Chiral and U(1)_A symmetries at finite temperature with overlap fermions

Sinya AOKI for JLQCD Collaboration

Yukawa Institute for Theoretical Physics, Kyoto University







High-precision QCD at low energy

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1. Introduction

Symmetries of QCD at high temperature

Restoration of non-singlet chiral symmetry

 $U(1)_B \otimes SU(N_f)_L \otimes SU(N_f)_R$

Theoretical questions

1. U(1)_A symmetry at high T?



2. Eigenvalue distribution of Dirac operator

 $ho(\lambda)$

 λ : eigenvalue of Dirac operator

Eigenvalue density

$$\rho(\lambda) = \sum_{n} \rho_n \frac{\lambda^n}{n!}$$

$$\lim_{m \to 0} \langle \bar{\psi} \psi \rangle = \pi \rho(0)$$

 $\rho(0) = \rho_0 = 0$ if chiral symmetry is restored.

Banks-Casher relation



$$\chi_{\Gamma}^{A} = \frac{1}{V} \int d^{4}x \langle M_{\Gamma}^{A}(x) M_{\Gamma}^{A}(0) \rangle \qquad \qquad N_{f} = 2$$

Susceptibility



 $U(1)_A$ susceptibilities

$$\chi^{\sigma-\eta} \equiv \chi^{\sigma} - \chi^{\eta} \qquad \qquad \chi^{\pi-\delta} \equiv \chi^{\pi} - \chi^{\delta} \qquad \qquad \chi^{\pi-\eta} \equiv \chi^{\pi} - \chi^{\eta}$$

If $U(1)_A$ is recovered, $\chi^{\sigma-\eta} = \chi^{\pi-\delta} = \chi^{\pi-\eta} = 0.$

2. Previous Theoretical Investigation

S.A, H. Fukaya, Y. Taniguchi,

"Chiral symmetry restoration, eigenvalue density of Dirac operator and axial U(1) anomaly at finite temperature", Phys. Rev D86(2012)114512.





Set up

Lattice regularization with Overlap fermion, 2-flavor

Exact "chiral" symmetry but explicit U(1)_A anomaly form Ginsparg-Wilson relation

$$D\gamma_5 + \gamma_5 D = a D R \gamma_5 D$$



Propagator

$$S(x,y) = \sum_{n} \left[\frac{\phi_n(x)\phi_n^{\dagger}(y)}{f_m \lambda_n - m} + \frac{\gamma_5 \phi_n(x)\phi_n^{\dagger}(y)\gamma_5}{f_m \overline{\lambda}_n - m} \right] - \sum_{k=1}^{N_{R+L}} \frac{1}{m} \phi_k(x)\phi_k^{\dagger}(y) + \sum_{K=1}^{N_D} \frac{Ra}{2} \phi_K(x)\phi_K^{\dagger}(y)$$

bulk modes(non-chiral)

zero modes(chiral)

doublers(chiral)

 $f_m = 1 + \frac{Rma}{2}$

$$P_m(A) = e^{-S_{YM}(A)} (-m)^{N_f N_{R+L}^A} \left(\frac{2}{Ra}\right)^{N_f N_D^A} \prod_{\Im \lambda_n^A > 0} \left(Z_m^2 \bar{\lambda}_n^A \lambda_n^A + m^2\right)$$
of zero modes
$$Z_m^2 = 1 - (ma)^2 \frac{R^2}{4}$$

positive definite and even function of $m \neq 0$ for even N_f

N_f=2 in this talk.

Chiral symmetry is restored

$$\lim_{m \to 0} \langle \delta^a \mathcal{O}_{n_1, n_2, n_3, n_4} \rangle_m = 0$$

$$\mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1} (S^a)^{n_2} (P^0)^{n_3} (S^0)^{n_4} \qquad S^a = \int d^4x \, S^a(x), \quad P^a = \int d^4x \, P^a(x)$$
scalar
pseudo-scalar

chiral rotation at N_f=2

$$\delta^a S^b = 2\delta^{ab} P^0, \delta^a P^b = -2\delta^{ab} S^0$$
$$\delta^a S^0 = 2P^a, \delta^a P^0 = -2S^a$$

Some assumptions



Note that this does not hold if the chiral symmetry is spontaneously broken.

Ex.
$$\lim_{V \to \infty} \frac{1}{V} \langle Q(A)^2 \rangle_m = m \frac{\Sigma}{N_f} + O(m^2)$$

topological charge

Assumption 3

eigenvalues density can be expanded as

$$\rho^{A}(\lambda) \equiv \lim_{V \to \infty} \frac{1}{V} \sum_{n} \delta \left(\lambda - \sqrt{\bar{\lambda}_{n}^{A} \lambda_{n}^{A}} \right) = \sum_{n=0}^{\infty} \rho_{n}^{A} \frac{\lambda^{n}}{n!} \qquad \text{at } \lambda = 0 \ \left(\lambda < \epsilon \right)$$

More precisely, configurations whose eigenvalue density can not be expanded at the origin are "measure zero" in the configuration space.

At finite lattice spacing, integrals over all eigenvalues are convergent, since

$$|\lambda| \le \frac{2}{Ra}$$

Analysis (some examples)

$$\frac{S_0}{V} = -\frac{1}{V} \sum_n \left[\frac{F(\lambda_n)}{f_m \lambda_n - m} + \frac{F(\bar{\lambda}_n)}{f_m \bar{\lambda}_n - m} \right] + \frac{N_{R+L}^A}{Vm}$$
$$V \to \infty$$
$$I_1 = \frac{1}{Z_m} \int_0^{\Lambda_R} d\lambda \, \rho^A(\lambda) g_0(\lambda^2) \frac{2m_R}{\lambda^2 + m_R^2} = \pi \rho_0^A + O(m)$$

source of m singularity

 $\langle \rho_0^A \rangle_m = O(m^2)$ $\lim_{V \to \infty} \left\langle \frac{N_{R+L}}{V} \right\rangle_m = O(m^2)$

$$\lim_{m \to 0} \chi^{\eta - \delta} = 0$$

$$\chi^{\eta-\delta} = \frac{1}{V} \langle P_0^2 - S_a^2 \rangle$$

$$\chi^{\eta-\delta} = N_f \left\langle \frac{1}{m^2 V} \{ 2N_{R+L} - N_f Q(A)^2 \} + \frac{1}{Z_m} \left(\frac{I_1}{m_R} + I_2 \right) \right\rangle_m \qquad \text{topological charge} \\ = 0 \\ \text{from others} \qquad I_2 = \frac{2}{Z_m} \int_0^{\Lambda_R} d\lambda \, \rho^A(\lambda) \, \frac{m_R^2 - \lambda^2 g_0(\lambda^2) g_m}{(\lambda^2 + m_R^2)^2}, \quad g_m = \frac{1}{Z_m^2} \left(1 + \frac{m^2}{2\Lambda_R^2} \right) \right\rangle_m$$

$$\frac{I_1}{m_R} + I_2 = \rho_0^A \left(\frac{\pi_m}{m} + \frac{2}{\Lambda_R}\right) + 2\rho_1^A + O(m),$$

$$\uparrow$$

$$\langle \rho_0^A \rangle_m = O(m^2)$$

$$\lim_{m \to 0} \frac{N_f^2 \langle Q(A)^2 \rangle_m}{m^2 V} = 2 \lim_{m \to 0} \langle \rho_1^A \rangle_m$$

repeat these analysis for higher susceptibilities.

Final Results

$$\lim_{m \to 0} \langle \rho^A(\lambda) \rangle_m = \lim_{m \to 0} \langle \rho_3^A \rangle_m \frac{|\lambda|^3}{3!} + O(\lambda^4)$$

No constraints to higher $\langle \rho_n^A \rangle_m$

 $\langle \rho_3^A \rangle_m \neq 0$ even for "free" theory.

$$\langle \rho_0^A \rangle_m = 0$$
$$\lim_{V \to \infty} \frac{1}{V^k} \langle (N_{R+L}^A)^k \rangle_m = 0, \quad \lim_{V \to \infty} \frac{1}{V^k} \langle Q(A)^{2k} \rangle_m = 0$$

Consequences

Singlet susceptibility at high T

$$\lim_{V \to 0} \chi^{\pi - \eta} = \lim_{m \to 0} \lim_{V \to \infty} \frac{N_f^2}{m^2 V} \langle Q(A)^2 \rangle_m = 0$$

This, however, does not mean U(1)_A symmetry is recovered at high T.

$$\lim_{m \to 0} \chi^{\pi - \eta} = 0 \qquad \Longrightarrow \qquad "m_{\pi} = m_{\eta}"$$

is necessary but NOT "sufficient" for the recovery of U(1)_A .

More general Singlet WT identities

$$\langle J^0 \mathcal{O} + \delta^0 \mathcal{O} \rangle_m = O(m)$$

anomaly(measure)

singlet rotation

We can show for $\mathcal{O} = \mathcal{O}_{n_1, n_2, n_3, n_4} = (P^a)^{n_1} (S^a)^{n_2} (P^0)^{n_3} (S^0)^{n_4}$

$$\lim_{V \to \infty} \frac{1}{V^k} \langle J^0 \mathcal{O} \rangle_m = \lim_{V \to \infty} \left\langle \frac{Q(A)^2}{mV} \times O(V^0) \right\rangle_m = 0$$

where k is the smallest integer which makes the $V \to \infty$ limit finite.

 $S^0 \sim O(V), P^a, S^a, P^0 \sim O(V^{1/2})$



$$\lim_{m \to 0} \lim_{V \to \infty} \frac{1}{V^k} \langle \delta^0 \mathcal{O} \rangle_m = 0$$

Breaking of U(1)_A symmetry is invisible for these "bulk quantities".

 $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{Z}_4$

Remarks

Important conditions

Large volume limit $V \to \infty$

chiral limit $m \rightarrow 0$

lattice chiral symmetry Ginsparg-Wilson relation $D\gamma_5 + \gamma_5 D = aDR\gamma_5 D$

Fractional power for the eigenvalue density

 $\rho^A(\lambda) \simeq c_A \lambda^\gamma, \ \gamma > 0$ non-singlet chiral symmetry is recovered.

$$\gamma \leq 2 \text{ is excluded.} \qquad \qquad \gamma > 2$$

consistent with the integer case (n > 2)

3. Recent Numerical Results

A. Tomiya et al. (JLQCD), Lat2015 G. Cossu et al. (JLQCD), Lat2015 **Eigenvalue densities**

$$\rho(\lambda) = \lim_{V \to \infty} \frac{1}{V} \sum_{n} \delta(\lambda - \lambda_n)$$

Cossu *et al.* (JLQCD), Overlap Phys. Rev. D87 (2013) 114514



Gap seems to open at smaller quark mass.

 $T_c \simeq 180 \text{ MeV}$

Buchoff et al. (LLNL/RBC), DomainWall, Phys. Rev. D89 (2014) 5,054514



Small eigenvalues appear.



Gap seems to close at or above critical temperature

 $T_c \simeq 180 \text{ MeV}$



Summary of recent results from chiral fermions.

Group	Fermion	Size	Gap in the spectrum	U _A (1) Correlator	$U(1)_A$ $T\gtrsim Tc$
JLQCD (2013)	Overlap (Top. fixed)	2 fm	Gap	Degenerate	Restored
TWQCD (2013)	Optimal domain-wall	3 fm	No gap	Degenerate	Restored
LLNL/RBC, Hot QCD (2013,2014)	(Möbius)- Domain-wall (W/ ov)	2, 4, 11 fm	No gap	No degeneracy	Violated
Viktor Dick et al (2015)	OV on HISQ sea	3, 4 fm	No gap	No degeneracy	Violated

What causes this difference ?

volume ? quark mass ? lattice chiral symmetry ?

JLQCD collaboration

Overlap: exact GW relation

LLNL/RBC collaborations

DomainWall: approximated GW relation

Recent study by A. Tomiya et al. for JLQCD collaboration

Preliminary

generate gauge configurations with an improved DomainWall quarks very small violation of GW relation

(0) calculate eigenvalue distribution of DW operator on these configurations

original

(1) calculate eigenvalue distribution of overlap operator on these configurations

partially quenched

(2)reweighting factor from the improved DW to Overlap is introduced to obtain the full eigenvalue distribution **full Overlap**





After the reweighting, small eigenvalues in PQ disappear, and the gap seems to open in full Overlap.

An exact lattice chiral symmetry is essential. A tiny violation of the chiral symmetry may destroy the theoretically expected relation.



DomainWall

reweighted Overlap

G. Cossu et al. (JLQCD), Lat2015

REWEIGHTING IS CRUCIAL



Point: Reweighting is crucial

Partially quenched results show accumulation of unphysical near zero modes

If the gap opens, the effective symmetry is

 $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_A$

S. Sharma, V. Dick, F. Karsch, E. Laermann, S. Mukherjee, Lattice2015 Eigenvalues density of Overlap on DomainWall (partially quenched !)

From Sharma's talk@Lat2015,

- General features: Near zero mode peak +bulk
- We fit to the ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2 + A} + B\lambda^{\gamma}$
- Bulk rises linearly as λ , no gap seen.
- No gap even when quark mass reduced!



4. Discussion

Possible loopholes for the theoretical argument?

Assumption 2
$$\langle \mathcal{O}(A) \rangle_m = f(m^2) \quad f(x) \text{ is analytic at } x = 0$$

May this be violated ?

Note that non-analyticity comes from valence quarks, not from determinant, even at zero temperature, where non-analyticity appears due to the spontaneous chiral symmetry breaking.

What kind of physics implies non-analyticity at m=0?

Assumption 3 eigenvalues density can be expanded at the origin ?

Ex. claim by LLNL/RBC: accumulation of near-zero modes leads to

$$\begin{split} \rho(\lambda) \simeq c_m \delta(\lambda) + \cdots & \# \text{ of near zero-modes } \propto VL = (V)^{5/4} \\ & & \uparrow \\ \pi \delta(\lambda) \sim \frac{1/L}{\lambda^2 + 1/L^2} & & \uparrow \\ & & \downarrow \\ &$$

Order of phase transition in 2-flavor QCD



Conformal bootstrap method predicts IR fixed point for these cases.

Even if the phase transition is of 2nd order, its universality class should be different from O(4).