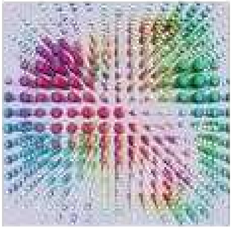
CHIRAL PERTURBATION AND THE LATTICE



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Logos Lund University

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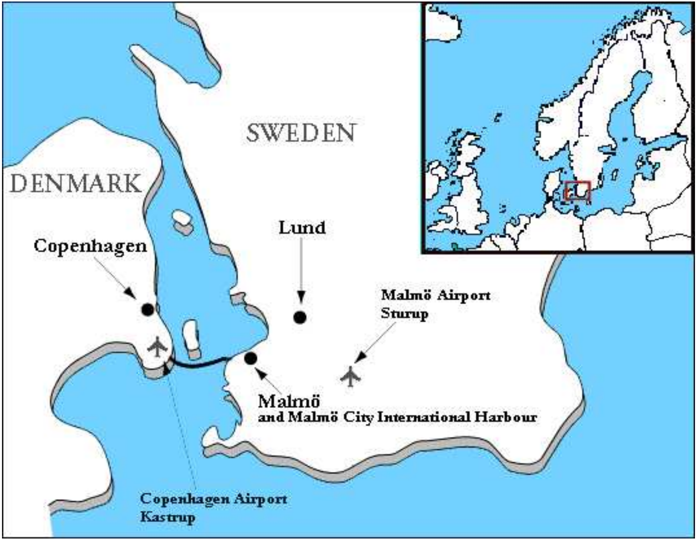
+ in Black, negative and Pantomine colour system

Where is Lund?



Lund-Benasque ≈ Lund-North of Sweden

Lund is known for:

- Pythia and several more Lund model Monte Carlos
- Rydberg (the famous constant)
- MAX IV (Fourth generation syncrotron ring, starts 2016
- ESS European spallation source, building started, first neutrons 2019, 25 instruments ready 2025
- Tetra pak, Sony-Ericsson, Gambro, Axis
- ...
- Chiral Perturbation Theory

Lund is known for:


Overview

The three seminal ChPT papers are cited by 5472 papers (2633+3375+3225)

Exploring the consequences of
the chiral symmetry of QCD
and its spontaneous breaking
using effective field theory techniques

Derivation from QCD:
H. Leutwyler,
On The Foundations Of Chiral Perturbation Theory,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For references to lectures see:
http://www.thep.lu.se/~bijnens/chpt/

Chiral Perturbation Theory

A general Effective Field Theory:

- Relevant degrees of freedom
- A powercounting principle (predictivity)
- Has a certain range of validity

Chiral Perturbation Theory:

- Degrees of freedom: Goldstone Bosons from spontaneous breaking of chiral symmetry
- Powercounting: Dimensional counting in momenta/masses
- Breakdown scale: Resonances, so about M_ρ.

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Goldstone Bosons

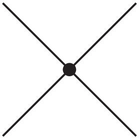
Spontaneous breakdown

- $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken $\Longrightarrow$ 8 massless degrees of freedom and interaction vanishes at zero momentum

Goldstone Bosons

Power counting in momenta: Meson loops, Weinberg powercounting

rules

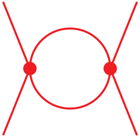


p^2

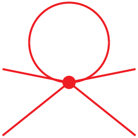
$1/p^2$

$\int d^4p$ $\qquad$ p^4

one loop example



$$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$$



$$(p^2)\,(1/p^2)\,p^4 = p^4$$

Chiral Perturbation Theories

- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \ldots$ and extensions to (partially) quenched
- Or beyond QCD
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite T
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- My general belief: if it involves soft pions (or soft K, η) some version of ChPT exists

Lagrangians: Lowest order

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix}.$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}$,

$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$,
left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$ quark masses via scalar density: $s = \mathcal{M} + \cdots$

$\langle A \rangle = Tr_F\,(A)$

Lagrangians: Lagrangian structure (mesons, strong)

	2 flavour		3 flavour		PQChPT/N_f flavour	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- L_i LEC = Low Energy Constants = ChPT parameters
- H_i: contact terms: value depends on definition of currents/densities
- Finite volume: no new LECs
- Other effects: (many) new LECs

Mesons: which Lagrangians are known ($n_f = 3$)

Loops	$\mathcal{L}_{\text{order}}$	LECs	effects included
$L = 0$	$\mathcal{L}_{p^2}$	2	strong (+ external W, γ)
	$\mathcal{L}_{e^2 p^0}$	1	internal γ
	$\mathcal{L}^{\Delta S=1}_{G_F p^2}$	2	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_8 e^2 p^0}$	1	nonleptonic weak+internal γ
	$\mathcal{L}^{\text{odd}}_{p^4}$	0	WZW, anomaly
$L \leq 1$	$\mathcal{L}_{p^4}$	10	strong (+ external W, γ)
	$\mathcal{L}_{e^2 p^2}$	13	internal γ
	$\mathcal{L}^{\Delta S=1}_{G_8 F p^4}$	22	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_{27} p^4}$	28	nonleptonic weak
	$\mathcal{L}^{\Delta S=1}_{G_8 e^2 p^0}$	14	nonleptonic weak+internal γ
	$\mathcal{L}^{\text{odd}}_{p^4}$	23	WZW, anomaly
	$\mathcal{L}^{\text{leptons}}_{e^2 p^2}$	5	leptons, internal γ
$L \leq 2$	$\mathcal{L}_{p^6}$	90	strong (+ external W, γ)

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars/axial currents
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)
- Unitarity included perturbatively

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots$$

$$M^2 = 2B\hat{m}$$

LECs and μ

$l_3^r(\mu)$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i}\, l_i^r(\mu) - \log\frac{M_\pi^2}{\mu^2}\,.$$

is independent of the scale μ.

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

Choice of μ :

- m_π, m_K: chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$
  what about large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exist
- Express orders in terms of physical masses and quantities (F_π, F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering
- Note: remaining μ dependence can occur at a given order
- Can make quite some difference in the expansion

I prefer physical masses

- Thresholds correct
- Chiral logs are from physical particles propagating
- but sometimes too many masses so very ambiguous

Extensions for the lattice

- No new parameters:
  - Finite temperature
  - Finite volume (including ϵ regime)
  - Twisted mass
  - Boundary conditions: twisted,...
- A few new parameters
  - Partially quenched ($2\to 2$,$10\to 11$, $90\to 112$)
- Many new parameters
  - Wilson ChPT ($2\to 3$,$10\to 18$)
  - Staggered ChPT ($2\to 10$,$10\to 126$ (but dependencies))
  - Mixed actions
- Other operators
  - Local object with well defined chiral properties: include via spurion techniques
  - Examples: tensor current, energy momentum tensor,...

Many LECs

- Is this too many parameters to do something?
- But if analytic in quark masses added in the fit not much extra
- Example: meson masses at NNLO have only the possible analytic quark mass dependence and the NLO meson-meson scattering parameters as input

Program availability

Making the programs more accessible for others to use:

- Two-loop results have very long expressions
- Many not published but available from http://www.thep.lu.se/∼bijnens/chpt/
- Many programs available on request from the authors
- Idea: make a more general framework
- CHIRON:

JB,
"CHIRON: a package for ChPT numerical results at two loops,"
Eur. Phys. J. C **75** (2015) 27 [arXiv:1412.0887]
http://www.thep.lu.se/∼bijnens/chiron/

Program availability: CHIRON

- Present version: 0.53
- Classes to deal with $L_i, C_i,\ L_i^{(n)}, K_i$, standardized in/output, changing the scale,...
- Loop integrals: one-loop and sunsetintegrals
- Included so far (at two-loop order):
  - Masses, decay constants and $\langle\bar{q}q\rangle$ for the three flavour case
  - Masses and decay constants at finite volume in the three flavour case
  - Masses and decay constants in the partially quenched case for three sea quarks
  - Masses and decay constants in the partially quenched case for three sea quarks at finite volume
- A large number of example programs is included
- Manual has already reached 82 pages
- I am continually adding results from my earlier work

LEC determination: (Partial) History/References

- Original determination at p^4: Gasser, Leutwyler, Annals Phys.158 (1984) 142, Nucl. Phys. B250 (1985) 465
- p^6 3 flavour: Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [hep-ph/0101127]
- Review article two-loops: JB, Prog. Part. Nucl. Phys. 58 (2007) 521 [hep-ph/0604043]
- Update of fits + new input: JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945]
- Recent review with more p^6 input: JB, Ecker, Ann. Rev. Nucl. Part. Sci. **64** (2014) 149 [arXiv:1405.6488]
- Review Kaon physics: Cirigliano, Ecker, Neufeld, Pich, Portoles, Rev.Mod.Phys. 84 (2012) 399 [arXiv:1107.6001]
- Lattice: FLAG reports:
  Colangelo et al., Eur.Phys.J. C71 (2011) 1695 [arXiv:1011.4408]
  Aoki et al., Eur. Phys. J. C **74** (2014) 9, 2890 [arXiv:1310.8555]

Three flavour LECs: uncertainties

- $m_K^2, m_\eta^2 \gg m_\pi^2$
- Contributions from p^6 Lagrangian are larger
- Reliance on estimates of the C_i much larger
- Typically: C_i^r: (terms with)
  kinematical dependence $\equiv$ measurable
  quark mass dependence $\equiv$ impossible (without lattice)
  100% correlated with L_i^r
- How suppressed are the $1/N_c$-suppressed terms?
- Are we really testing ChPT or just doing a phenomenological fit?

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Testing if ChPT works: relations

Yes: JB, Jemos, Eur.Phys.J. C64 (2009) 273-282 [arXiv:0906.3118]

Systematic search for relations between observables that do not depend on the C_i^r

Included:

- m_M^2 and F_M for π, K, η.
- 11 $\pi\pi$ threshold parameters
- 14 πK threshold parameters
- 6 $\eta \to 3\pi$ decay parameters,
- 10 observables in $K_{\ell 4}$
- 18 in the scalar formfactors
- 11 in the vectorformfactors
- Total: 76

We found 35 relations

Relations at NNLO: summary

- We did numerics for $\pi\pi$ (7), πK (5) and $K_{\ell 4}$ (1)
  13 relations
- $\pi\pi$: similar quality in two and three flavour ChPT
  The two involving a_3^- significantly did not work well
- πK: relation involving a_3^- not OK
  one more has very large NNLO corrections
- The relation with $K_{\ell 4}$ also did not work: related to that ChPT has trouble with curvature in $K_{\ell 4}$
- Conclusion: Three flavour ChPT "sort of" works

Fits: inputs

Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [hep-ph/0101127] (ABT01)

JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945] (BJ12)

JB, Ecker, arXiv:1405.6488, Ann. Rev. Nucl. Part. Sci .64 (2014) 149-174 (BE14)

- $M_\pi, M_K, M_\eta, F_\pi, F_K/F_\pi$
- $\langle r^2\rangle^\pi_S$, c^π_S slope and curvature of F_S
- $\pi\pi$ and πK scattering lengths a^0_0, a^2_0, $a^{1/2}_0$ and $a^{3/2}_0$.
- Value and slope of F and G in $K_{\ell 4}$
- $\frac{m_s}{\hat{m}} = 27.5$ (lattice)
- $\bar{l}_1, \ldots, \bar{l}_4$
- more variation with C^r_i, a penalty for a large p^6 contribution to the masses
- 17+3 inputs and 8 L^r_i+34 C^r_i to fit

Main fit

	ABT01	BJ12	L_4^r free	BE14
	old data			
$10^3 L_1^r$	0.39(12)	0.88(09)	0.64(06)	0.53(06)
$10^3 L_2^r$	0.73(12)	0.61(20)	0.59(04)	0.81(04)
$10^3 L_3^r$	−2.34(37)	−3.04(43)	−2.80(20)	−3.07(20)
$10^3 L_4^r$	$\equiv 0$	0.75(75)	0.76(18)	$\equiv 0.3$
$10^3 L_5^r$	0.97(11)	0.58(13)	0.50(07)	1.01(06)
$10^3 L_6^r$	$\equiv 0$	0.29(8)	0.49(25)	0.14(05)
$10^3 L_7^r$	−0.30(15	−0.11(15)	−0.19(08)	−0.34(09)
$10^3 L_8^r$	0.60(20)	0.18(18)	0.17(11)	0.47(10)
χ^2	0.26	1.28	0.48	1.04
dof	1	4	?	?
F_0 [MeV]	87	65	64	71

$? = (17 + 3) - (8 + 34)$

Main fit: Comments

- All values of the C_i^r we settled on are "reasonable"
- Leaving L_4^r free ends up with $L_4^r \approx 0.76$
- keeping L_4^r small: also L_6^r and $2L_1^r - L_2^r$ small (large N_c relations)
- Compatible with lattice determinations
- Not too bad with resonance saturation both for L_i^r and C_i^r, including from the scalars
- decent convergence (but enforced for masses)
- Many prejudices went in: large N_c, resonance model, quark model estimates,...

Some results of this fit

Mass:

$$m_\pi^2/m_{\pi phys}^2 = 1.055(p^2) - 0.005(p^4) - 0.050(p^6)\,,$$
$$m_K^2/m_{K phys}^2 = 1.112(p^2) - 0.069(p^4) - 0.043(p^6)\,,$$
$$m_\eta^2/m_{\eta phys}^2 = 1.197(p^2) - 0.214(p^4) + 0.017(p^6)\,,$$

Decay constants:

$$F_\pi/F_0 = 1.000(p^2) + 0.208(p^4) + 0.088(p^6)\,,$$
$$F_K/F_\pi = 1.000(p^2) + 0.176(p^4) + 0.023(p^6)\,.$$

Scattering:

$$a_0^0 = 0.160(p^2) + 0.044(p^4) + 0.012(p^6)\,,$$
$$a_0^{1/2} = 0.142(p^2) + 0.031(p^4) + 0.051(p^6)\,.$$

Update on $K_{\ell 3}$

- Take Bijnens-talavera 2003 result but update for BE14 parameters
- $f_+^{K^0\pi^-}(0) = 1 - 0.02276 - 0.00754 = 0.970 \pm 0.008$
- in good agreement with the latest lattice numbers

Charged pion polarizabilities: experiment

An example where ChPT triumphed

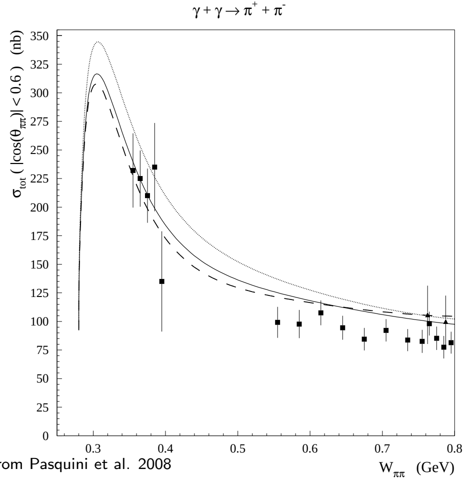
Review: Holstein, Scherer, Ann. Rev. Nucl. Part. Sci. **64** (2014) 51 [1401.0140]

- Expand $\gamma\pi^\pm \to \gamma\pi^\pm$ near threshold: $(z_\pm = 1 \pm \cos\theta_{\rm cm})$

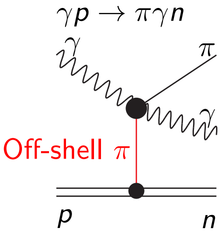
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d_{\Omega}}_{\rm Born} - \frac{\alpha m_\pi^3 \left((s - m_\pi^2\right)^2}{4s^2\left(sz_+ + m_\pi^2 z_-\right)}\left(z_-^2(\alpha - \beta) + \frac{s^2}{m_\pi^4} z_+^2(\alpha + \beta)\right)$$

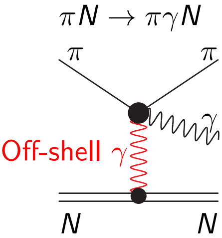
- Three ways to measure: (all assume $\alpha + \beta = 0$)
  - $\pi\gamma \to \pi\gamma$ (Primakoff, high energy pion beam)
    Dubna (1985) $\alpha = (6.8 \pm 1.4)\ 10^{-4}\ \text{fm}^3$
    Compass (CERN, 2015) $\alpha = (2.0 \pm 0.6 \pm 0.7)\ 10^{-4}\ \text{fm}^3$
  - $\gamma\pi \to \pi\gamma$ (via one-pion exchange)
    Lebedev (1986) $\alpha = (20 \pm 12)\ 10^{-4}\ \text{fm}^3$
    Mainz (2005) $\alpha = (5.8 \pm 0.75 \pm 1.5 \pm 0.25)\ 10^{-4}\ \text{fm}^3$
  - $\gamma\gamma \to \pi\pi$ (in $e^+e^- \to e^+e^-\pi^+\pi^-$)
    MarkII data analyzed (1992) $\alpha = (2.2 \pm 1.1)\ 10^{-4}\ \text{fm}^3$
- Extrapolation and subtraction: difficult experiments

Polarizabilities: extrapolations needed



from Pasquini et al. 2008





Charged pion polarizabilities: theory

- ChPT:
  - One-loop JB, Cornet, 1986, Donoghue-Holstein 1989
    $\alpha + \beta = 0$, $\alpha = (2.8 \pm 0.2)\ 10^{-4}$ fm^3
    input $\pi \to e\nu\gamma$ (error only from this)
  - Two-loop Bürgi, 1996, Gasser, Ivanov, Sainio 2006
    $\alpha + \beta = 0.16\ 10^{-4}$ fm^3,$\alpha = (2.8 \pm 0.5)\ 10^{-4}$ fm^3
- Dispersive analysis from $\gamma\gamma \to \pi\pi$:
  - Fil'kov-Kashevarov, 2005 $(\alpha_1 - \beta_1) = (13.0^{+2.6}_{-1.9}) \cdot 10^{-4}\mathrm{fm}^3$
  - Critized by Pasquini-Drechsel-Scherer, 2008
    "Large model dependence in their extraction"
    "Our calculations. . . are in reasonable agreement with ChPT for charged pions"
    $(\alpha_1 - \beta_1) = (5.7) \cdot 10^{-4}\mathrm{fm}^3$ perfectly possible

Finite volume

- Lattice QCD calculates at different quark masses, volumes boundary conditions,. . .
- A general result by Lüscher: relate finite volume effects to scattering (1986)
- Chiral Perturbation Theory is also useful for this
- Start: Gasser and Leutwyler, Phys. Lett. B184 (1987) 83, Nucl. Phys. B 307 (1988) 763
  M_π, F_π, $\langle\bar{q}q\rangle$ one-loop equal mass case
- I will stay with ChPT and the p regime ($M_\pi L >> 1$)
- $1/m_\pi = 1.4$ fm
  may need to go beyond leading $e^{-m_\pi L}$ terms
- Convergence of ChPT is given by $1/m_\rho \approx 0.25$ fm

Finite volume: selection of ChPT results

- masses and decay constants for π, K, η one-loop
  Becirevic, Villadoro, Phys. Rev. D 69 (2004) 054010
- M_π at 2-loops (2-flavour)
  Colangelo, Haefeli, Nucl.Phys. B744 (2006) 14 [hep-lat/0602017]
- $\langle \bar{q}q \rangle$ at 2 loops (3-flavour)
  JB, Ghorbani, Phys. Lett. B636 (2006) 51 [hep-lat/0602019]
- Twisted mass at one-loop
  Colangelo, Wenger, Wu, Phys.Rev. D82 (2010) 034502 [arXiv:1003.0847]
- Twisted boundary conditions
  Sachrajda, Villadoro, Phys. Lett. B 609 (2005) 73 [hep-lat/0411033]
- This talk:
  - Twisted boundary conditions and some funny effects
  - Some results on masses 3-flavours at two loop order

Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i/L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions: $q(x^i + L) = e^{i\theta^i_q} q(x^i)$
- Then momenta are $p^i = \theta^i/L + 2\pi n^i/L$. Allows to map out momentum space on the lattice much better Bedaque,...
- But:
  - Box: Rotation invariance $\rightarrow$ cubic invariance
  - Twisting: reduces symmetry further

  Consequences:
  - $m^2(\vec{p}^2) = E^2 - \vec{p}^2$ is not constant
  - There are typically more form-factors
  - In general: quantities depend on more (all) components of the momenta
  - Charge conjugation involves a change in momentum

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Twisted boundary conditions: Two-point function

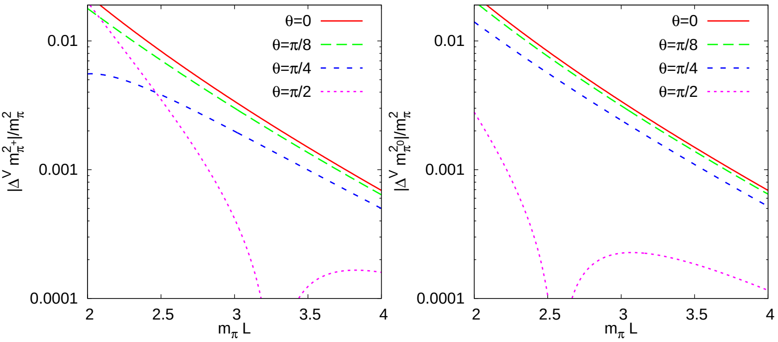
JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

- $\int_V \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2 - m^2} \neq 0$
- $\langle \bar{u}\gamma^\mu u \rangle \neq 0$
- $j_\mu^{\pi^+} = \bar{d}\gamma_\mu u$
  satisfies $\partial^\mu \left\langle T(j_\mu^{\pi^+}(x) j_\nu^{\pi^-}(0)) \right\rangle = \delta^{(4)}(x) \left\langle \bar{d}\gamma_\nu d - \bar{u}\gamma_\nu u \right\rangle$
- $\Pi_{\mu\nu}^a(q) \equiv i \int d^4 x e^{iq\cdot x} \left\langle T(j_\mu^a(x) j_\nu^{a\dagger}(0)) \right\rangle$
  Satisfies WT identity. $q^\mu \Pi_{\mu\nu}^{\pi^+} = \left\langle \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d \right\rangle$
- ChPT at one-loop satisfies this
  see also Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]

Twisted boundary conditions: volume correction masses

JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

$m_\pi L = 2$, $\vec{\theta}_u = (\theta, 0, 0)$, $\vec{\theta}_d = \vec{\theta}_s = 0$

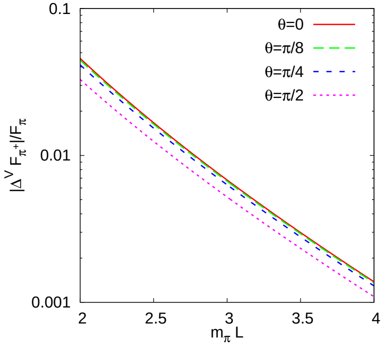


Charged pion mass — Neutral pion mass

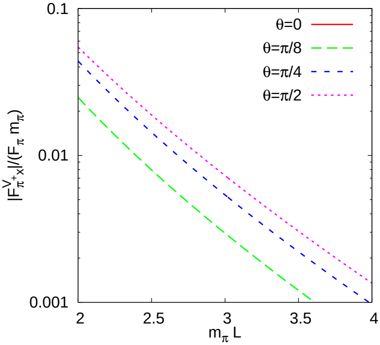
$\Delta^V X = X^V - X^\infty$ (dip is going through zero)

Volume correction decay constants: F_{π^+}

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
- $\left\langle 0 | A_\mu^M | M(p) \right\rangle = i\sqrt{2} F_M p_\mu + i\sqrt{2} F_{M\mu}^V$
- Extra terms are needed for Ward identities



relative for F_π



Extra for $\mu = x$

Volume correction electromagnetic formfactor

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]
  earlier two-flavour work:
  Bunton, Jiang, Tiburzi, Phys.Rev. D74 (2006) 034514 [hep-lat/0607001]
- $\langle M'(p')|j_\mu|M(p)\rangle = f_\mu = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu$
- Extra terms are again needed for Ward identities
- Note that masses have finite volume corrections
  - q^2 for fixed $\vec{p}$ and $\vec{p}'$ has corrections
    small effect
  - This also affects the ward identities, e.g.
    $q^\mu f_\mu = (p^2 - p'^2)f_+ + q^2 f_- + q^\mu h_\mu = 0$
    is satisfied but all effects should be considered

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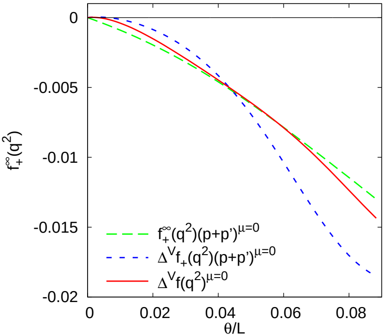
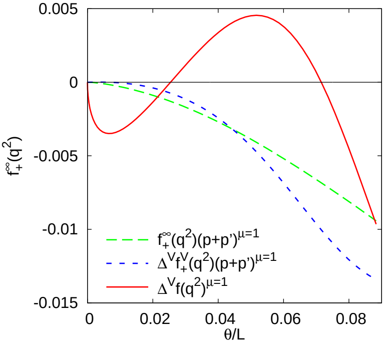
Volume correction electromagnetic formfactor

- $f_\mu = -\frac{1}{\sqrt{2}}\langle\pi^0(p')|\bar{d}\gamma_\mu u|\pi^+(p)\rangle$
  $= \left(1 + f_+^\infty + \Delta^V f_+\right)(p+p')_\mu + \Delta^V f_- q_\mu + \Delta^V h_\mu$
- Pure loop plotted: $f_+^\infty(p+p'), \Delta^V f_+(p+p')$ and $\Delta^V f_\mu$



$\mu = t$

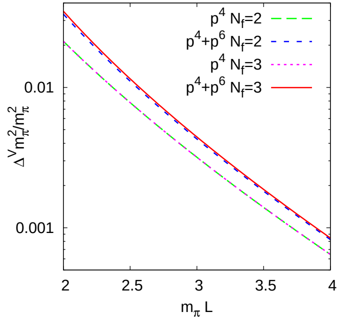


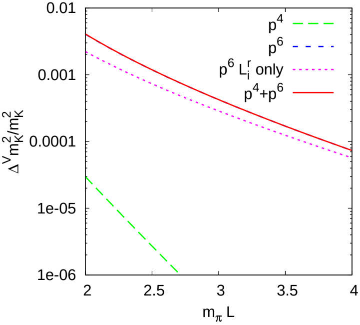
$\mu = x$

Finite volume corrections large, different for different μ

Masses at two-loop order

- Sunset integrals at finite volume done
  JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]
- Loop calculations:
  JB, Rössler, JHEP **1501** (2015) 034 [arXiv:1411.6384]





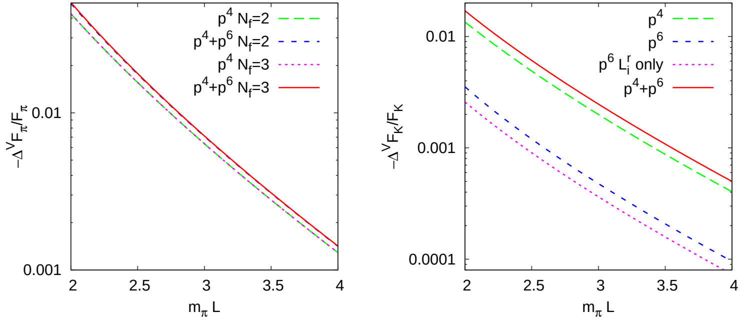
- Agreement for $N_f = 2, 3$ for pion
- K has no pion loop at LO

Decay constants at two-loop order

- Sunset integrals at finite volume done

  JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]
- Loop calculations:

  JB, Rössler, JHEP **1501** (2015) 034 [arXiv:1411.6384]



- Agreement for $N_f = 2,3$ for pion
- K now has a pion loop at LO

Other stuff I work/want to do

- Partially quenched finite volume mass and decay constant work done, paper being written
- QCD-like theories: partially quenched and finite volume in progress
- Twisted (thus finite volume) and partially quenched: $K_{\ell 3}$
- Leading logarithms: another talk
- Get our quark mass isospin breaking at NNLO calculations in an updated shape + combine with em
- Any more suggestions?

Conclusions

- ChPT and all the extensions I talked about can be applied (and have often been) to baryons, heavy mesons,. . .
- Gave you some examples of the uses of ChPT
- Future:
  - A tool for studying lattice artefacts, finite volume,. . .
  - Combine with other methods, dispersion relations already heavily done