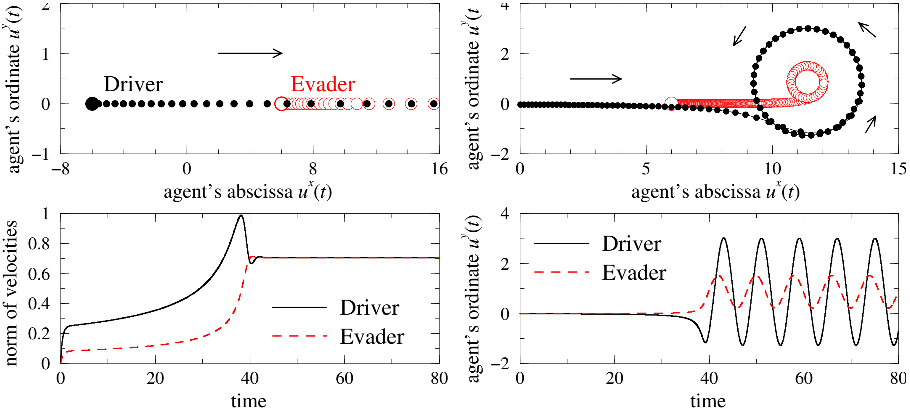

Figure: The two operating modes of the system. Left panels: pursuit mode ($\kappa = 0$). Right panels: circumvention mode ($\kappa = 1$). Initial configuration, in both cases: $\vec{u}_p = (-6, 0)$, $\vec{u}_e = (6, 0)$ with zero initial velocities. Upper panels: agents' trajectories. Left-bottom panel: time variation of the norm of the velocities $\|\vec{v}_{p,e}(t)\|$. Right-bottom panel: time variation of agents' ordinates.

Driving the evader

Let $\kappa_\tau(t)$: $\mathbb{R} \to \{-1, 0, 1\}$ be the following step-function in the time interval $[t_0, t_f]$, wiht t_f sufficiently large.

$$\kappa_\tau(t) = \begin{cases} \kappa_0 & \text{if } t < \tau \\ 0 & \text{if } t \geq \tau \end{cases}, \tag{7}$$

where $\kappa_0 = \pm 1$ is the initial value at time t_0: $\kappa_\tau(t_0) = \kappa_0$. Then, there exists an interval $[\tau_\alpha, \tau_\omega] \subset (t_0, t_f)$ such that for all $\tau \in [\tau_\alpha, \tau_\omega]$, there exists a time $t \in (t_0, t_f)$ for which the evader is in the interior of the ball of radius r centered in the target T. That is:

$$\forall\, \tau \in [\tau_\alpha, \tau_\omega],\ \exists\, t \in (t_0, t_f) \text{ such that } \|\vec{u}_e(t) - \vec{u}_T\| < r. \tag{8}$$

Moreover, if $r \to 0$, then the interval $[\tau_\alpha, \tau_\omega]$ shrinks to a single point τ^* such that there exists a time $t \in (t_0, t_f)$ for which $\vec{u}_e(t) = \vec{u}_T$.

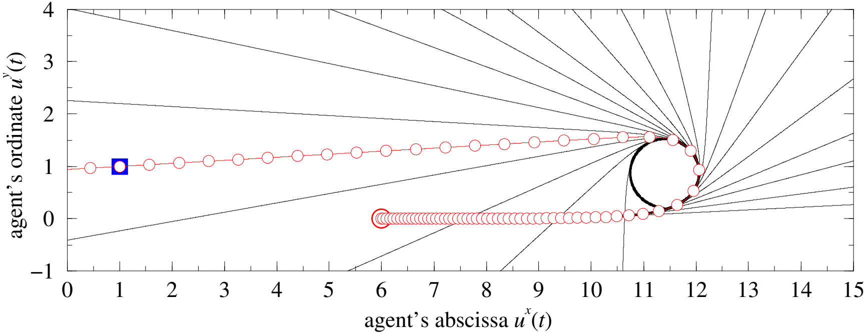

Figure: Filled (blue) square denotes target's position $\vec{u}_T = (1, 1)$, empty (red) circle evader's initial position $\vec{u}_e(0) = (6, 0)$; driver's initial position $\vec{u}_p(0) = (-6, 0)$ is not depicted. Line with (red) circles denote the trajectory of the evader for the optimal value $\tau^* = 41.15$. Wide (black) line denotes the accumulation circle around which the evader turns if the control is kept to one. Thin (black) lines denote trajectories of the evader for the following non-equispaced values of τ: 34, 36, 37, 37.5, 38, 38.4, 38.7, 39, 39.3, 39.6, 39.9, 40.2, 40.4, 40.6, 40.8, 41, 41.3, 41.6, 42 and 43.

Simple shooting method to find τ

1. We choose a time instant t and we evaluate the instantaneous alignment $\alpha(t)$ of the velocity vector of the evader $\vec{v}_e(t)$ with respect to the target point $\vec{u}_T$:

$$\alpha(t) = \left(u_T^y - u_e^y(t)\right) v_e^x(t) - \left(u_T^x - u_e^x(t)\right) v_e^y(t). \tag{9}$$

   Then, if $\alpha(t) < 0$, take a smaller value of $t_{\texttt{OFF}}$ and shoot again.
2. If at time t_f the velocity vector of the evader is still pointing above the target, that is, $\alpha(t_f) > 0$, then take a larger value of $t_{\texttt{OFF}}$ and shoot again.

The new value of $t_{\texttt{OFF}}$ for the next shoot can be selected with a simple method (*e.g.*, bisection).
Stop when $|\alpha(t)| < \epsilon$, for a small value of the tolerance ϵ; the value of $t^*_{\texttt{OFF}}(t_{\texttt{ON}})$ has been found, proceed to the next value of $t_{\texttt{ON}}$.

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Thank you very much for your attention!