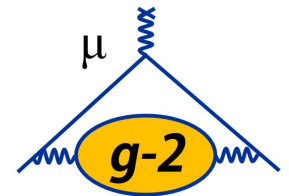


Hadronic Vacuum Polarisation in $(g-2)_\mu$



Thomas Teubner

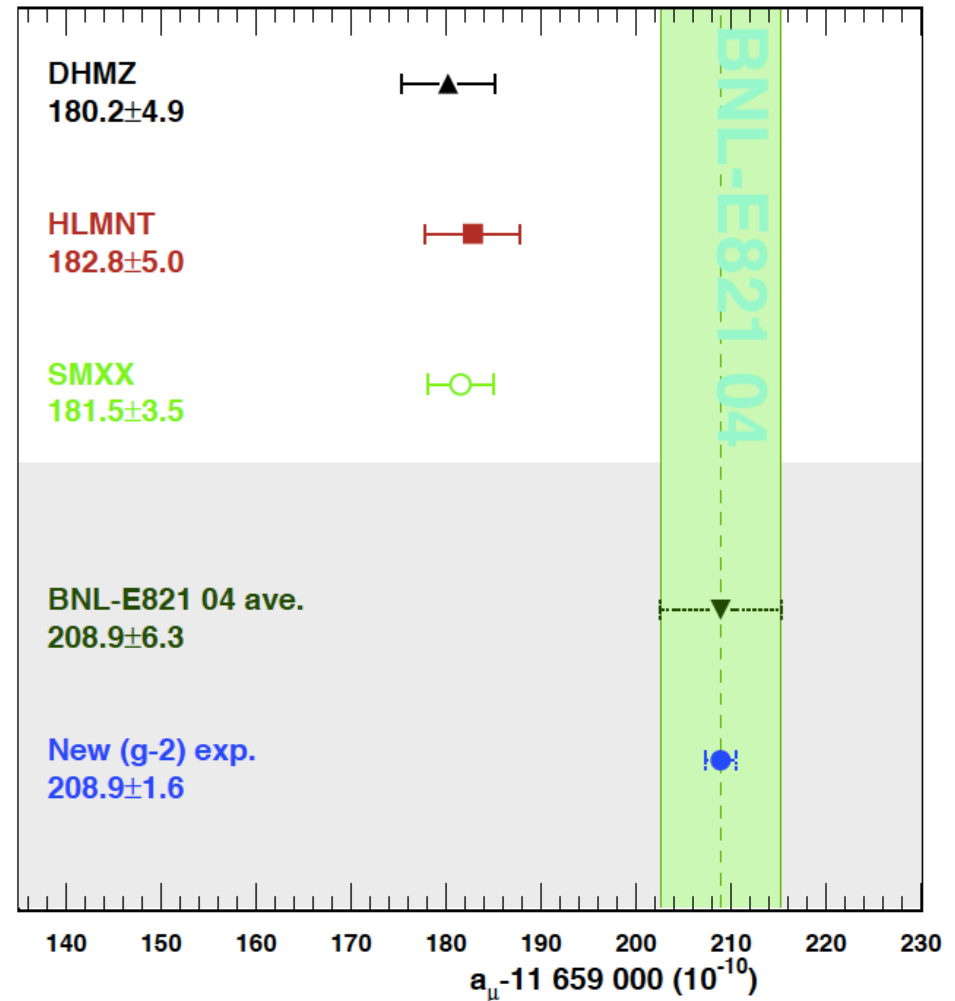


- Introduction
- Radiative corrections; VP and FSR
- Data combination, fits, models
- Ongoing puzzles, new data and new results
- Outlook, discussion

a_μ : Status and future projection → charge for TH

E989: 1st results ~ end 2017,
1-2x BNL stats early 2019...

- if mean values stay and with **no** a_μ^{SM} improvement:
5 σ discrepancy
- if also EXP+TH can improve a_μ^{SM}
'as expected' (consolidation of
L-by-L on level of Glasgow
consensus, about factor 2 for
HVP): NP at 7-8 σ
- or, if mean values get closer, very
strong exclusion limits on many
NP models (extra dims, new dark
sector, xxxSSSM)...



a_μ^{SM} : overview, numbers

- Several groups have produced hadronic compilations over the years.
- Here: Hagiwara+Liao+Martin+Nomura+T, numerics ($\sim 3.x \sigma$) still valid
- Many more precise data in the meantime and more expected for near future
- At present HVP still dominates the SM error:

QED contribution	11 658 471.808 (0.015) $\times 10^{-10}$	Kinoshita & Nio, Aoyama et al
EW contribution	15.4 (0.2) $\times 10^{-10}$	Czarnecki et al
Hadronic contribution		
LO hadronic	694.9 (4.3) $\times 10^{-10}$	HLMNT11
NLO hadronic	-9.8 (0.1) $\times 10^{-10}$	HLMNT11
light-by-light	10.5 (2.6) $\times 10^{-10}$	Prades, de Rafael & Vainshtein
Theory TOTAL	11 659 182.8 (4.9) $\times 10^{-10}$	
Experiment	11 659 208.9 (6.3) $\times 10^{-10}$	world avg
Exp – Theory	26.1 (8.0) $\times 10^{-10}$	3.3 σ discrepancy

(Numbers taken from HLMNT11, arXiv:1105.3149)

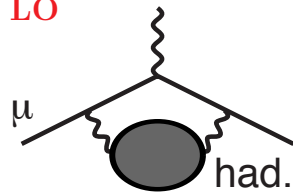
$a_\mu^{\text{had, VP}}$: Hadronic Vacuum Polarisation

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{NP?}}$$

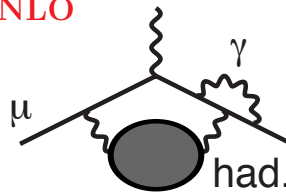
- QED: ✓
- EW: ✓
- Hadronic: **the limiting factor of the SM prediction** ✗

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$

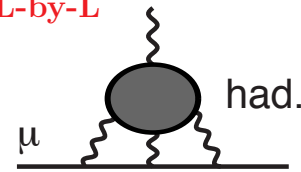
LO



NLO



L-by-L



HVP: - most precise prediction by using e^+e^- hadronic cross section (+ tau) data and a dispersion integral

- done at LO and NLO (see graphs)

- now even at NNLO [Steinhauser et al., PLB734(2014)144] →

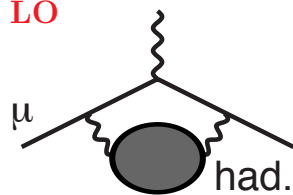
- alternative: lattice QCD, but also need QED corrections; systematics <1% ?

$a_\mu^{\text{had, VP}}$: Hadronic Vacuum Polarisation

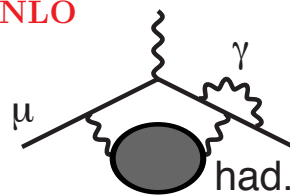
- HVP: **still the largest error in the SM prediction** ✗

$$a_\mu^{\text{had}} = a_\mu^{\text{had, VP LO}} + a_\mu^{\text{had, VP NLO}} + a_\mu^{\text{had, Light-by-Light}}$$

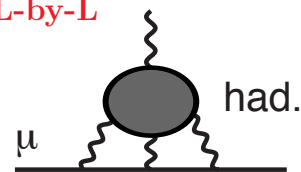
LO



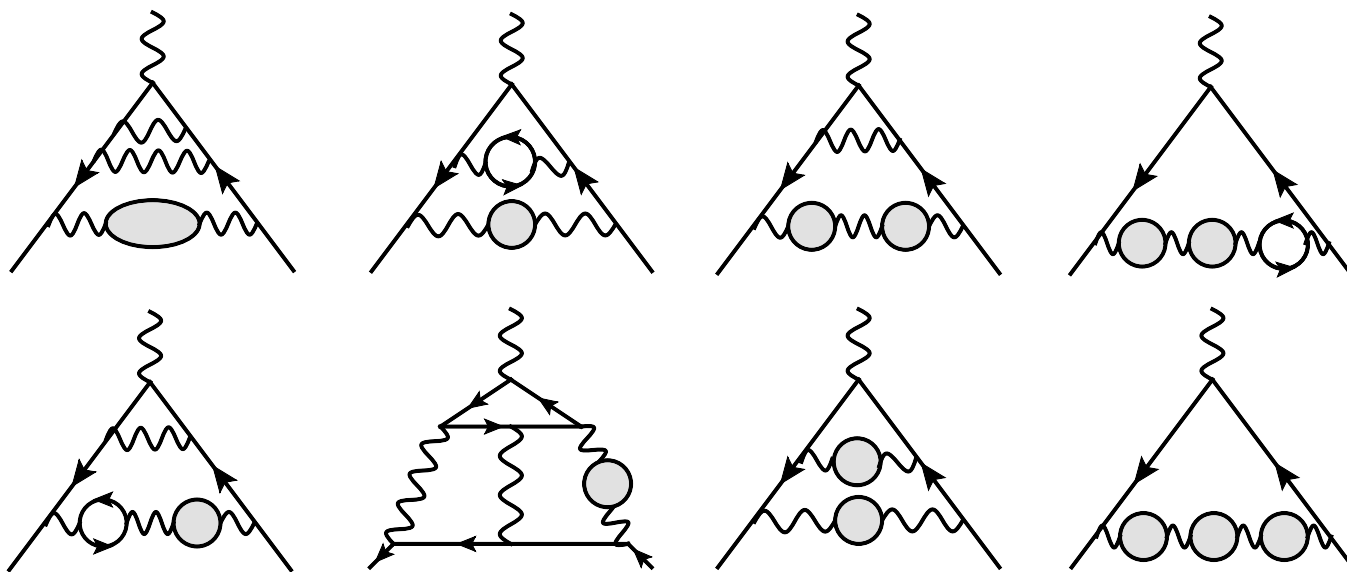
NLO



L-by-L

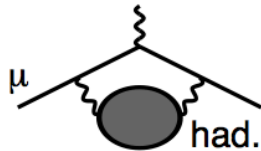


HVP at NNLO by Steinhauser et al.: $a_\mu^{\text{HVP, NNLO}} = + 1.24 \times 10^{-10}$



Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:



pQCD not useful. Use the **dispersion relation** and the **optical theorem**.

$$\text{had.} = \int \frac{ds}{\pi(s-q^2)} \text{Im had.}$$

$$2 \text{Im had.} = \sum_{\text{had.}} \int d\Phi \left| \text{had.} \right|^2$$

$$a_{\mu}^{\text{had,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$
 \Rightarrow **Lower** energies **more important**
 $\Rightarrow \pi^+\pi^-$ channel: 73% of total $a_{\mu}^{\text{had,LO}}$

How to get the most precise σ_{had}^0 ? **e^+e^- data:**

- Low energies: **sum** \sim 25 **exclusive channels**, $2\pi, 3\pi, 4\pi, 5\pi, 6\pi, KK, KK\pi, KK\pi\pi, \eta\pi, \dots$, use **iso-spin relations** for **missing** channels
- Above ~ 1.8 GeV: can start to use **pQCD** (away from flavour thresholds), supplemented by **narrow resonances** ($J/\psi, \Upsilon$)
- Challenge of **data combination (locally in \sqrt{s})**: from many experiments, in different energy bins, errors from different sources, **correlations**; must avoid **inconsistencies/bias**
- traditional '**direct scan**' (tunable e^+e^- beams) vs. '**Radiative Return**' [+ τ spectral functions]
- σ_{had}^0 means 'bare' σ , but WITH FSR: **RadCorrs** [HLMNT: $\delta a_{\mu}^{\text{had, RadCor VP+FSR}} = 2 \times 10^{-10}$!]

Radiative Corrections: HVP for running $\alpha(q^2)$

- Dyson summation of Real part of one-particle irreducible blobs Π into the effective, real running coupling α_{QED} :

$$\Pi = \text{wavy line } \gamma^* \text{ with momentum } q \text{ entering a shaded blob } \Pi \text{ and another wavy line exiting}$$

Full photon propagator $\sim 1 + \Pi + \Pi \cdot \Pi + \Pi \cdot \Pi \cdot \Pi + \dots$

$$\rightsquigarrow \alpha(q^2) = \frac{\alpha}{1 - \text{Re}\Pi(q^2)} = \alpha / (1 - \Delta\alpha_{\text{lep}}(q^2) - \Delta\alpha_{\text{had}}(q^2))$$

- The Real part of the VP, $\text{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\text{Im}\Pi \sim \sigma(e^+e^- \rightarrow \text{hadrons})$:

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{q^2}{4\pi^2\alpha} \text{P} \int_{m_\pi^2}^{\infty} \frac{\sigma_{\text{had}}^0(s) ds}{s - q^2}, \quad \sigma_{\text{had}}(s) = \frac{\sigma_{\text{had}}^0(s)}{|1 - \Pi|^2}$$

[$\rightarrow \sigma^0$ requires 'undressing', e.g. via $\cdot(\alpha/\alpha(s))^2 \rightsquigarrow$ iteration needed]

- Observable cross sections σ_{had} contain the |full photon propagator|², i.e. |infinite sum|².
 \rightarrow To include the subleading Imaginary part, use dressing factor $\frac{1}{|1 - \Pi|^2}$.

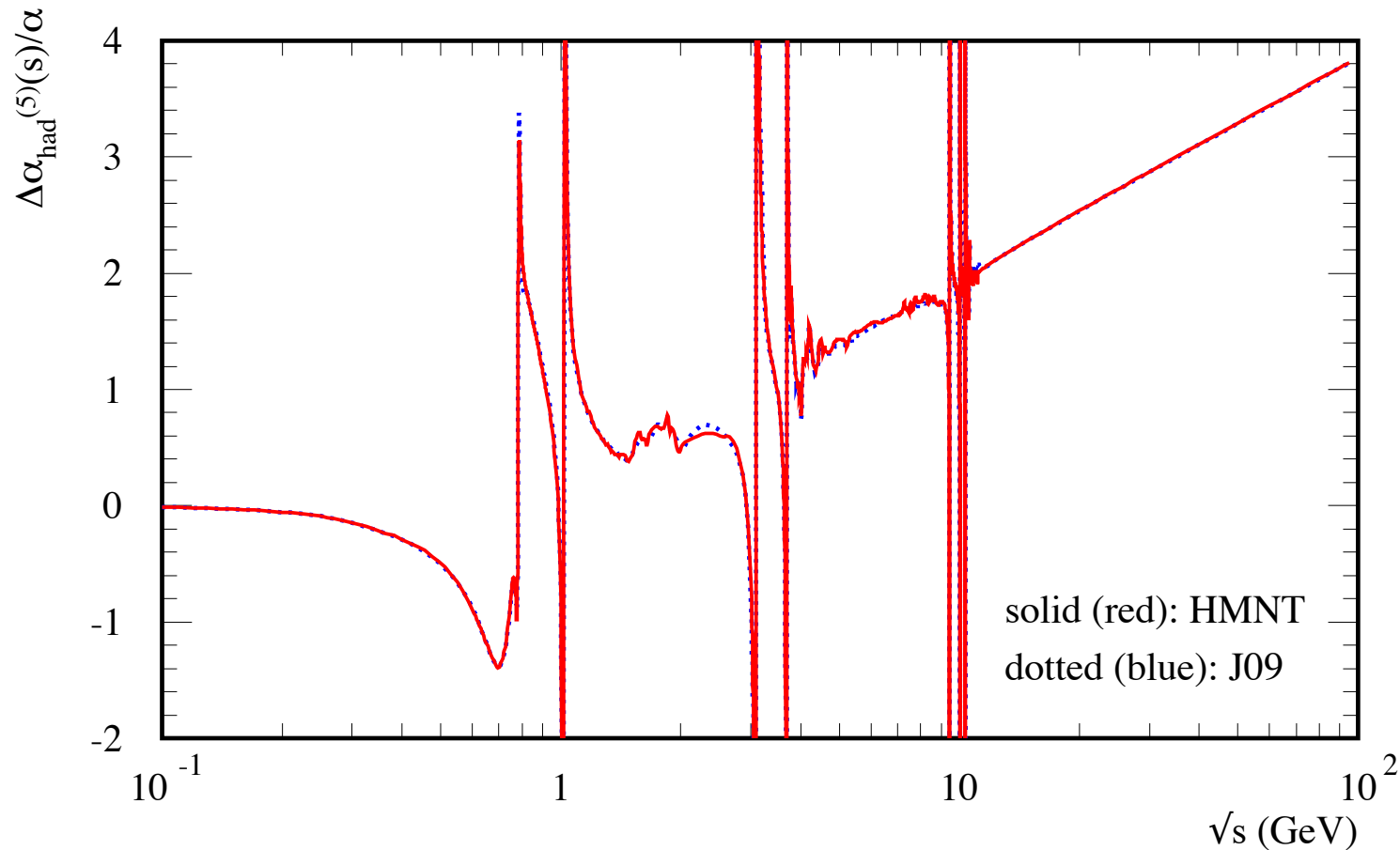
Radiative Corrections: HVP for running $\alpha(q^2)$

Parametrisations/routines based on 'global' data compilations available from a few groups:

- Novosibirsk: <http://cmd.inp.nsk.su/~ignatov/vpl/> tabulation with ROOT package
- Davier et al: [HVPTools](#) (status of distribution? still in preparation?)
- Fred Jegerlehner's package: <http://www-com.physik.hu-berlin.de/~fjeger/software.html>
 - set of routines with analytic codes and tabulations
 - uses rhad from Harlander+Steinhauser for Im part
 - regular updates (last 5.4.2012)
- HLMNT routine
 - provided upon request by authors (Daisuke Nomura or TT)
 - standalone Fortran, partly analytic, partly tabulation
 - current version is VP_HLMNT_v2_2 (20.5.2015, based mainly on HLMNT11 status)
 - flag to control if narrow resonances included or not, but Φ and higher Υ always included through direct data integration

Radiative Corrections: HVP for running $\alpha(q^2)$

- $\Delta\alpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner's new routines



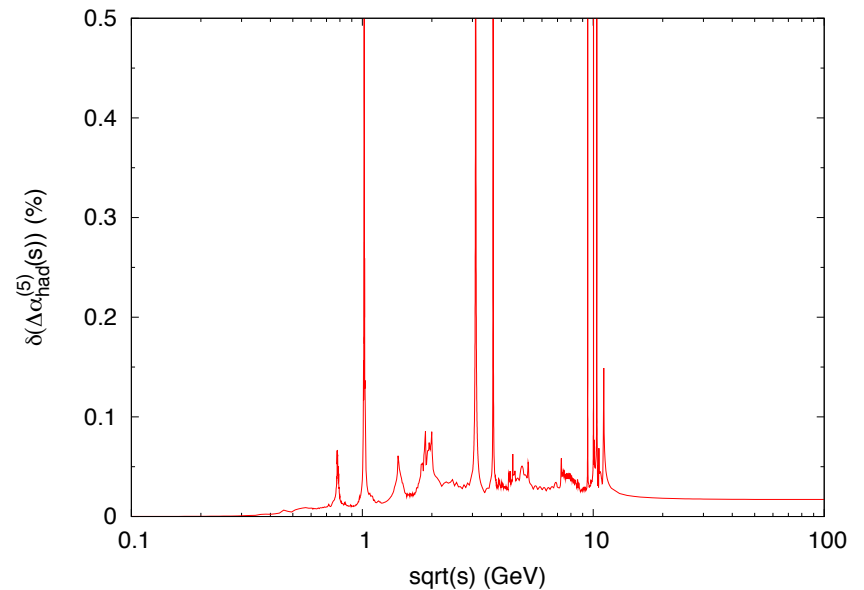
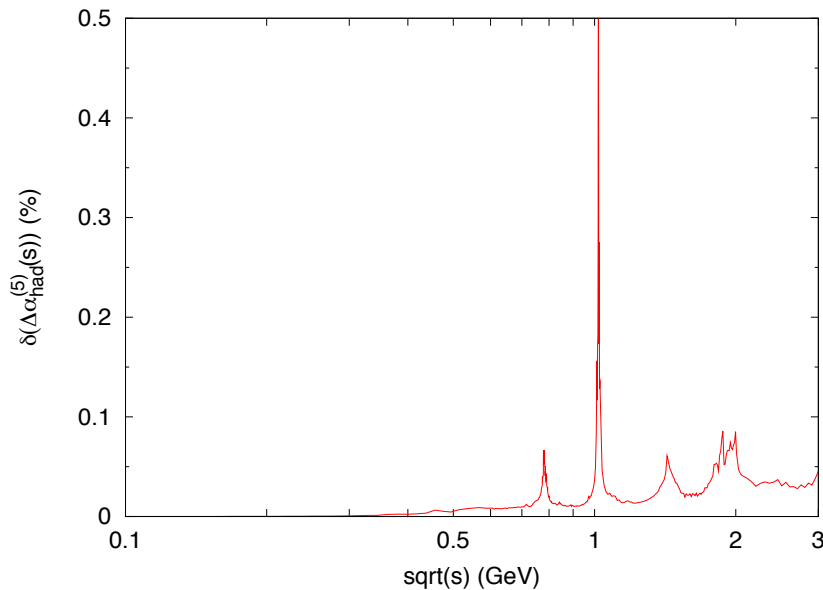
→ with new version big differences (with 2003 version) gone

— smaller differences remain and reflect different choices, smoothing etc.

Radiative Corrections: HVP for running $\alpha(q^2)$

- Typical accuracy $\delta \left(\Delta\alpha_{\text{had}}^{(5)}(s) \right)$

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):



→ Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.

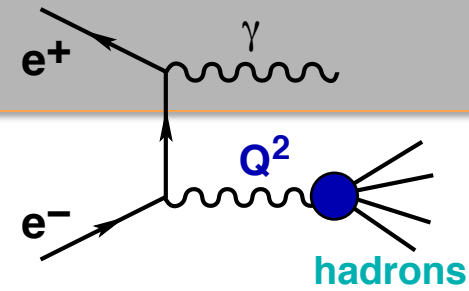
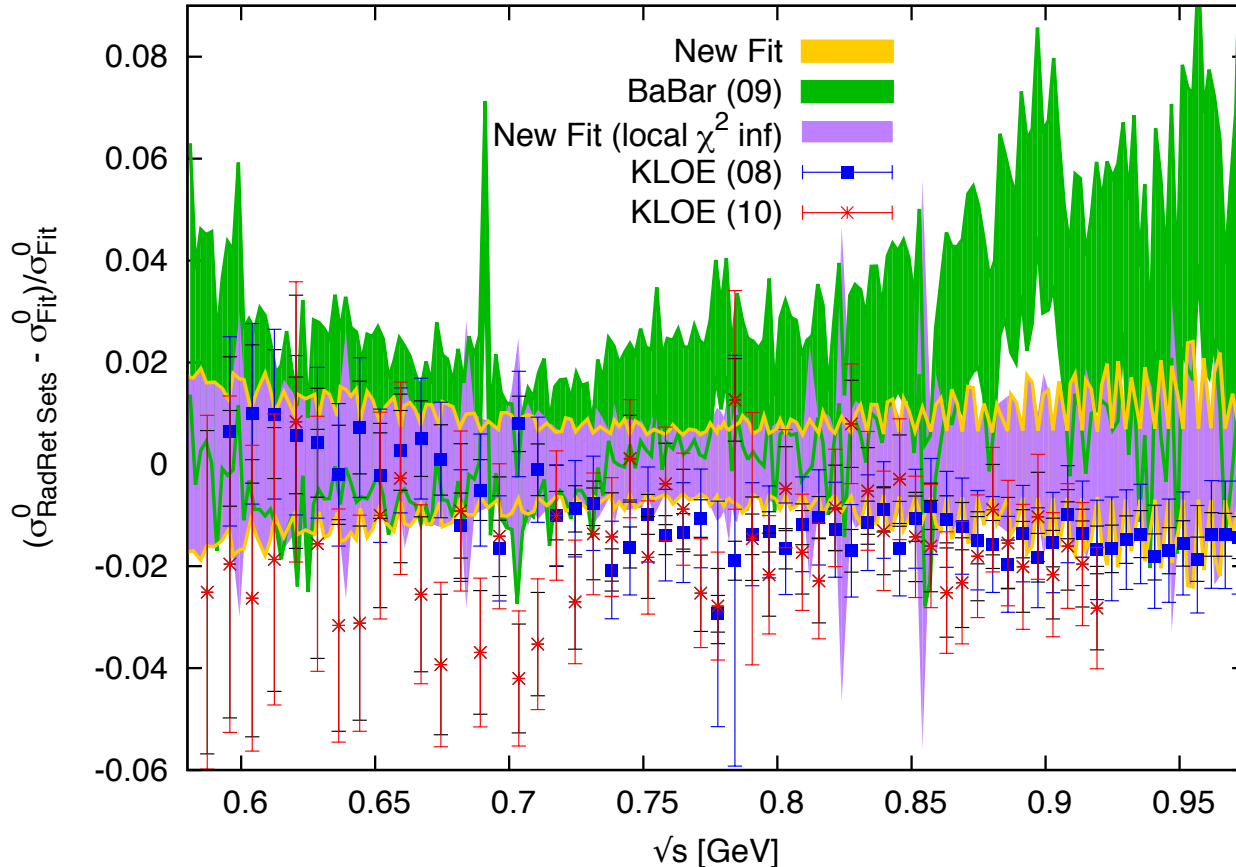
Enough in the long term? Need for more work in resonance regions.

Radiative Corrections: Final State Radiation

- Real+virtual; must be included in dispersion integral, but some events with real radiation will have been cut-off by experimental analyses.
(No problem if γ just missed but event counted! Possible problem of mis-identifies)
- Experiments (now) account for this and add some FSR back;
 - based on MC and sQED for pions (checked to work well),
 - including some uncertainties in their systematics
- Note: at low energies, hard radiation limited by phase space
- HMNT (and other groups) include large, conservative Rad. Corr. error, i.e.
HLMNT11: $\delta a_{\mu}^{\text{had, RadCor VP+FSR}} = 2 \times 10^{-10}$
- Work in progress: this error estimate is too conservative (especially in K^+K^-), and
- use of more modern data sets, where FSR has been part of the analysis, will bring this down.
- However: More scrutiny needed for Rad. Ret. EXPs where this is part of the 'LO'.

Data 'puzzle' in the $\pi^+\pi^-$ channel

Radiative Return data in the combined fit of HLMNT 11



ISR

- 2π fit: overall $\chi^2_{\min}/\text{d.o.f.} \sim 1.5$ needs error inflation, limited gain in error
- Latest KLOE12 data confirm this tension, see below for new fits also including latest BESIII data

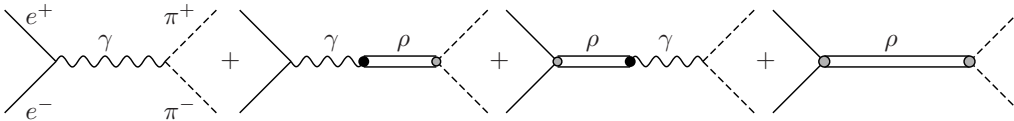
Note: $a_{\mu}^{\pi\pi}$, w/out Rad Ret = 498.7 ± 3.3 BUT $a_{\mu}^{\pi\pi}$, with Rad Ret = 504.2 ± 3.0

→ i.e. a shift of +5.5 in HLMNT [DHMZ: $a_{\mu}^{\pi\pi}$ even higher by 2.1 units]

Another 'puzzle': Use of **tau** spectral function data?

- Use CVC (iso-spin symmetry) to connect $\tau^- \rightarrow \pi^0 \pi^- \nu_\tau$ spectral functions to $e^+e^- \rightarrow \omega, \rho \rightarrow \pi^+\pi^-$ but have to apply **iso-spin corrections**
- Early calculations by **Alemay, Davier, Hoecker**: use of τ data complementing e^+e^- data originally resulted in an improvement w.r.t. use of e^+e^- data alone; discrepancy smaller with tau data; later increased tension between e^+e^- and τ
- Recent compilation by **Davier et al** (Fig. from PRD86, 032013):

- **Jegerlehner+Szafron**: crucial role of **γ - ρ mixing**:

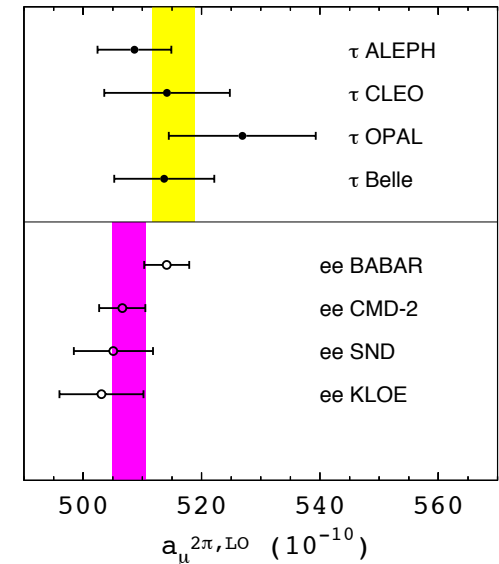


- They found discrepancy gone but τ data improves e^+e^- analysis only marginally

- Analyses by **Benayoun et al**: combined fit of e^+e^- and τ based on **Hidden Local Symmetry (HLS)**:
no big tension betw. e^+e^- and τ , but w. **BaBar, hence not used**; increased Δa_μ : of $> \approx 4.5\sigma$

- **Davier+Malaescu** refute criticism, claim fair agreement betw. BaBar and their τ comp.

- **HLMNT**: stick to e^+e^- (and do not use τ data). With e^+e^- (incl. BaBar) discrepancy of **3-3.5 σ**



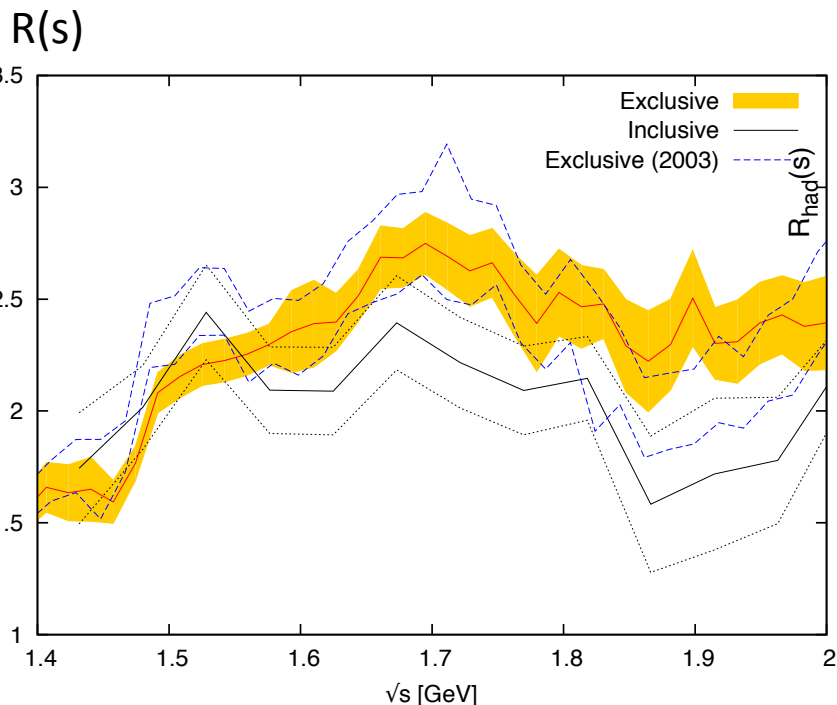
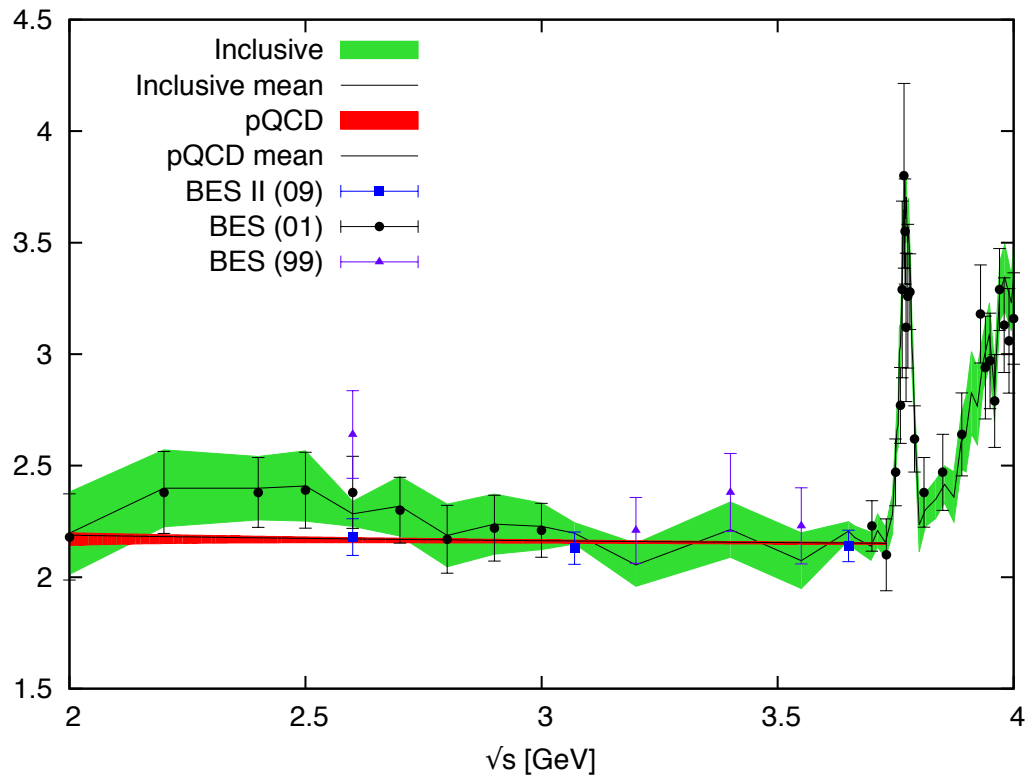
σ_{had} at higher energies

2011 status to be improved; BESIII

Inclusive vs. $\Sigma_{\text{exclusive}}$ + 'missing'

More from SND, CMD3, Belle, BaBar, BESIII

Inclusive data or perturbative QCD



More from BESIII

- Exclusive data recently improved mainly due to many Radiative Return data from BaBar
- BESII data (blue markers) in perfect agreement w. pQCD; data-based $a_{\mu}^{\text{incl}} > a_{\mu}^{\text{pQCD}}$
- Different data and data vs. pQCD choices give slightly different a_{μ} (within errors)

- Fair agreement between different e^+e^- analyses, including recent updates: (all numbers in 10^{-10})

HLMNT (11): 694.9 ± 3.7 (exp) ± 2.1 (rad)

Jegerlehner (11): 690.8 ± 4.7

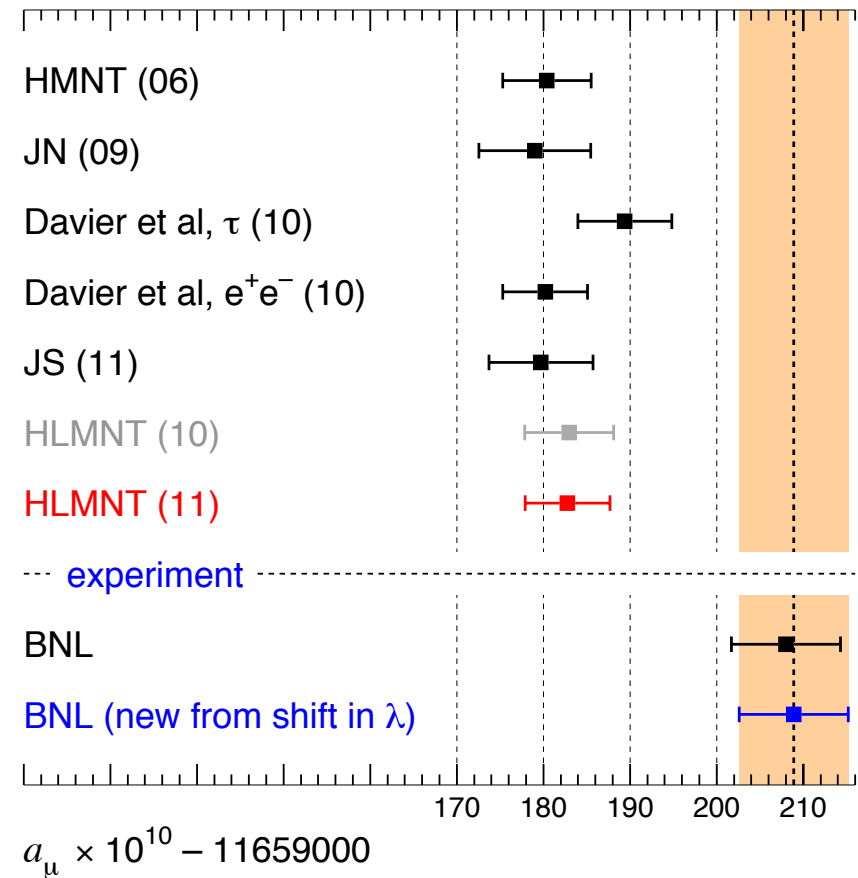
Davier et al (11): 692.3 ± 4.2

- The 'extremes' (both with τ data):

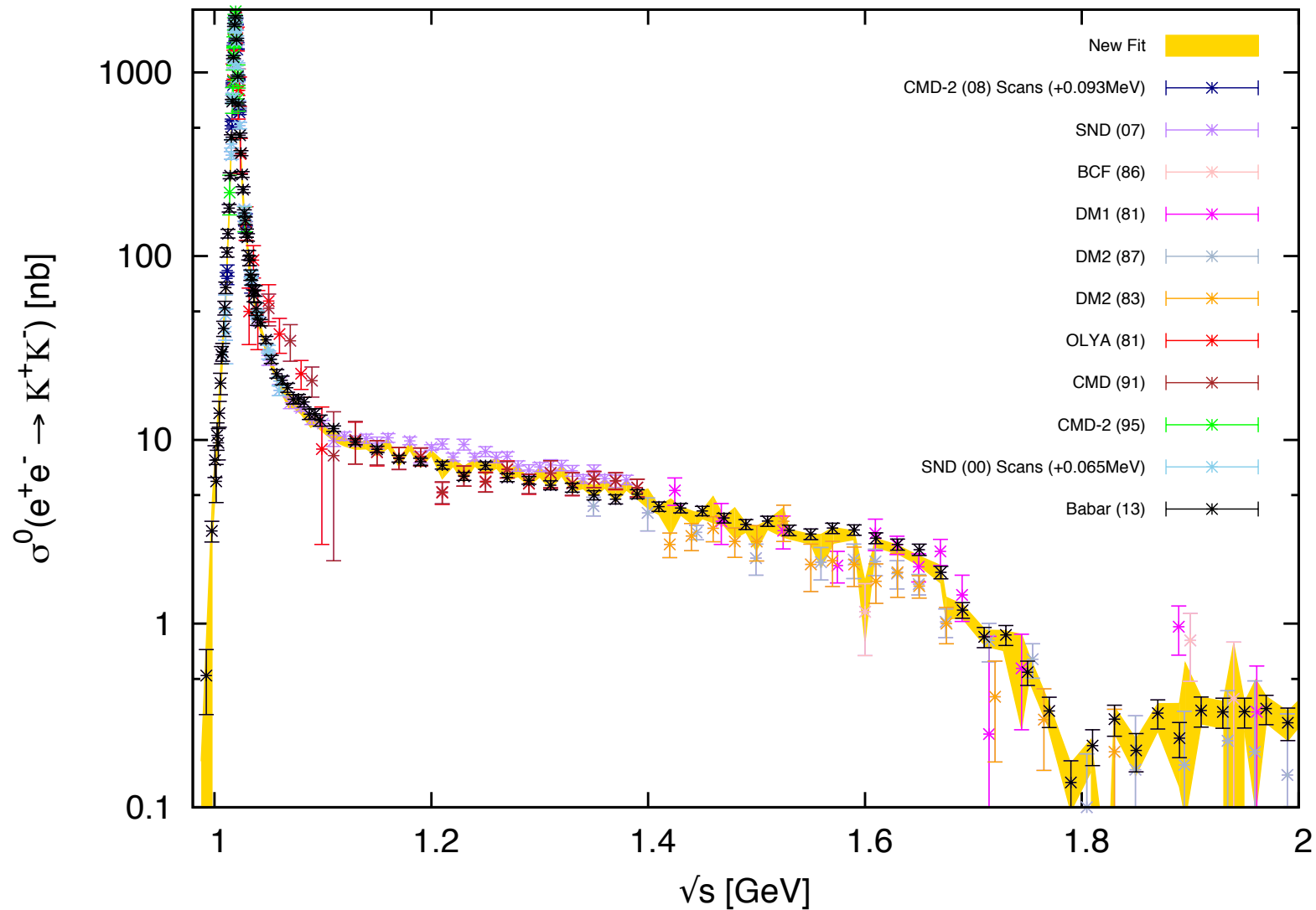
Davier et al (11): 701.5 ± 4.7 (+ ~ 1.5 shift from 2013 τ re-analysis EPJC74,3,2803)

Benayoun et al (12): 681.2 ± 4.5

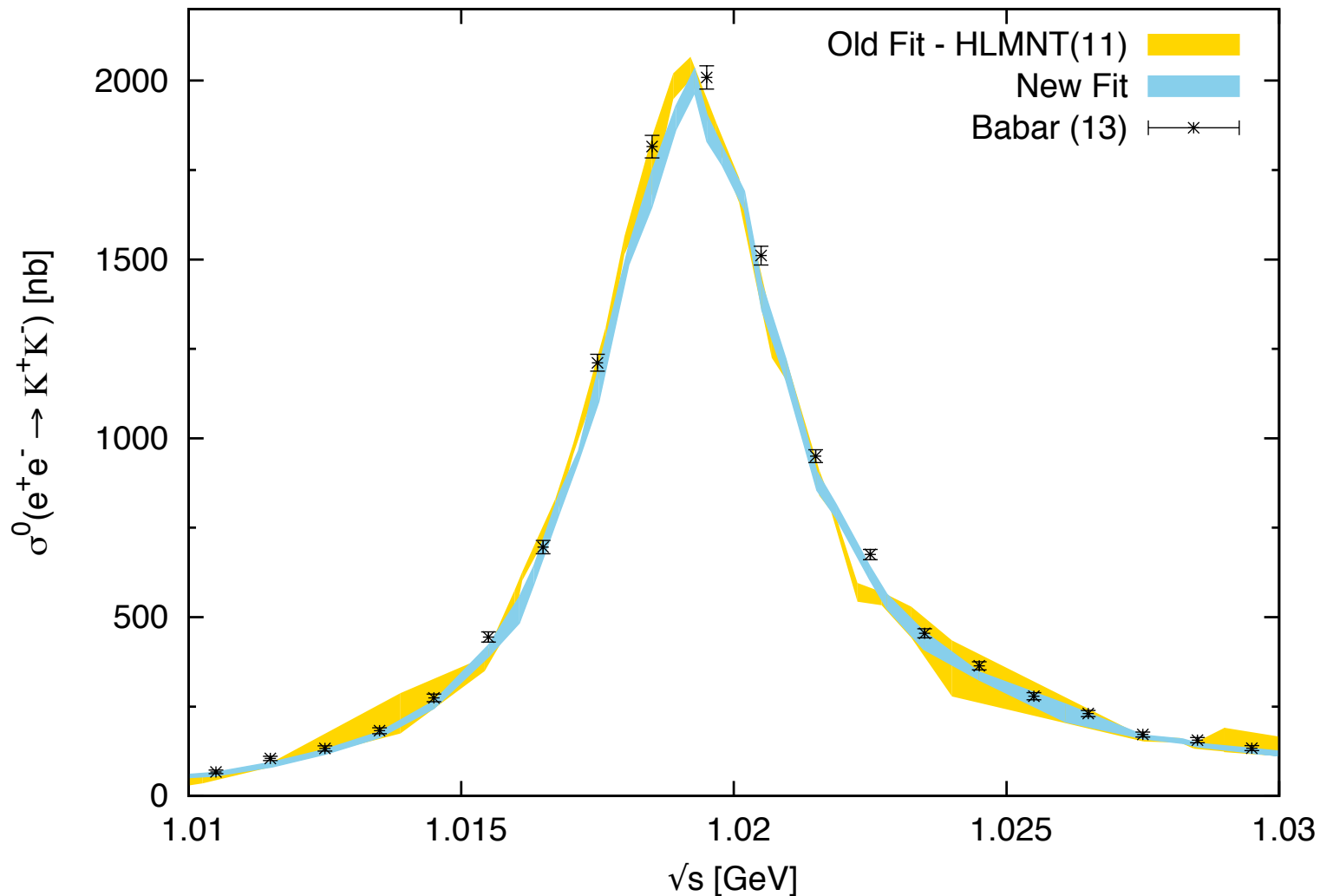
- New data available already do not shift the mean value strongly, but are incrementally improving the determination of a_μ^{HVP}



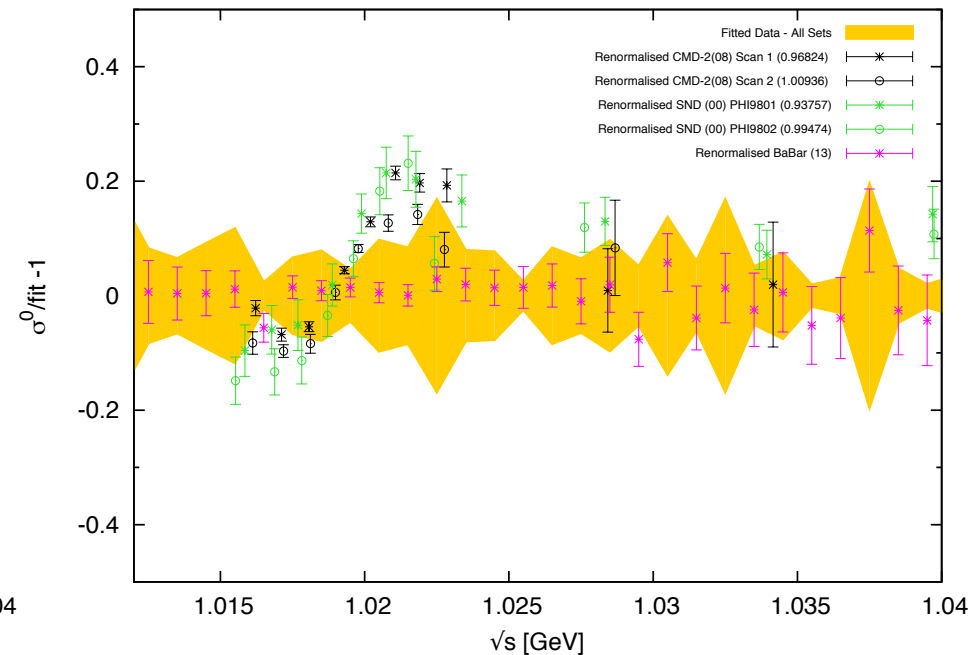
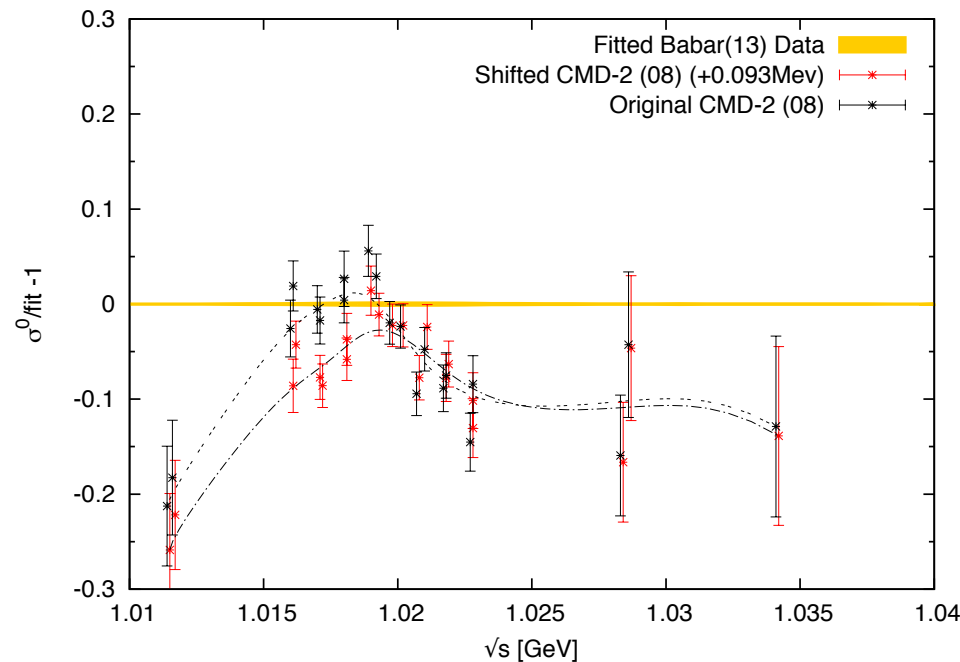
Work on new combination with A. Keshavarzi: PRELIMINARY results



Work on new combination with A. Keshavarzi: PRELIMINARY results



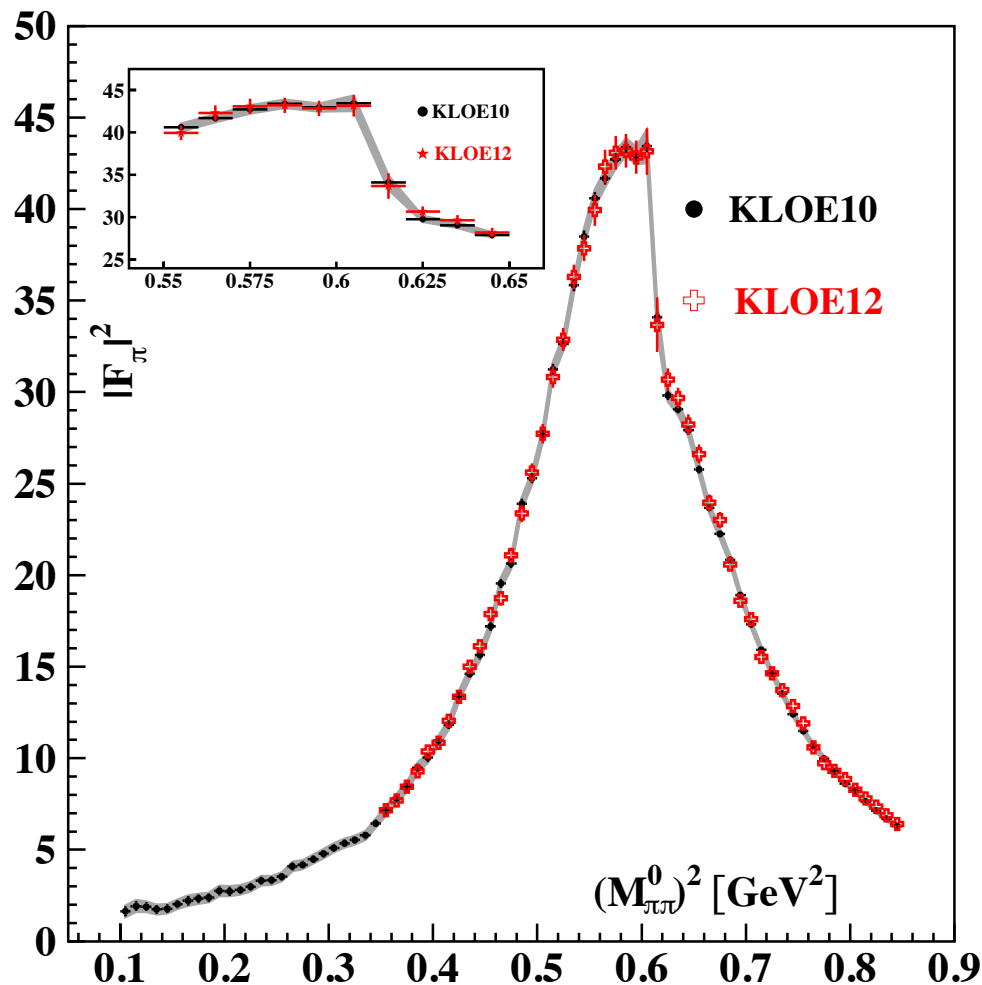
Problem of data combination between BaBar and older sets: another puzzle?



- Energy dependent difference; **fitted Φ mass is different** betw. BaBar and CMD-2, SND
- study to **combine including energy** (and normalisation) **shifts**
- $\chi^2_{\text{min}}/\text{dof}$ improved through energy shifts, but still not very good;
(locally) inflated error $\Delta a_{\mu}^{\text{KK}}$ only marginally improved when BaBar data are included, and worse for all sets combined compared to BaBar only ($\rightarrow \chi^2_{\text{min}}$ inflation of error)
- In total we get $a_{\mu}^{\text{K}^+\text{K}^-} = 21.74 \pm 0.45$ (HLMNT11) $\rightarrow 21.69 \pm 0.36$ (preliminary!)

$\pi^+\pi^-$: recent data from KLOE

KLOE with $\sigma_{\mu\mu}$ normalisation [PLB720(2013)336]:



- confirm previous KLOE measurements
- do not decrease tension with BaBar, slightly increase KLOE's significance
- Open question: Why are BaBar's data so different from KLOE's?

Are there any issues with the MCs or analysis techniques used?

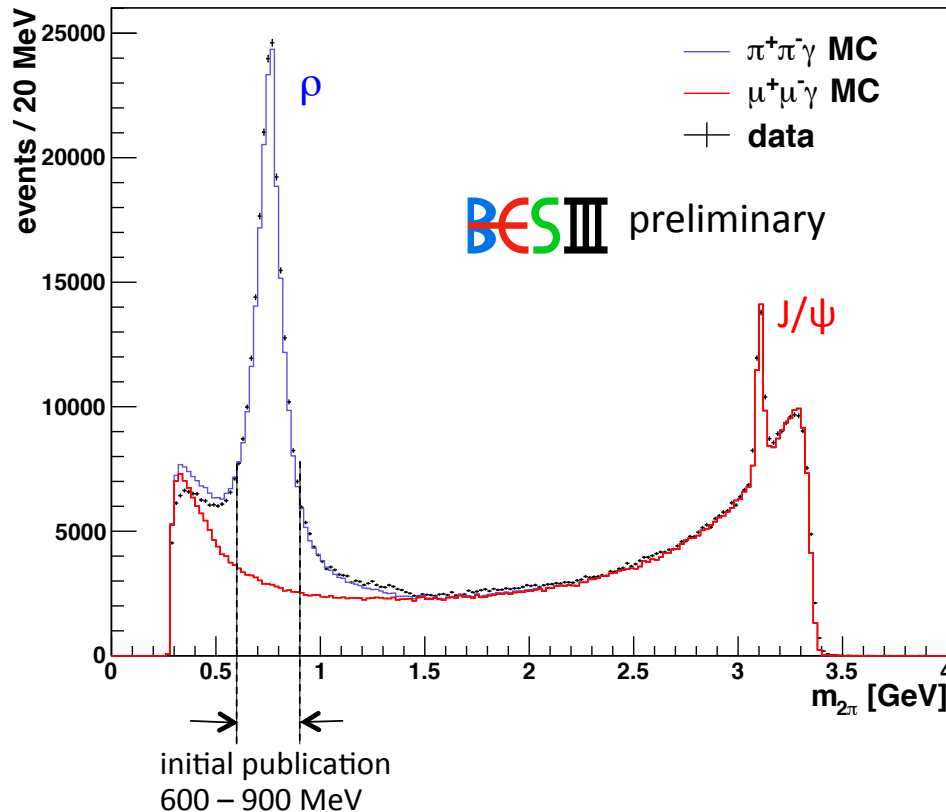
- Soon more from Novosibirsk, already now from **BESIII**:

1

Flagship Analysis: $e^+e^- \rightarrow \pi^+\pi^-\gamma_{ISR}$



Event yield after basic event selection (acceptance only!)

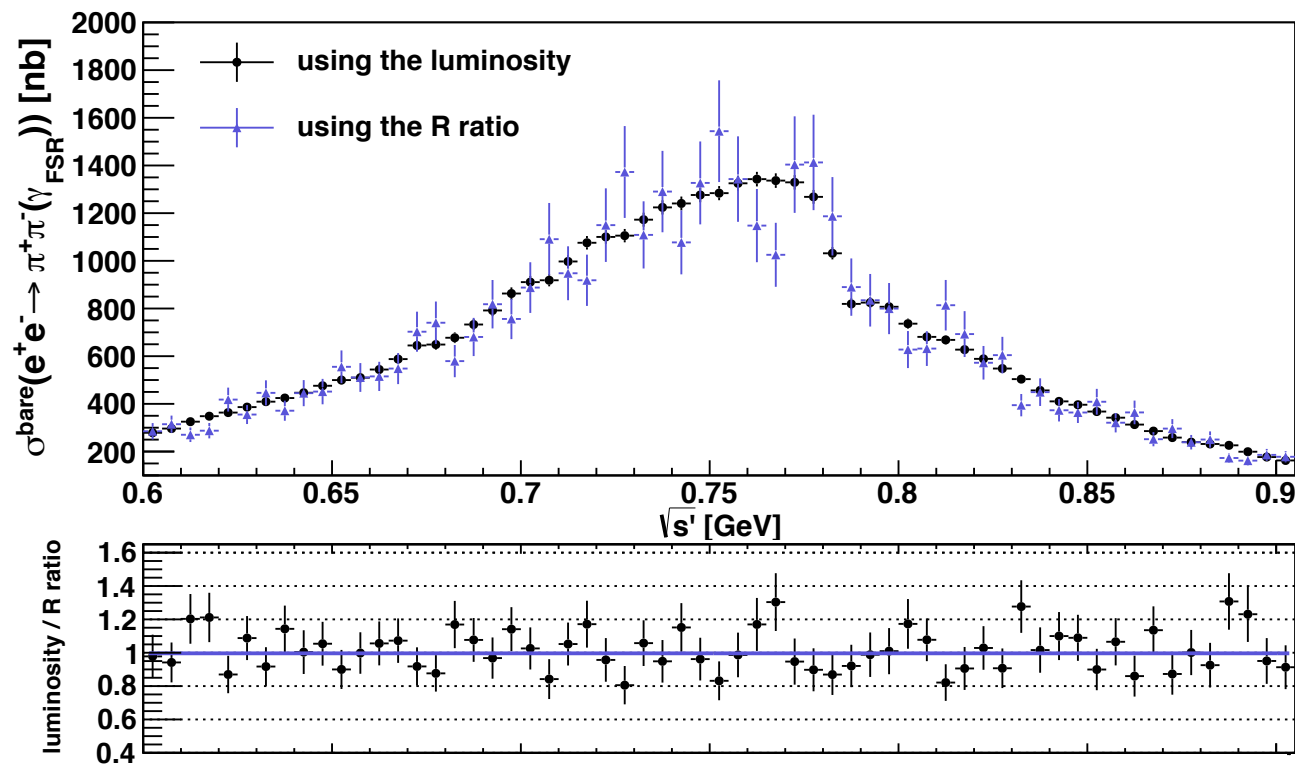


Features:

- $\psi(3770)$ data only (2.9 fb^{-1})
- no background subtraction
- PHOKHARA event generator
- tagged ISR photon

- large statistics of $\pi\pi\gamma$ events
- background dominated by $\mu\mu\gamma$
- data – MC differences observed

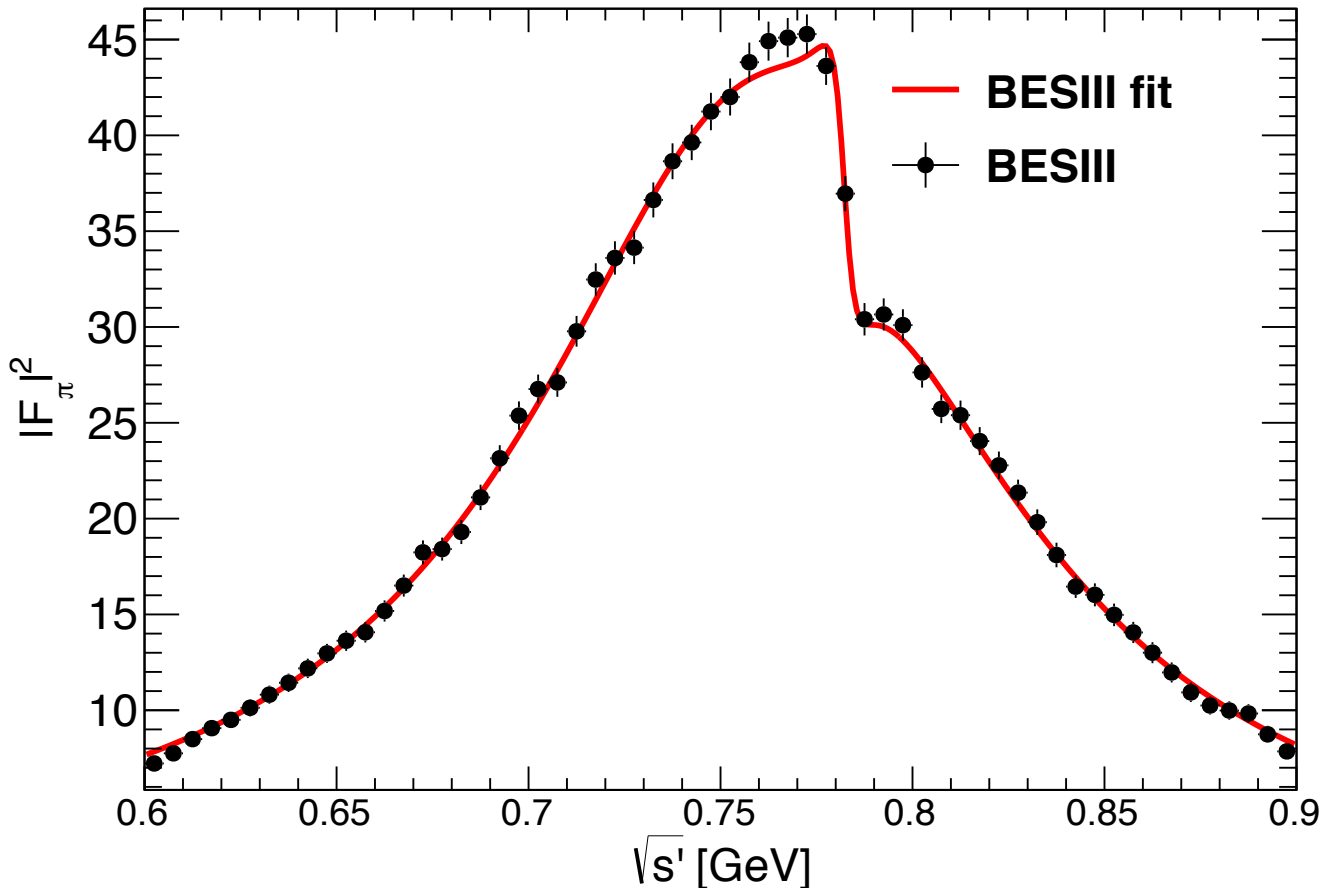
$\pi^+\pi^-$: new data from BESIII; arXiv:1507.08188



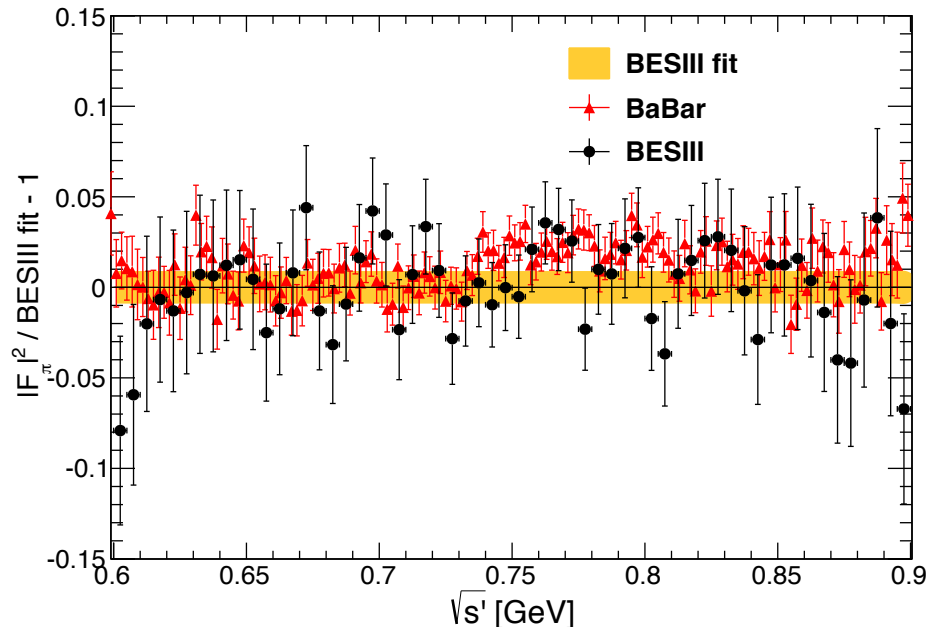
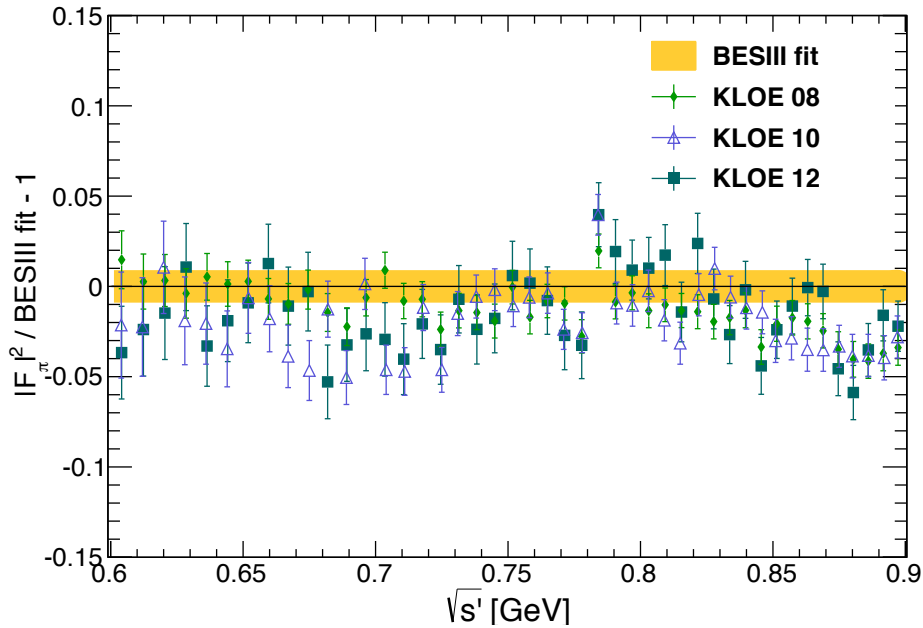
Two methods used for the normalisation:

- Bhabha for the luminosity vs. $\mu^+\mu^-(\gamma)$, where the luminosity, the radiator function and the VP corrections are cancelling out.
- Much higher stats for Bhabha, but $\mu^+\mu^-(\gamma)$ important consistency check.

Form Factor plot with fit result:



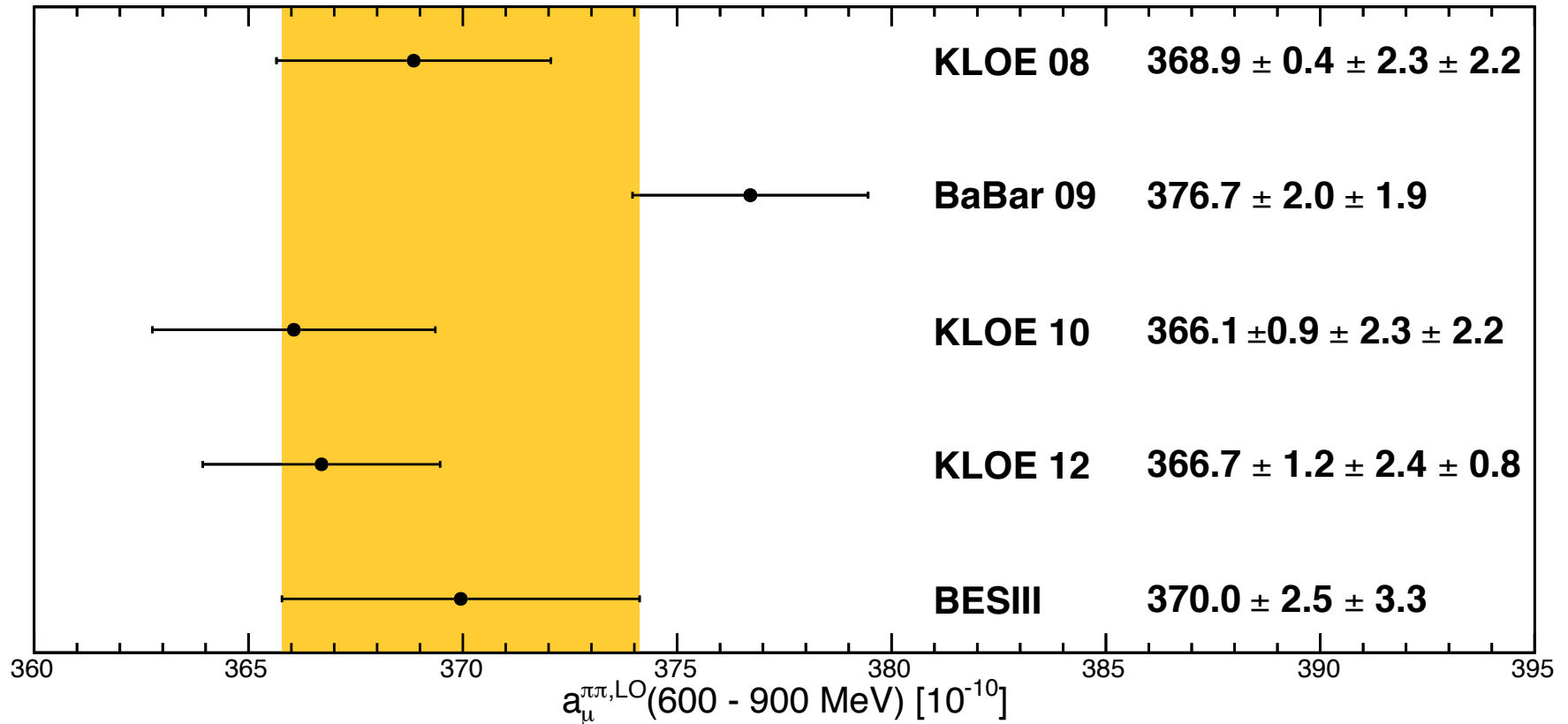
$\pi^+\pi^-$: new data from BESIII; arXiv:1507.08188



Good agreement with previous KLOE results, marginally consistent with BaBar;

BESIII 'interpolates' between the two:

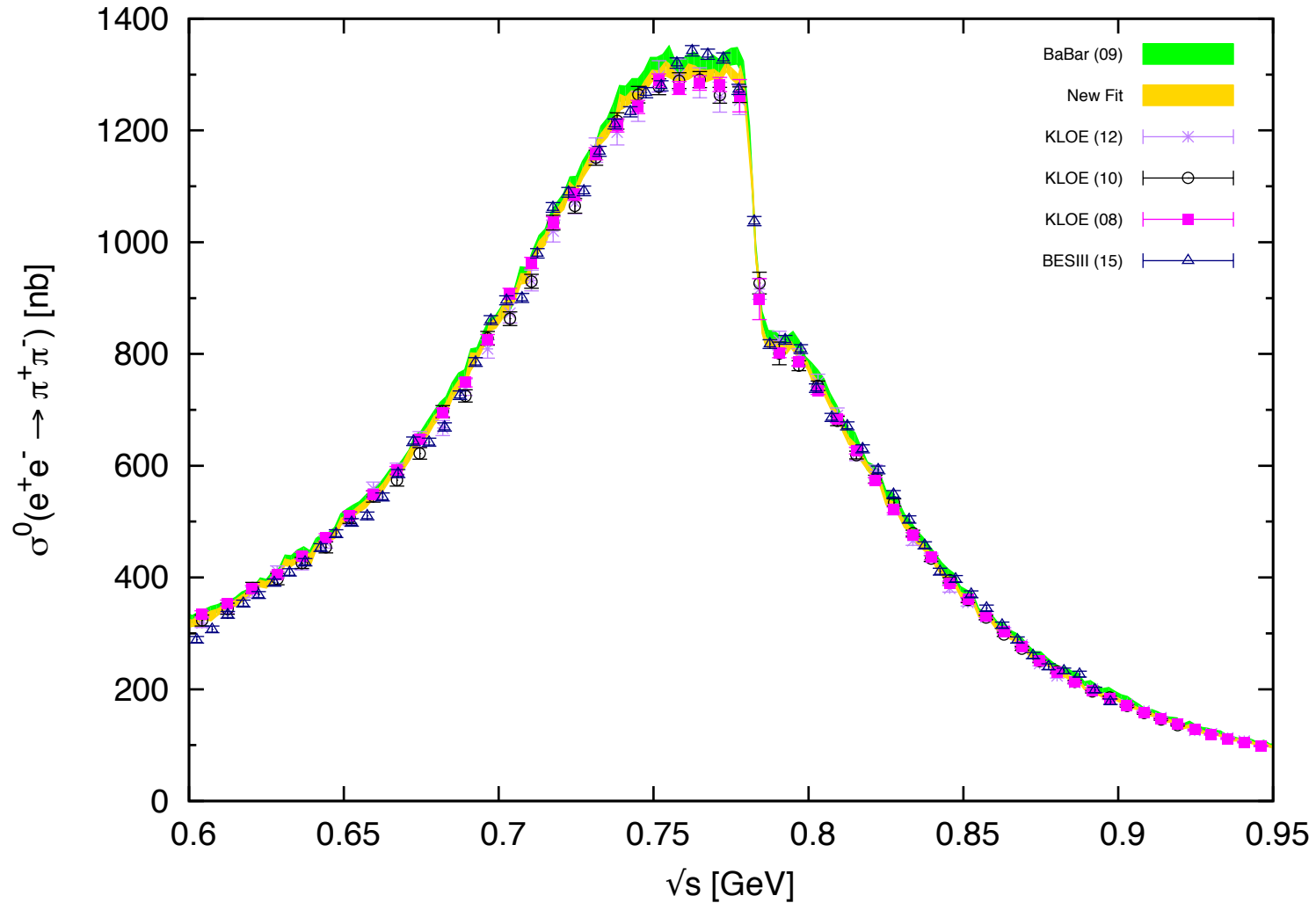
$\pi^+\pi^-$: new data from BESIII; arXiv:1507.08188v2



[g-2 integral and plot corrected in v2 after us finding a value different from the one of v1]

$\pi^+\pi^-$: Effect of KLOE12 + BESIII data in 'global' 2π fit

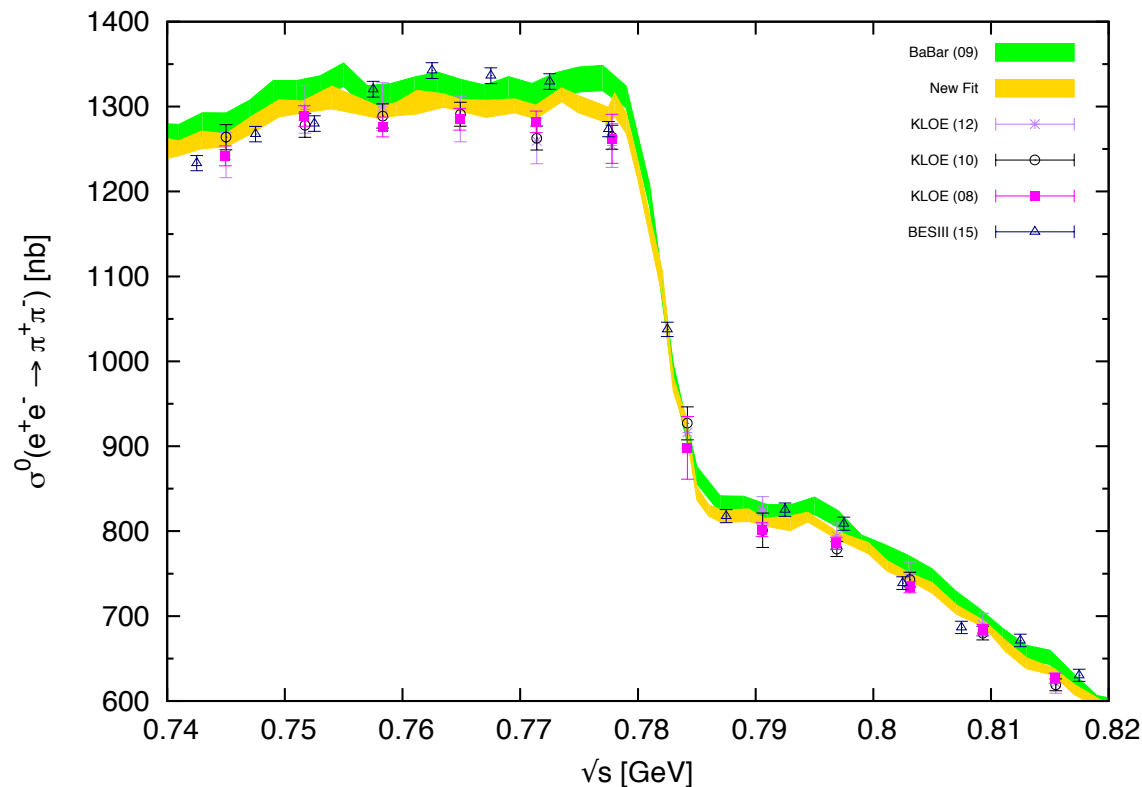
Very preliminary first results (this Monday) from my PhD student Alex Kheshavarzi:
ISR data comparison in ρ peak region and new fit band:



$\pi^+\pi^-$: Effect of KLOE12 + BESIII data in 'global' 2π fit

Very preliminary first results (this Monday) from my PhD student Alex Kheshavarzi:

Zoom into the ρ - ω interference region:



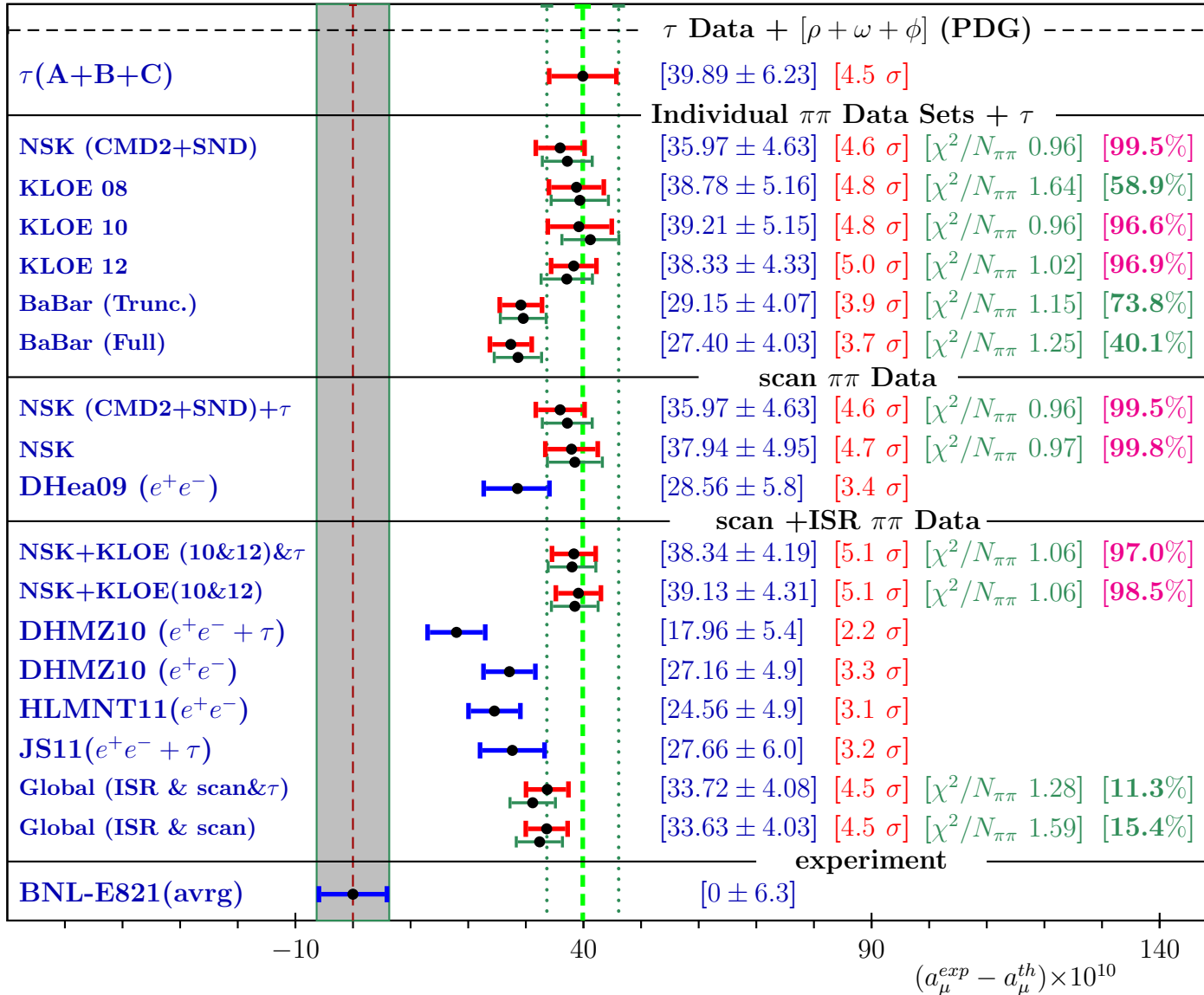
Very prel. numbers,
for indication only:

(range 0.305 ... 2 GeV,
units of 10^{-10})

- all sets and fit as in HLMNT11:
 505.8 ± 3.1
- including now also KLOE12 and the new BESIII data:
 504.3 ± 2.6

-> improvement in error, but numbers will still change.

Hot from the arXiv: Benayoun et al., 1507.02943



Updated anal.
(now with
fitted normal.
factors) in
global fit
based on HLS
model:

← 'preferred'
has 5 σ already,
from both
aggressive
errors and
shifts due to
the HLS model
(and the fit?)

Hot from the arXiv: Benayoun et al., 1507.02943

Table 4 from their paper showing shifts and error improvements:

Channel	$A = m$	$A = M_0$	$A = M(\vec{a})$ (variable)	Exp. Value
$\pi^+\pi^-$	494.57 ± 1.48	494.01 ± 1.11	493.80 ± 1.00	496.38 ± 3.13
$\pi^0\gamma$	4.53 ± 0.04	4.54 ± 0.04	4.54 ± 0.04	3.67 ± 0.11
$\eta\gamma$	0.64 ± 0.01	0.64 ± 0.01	0.64 ± 0.01	0.56 ± 0.02
$\pi^+\pi^-\pi^0$	40.83 ± 0.58	40.85 ± 0.58	40.87 ± 0.57	43.54 ± 1.29
$K_L K_S$	11.56 ± 0.08	11.56 ± 0.08	11.56 ± 0.08	12.21 ± 0.33
$K^+ K^-$	16.78 ± 0.20	16.77 ± 0.21	16.77 ± 0.20	17.72 ± 0.52
Total	569.19 ± 1.60	568.37 ± 1.27	568.17 ± 1.17	574.08 ± 3.45

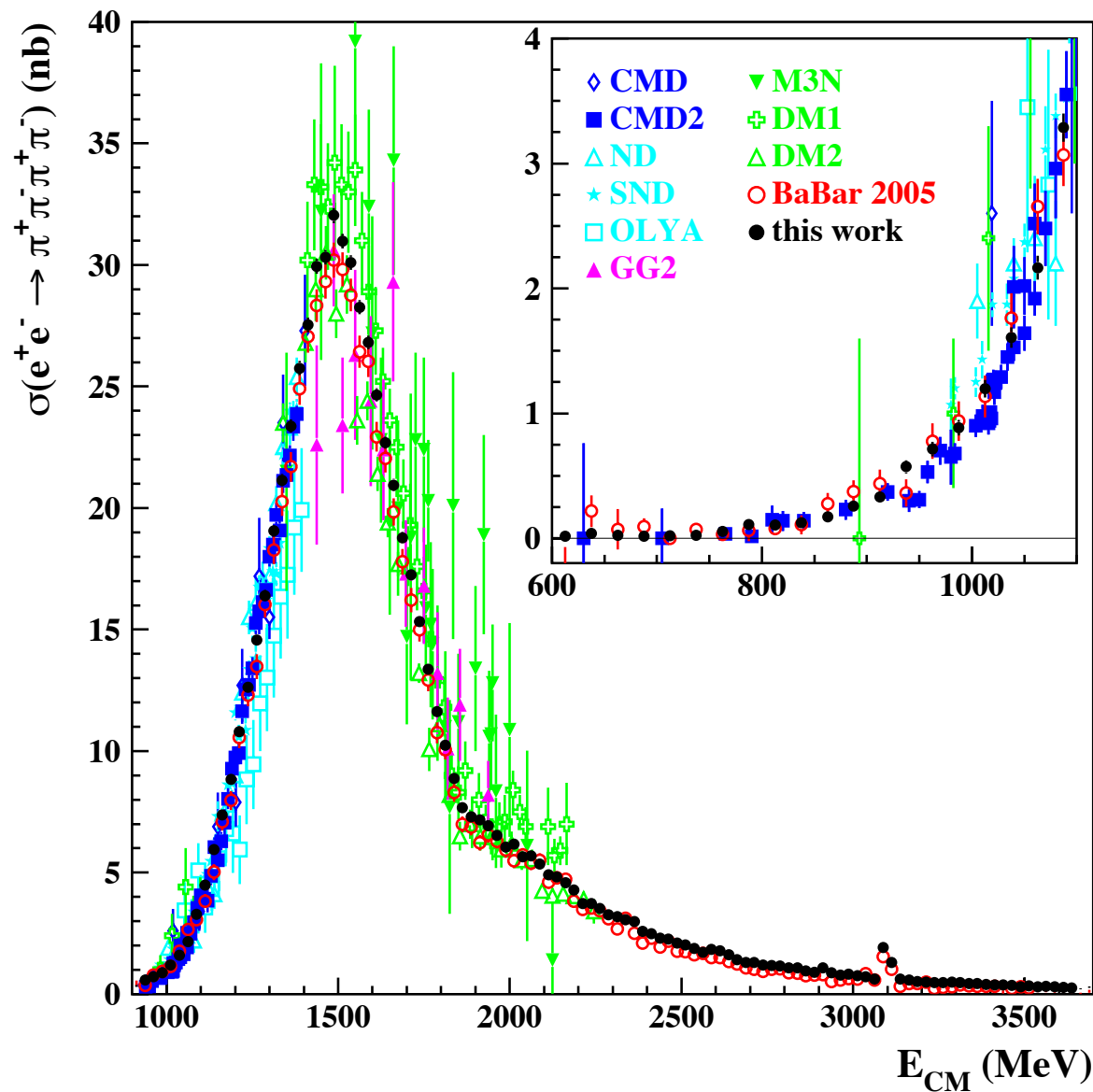
Table 4: The contributions to the muon LO–HVP from the various channels covered by BHLS from their respective thresholds to 1.05 GeV in units of 10^{-10} at start and after iteration. The last column displays the direct numerical integration of the various spectra used within BHLS. The $\pi^+\pi^-$ data samples considered are those flagged by "Combination 2" in Table 2.

- 2π data choices lead to very different results;
 - if BaBar is used (untruncated) then their fit is worse (incompatibilities mainly between BaBar and other data), their mean and error are larger and the discrepancy goes to 3.7σ .

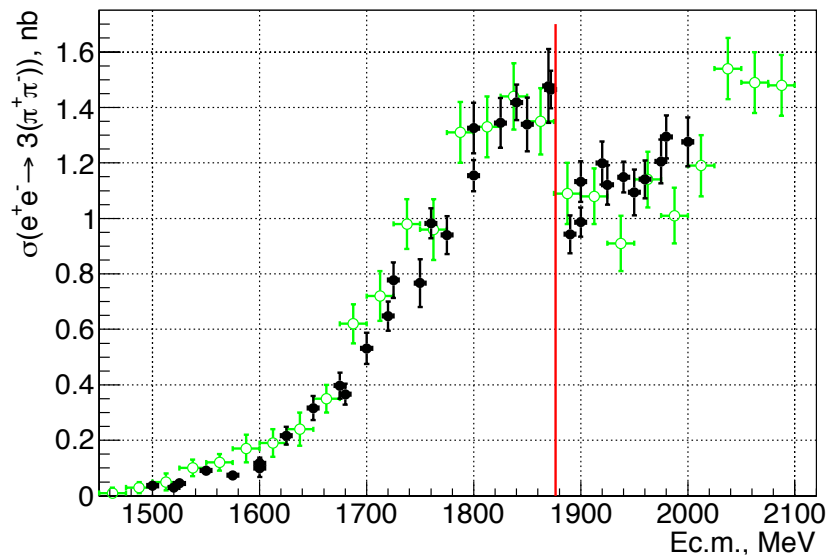
σ_{had} : recent new data: $2\pi^+2\pi^-(\gamma)$ from BaBar

PRD85(2012)112009

- shift of $+0.3 \times 10^{-10}$ for a_μ
- error down to a third
- combination?

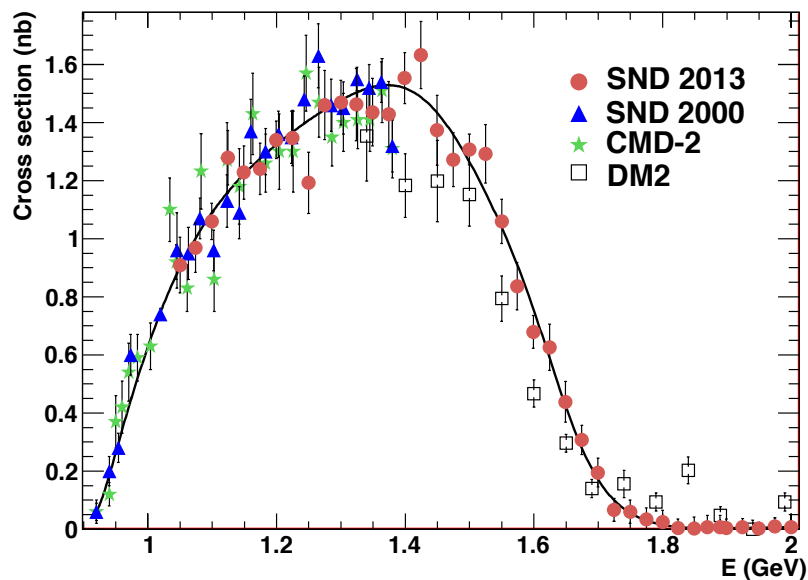


σ_{had} : recent new data from Novosibirsk



CMD-3 6π charged, PLB723(2013)82

- solid black: CMD-3, open green: BaBar
- full analysis will include $2(\pi^+\pi^-\pi^0)$



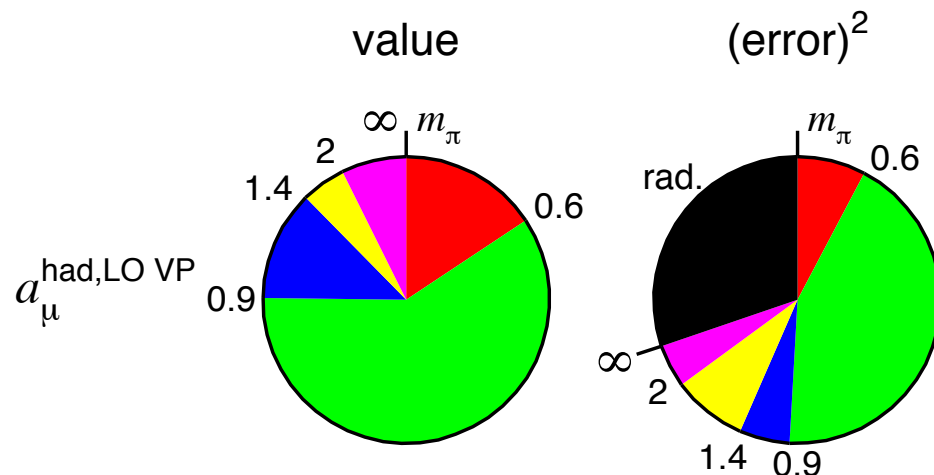
SND $\omega\pi^0$, PRD88(2013)054013

- many more analyses reported with preliminary results, incl. 3π , $4\pi(2n)$
- looking forward to rich harvest from SND and CMD-3

Outlook: Future improvements for a_μ^{HVP}

- Most important 2π :
 - close to threshold important; possible info also from space-like
 - more data already now and more expected well before new $g-2$ EXPs
 - understand discrepancy between sets, especially 'BaBar puzzle'?!
 - possibility of direct scan & ISR in the same experiment? Lattice?!
- $\sqrt{s} > 1.4$ GeV:
 - higher energies will improve with input from SND, CMD-3, BESIII, BaBar
- With channels more complete, test/replace iso-spin corrections
- Very good prospects to significantly squeeze the dominant HLO error!

Pie diagrams from HLMNT 11:



3-5 year plan:

- 2π : error down by about 30-50%
- subleading channels: by factor 2-3
- $\sqrt{s} > 2$ GeV: by about a factor 2

→ I believe we can half the HVP error in time for the new $g-2$ EXPs.

Conclusions:

- All sectors of the Standard Model prediction of $g-2$ have been scrutinised a lot in recent years
- No major changes recently, but discrepancy $> 3\sigma$ consolidated
- With the next round of hadronic data it should be possible to push the error down significantly, hopefully halving $\Delta a_\mu^{\text{HVP}}$
- For this much more work will be needed (solve some puzzles)
- though Benayoun et al. already claim 5σ now
- But where is the NP?



Extras

Channels with biggest errors. PQCD at 2 GeV?

Importance of various 'channels'

[Numbers from HLMNT, 'local error infl.', $\cdot 10^{-10}$]

- Errors contributions to a_μ from leading and subleading channels (ordered) up to 2 GeV

Purely from data:

'Higher multiplicity' region from 1.4 to 2 GeV

with use of isospin relations for some channels:

[Use of old inclusive data disfavoured.]

channel	error
$\pi^+\pi^-$	3.09
$\pi^+\pi^-\pi^0\pi^0$	1.26
3π	0.99
$2\pi^+2\pi^-$	0.47
K^+K^-	0.46
$2\pi^+2\pi^-2\pi^0$	0.24
$K_S^0K_L^0$	0.16

Channel	contr. \pm error
$K\bar{K}2\pi$	3.31 ± 0.58
$\pi^+\pi^-4\pi^0$	0.28 ± 0.28
$\eta\pi^+\pi^-$	0.98 ± 0.24
$K\bar{K}\pi$	2.77 ± 0.15
$2\pi^+2\pi^-\pi^0$	1.20 ± 0.10

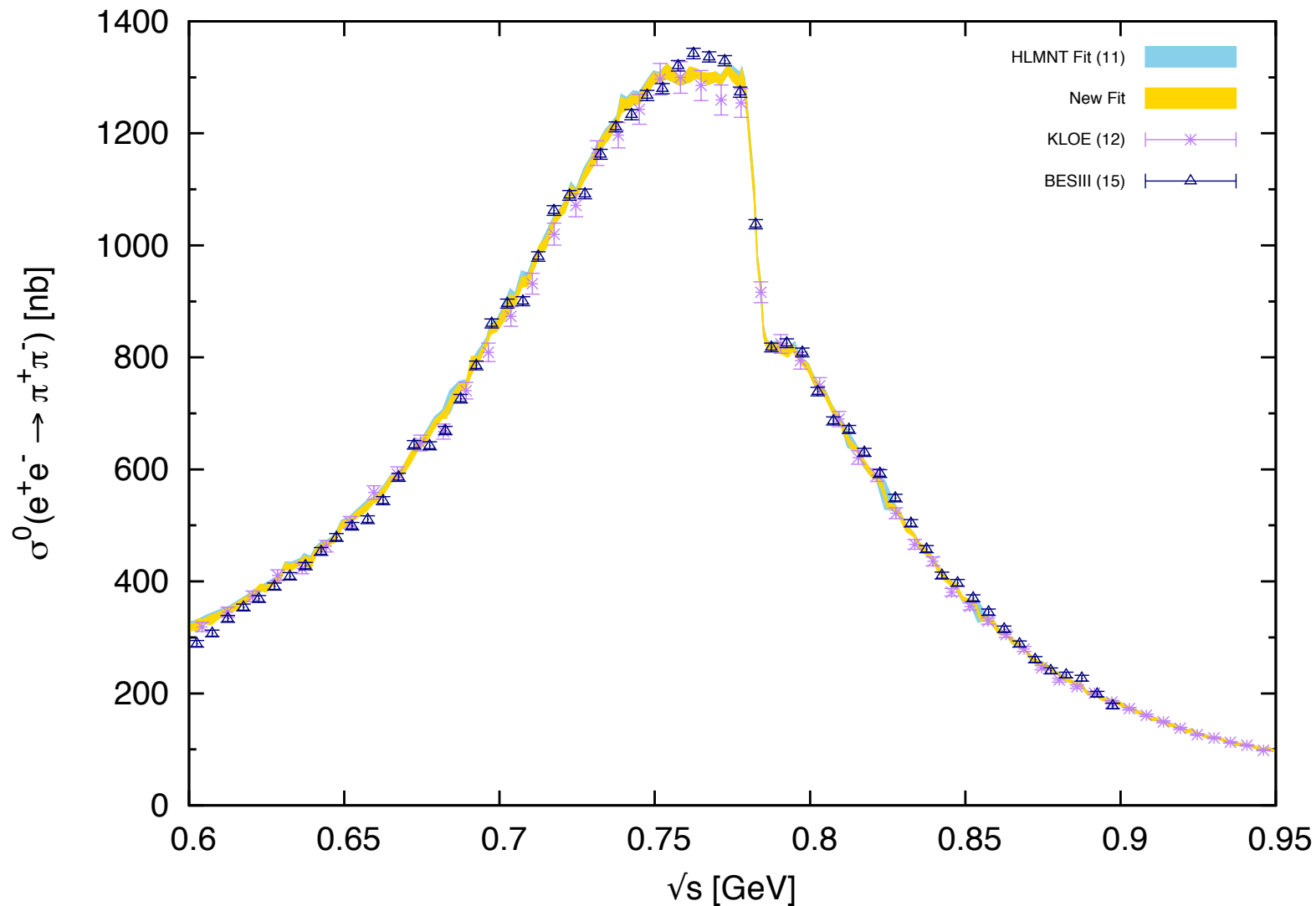
- 'Inclusive' region from 2 to ~ 11 GeV: **41.19 ± 0.82**

Can be 'squeezed' by using pQCD (done by DHMZ from 1.8 GeV);

region from 2 to 2.6 GeV: $15.69 \pm 0.63 \rightarrow 14.49 \pm 0.13$, only small changes for higher energies.

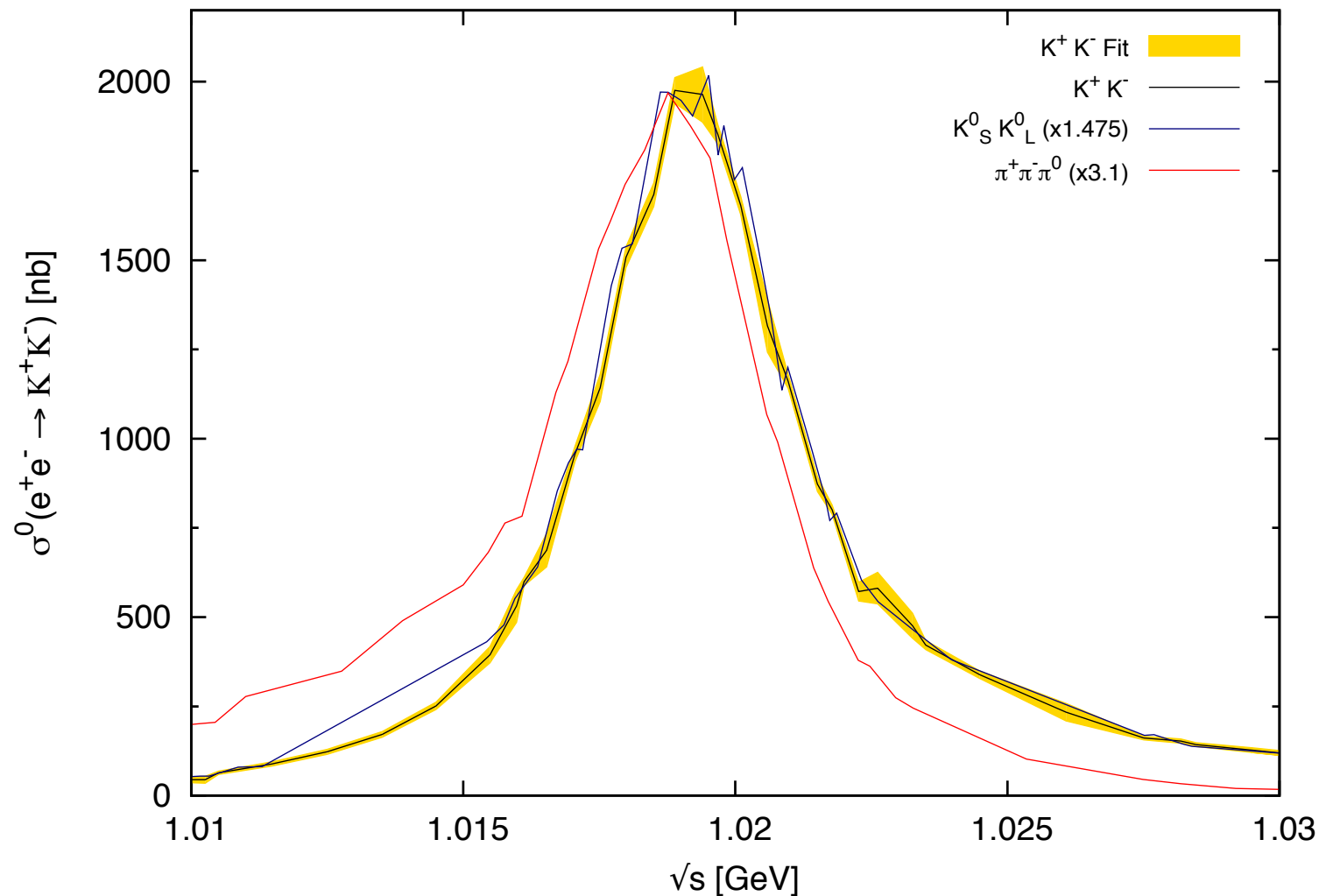
$\pi^+\pi^-$: Effect of KLOE12 + BESIII data in 'global' 2π fit

Comparison old vs. new fit, with new ISR data:



$\sigma_{\text{had}}: \Phi$ in different final states K^+K^- , $K_S^0K_L^0$, $\pi^+\pi^-\pi^0$

Why we prefer to integrate data directly, or: **How to not fit a resonance**



Fitting The Data

Use a non-linear non-linear χ^2 -function [HLMNT, 2012]

$$\chi^2(R_m, f_k) = \sum_{k=1}^{N_{exp}} \left(\frac{1 - f_k}{df_k} \right)^2 + \left\{ \sum_{m=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \left(\frac{R_i^{(k,m)} - f_k R_m}{d\tilde{R}_i^{(k,m)}} \right)^2 \right\}_{\text{w/o cov. mat}}$$
$$+ \left\{ \sum_{m=1}^{N_{clu}} \sum_{i=1}^{N^{(k,m)}} \sum_{j=1}^{N^{(k,n)}} (R_i^{(k,m)} - f_k R_m) C^{-1}(m_i, n_j) (R_j^{(k,n)} - f_k R_n) \right\}$$

where N_{clu} , N_{exp} are total number of clusters, experiments.

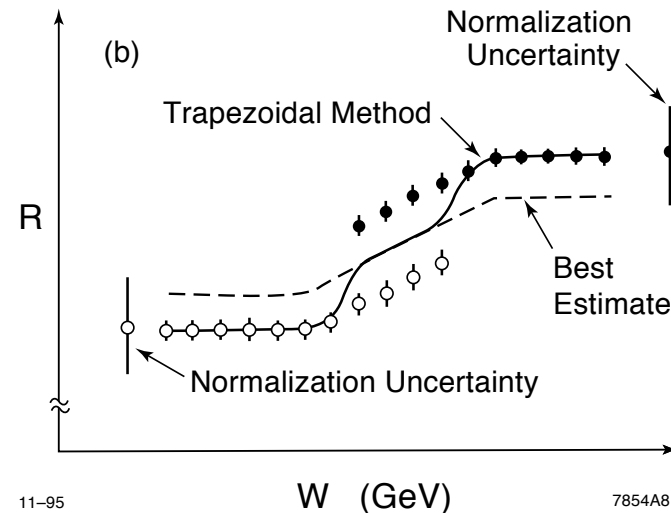
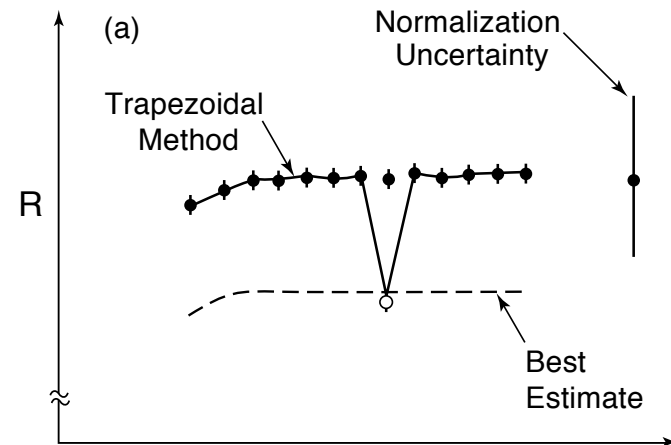
- Treat the statistical/systematic errors according to experimental data.
- Input covariance matrices where provided (last term!).

HLMNT fit strategy:

Processing the hadronic data: Clustering

- Need: combination of data from different experiments (for the same channel) with very different stat.+sys. errors and different energy ranges.
- Aims:
 - Make maximal use of (normalization of) precise data.
 - Don't suppress shape info. of older data.
 - Have as few theor. constraints on R as possible (like pQCD, BW resonance shapes, fit by polynomials...).
- Solution: 'A fit that's not a fit' \rightsquigarrow

Figure from M.L. Swartz:



HLMNT fit strategy:

- ▶ Our fit-model: piecewise constant R within a *Cluster* of a given (min.) size.
- ▶ Realization: Non-linear (numerical, iterative) χ^2 -minimalization of:

$$\chi^2(\bar{R}_m, f_k) = \sum_{k=1}^{\#Exp} [(1 - f_k) / df_k]^2 + \sum_{m=1}^{\#Cl} \sum_{i=1}^{N_{\{k,m\}}} \left[\left(R_i^{\{k,m\}} - f_k \bar{R}_m \right) / dR_i^{\{k,m\}} \right]^2$$

Given a binning of all R data $R_i^{\{k,m\}} \pm dR_i^{\{k,m\}} \pm df_k \cdot R_i^{\{k,m\}}$ (from k exp., in m clusters), the fit returns the mean values \bar{R}_m (and the renormalizations f_k of $R_i^{\{k,m\}} \pm dR_i^{\{k,m\}}$).

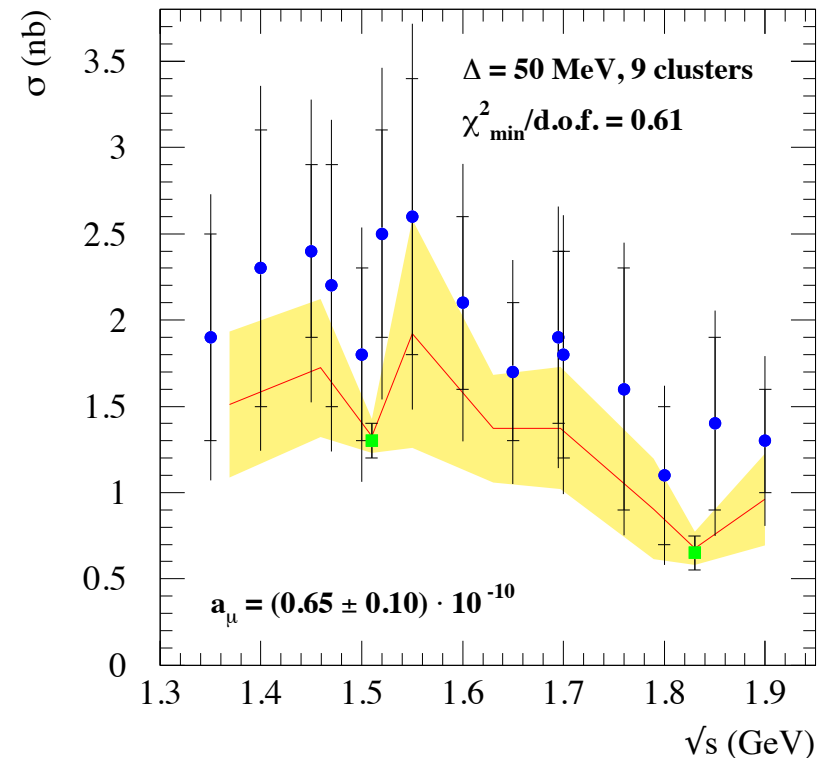
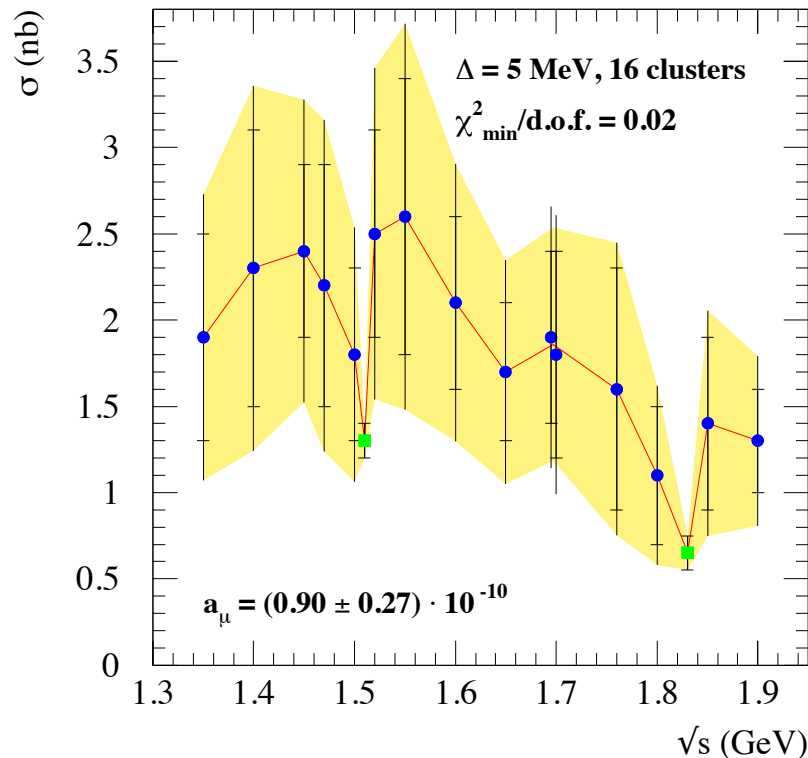
Advantages:

- No prejudice from TH-inspired modelling of R .
- Reliable error estimate using the complete covariance matrix (taking into account **statistical and systematic** (p.t.p. and overall) errors from all different experiments)
↪ **correlations** over different energies important.
- Automatic check of data-consistency and fit-quality;
minimal $\chi_{\min}^2 / (d.o.f.) (\delta)$ determines choice of cluster size δ .
- Amazing **stability** of $a_{\mu}^{\text{had,LO}}$ and its **error**.

HLMNT fit strategy:

How does it work? A few examples:

- Artificial 'demo' data for illustration:

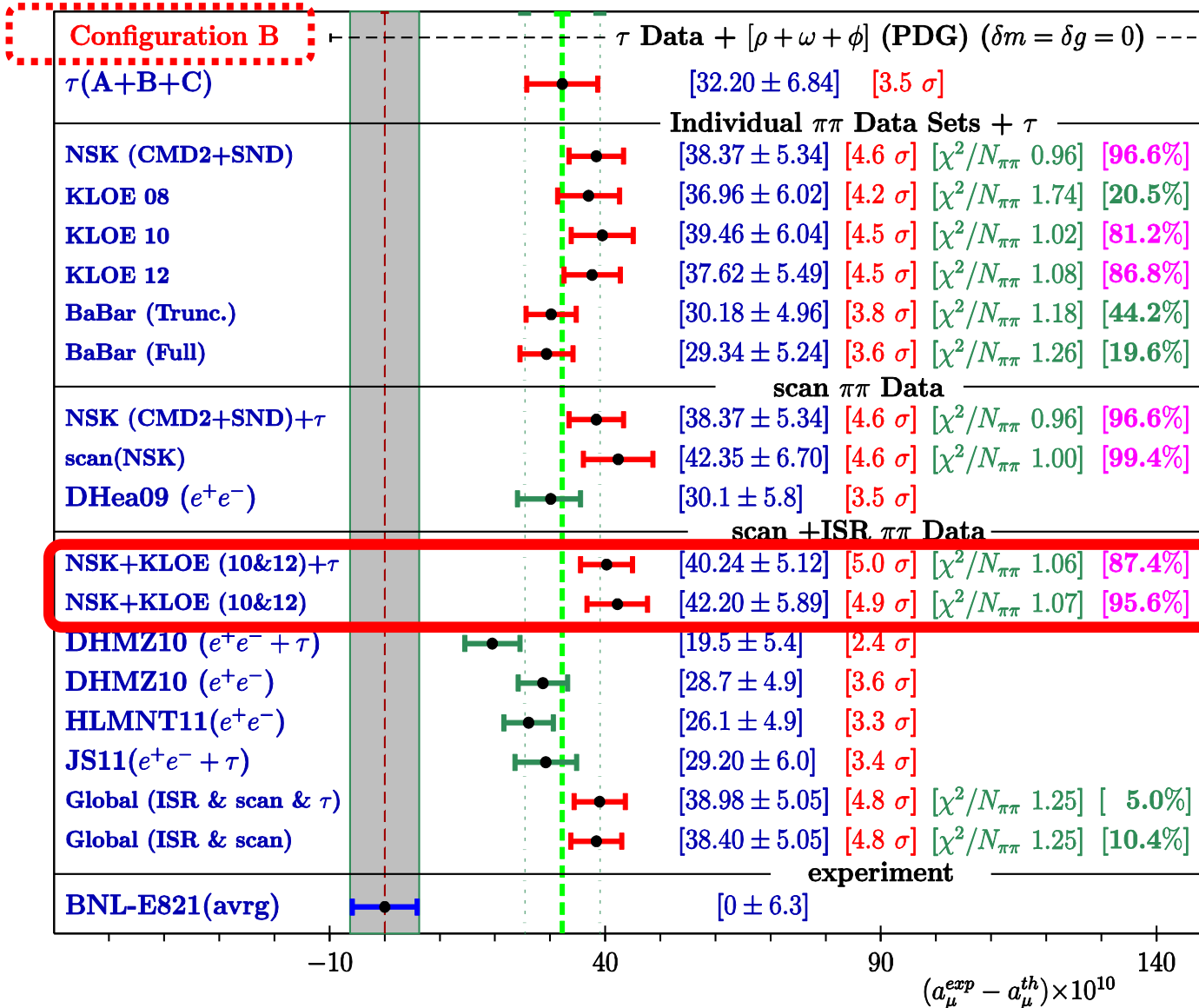


The two green 'high quality' points have 1% sys. error, the other set 30%

↪ 'maximal' adjustment of normalization of the latter by 1/1.35;

↪ smaller a_μ value and error, fit still good, $\chi^2_{\min}/d.o.f. < 1$.

$\pi^+\pi^-$: HLS fit-based results from M Benayoun et al.



Results for g-2 and the discrepancy as presented by M. Benayoun at the Mainz meeting April 2014, but now 'superseeded' by new 1507.02943.



MB: Preferred fits discard BaBar 2pi data (red framed)

$\pi^+\pi^-$: diff. betw. HLMNT & Benayoun et al (as of 2014)

- Taking only direct scan as baseline:
- **Benayoun et al**: -3.1 from HLS-based fit, -4.3 from KLOE10+12
- **HLMNT**: +5.5 from KLOE and BaBar (compared to scan only)

- So the big difference ($\sim 13 \times 10^{-10}$, $3.3 \rightarrow 5\sigma$) comes to a big part from the data input, i.e. if BaBar's 2π is used or not.
(If used: error relatively poor despite stats due to inflation)

- Future SND, CMD-3, BELLE and BESIII 2π data may dilute the strong significance of BaBar
[also more data from BaBar to be analysed!]

- Ideally find out why the different data sets are not consistent.
If this could be achieved the 2π channel would be great!