Electric polarizability and magnetic moment in external electric field

Eigo Shintani (Mainz->AICS)

Collaboration with Hartmut Wittig (Mainz)

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1. Introduction

First motivation

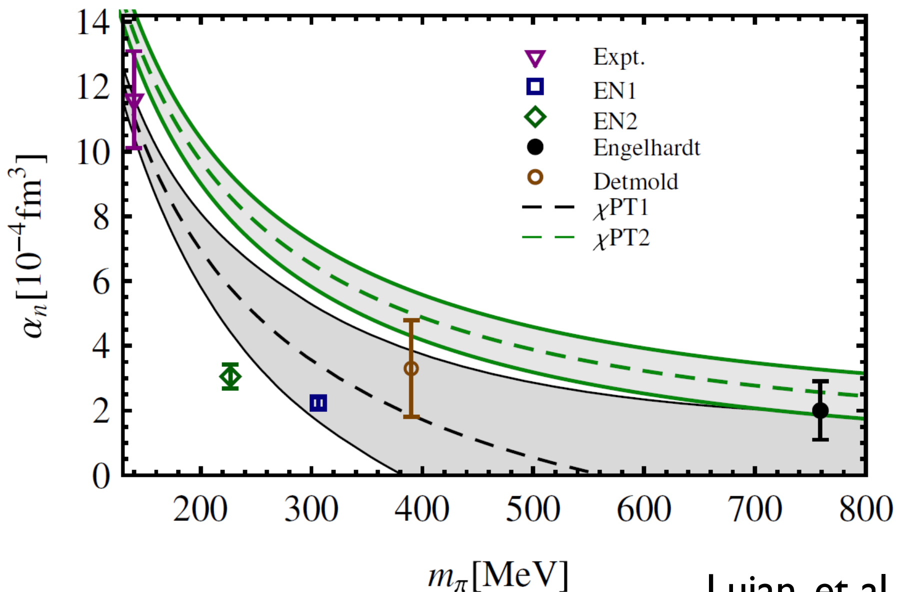
- Electric polarization

$$H_{\rm em} = -\vec{d}\cdot\vec{E} - \frac{1}{2}\alpha_N \vec{E}^2 + \mathcal{O}(E^3)$$

α_N : electric polarizability
d: electric dipole moment

- Electric polarizability is given from quadratic term of E field in energy shift.
- The EDM term appears in the leading order of E field in CPV vacuum.

Aoki, et al. (1992), ES, et.al. (2008)



Lujan, et al., PRD89 (2014)

- Recent lattice works indicate that polarizability of neutron may have strong pion mass dependence.
- In m_π > 210 MeV, there is large discrepancy from experiment.
- Finite size correction is also appearance.
- Light pion is necessary ?
- Need systematic study with various parameters.

1. Introduction

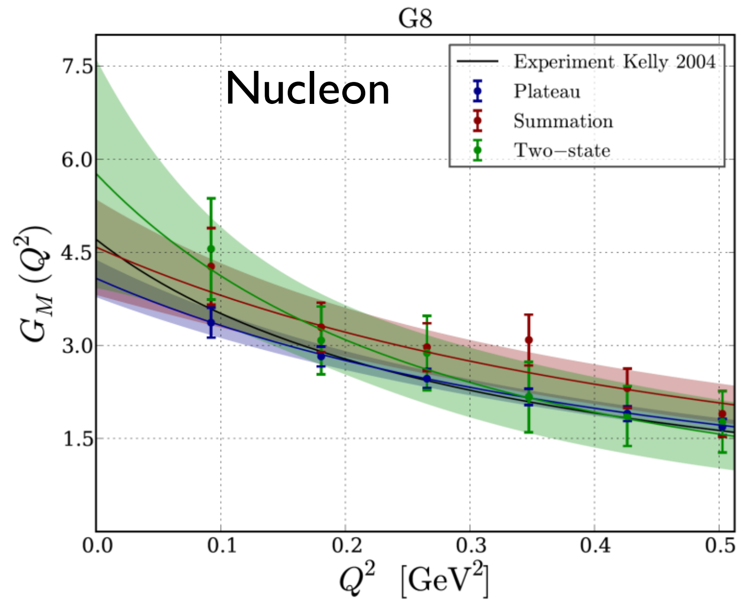
Second motivation

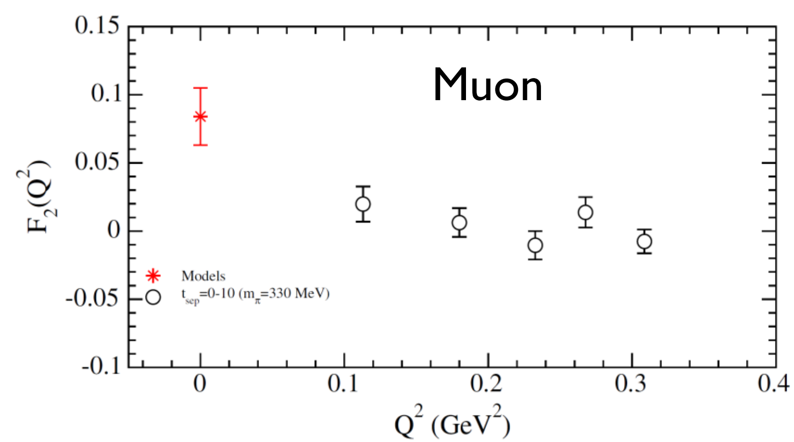
- Magnetic moment
  - Form factor computation
    - $\mu = G_M(0)/G_E(0)$
      $(G_M(0) = F_1(0) + F_2(0))$
  - Extrapolation to $Q^2=0$ for $G_M(Q^2)$
  - Search for direct measurement
    - Many application
    - Nucleon charge radius

$$\langle r_M^2 \rangle = -\frac{6}{G_M(0)} \frac{d}{dQ^2} G_M(Q^2)\Big|_{Q^2=0}$$

    - muon g-2 in QED+QCD system

Mainz (2015)





Blum et al. (2014)

1. Introduction

Magnetic moment in external B field

- Energy shift in B field

Aubin et al. PRD79(2009), Primer et al., PRD89(2014)

$$H_{\rm em}(B) = m - \vec{\mu}\vec{B} + \frac{e|B|}{2m} - 2\pi\beta B^2 + \mathcal{O}(B^2)$$

β: magnetic polarizability

- Magnetic moment appears in the leading term of B.
- Background B field, which is along z axis, for instance,

$$B_z = \partial_x A_y - \partial_y A_x, \quad A_x = -By,\ A_y(x,y) = \begin{cases} 0 & (y/a < N_y - 1) \\ N_y B x & (y/a = N_y - 1) \end{cases}$$

This magnetic field is made uniform by handling discontinuity at boundary, to be consistent with $B=F_{xy}$, and therefore it requires the quantization condition due to issue of double boundary

$$qBa^2 = \frac{2\pi n}{N_x N_y}$$

Link variable involving B field is violating periodicity either → may distort baryon. Large volume is required, but increase of spatial volume is difficult.

Method

- Non-relativistic equation in E field
  - Pauli-Dirac equation (Minkowski space) e.g. Lujan, et al., PRD89 (2014)

$$\left[i\gamma_\mu(\partial^\mu - q_e A^\mu) - m - \frac{\mu}{2m}F_{\mu\nu}\sigma^{\mu\nu}\right]\psi = 0$$

μ : magnetic moment

Using non-relativistic approximation, $\psi^t \simeq e^{-imt}(\phi, \chi)$

$$i\partial_t\phi = \left[\frac{p^2}{2m} + \frac{1}{2m}\left(-\frac{\mu^2}{m^2} + q_e^2 t^2\right)\vec{E}^2 + \frac{iq_e t}{m}\vec{p}\cdot\vec{E} - \frac{1}{m^2}\vec{\mu}\cdot(\vec{p}\times\vec{E})\right]\phi$$

- Assuming that there is electric field only, $F_{0i} = E_i,\ F_{ij} = 0,\ A_\mu = (0, tE_i)$
- q_e = 0 (nuetron), q_e = e (proton).
- Magnetic moment is in the last term proportional to cross-product of momentum and electric field.
- Compatible with form factor measurement.

2. External electric field

E field on the lattice

- Euclidean (imaginary) electric field

$$U_i(\vec{x}, t) \rightarrow U_i(\vec{x}, t) e^{ie_q \varepsilon_i t}$$

e_q : quark charge

- ε_i is now defined in Euclidean space, which is related to iE_i (E_i : real E field).
- Measurement of spin-projected nucleon two-point function.
- Solve u,d quark field and perform quark contraction.
- Periodic boundary condition for $U_i(x,t)$ at temporal direction is broken.
  → to suppress it, source location is set to half position of t extension.
- Temporal extension is usually larger than spatial one, and preserving periodicity → may avoid distortion of baryon.

➢ Measurement of energy shift → polarizability

➢ Ratio of each nucleon spin-component → magnetic moment

$$\mathrm{tr}[P_+\langle NN\rangle(i\vec{E}, t, \vec{\sigma}, \theta)] \simeq Z \exp\left[-(E_N + H_N^\theta)t\right],$$

$$H_N^\theta = \frac{1}{2}\alpha_N \vec{E}^2 - \frac{q_e t}{m}\vec{p}\cdot\vec{E} - i\frac{\mu}{2m^2}(\vec{p}\times\vec{E})\cdot\vec{\sigma} + id\vec{\sigma}\cdot\vec{E},\ \alpha_N = \alpha_c - \frac{\mu^2}{2m^3}$$

α_c : Compton polarizability

3. Numerical test
N_f=2 Wilson-clover

- Source location is maximally separated from t boundary as $t_{src}=N_t/2$
- $\varepsilon_x = (2\pi/N_t)c_E$, $c_E = \pm 0.03, \pm 0.06$
- $(e_u, e_d) = (2/3, -1/3)$
- Gauge invariant gaussian smeared-smeared source-sink.
- Deflation + SAP + GCR
- Computation:

  1 (non-E field) + 2 (u quark + d quark) × 4 (E field) = 9 props per 1 config.

	Lattice	*a* (fm)	m_π (GeV)	N_G	#conf	#meas
E5	64×32^3 $(2.0\ \mathrm{fm})^3$	0.063	0.456 $(m_\pi L=4.7)$	32	488	15,616
F7	96×48^3 $(3.0\ \mathrm{fm})^3$	0.063	0.277 $(m_\pi L=4.2)$	64	72	4,608

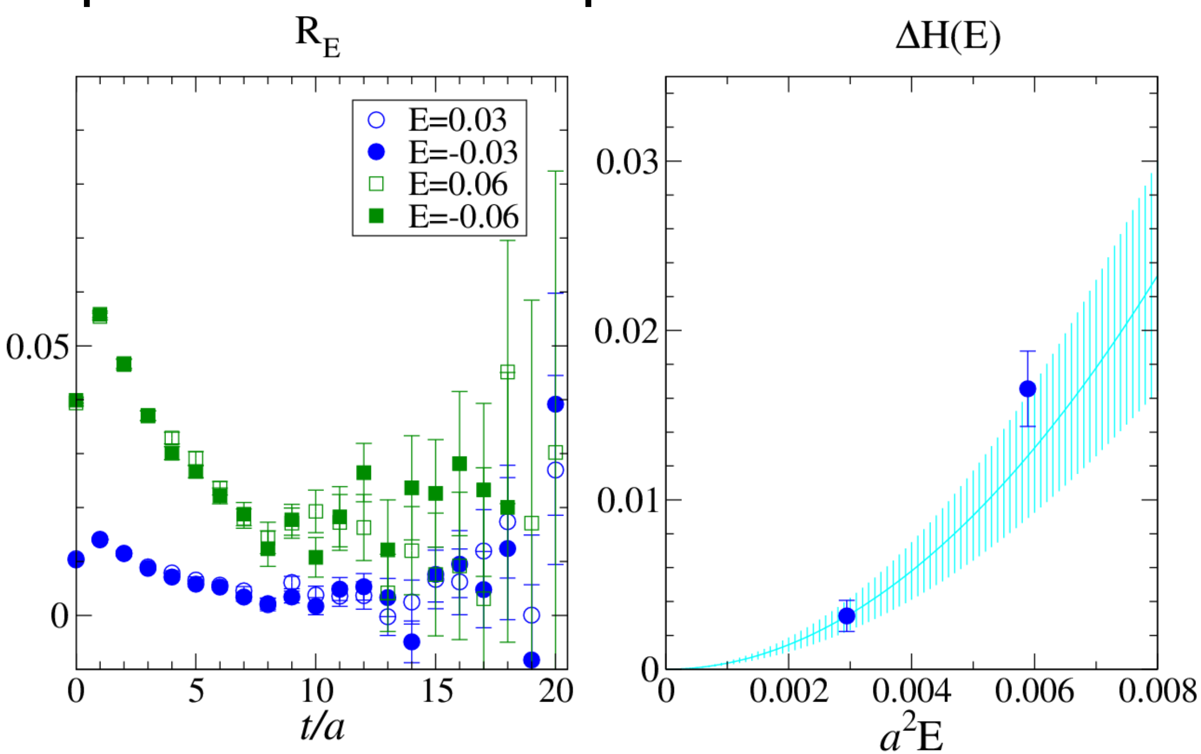
3. Numerical test

Electric polarizability

$$H_N = \boxed{\frac{1}{2}\alpha_N \vec{E}^2} - \frac{q_e t}{m}\vec{p}\cdot\vec{E} - i\frac{\mu}{2m^2}(\vec{p}\times\vec{E})\cdot\vec{\sigma}$$

$$R_E = \langle NN\rangle(E)/\langle NN\rangle(E=0) \simeq \exp\Big[-\Delta H t\Big],\ \Delta H = \frac{1}{2}\alpha_N E^2$$

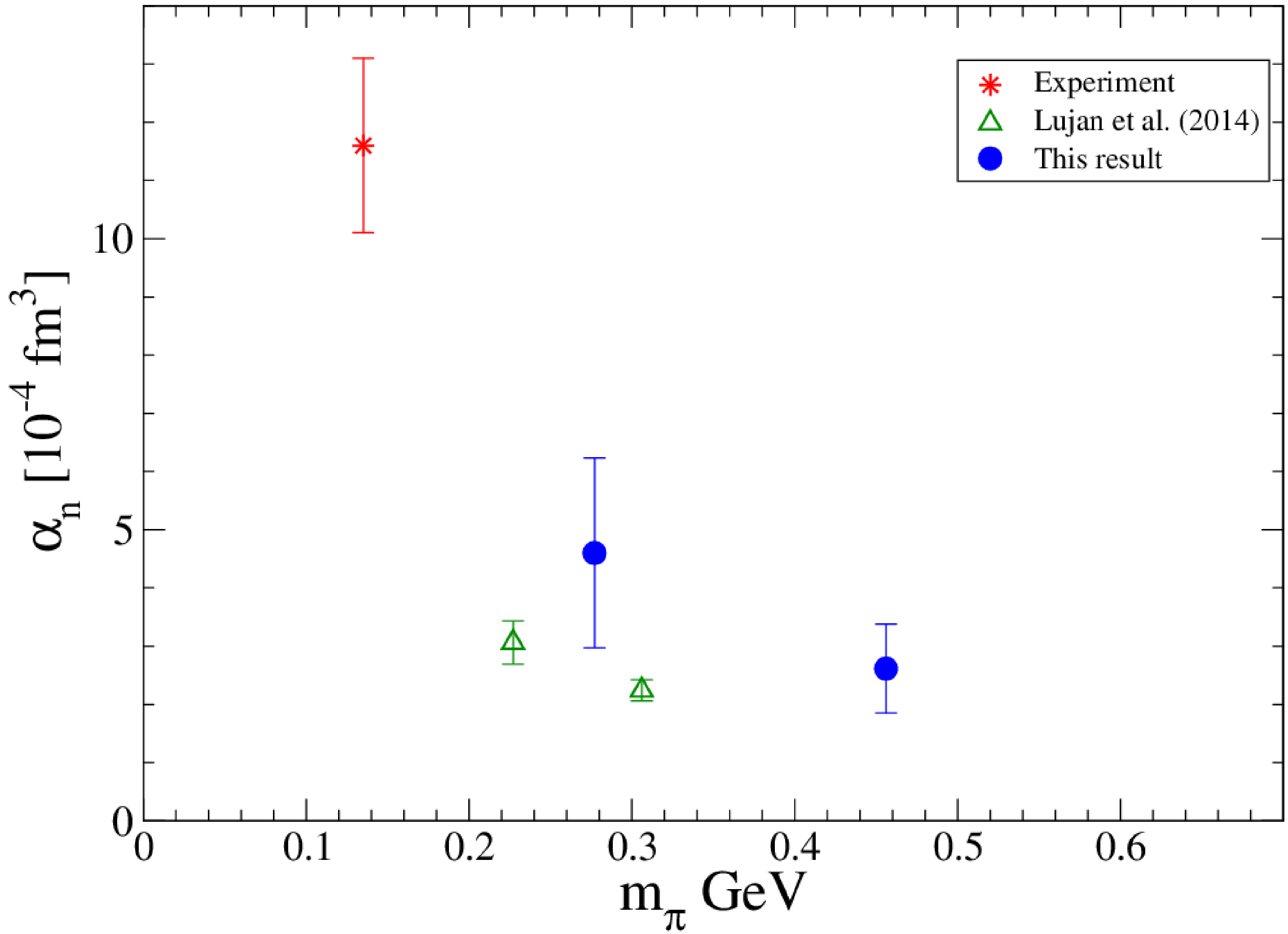
➢ p=0, effective mass plot for neutron



- Plateau is [10,15]
- Positive E and negative E have consistent signal.
  → expected signal
- E^2 dependence appears.

3. Numerical test

Comparison of neutron polarizability

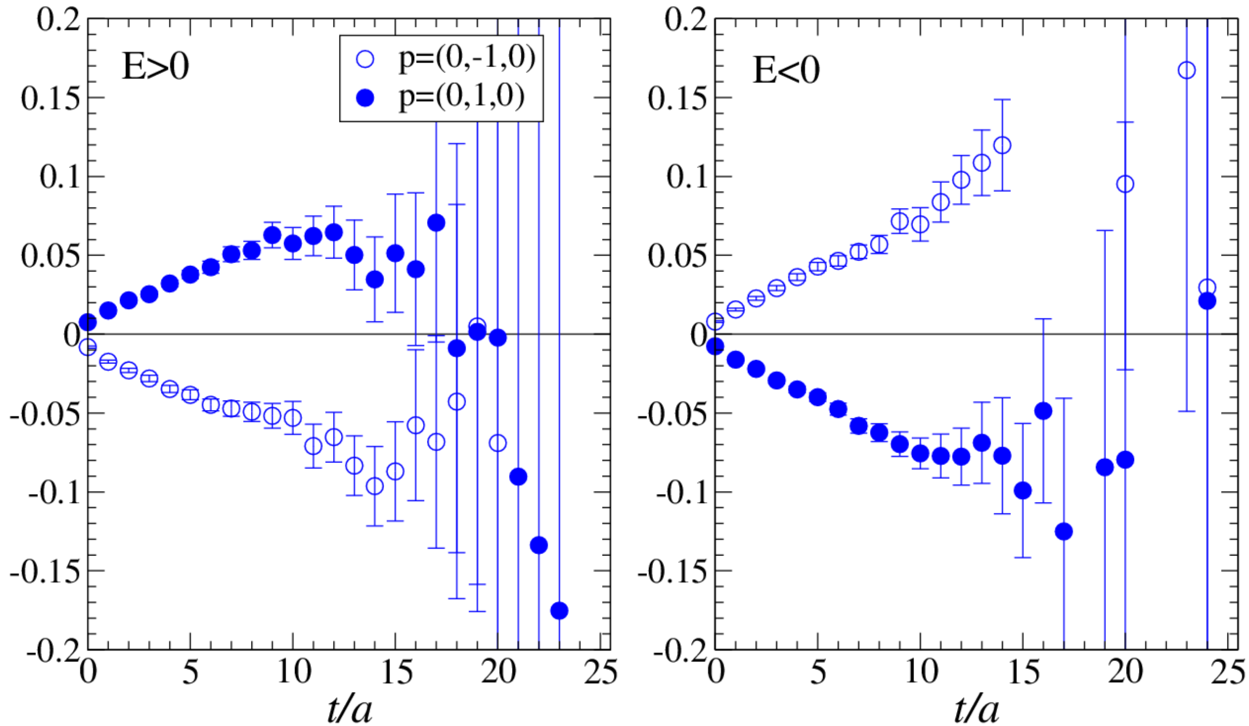


Ratio of correlator of spin-component

$$H_N = \frac{1}{2}\alpha_N \vec{E}^2 - \frac{q_e t}{m}\vec{p}\cdot\vec{E} \boxed{- i\frac{\mu}{2m^2}(\vec{p}\times\vec{E})\cdot\vec{\sigma}}$$

$$\mathrm{Im}[\langle NN\rangle(E,\sigma_3)/\langle NN\rangle(E,-\sigma_3)] \simeq \sin\Big[-\mu p_y \varepsilon_x t/m^2\Big]$$

E5, Proton, c_E=0.03, spin ratio

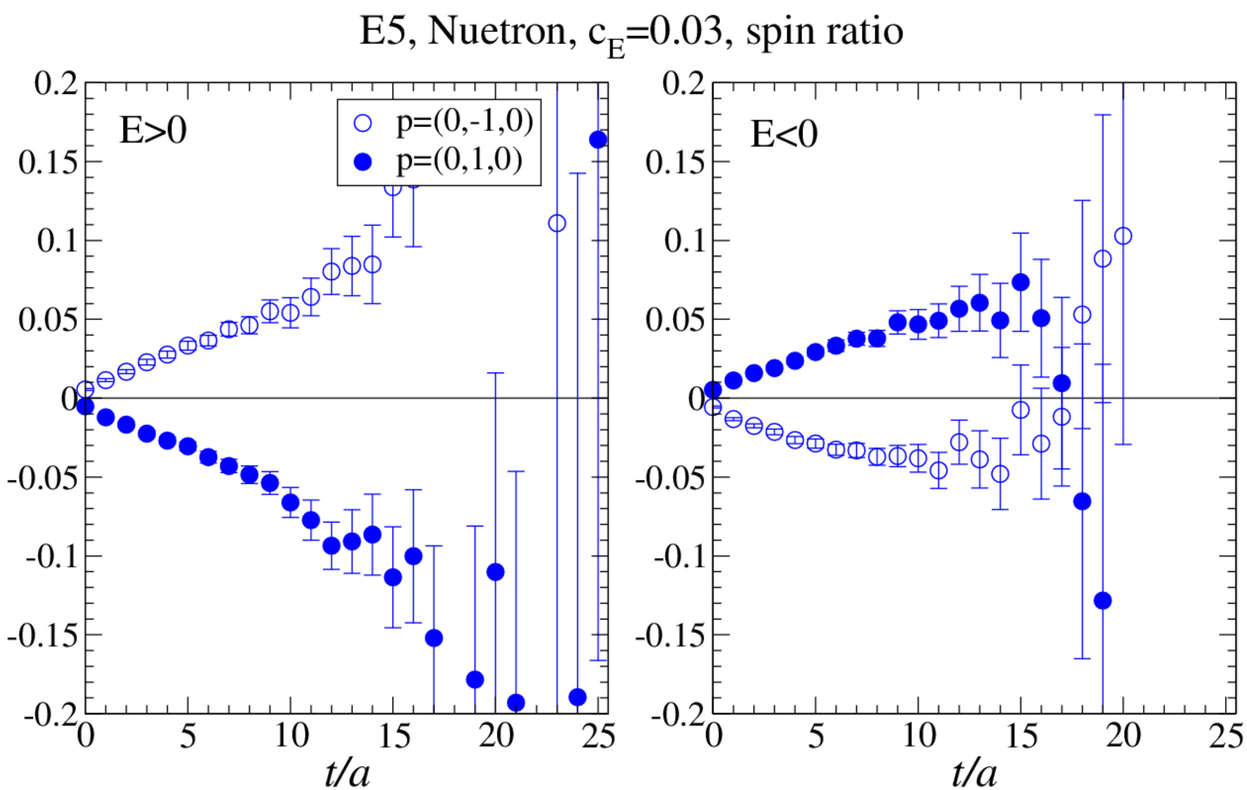


3. Numerical test

Ratio of correlator of spin-component

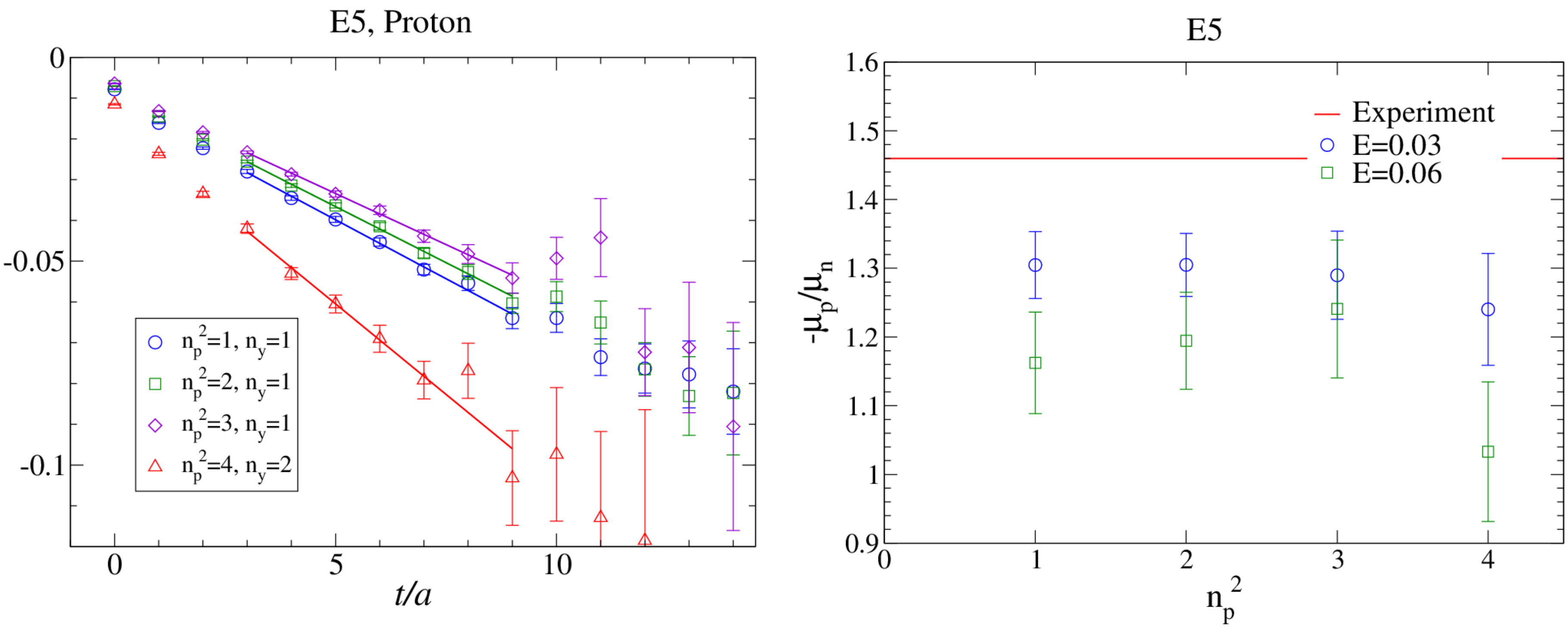
$$H_N = \frac{1}{2}\alpha_N \vec{E}^2 - \frac{q_e t}{m}\vec{p}\cdot\vec{E} \boxed{- i\frac{\mu}{2m^2}(\vec{p}\times\vec{E})\cdot\vec{\sigma}}$$

$$\mathrm{Im}[\langle NN\rangle(E,\sigma_3)/\langle NN\rangle(E,-\sigma_3)] \simeq \sin\left[-\mu p_y \varepsilon_x t/m^2\right]$$



3. Numerical test

Extraction of magnetic moment

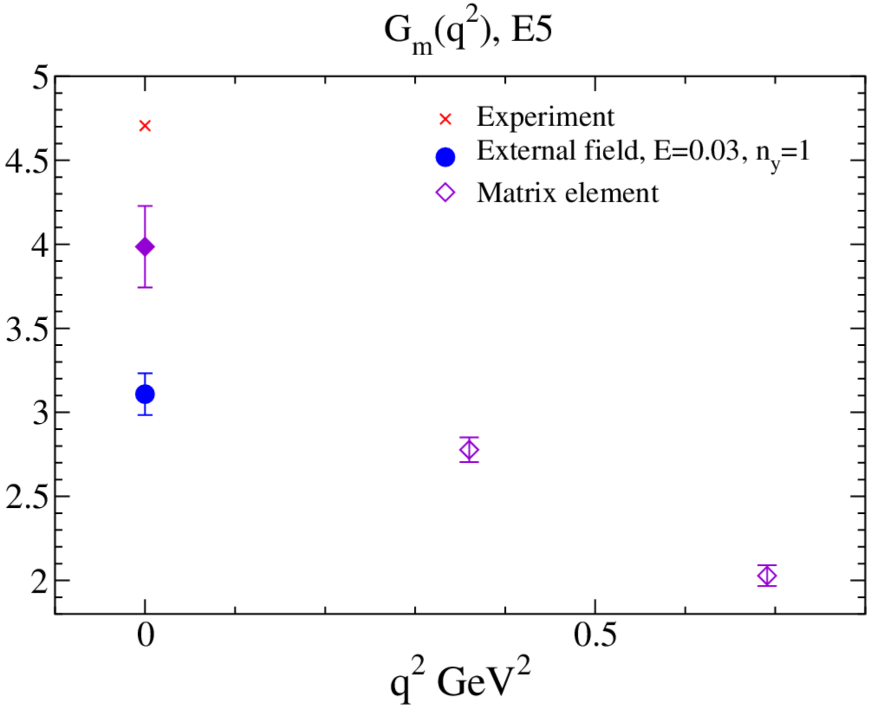


- Average over sign of E and p assignment.
- Check of consistency with μ extracted from different value of p^2 and E.
- The ratio of μ in p and n is in agreement between each n_p^2
- E=0.03 and 0.06 have slight tension → higher order effect ?

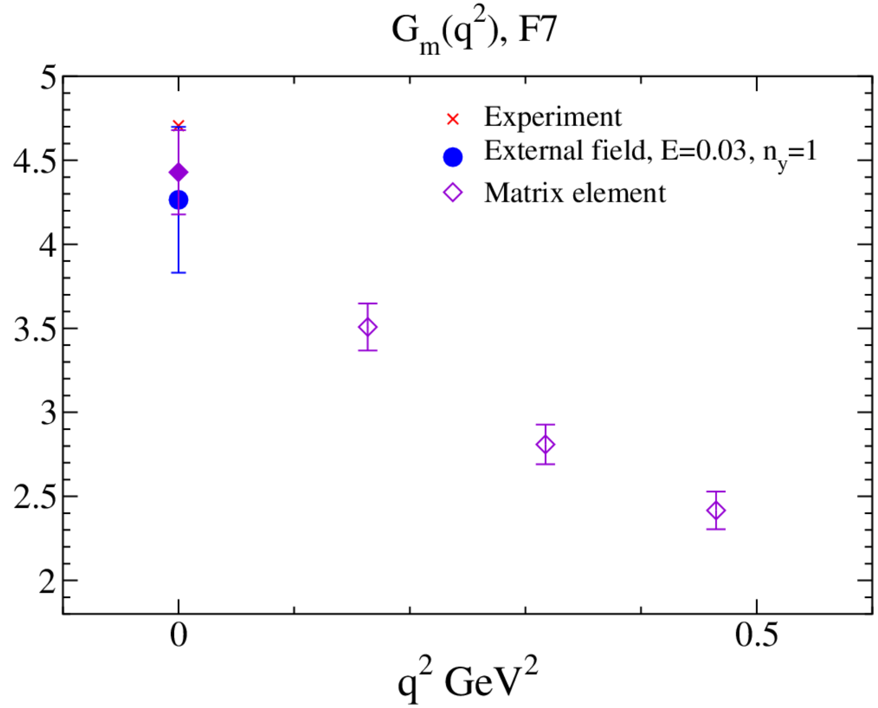
3. Numerical test

Comparison with matrix element

N_t=64, m_π=0.456 GeV, 2 fm^3 lattice



N_t=96, m_π=0.277 GeV, 3 fm^3 lattice

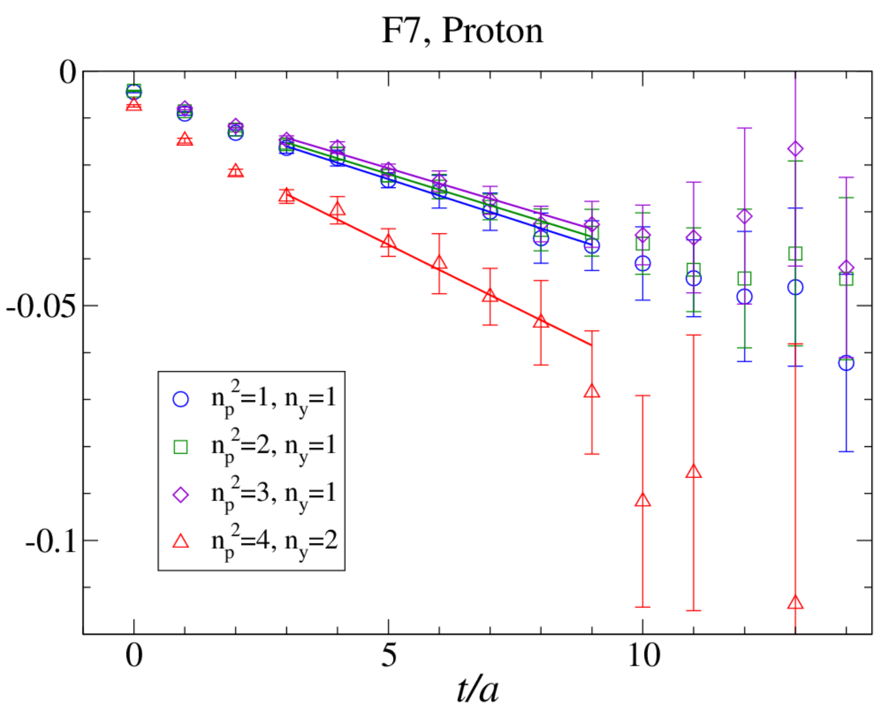


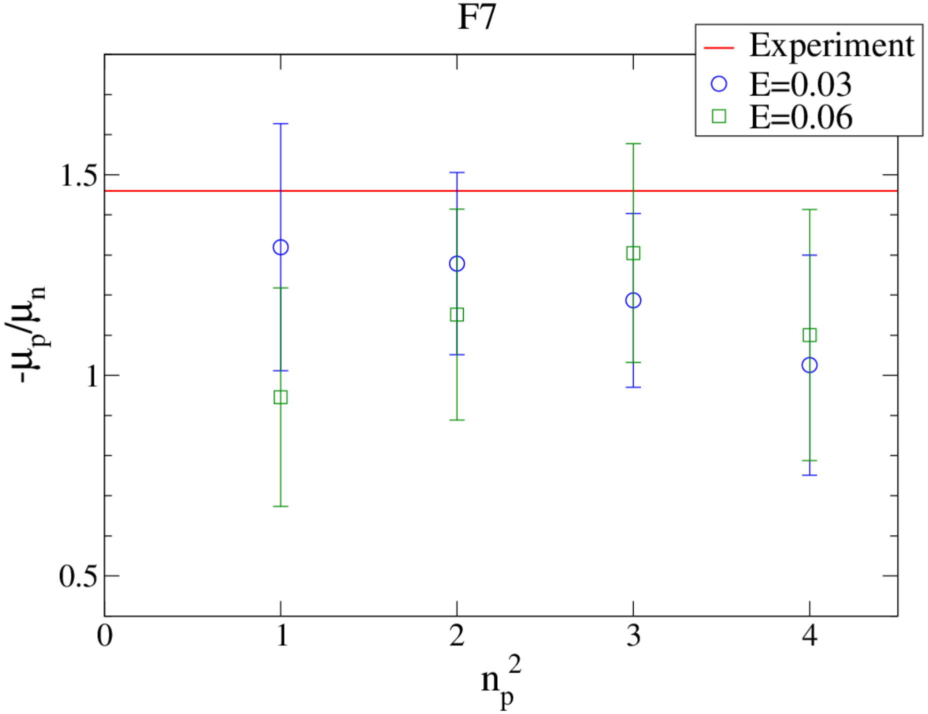
- Precision is comparable with form factor calculation, but statistics is much smaller than matrix element (about factor 5 and more).
- There is discrepancy from extrapolated value of $G_M(0)$ in small lattice.
- Larger lattice may improve such discrepancy → boundary correction ?

Summary and future work

- Compute polarizability and magnetic moment in external electric field (Euclidean).
- Although neutron polarizability is still large statistical fluctuation, its value is in agreement with other group.
- Magnetic moment is comparable with form factor.
- Boundary effect and higher order of E correction are not negligible. Large N_t computation is helpful.
- Cut-off effect, volume correction are still unknown.
- Electric dipole moment may also be measureable.
  (however many statistics are needed…)

Backup


F7, Proton
0
-0.05
-0.1
0
5
10
t/a
n_p^2=1, n_y=1
n_p^2=2, n_y=1
n_p^2=3, n_y=1
n_p^2=4, n_y=2


F7
Experiment
E=0.03
E=0.06
1.5
1
0.5
0
1
2
3
4
$-\mu_p/\mu_n$
n_p^2

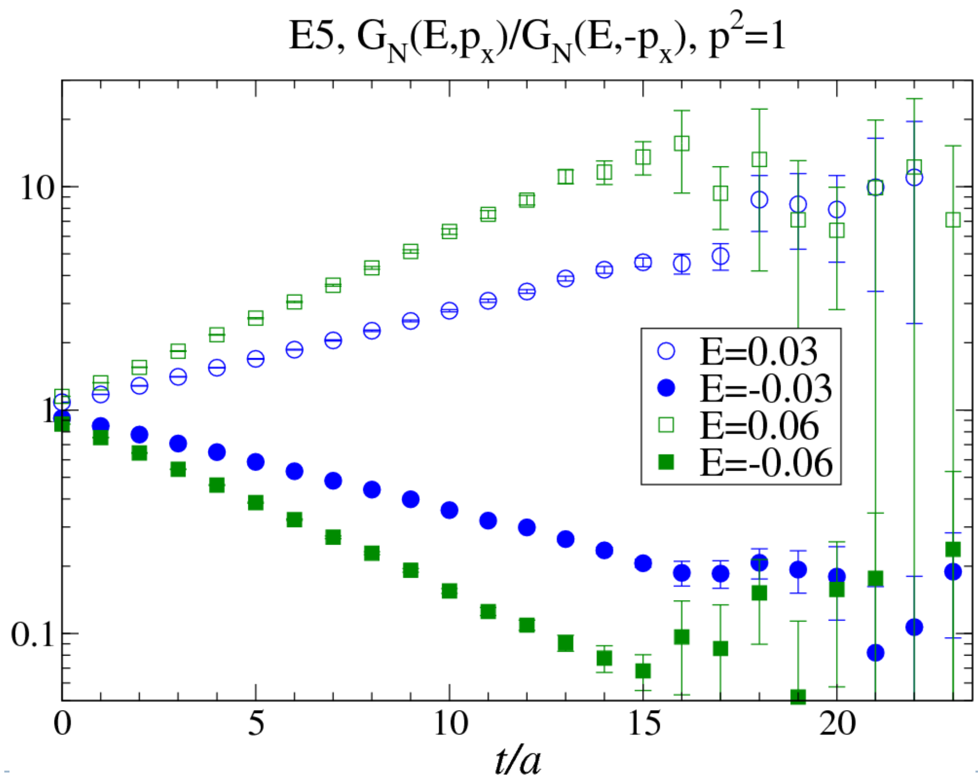
2. Euclidean electric field

Ratio of correlator in p and -p

$$H_N = \frac{1}{2}\Big(\alpha_N - \frac{\mu^2}{2m^3}\Big)\vec{E}^2 \boxed{-\frac{q_e t}{m}\vec{p}\cdot\vec{E}} - i\frac{\mu}{2m^2}(\vec{p}\times\vec{E})\cdot\vec{\sigma}$$

$$\langle NN\rangle(E,p_x)/\langle NN\rangle(E,-p_x) \simeq \exp\Big[-2\xi q_e p_x \varepsilon_x t^2\Big]$$

- $p_x \neq 0$



There appears linear response to E and p.

2. Euclidean electric field

Ratio of correlator in p and -p

$$H_N = \frac{1}{2}\left(\alpha_N - \frac{\mu^2}{2m^3}\right)\vec{E}^2 \boxed{- \frac{q_e t}{m}\vec{p}\cdot\vec{E}} - i\frac{\mu}{2m^2}(\vec{p}\times\vec{E})\cdot\vec{\sigma}$$

$$\langle NN\rangle(E,p_x)/\langle NN\rangle(E,-p_x) \simeq \exp\left[-2\xi q_e p_x \varepsilon_x t^2\right]$$

- $p_x \neq 0$, effective mass