

progress and prospects for heavy flavour physics on the lattice

Carlos Pena



why we care

theory-driven discovery of new physics through loops has an impressive track record from the 70s, 80s, 90s

- no $K_L \rightarrow \mu\mu \Rightarrow$ charm [Glashow, Iliopoulos, Maiani 1970]
- $\epsilon_K \Rightarrow$ 3rd generation [Kobayashi, Maskawa 1973]
- $\Delta m_K \Rightarrow m_c$ [Gaillard, Lee 1974; Vainshtein, Khriplovich 1973]
- $\Delta m_B \Rightarrow m_t$ large (with some hindsight...)
- ...

crucial to build and consolidate the SM!

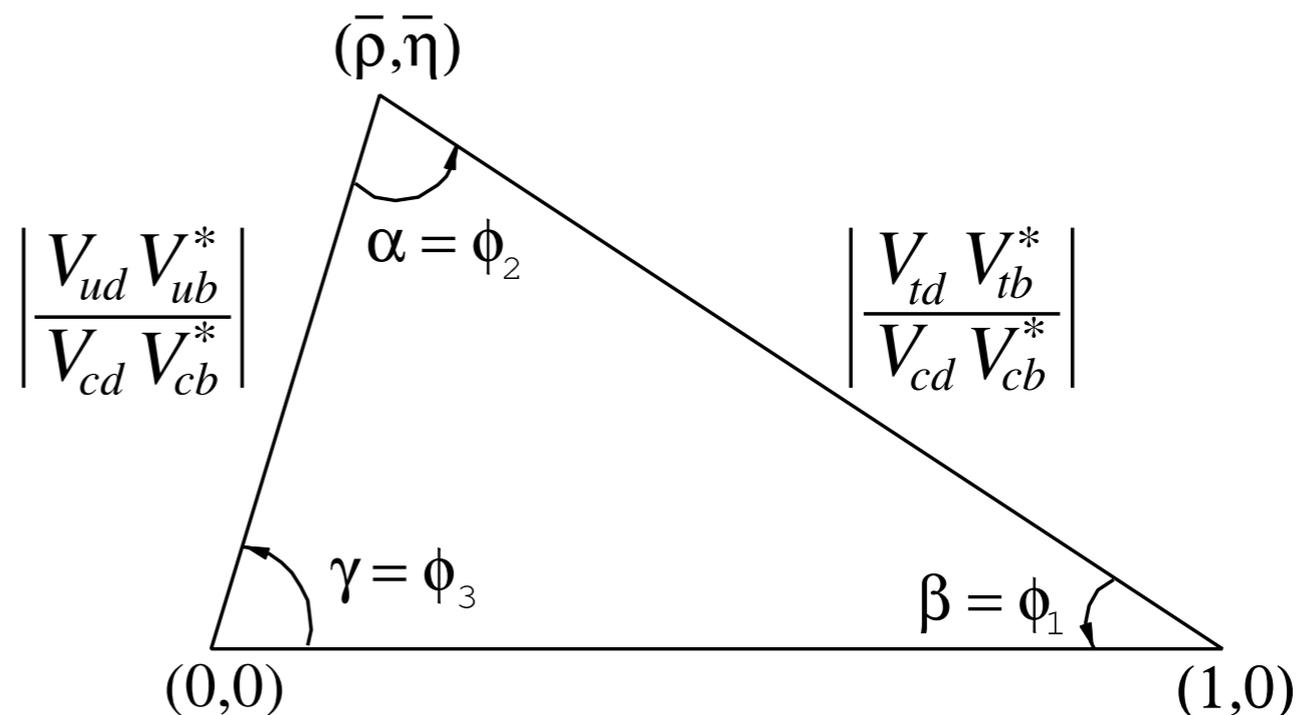
why we care

SM quark flavour dynamics neatly encoded in CKM matrix

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

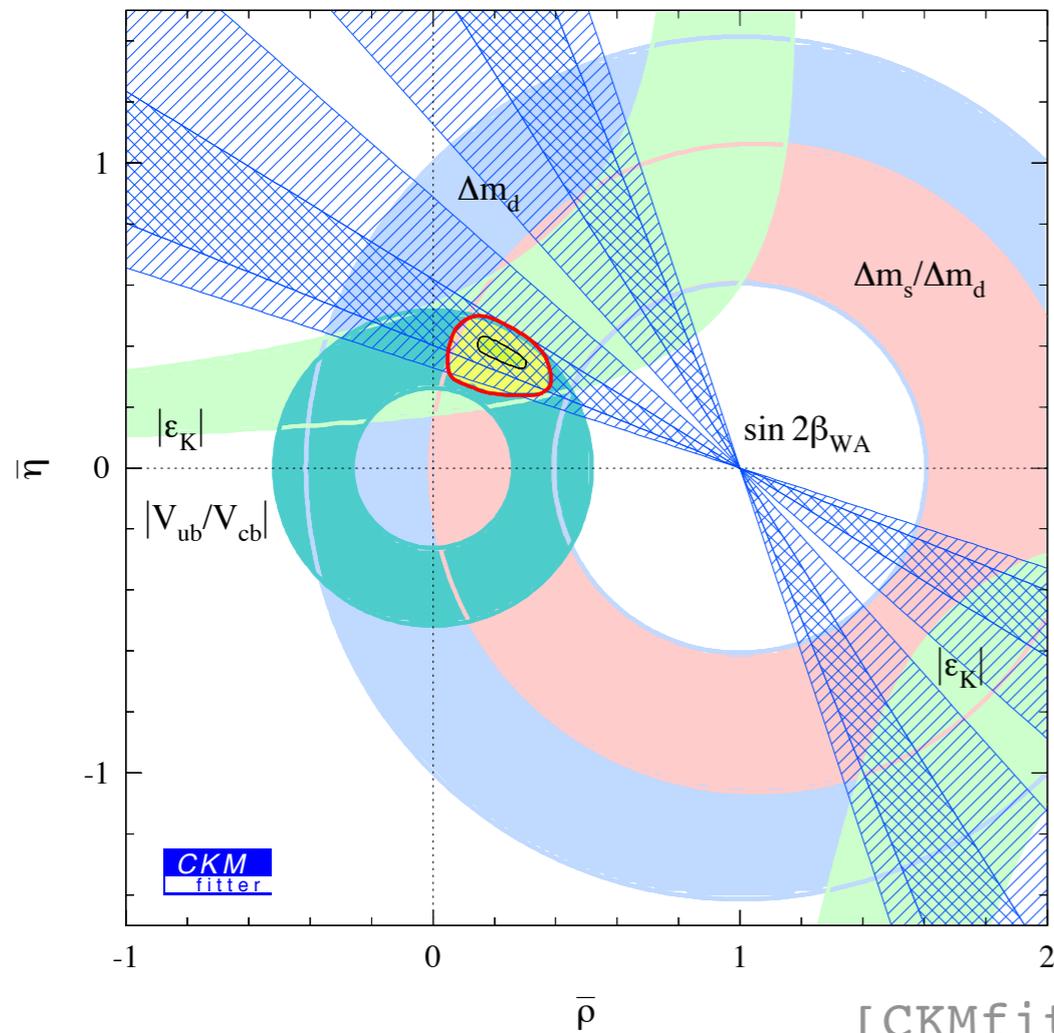
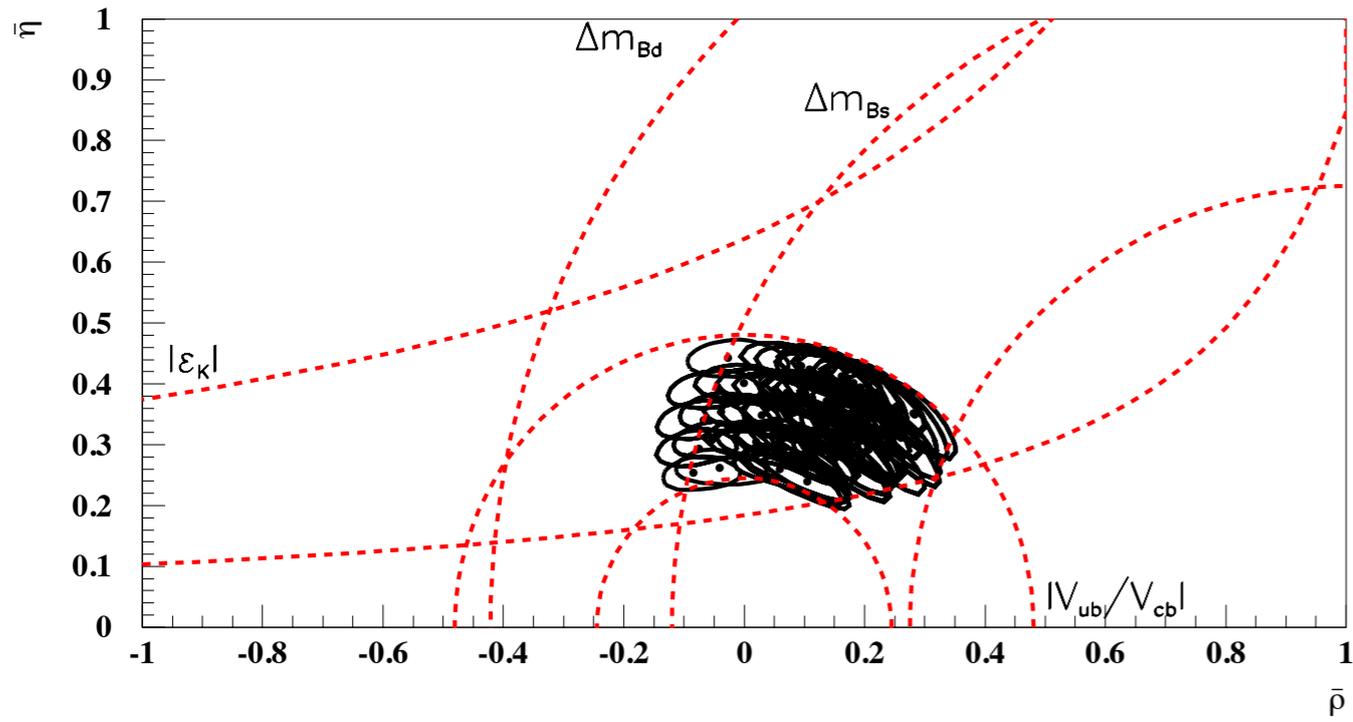
$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$V_{\text{CKM}}^\dagger V_{\text{CKM}} = \mathbf{1} \quad \Rightarrow$$

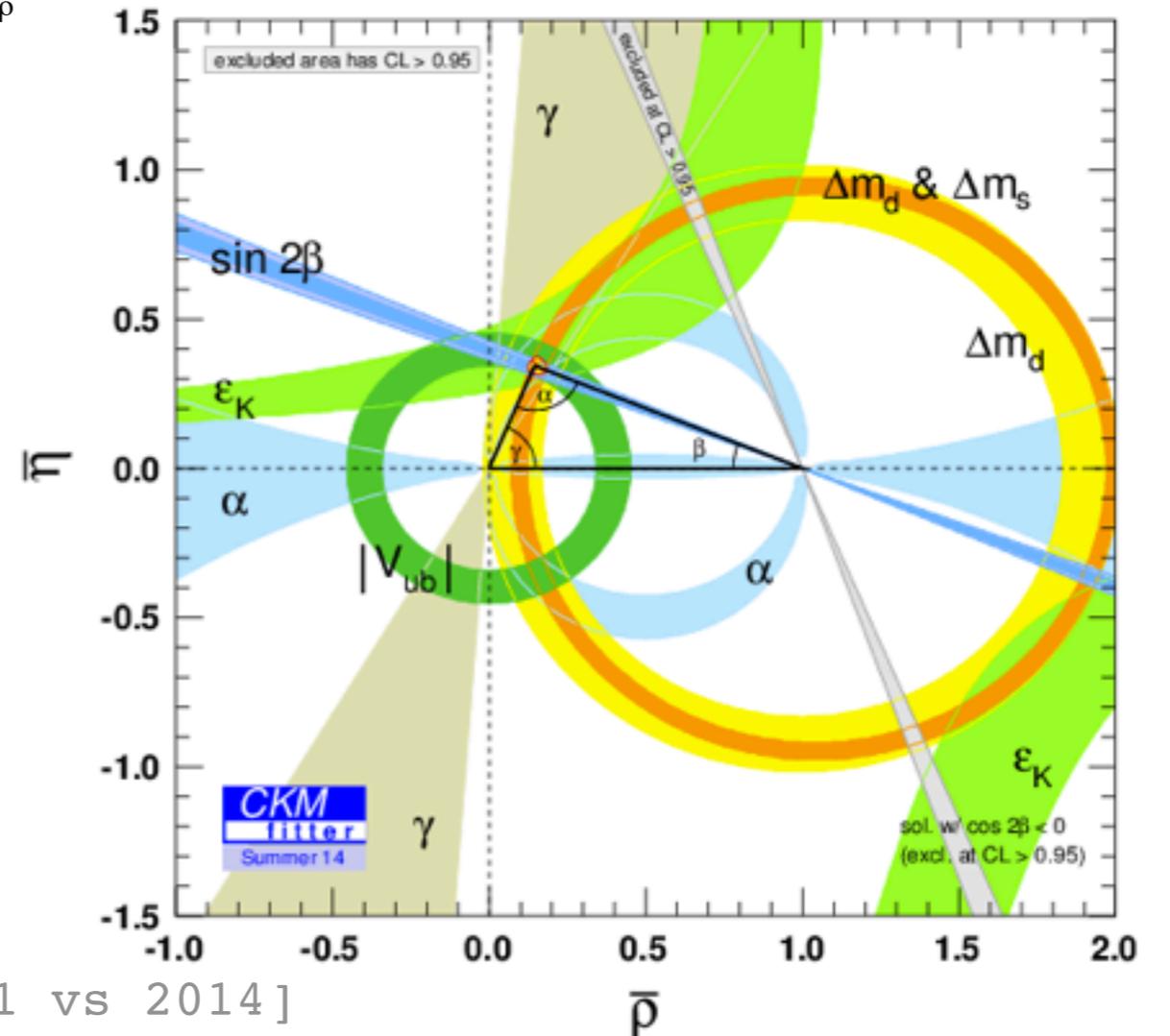


why we care

[BaBar Physics Book, 1999]



[CKMfitter 2001 vs 2014]



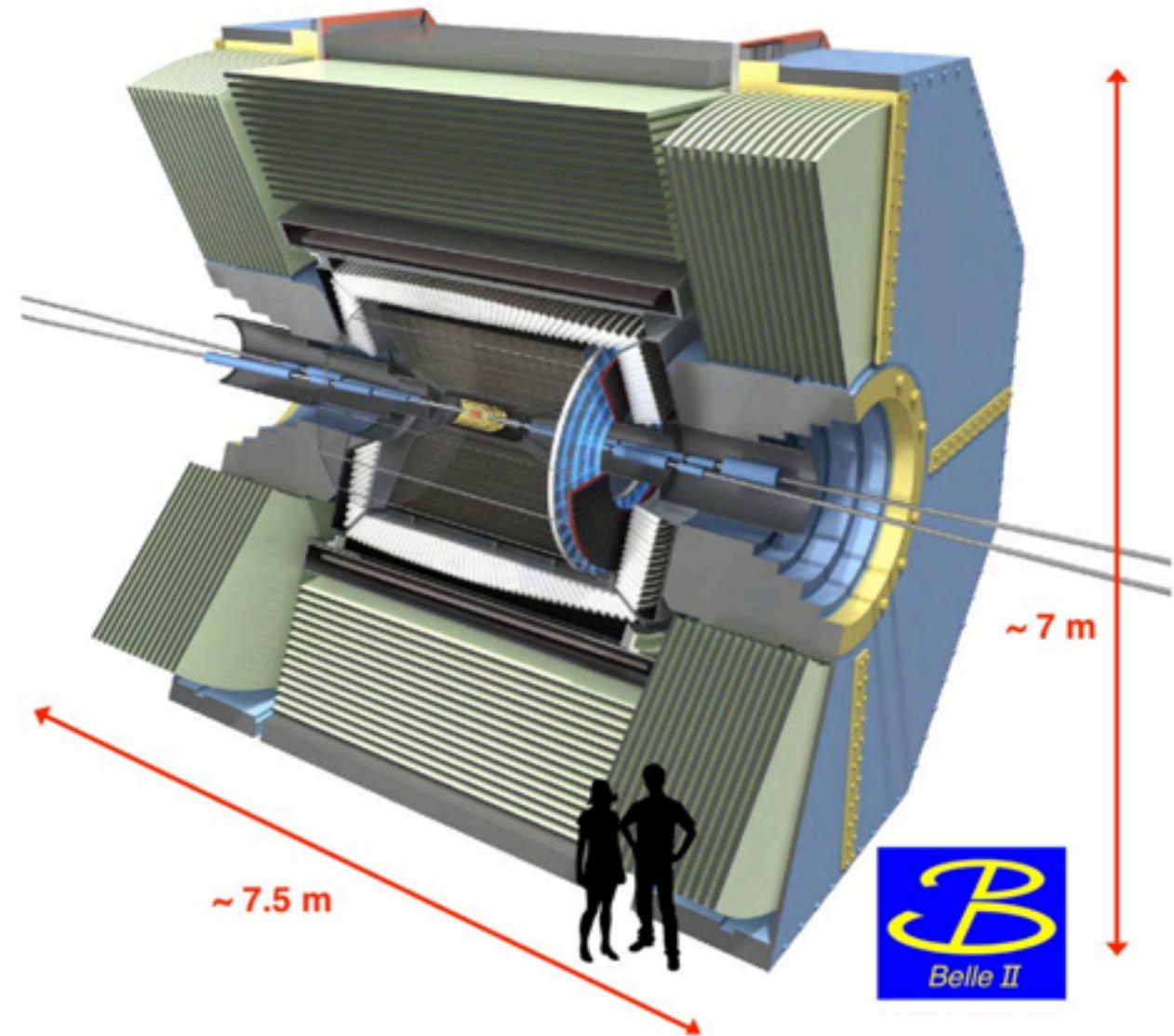
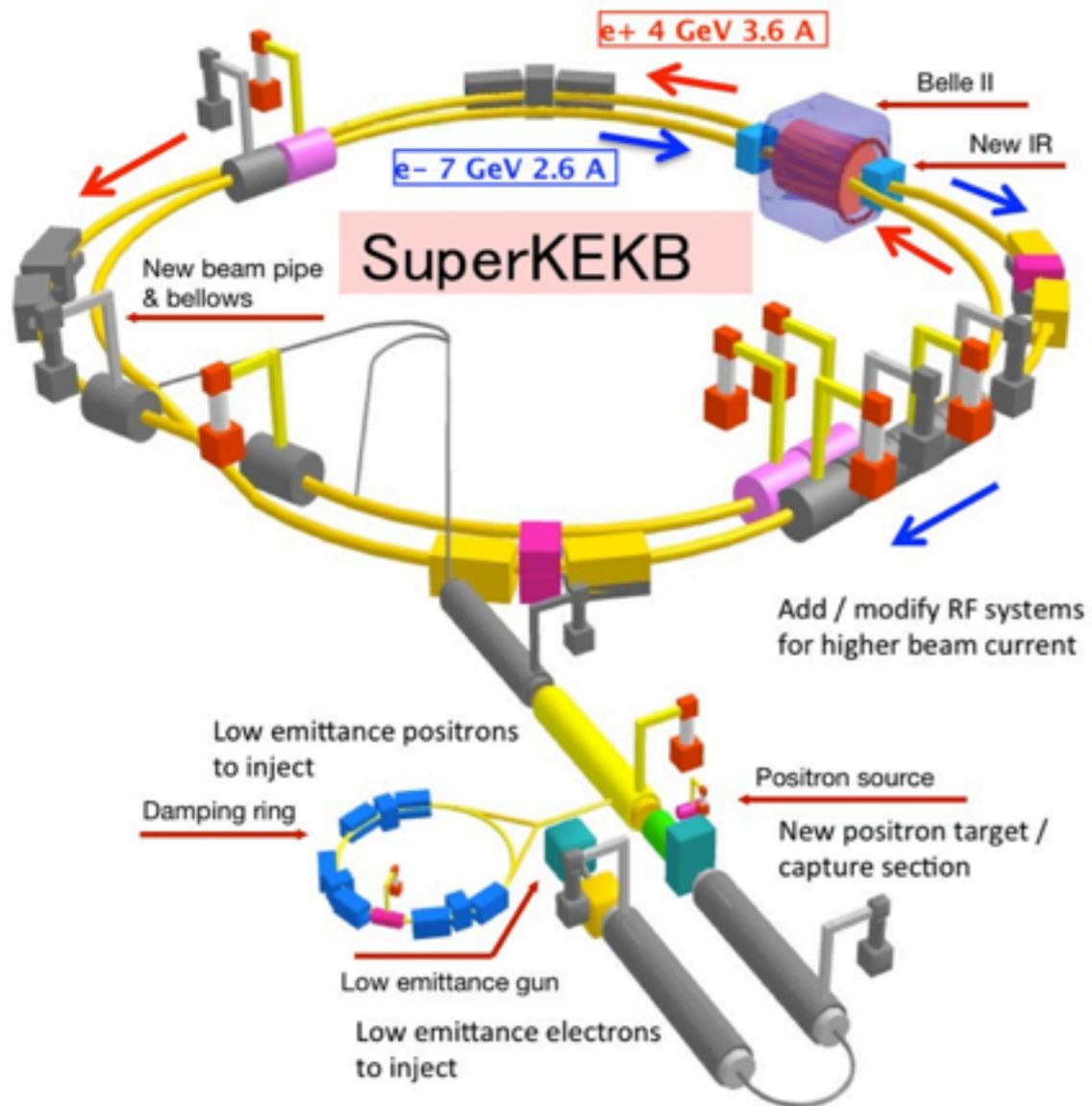
why we still care

- tensions/puzzles: exclusive vs inclusive $|V_{ub}|$ and $|V_{cb}|$, $b \rightarrow s$, lepton universality in $R(D)$, non-leptonic decays, D -mixing, ...
- FCNC (relatively) poorly constrained in several processes
- some complementary precision tests are expected to improve significantly (EDM by 10^2 - 10^3 , CLFV by at least 10^2 , ...)
- upcoming new generation of experimental results

$$\frac{\text{LHCb upgrade}}{\text{LHCb 1/fb}} \sim \frac{\text{Belle II}}{\text{Belle}} \sim \frac{\text{BaBar 2009}}{\text{CLEO 1999}} \sim 50$$

[quoted from Z Ligeti's TASI Lectures on Flavor Physics, arXiv:1502.01372]

Belle II projections

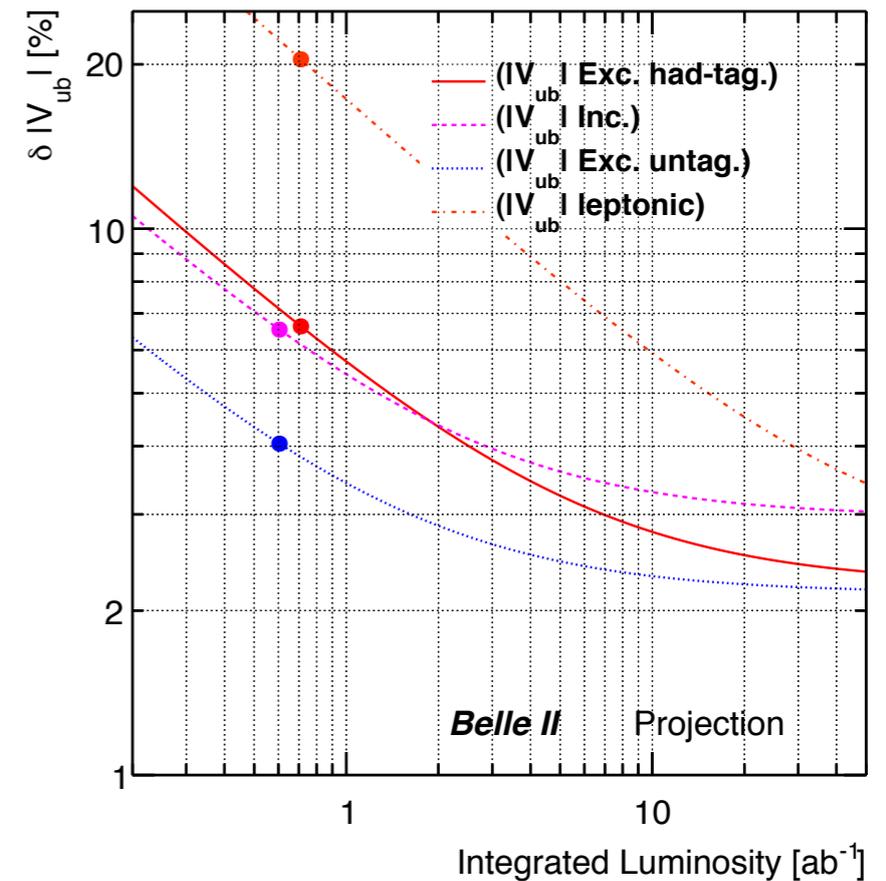
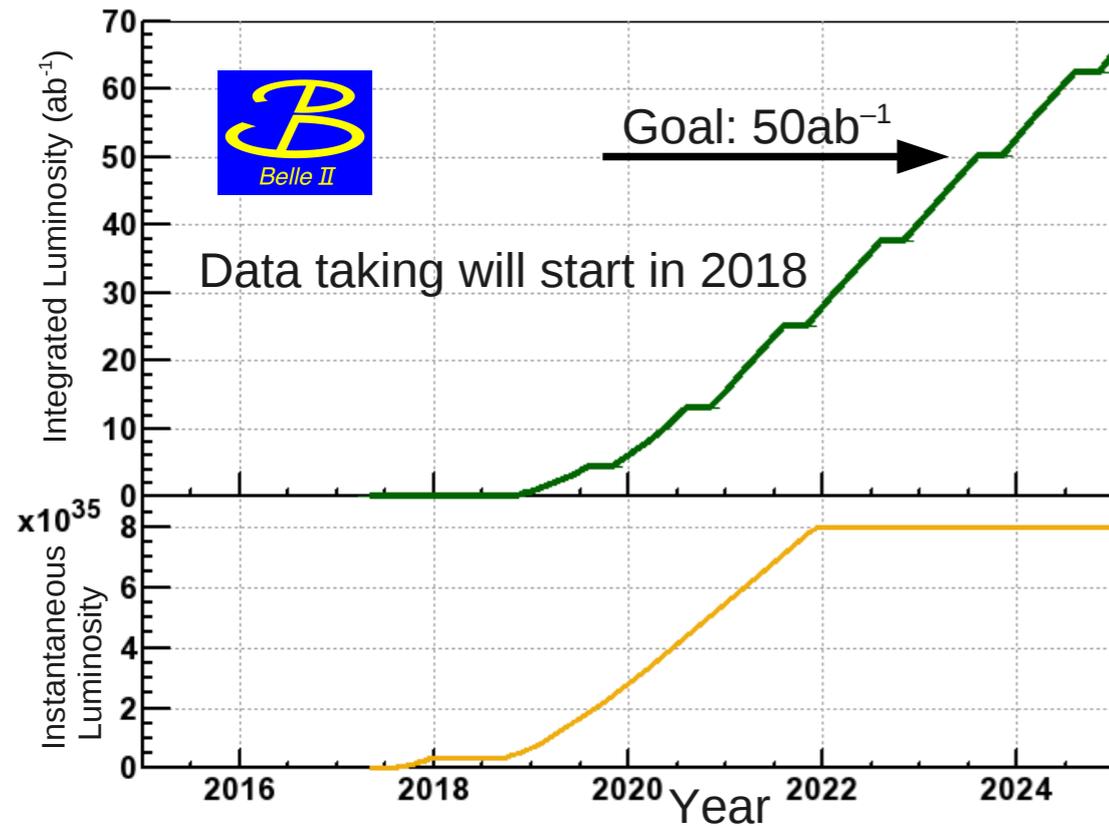


Belle II projections

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	± 0.012	± 0.008	6
α		$\pm 2^\circ$	$\pm 1^\circ$	
γ	$\pm 14^\circ$	$\pm 6^\circ$	$\pm 1.5^\circ$	
$S(B \rightarrow \phi K^0)$	$0.90^{+0.09}_{-0.19}$	± 0.053	± 0.018	>50
$S(B \rightarrow \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	± 0.028	± 0.011	>50
$S(B \rightarrow K_S^0 K_S^0 K_S^0)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
$ V_{cb} $ incl.	$\pm 2.4\%$	$\pm 1.0\%$		< 1
$ V_{cb} $ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
$ V_{ub} $ incl.	$\pm 6.5\%$	$\pm 3.4\%$	$\pm 3.0\%$	2
$ V_{ub} $ excl. (had. tag.)	$\pm 10.8\%$	$\pm 4.7\%$	$\pm 2.4\%$	20
$ V_{ub} $ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \rightarrow \tau\nu)$ [10 ⁻⁶]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \rightarrow \mu\nu)$ [10 ⁻⁶]	< 1.7	5σ	$\gg 5\sigma$	>50
$R(B \rightarrow D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \rightarrow D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
$\mathcal{B}(B \rightarrow K^{*+}\nu\bar{\nu})$ [10 ⁻⁶]	< 40		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow K^+\nu\bar{\nu})$ [10 ⁻⁶]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow X_s\gamma)$ [10 ⁻⁶]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \rightarrow X_s\gamma)$		± 0.01	± 0.005	8
$S(B \rightarrow K_S^0\pi^0\gamma)$	$-0.10 \pm 0.31 \pm 0.07$	± 0.11	± 0.035	> 50
$S(B \rightarrow \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	± 0.23	± 0.07	> 50
$C_7/C_9 (B \rightarrow X_s\ell\ell)$	$\sim 20\%$	10%	5%	
$\mathcal{B}(B_s \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 8.7	± 0.3		
$\mathcal{B}(B_s \rightarrow \tau^+\tau^-)$ [10 ⁻³]		< 2		

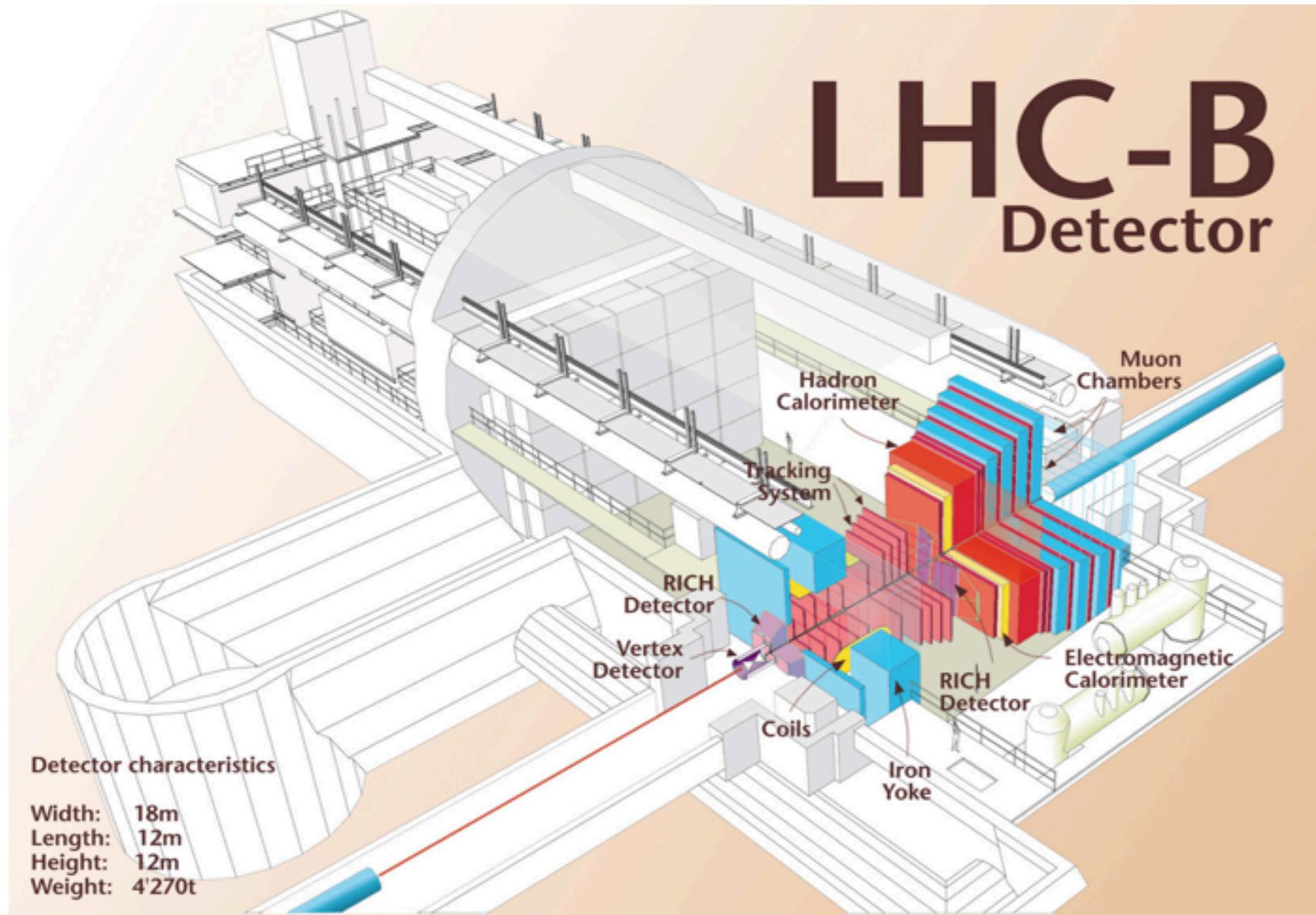
Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
$\mathcal{B}(D_s \rightarrow \mu\nu)$	$5.31 \times 10^{-3}(1 \pm 0.053 \pm 0.038)$	$\pm 2.9\%$	$\pm(0.9\%-1.3\%)$	> 50
$\mathcal{B}(D_s \rightarrow \tau\nu)$	$5.70 \times 10^{-3}(1 \pm 0.037 \pm 0.054)$	$\pm(3.5\%-4.3\%)$	$\pm(2.3\%-3.6\%)$	3-5
y_{CP} [10 ⁻²]	$1.11 \pm 0.22 \pm 0.11$	$\pm(0.11-0.13)$	$\pm(0.05-0.08)$	5-8
A_Γ [10 ⁻²]	$-0.03 \pm 0.20 \pm 0.08$	± 0.10	$\pm(0.03-0.05)$	7 - 9
$A_{CP}^{K^+K^-}$ [10 ⁻²]	$-0.32 \pm 0.21 \pm 0.09$	± 0.11	± 0.06	15
$A_{CP}^{\pi^+\pi^-}$ [10 ⁻²]	$0.55 \pm 0.36 \pm 0.09$	± 0.17	± 0.06	> 50
$A_{CP}^{\phi\gamma}$ [10 ⁻²]	± 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.56 \pm 0.19 \pm^{0.07}_{0.13}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.30 \pm 0.15 \pm^{0.05}_{0.08}$	± 0.08	± 0.05	15
$ q/p ^{K_S\pi^+\pi^-}$	$0.90 \pm^{0.16}_{0.15} \pm^{0.08}_{0.06}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [°]	$-6 \pm 11 \pm^4_5$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10 ⁻²]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K_S^0\pi^0}$ [10 ⁻²]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
	$\tau \rightarrow \mu\gamma$ [10 ⁻⁹]	< 45	< 14.7	< 4.7
	$\tau \rightarrow e\gamma$ [10 ⁻⁹]	< 120	< 39	< 12
	$\tau \rightarrow \mu\mu\mu$ [10 ⁻⁹]	< 21.0	< 3.0	< 0.3

Belle II projections



Lattice Quantity	CKM element	WA Expt. Error	Lattice error		
			2013 (Present)	2014	2018
$F(1) (B \rightarrow D^* \ell \nu)$	$ V_{cb} $	1.3	1.8	1.5	<1
$G(1) (B \rightarrow D \ell \nu)$	$ V_{cb} $	1.3	1.8	1.5	<1
$G_s(1) (B_s \rightarrow D_s^* \ell \nu)$	$ V_{cb} $	—	4.6	—	—
$\zeta(B \rightarrow \pi \ell \nu)$	$ V_{ub} $	4.1	8.7	4	2
$f_B (B \rightarrow \tau \nu, \mu \nu)$	$ V_{ub} $	9.0	2.5	1.5	<1
$R(D)(B \rightarrow D \tau \nu)$	—	13	4.3	4	< 2
Mixing $\zeta(\Delta m_d / \Delta m_s)$	$ V_{td} / V_{ts} $	0.4	4.0	—	< 1

LHCb Run 2 and upgrade projections



n.b.: LHCb outdoing expectations (e.g. baryon semileptonic decay)

LHCb Run 2 and upgrade projections

Type	Observable	LHC Run 1	LHCb 2018	LHCb upgrade	Theory
B_s^0 mixing	$\phi_s(B_s^0 \rightarrow J/\psi \phi)$ (rad)	0.049	0.025	0.009	~ 0.003
	$\phi_s(B_s^0 \rightarrow J/\psi f_0(980))$ (rad)	0.068	0.035	0.012	~ 0.01
	$A_{sl}(B_s^0)$ (10^{-3})	2.8	1.4	0.5	0.03
Gluonic penguin	$\phi_s^{\text{eff}}(B_s^0 \rightarrow \phi\phi)$ (rad)	0.15	0.10	0.018	0.02
	$\phi_s^{\text{eff}}(B_s^0 \rightarrow K^{*0}\bar{K}^{*0})$ (rad)	0.19	0.13	0.023	< 0.02
	$2\beta^{\text{eff}}(B^0 \rightarrow \phi K_S^0)$ (rad)	0.30	0.20	0.036	0.02
Right-handed currents	$\phi_s^{\text{eff}}(B_s^0 \rightarrow \phi\gamma)$ (rad)	0.20	0.13	0.025	< 0.01
	$\tau^{\text{eff}}(B_s^0 \rightarrow \phi\gamma)/\tau_{B_s^0}$	5%	3.2%	0.6%	0.2%
Electroweak penguin	$S_3(B^0 \rightarrow K^{*0}\mu^+\mu^-; 1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.04	0.020	0.007	0.02
	$q_0^2 A_{\text{FB}}(B^0 \rightarrow K^{*0}\mu^+\mu^-)$	10%	5%	1.9%	$\sim 7\%$
	$A_{\text{I}}(K\mu^+\mu^-; 1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.09	0.05	0.017	~ 0.02
	$\mathcal{B}(B^+ \rightarrow \pi^+\mu^+\mu^-)/\mathcal{B}(B^+ \rightarrow K^+\mu^+\mu^-)$	14%	7%	2.4%	$\sim 10\%$
Higgs penguin	$\mathcal{B}(B_s^0 \rightarrow \mu^+\mu^-)$ (10^{-9})	1.0	0.5	0.19	0.3
	$\mathcal{B}(B^0 \rightarrow \mu^+\mu^-)/\mathcal{B}(B_s^0 \rightarrow \mu^+\mu^-)$	220%	110%	40%	$\sim 5\%$
Unitarity triangle angles	$\gamma(B \rightarrow D^{(*)}K^{(*)})$	7°	4°	0.9°	negligible
	$\gamma(B_s^0 \rightarrow D_s^\mp K^\pm)$	17°	11°	2.0°	negligible
	$\beta(B^0 \rightarrow J/\psi K_S^0)$	1.7°	0.8°	0.31°	negligible
Charm	$A_\Gamma(D^0 \rightarrow K^+K^-)$ (10^{-4})	3.4	2.2	0.4	–
CP violation	ΔA_{CP} (10^{-3})	0.8	0.5	0.1	–

[LHCb-PUB-2014-040]

n.b.: LHCb outdoing expectations (e.g. baryon semileptonic decay)

why we still care

- B-physics: much better precision + new channels (e.g. much more information on rare decays)
- + contributions from ATLAS/CMS
- + dedicated charm physics (BESIII running from 2011, large charm production cross section at Belle II, ...)
- theory has to meet the challenge

plan

- starting point:

- thorough reviews on HQ decays/mixing (C Bouchard) and quark masses (F Sanfilippo) at Lattice 2014

[Bouchard arXiv:1501.03204]

[Sanfilippo arXiv:1505.02794]

- contributions to the Lattice 2015 Conference



- detailed coverage of results up to end 2013 in FLAG-2

[FLAG 2013, Eur J Phys C74 (2014) 2890, arXiv:1310.8555v2]

(unfortunately, no preliminary updates of FLAG averages for HQ quantities yet)

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- guidelines:

- CKM tests focus
- status overview for decay constants, B-mixing
- **strong progress + expt synergy in semileptonic decays**
- not covered: HQ masses, BSM/rare decays (exc.), processes involving resonances (exc.), D-mixing, spectroscopy, ...

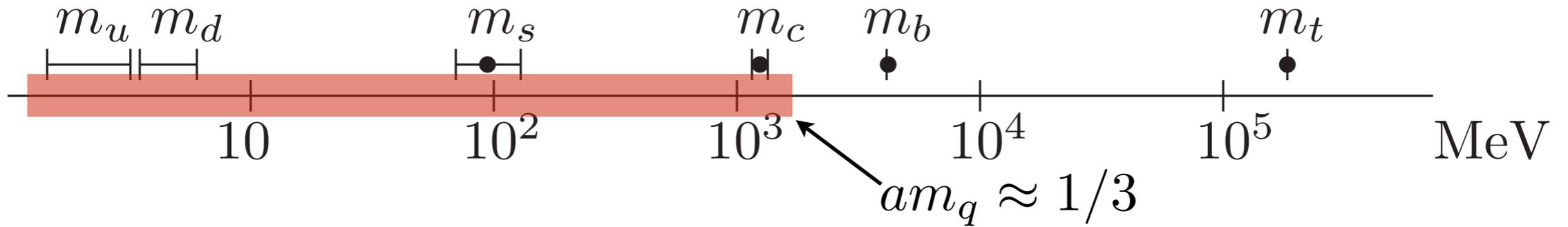
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 - ensembles used in HQ physics, reach
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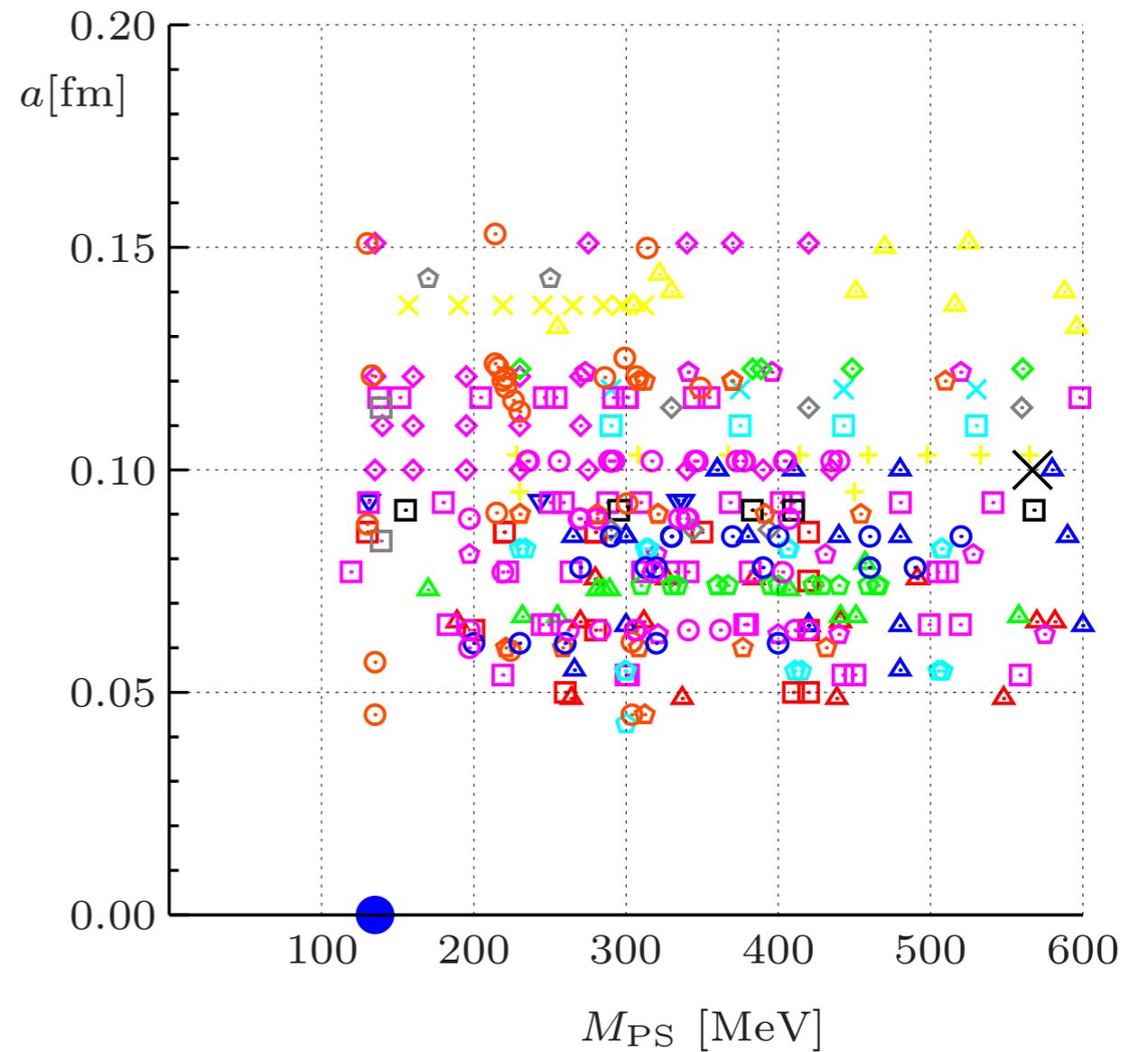
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ensembles/physics reach

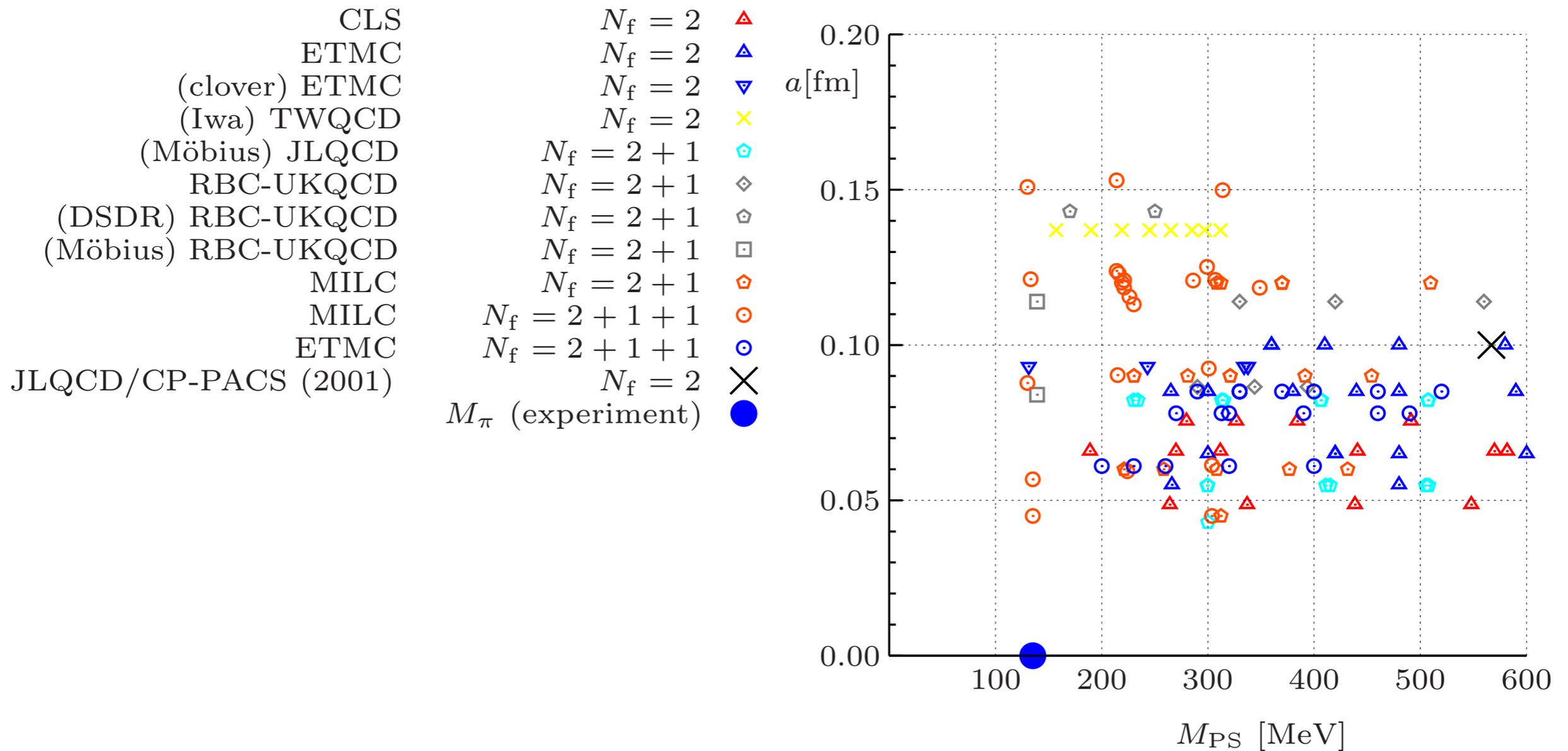


CLS	$N_f = 2$	\blacktriangle
ETMC	$N_f = 2$	\blacktriangle
(clover) ETMC	$N_f = 2$	\blacktriangledown
QCDSF	$N_f = 2$	\blacktriangle
BGR	$N_f = 2$	\blacktriangle
JLQCD	$N_f = 2$	\times
(plaq) TWQCD	$N_f = 2$	$+$
(Iwa) TWQCD	$N_f = 2$	\times
(HEX) BMW	$N_f = 2 + 1$	\square
(stout) BMW	$N_f = 2 + 1$	\diamond
(stout-stag) BMW	$N_f = 2 + 1$	\diamond
CLS	$N_f = 2 + 1$	\square
HSC	$N_f = 2 + 1$	\diamond
PACS-CS	$N_f = 2 + 1$	\square
QCDSF	$N_f = 2 + 1$	\diamond
JLQCD	$N_f = 2 + 1$	\square
(Möbius) JLQCD	$N_f = 2 + 1$	\diamond
RBC-UKQCD	$N_f = 2 + 1$	\diamond
(DSDR) RBC-UKQCD	$N_f = 2 + 1$	\diamond
(Möbius) RBC-UKQCD	$N_f = 2 + 1$	\square
MILC	$N_f = 2 + 1$	\diamond
MILC	$N_f = 2 + 1 + 1$	\circ
ETMC	$N_f = 2 + 1 + 1$	\circ
BMW	$N_f = 1 + 1 + 1 + 1$	\circ
JLQCD/CP-PACS (2001)	$N_f = 2$	\times
M_π (experiment)		\bullet



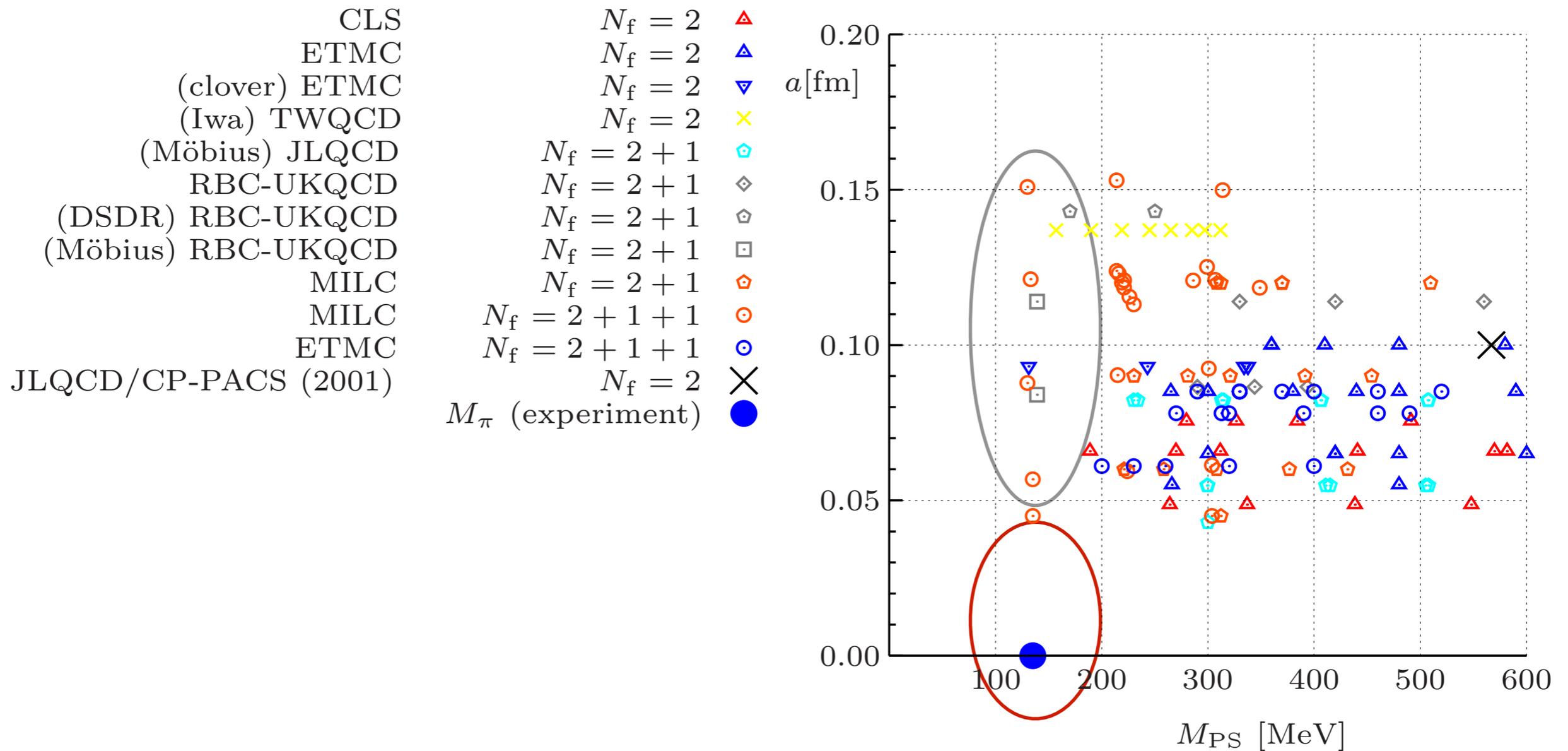
ensembles/physics reach

- charm physics directly accessible for some time now
- fraction of available ensembles used for HQ physics still limited



ensembles/physics reach

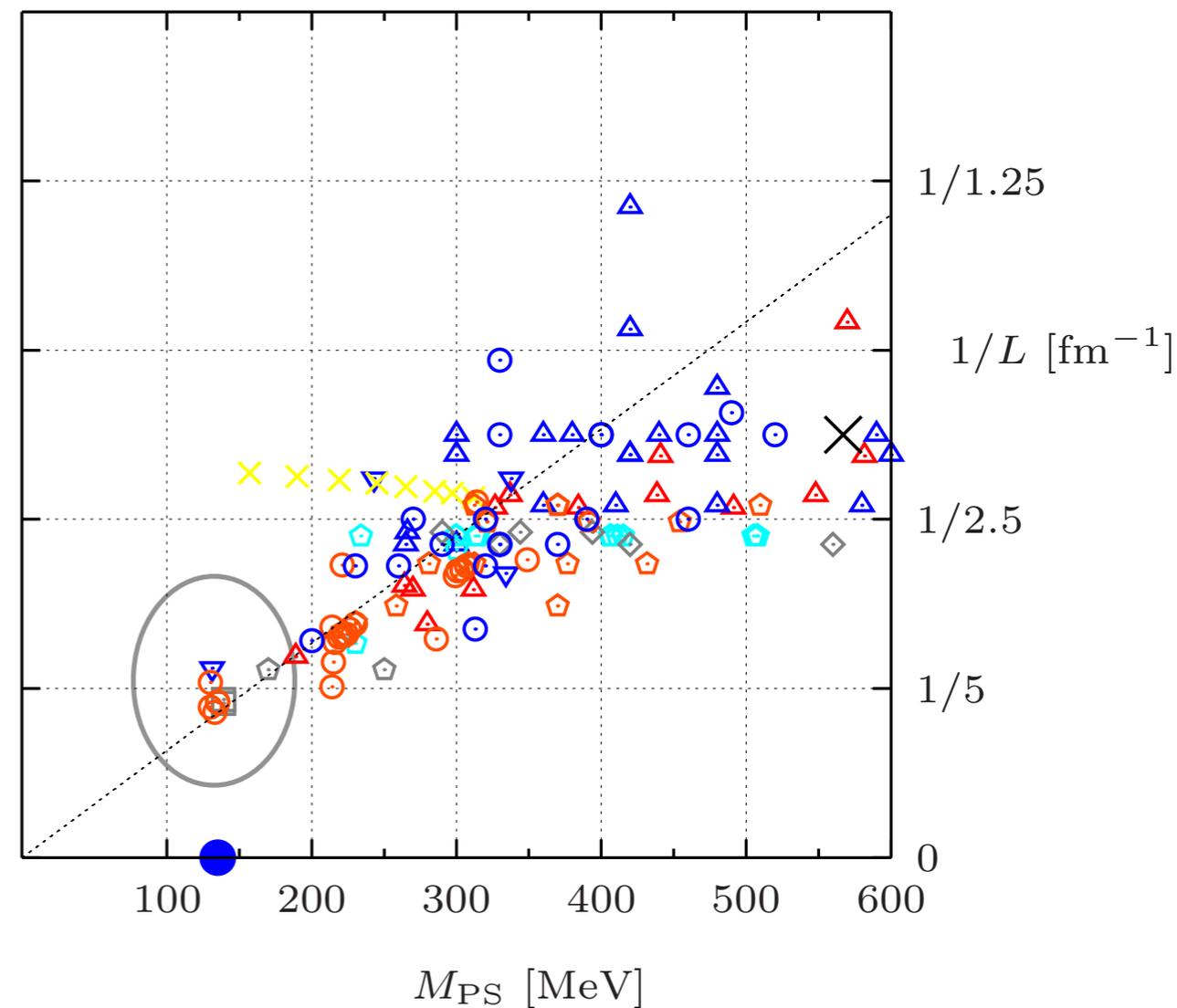
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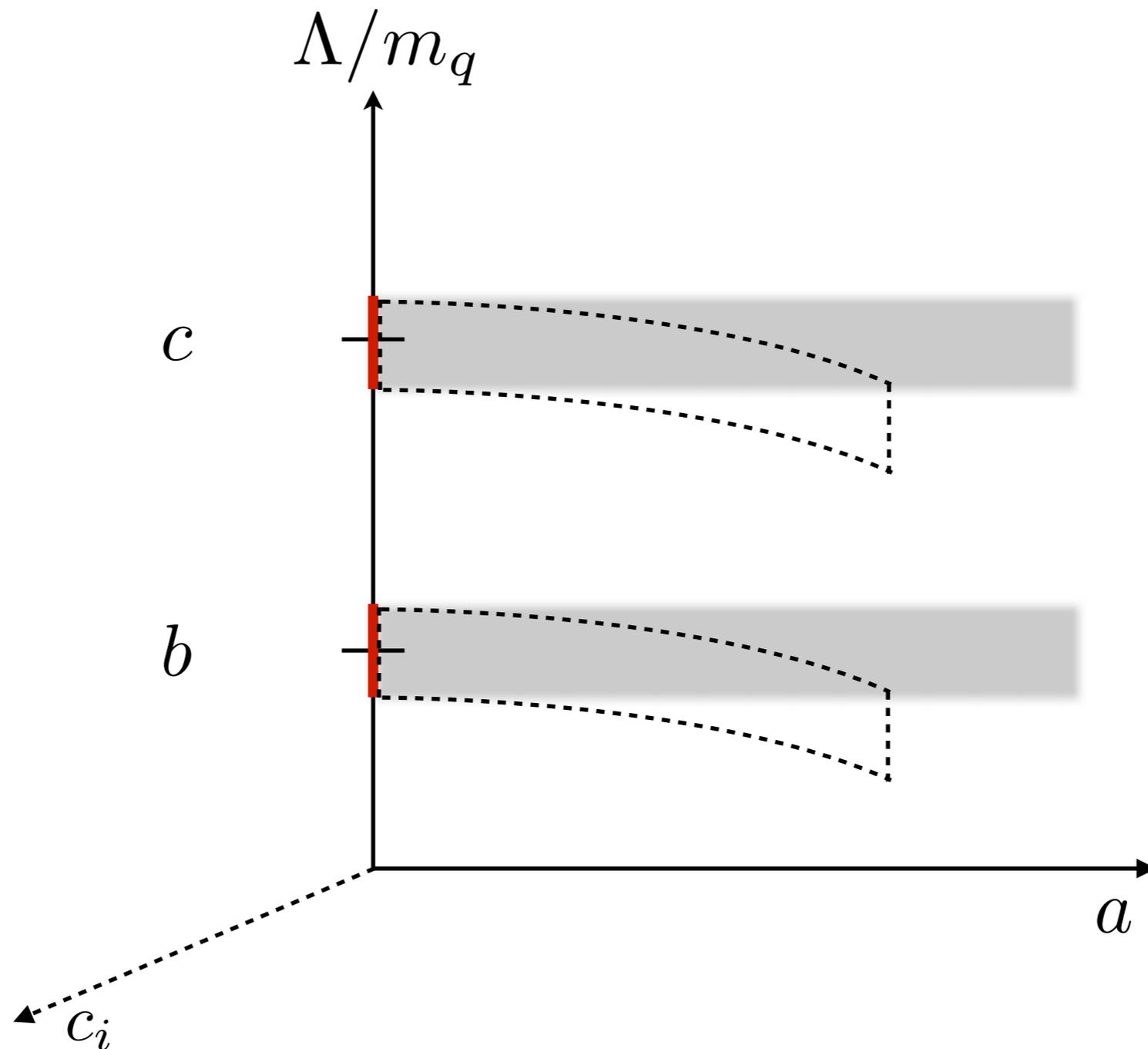
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CLS	$N_f = 2$	▲
ETMC	$N_f = 2$	▲
(clover) ETMC	$N_f = 2$	▼
(Iwa) TWQCD	$N_f = 2$	×
(Möbius) JLQCD	$N_f = 2 + 1$	◊
RBC-UKQCD	$N_f = 2 + 1$	◊
(DSDR) RBC-UKQCD	$N_f = 2 + 1$	◊
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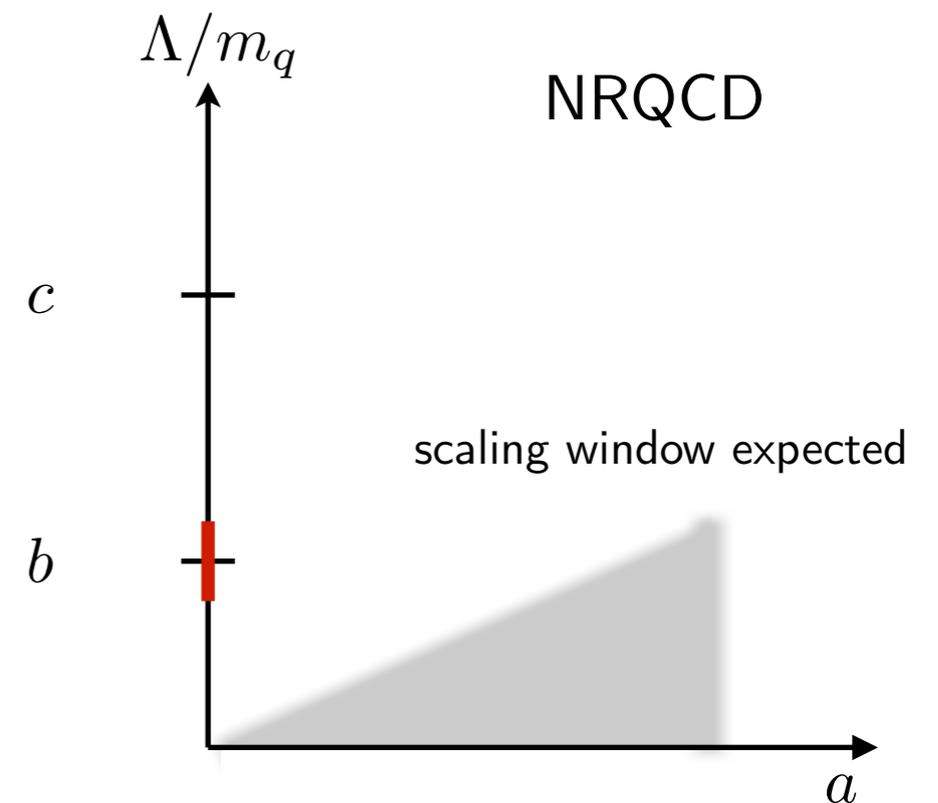
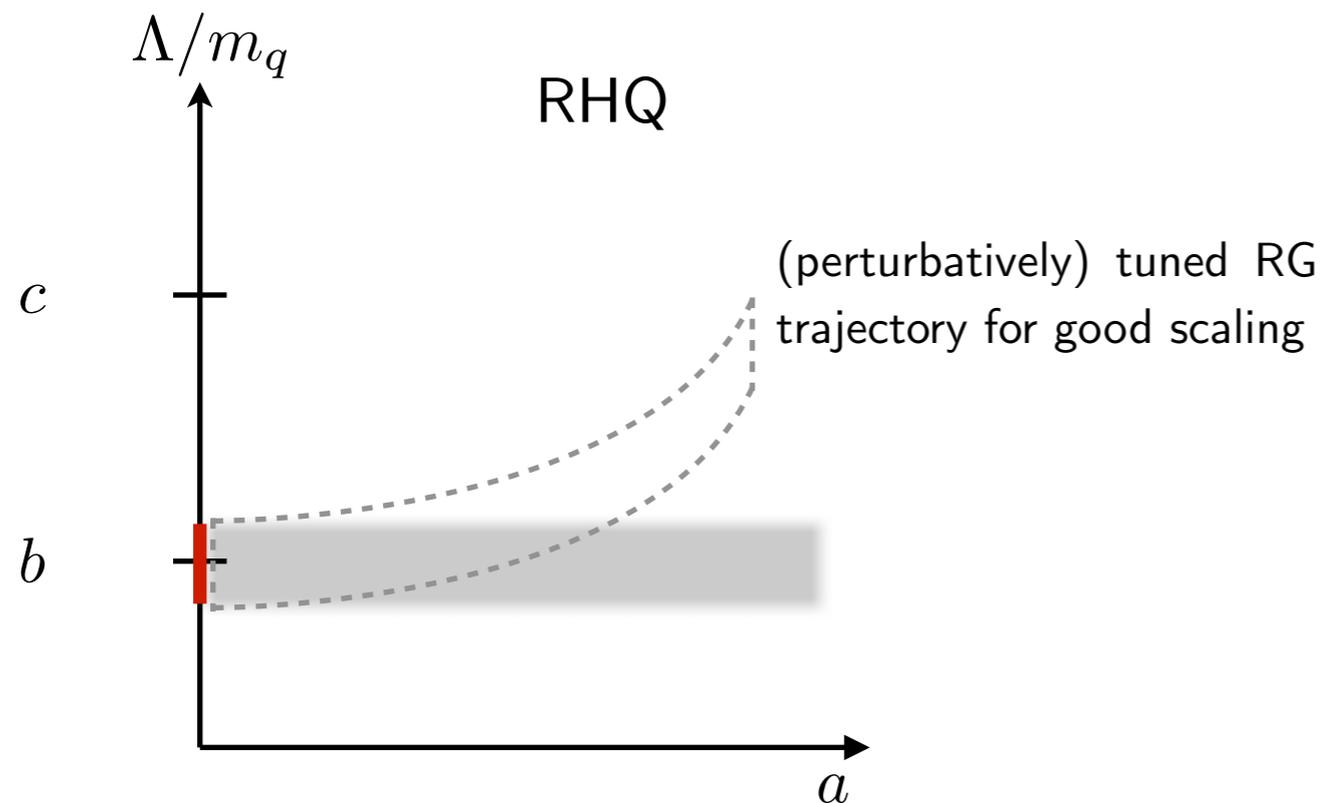
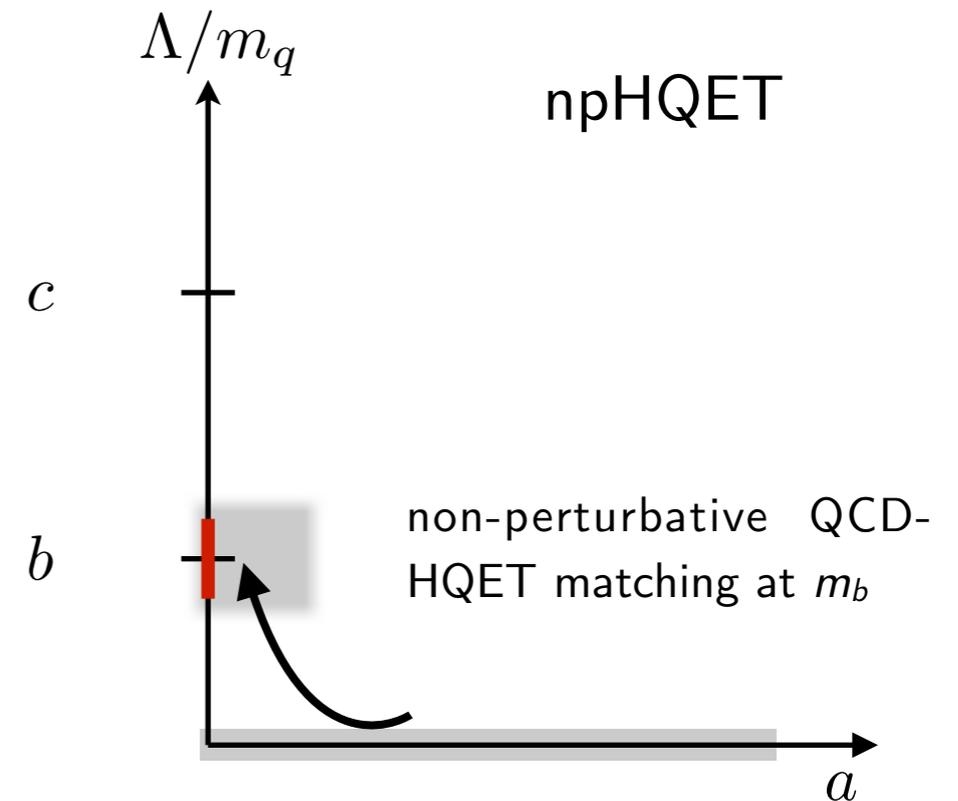
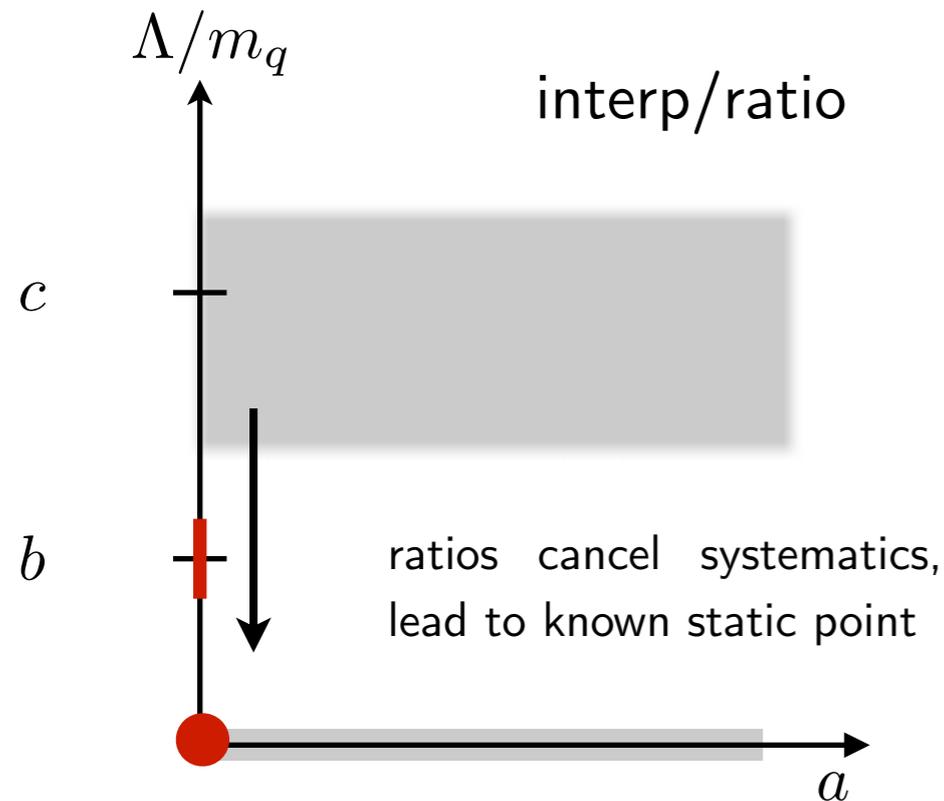
approaches to B physics

what one would like to do



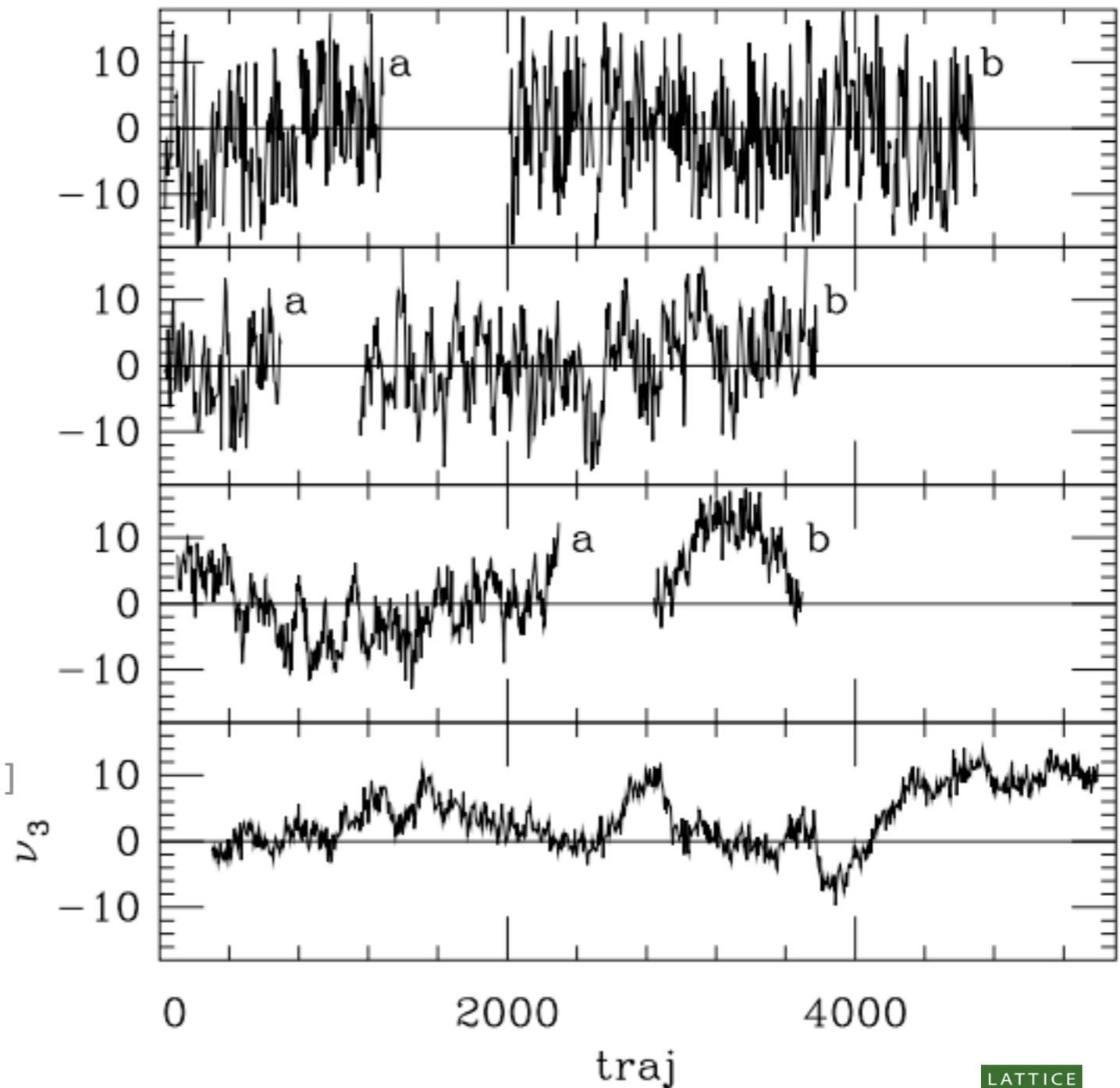
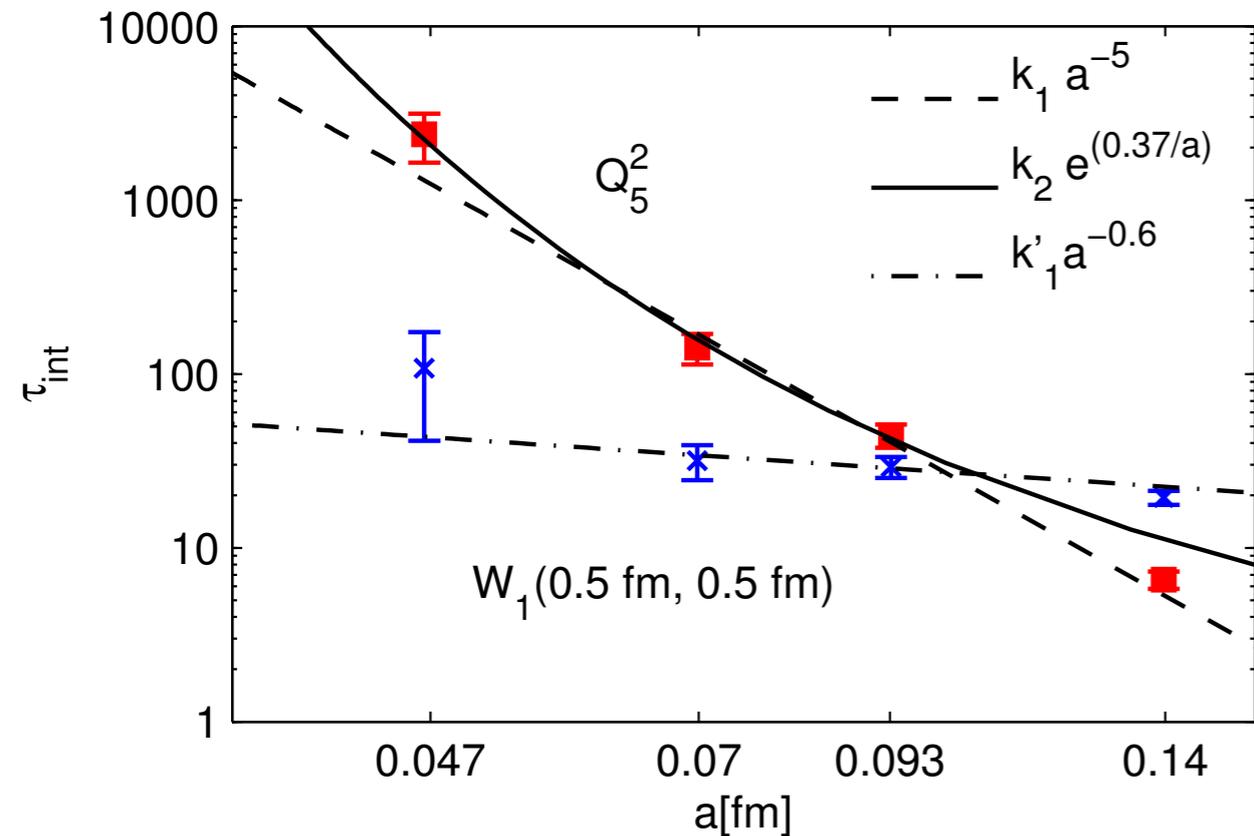
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effective theory used differently, different pros/cons balance: **crosschecks crucial**



towards a fully relativistic b

crucial issue: strong lattice space dependence of autocorrelations



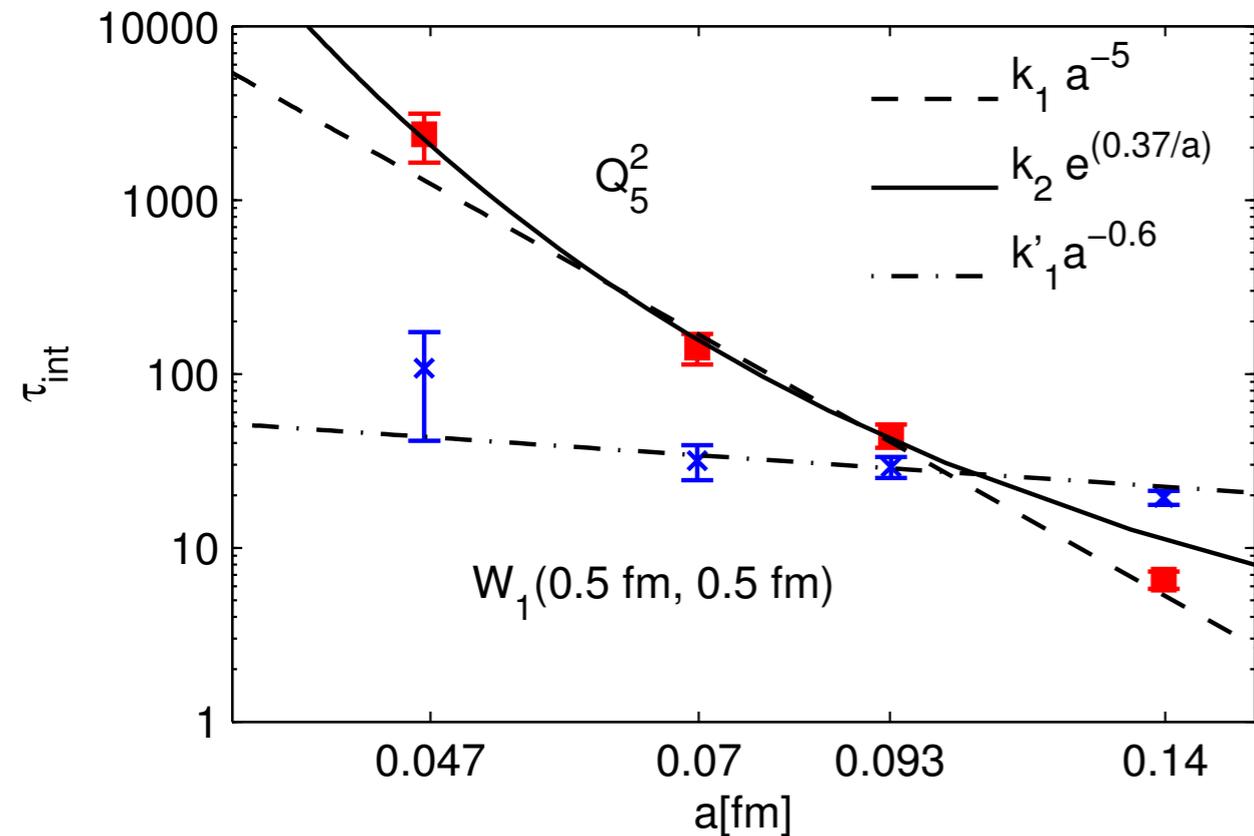
[Del Debbio, Panagopoulos, Vicari 2002]
[Schaefer, Sommer, Virotta 2010]
[Lüscher, Schaefer 2011]
[CLS $N_f=2+1$ obc programme]

[S Gottlieb, Tue 18:10]

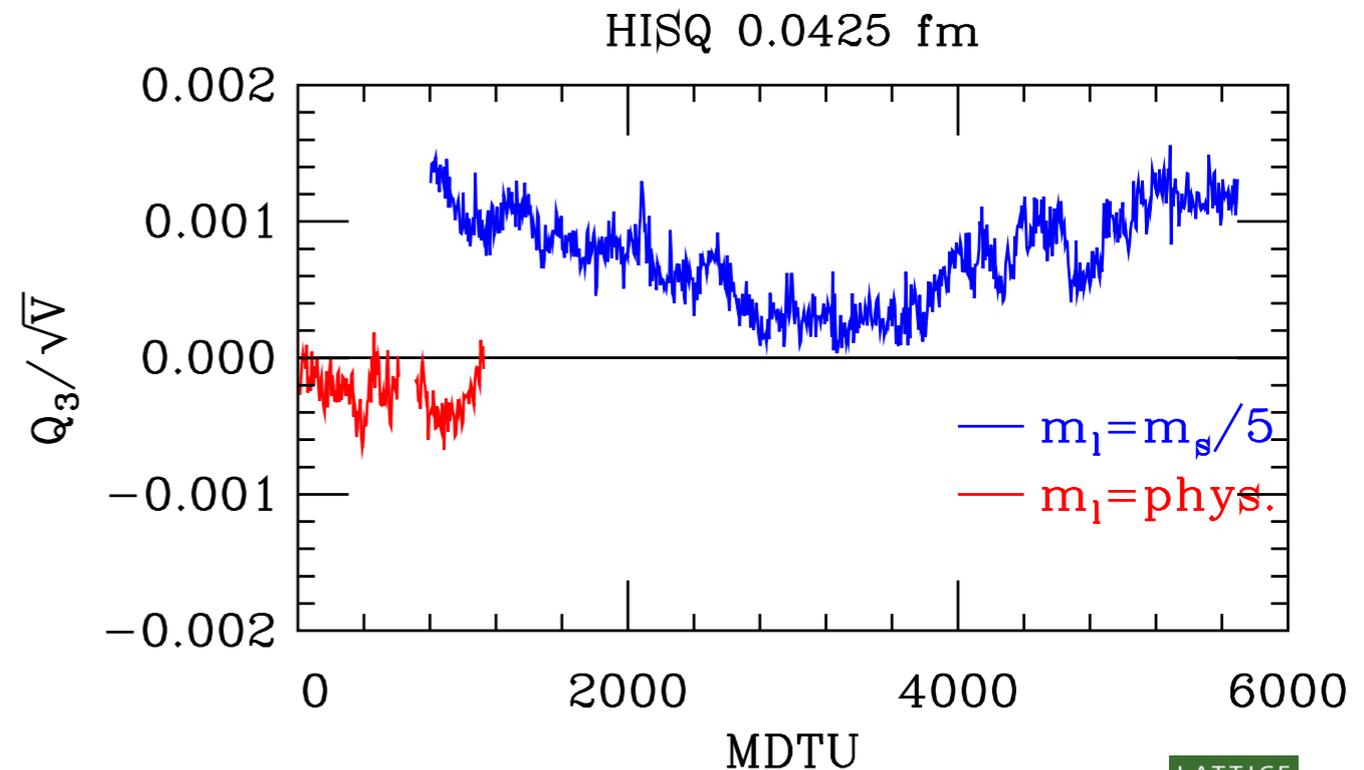
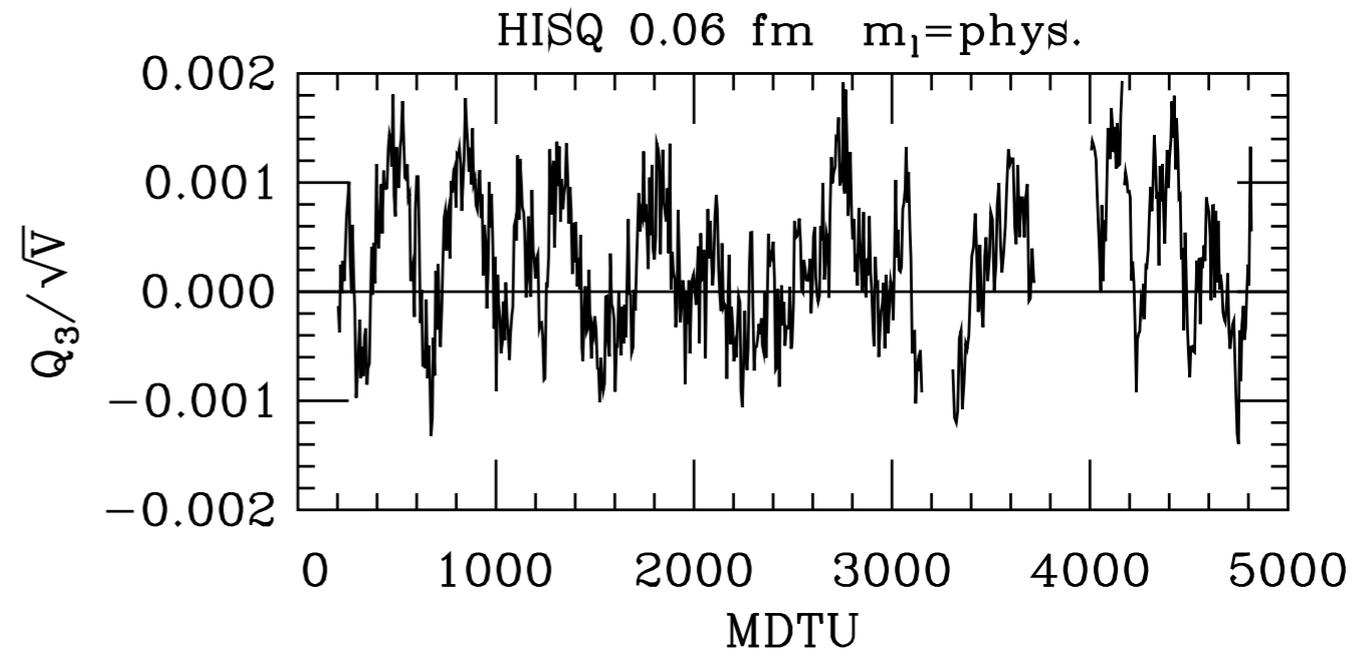


towards a fully relativistic b

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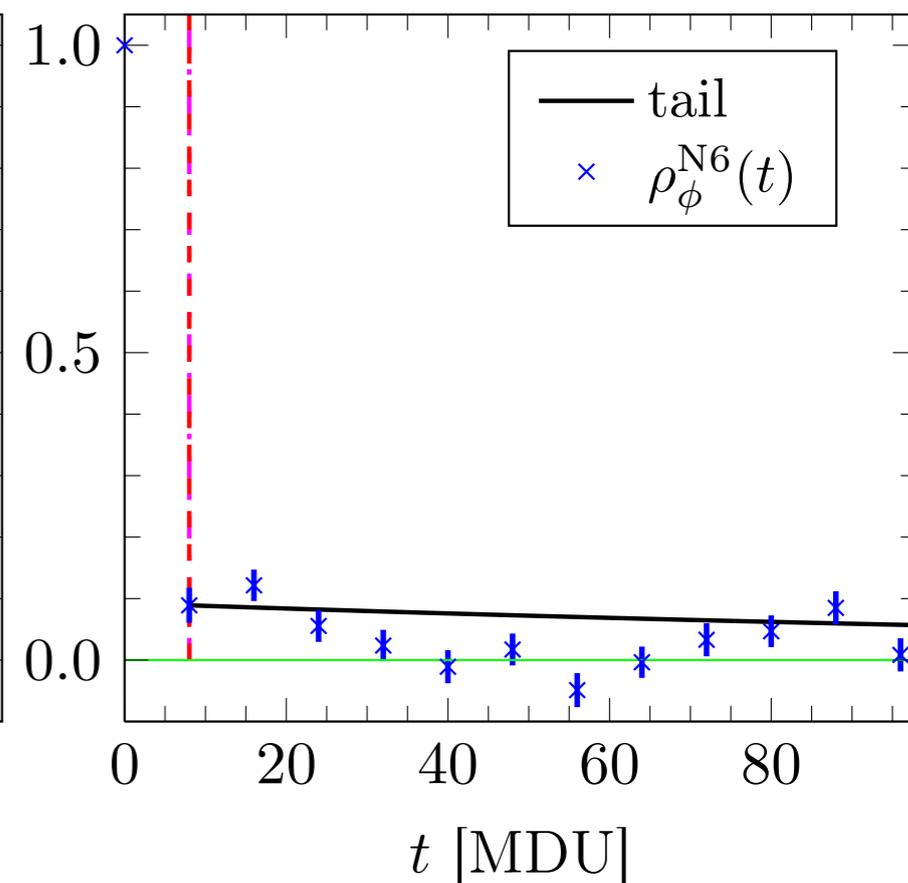
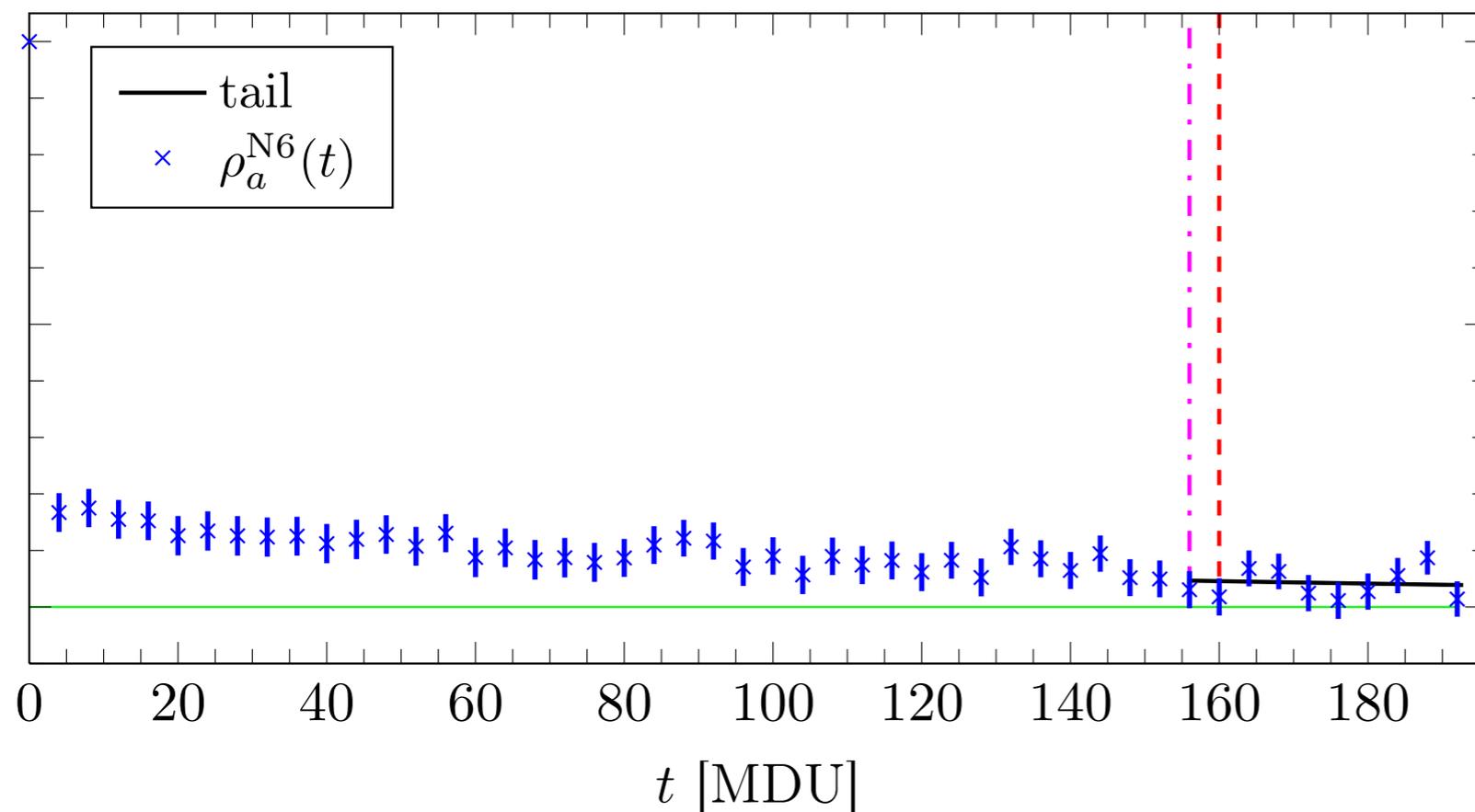
[C DeTar, Thu 8:50]



towards a fully relativistic b

crucial issue: strong lattice space dependence of autocorrelations

$a = 0.048 \text{ fm}$, $m_\pi = 340 \text{ MeV}$

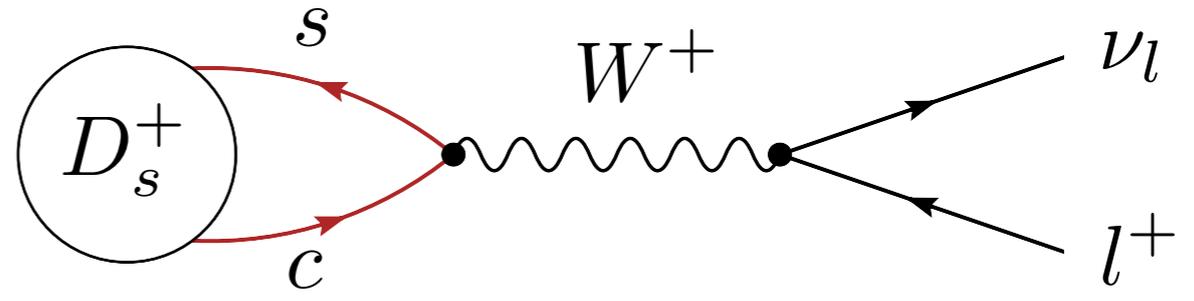
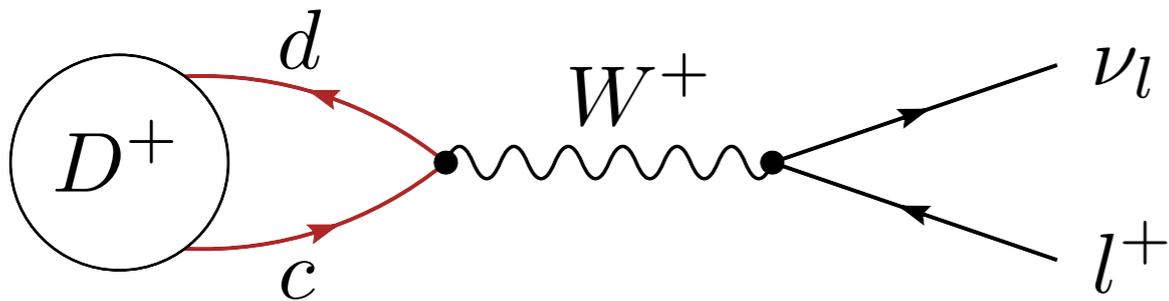


[ALPHA f_B , arXiv:1404.3590]

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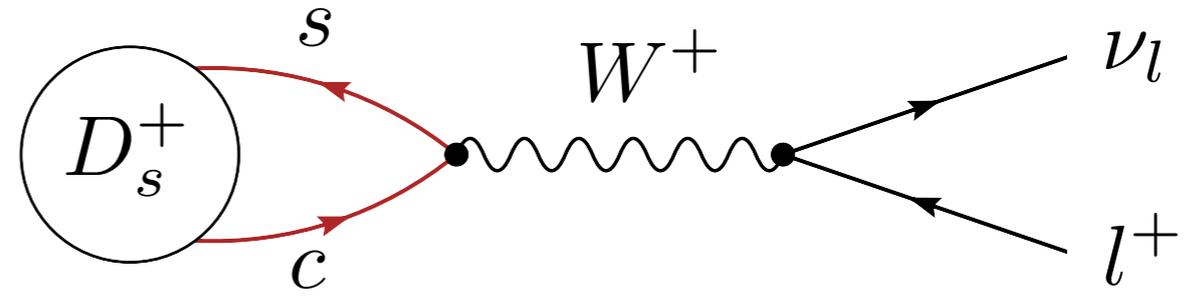
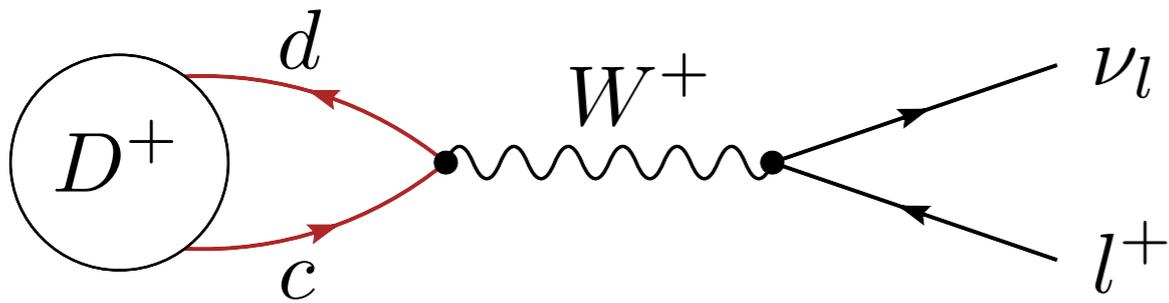
charm leptonic decay



$$\frac{\mathcal{B}(D_{(s)} \rightarrow l\nu_l)}{\tau_{D_{(s)}}} = \frac{G_F^2}{8\pi} m_{D_{(s)}} m_l^2 \left(1 - \frac{m_l^2}{m_{D_{(s)}}^2}\right)^2 |V_{cq}|^2 f_{D_{(s)}}^2 \quad [+ \text{h.o. OPE}]$$

$$\langle 0 | \bar{c} \gamma^\mu \gamma^5 q | D_q(p) \rangle = f_{D_q} p^\mu$$

charm leptonic decay



$$\frac{\mathcal{B}(D_{(s)} \rightarrow l\nu_l)}{\tau_{D_{(s)}}} = \frac{G_F^2 m_{D_{(s)}} m_l^2 \left(1 - \frac{m_l^2}{m_{D_{(s)}}^2}\right)^2 |V_{cq}|^2 f_{D_{(s)}}^2}{8\pi} \quad [+ \text{h.o. OPE}]$$

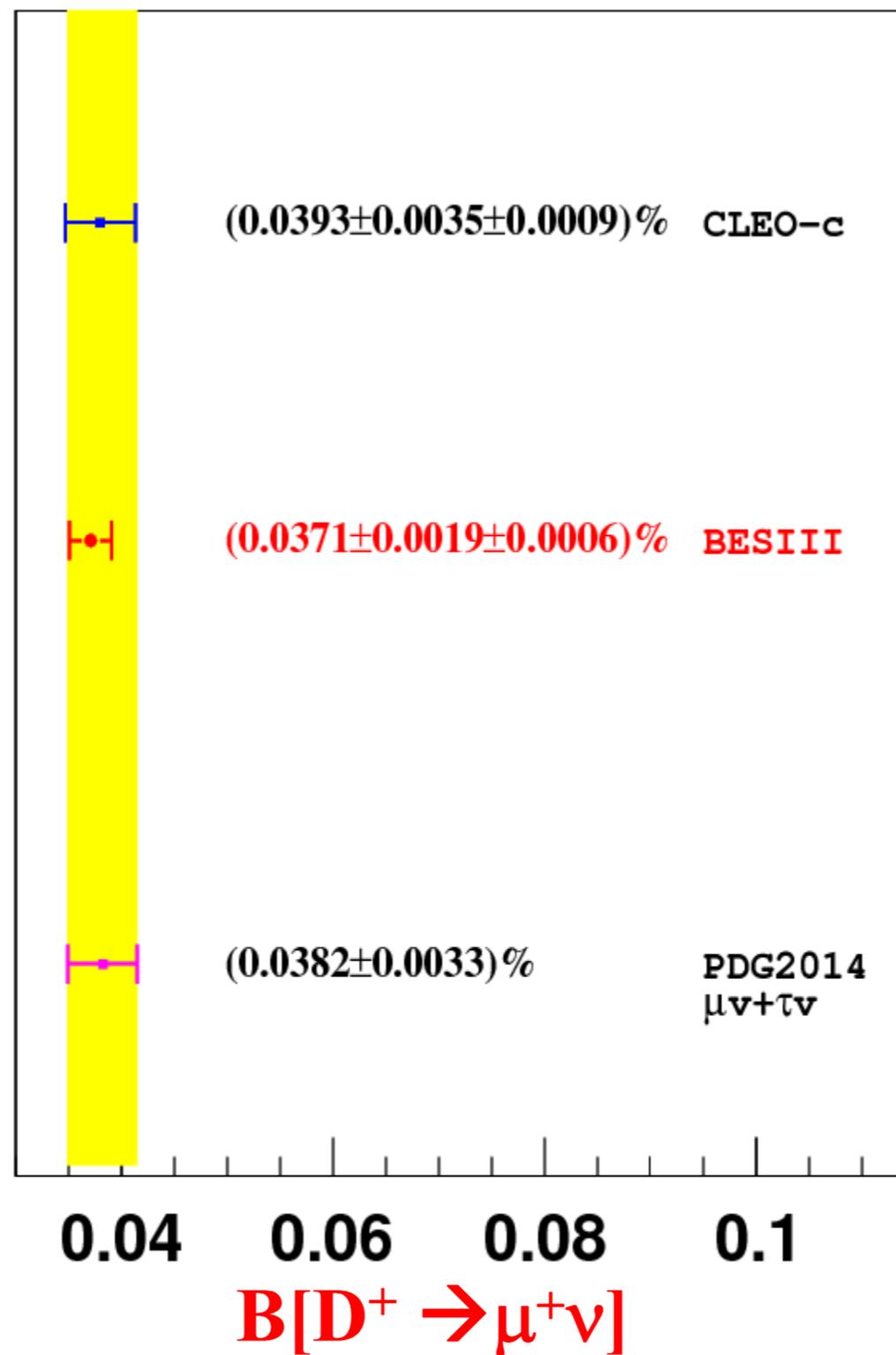
$$\Delta \sim 10^{-3} (m_{D_{(s)}})$$

$$\Delta = 0.7\% (D), 1.4\% (D_s)$$

(+ e.m. corrections!)

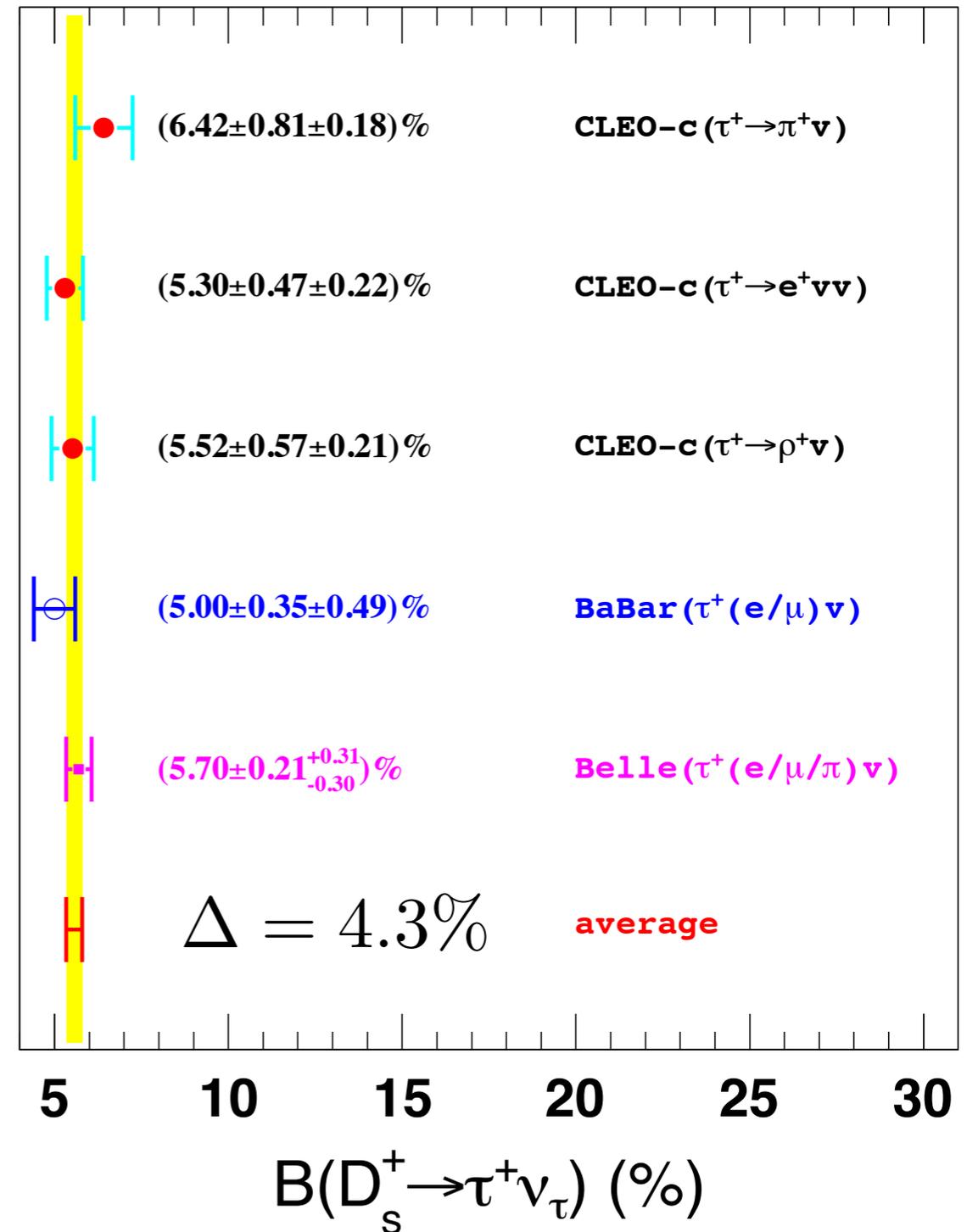
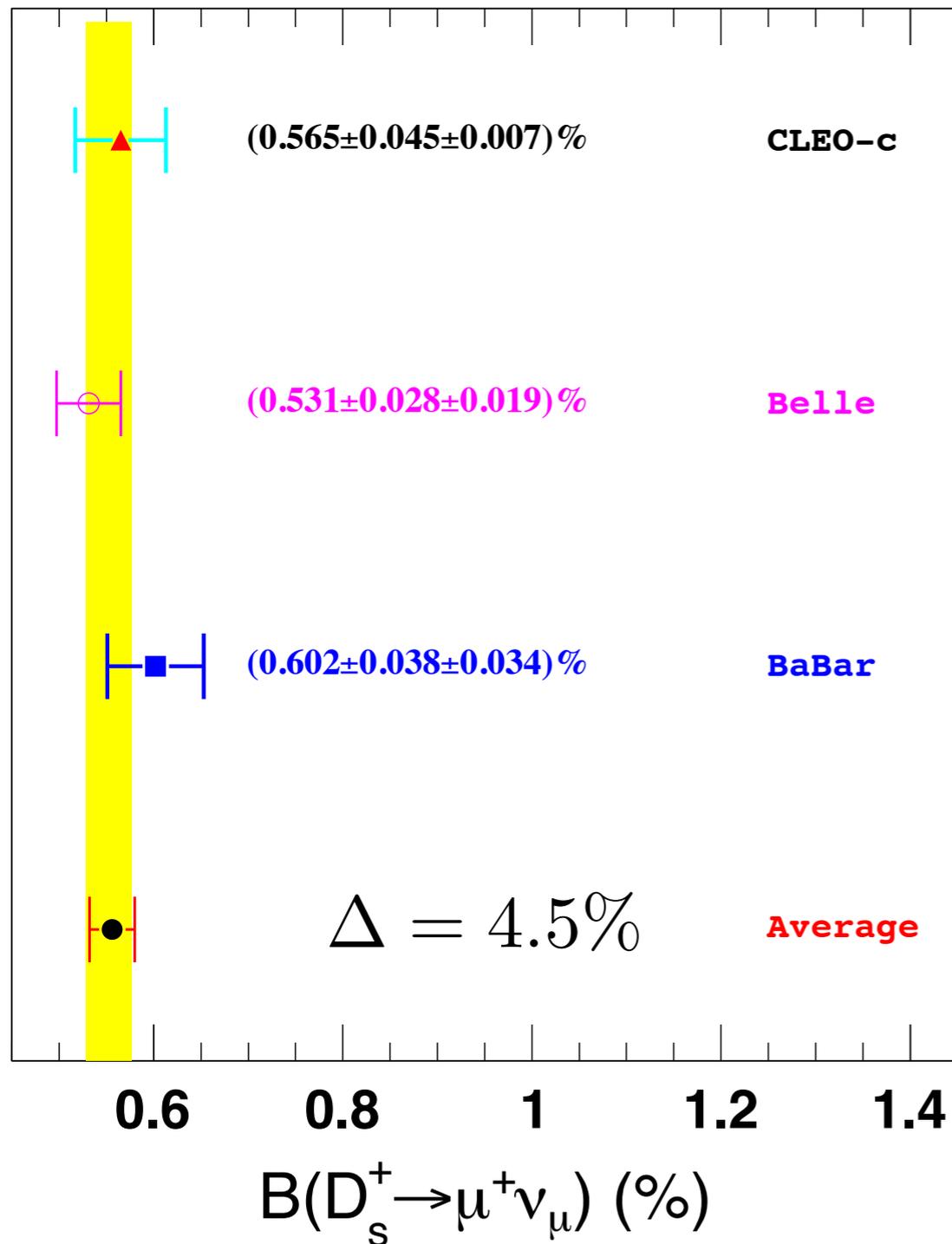
$$\Delta \sim \mathcal{O}\left(\frac{m_{D_{(s)}}^2}{M_W^2}\right) \sim 10^{-4}$$

charm leptonic decay



[from H Ma's talk on behalf of BESIII at CHARM 2015]

charm leptonic decay

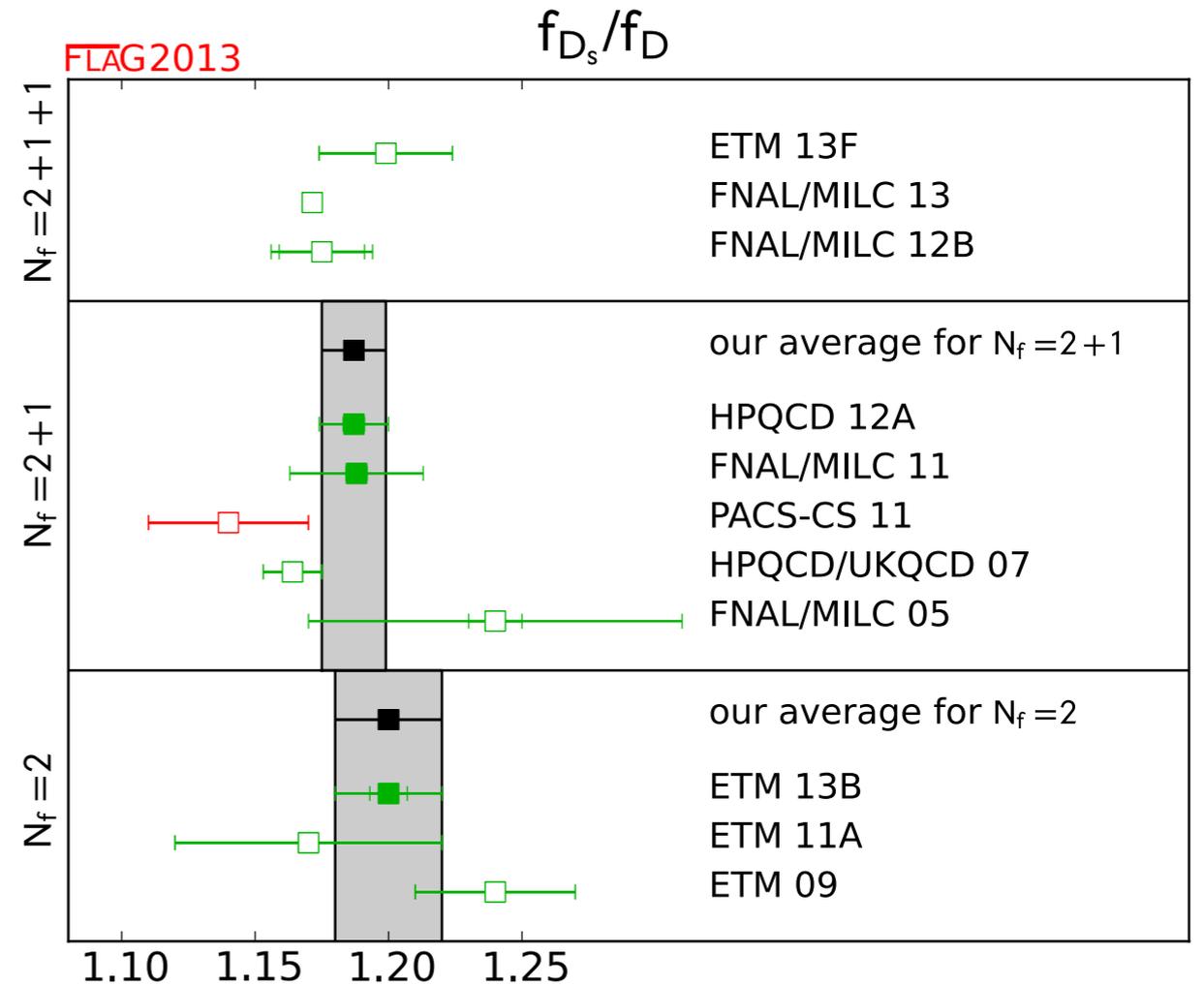
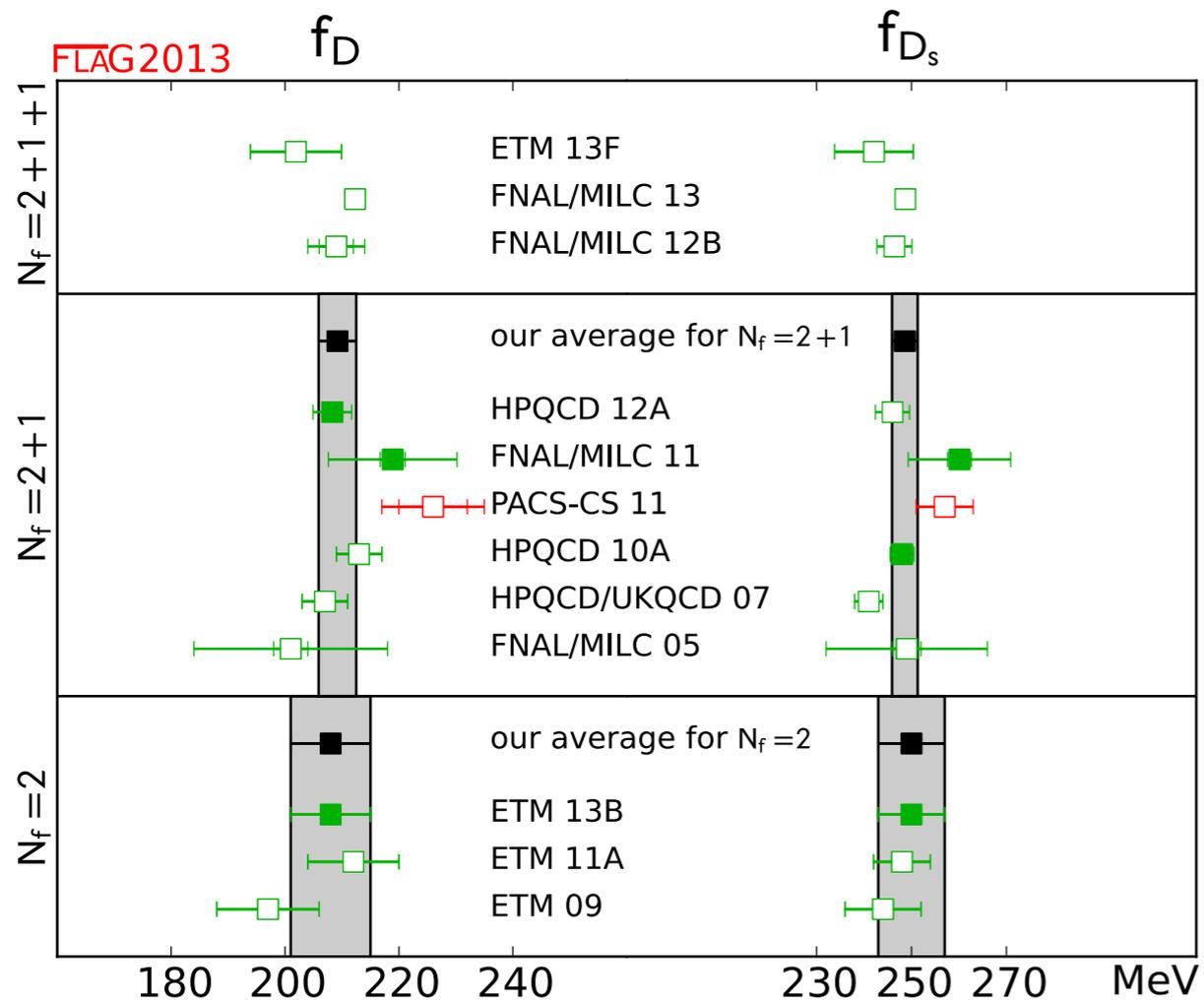


Belle II projections

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	± 0.012	± 0.008	6
α		$\pm 2^\circ$	$\pm 1^\circ$	
γ	$\pm 14^\circ$	$\pm 6^\circ$	$\pm 1.5^\circ$	
$S(B \rightarrow \phi K^0)$	$0.90^{+0.09}_{-0.19}$	± 0.053	± 0.018	>50
$S(B \rightarrow \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	± 0.028	± 0.011	>50
$S(B \rightarrow K_S^0 K_S^0 K_S^0)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
$ V_{cb} $ incl.	$\pm 2.4\%$	$\pm 1.0\%$		< 1
$ V_{cb} $ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
$ V_{ub} $ incl.	$\pm 6.5\%$	$\pm 3.4\%$	$\pm 3.0\%$	2
$ V_{ub} $ excl. (had. tag.)	$\pm 10.8\%$	$\pm 4.7\%$	$\pm 2.4\%$	20
$ V_{ub} $ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \rightarrow \tau\nu)$ [10 ⁻⁶]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \rightarrow \mu\nu)$ [10 ⁻⁶]	< 1.7	5σ	$\gg 5\sigma$	>50
$R(B \rightarrow D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \rightarrow D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
$\mathcal{B}(B \rightarrow K^{*+}\nu\bar{\nu})$ [10 ⁻⁶]	< 40		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow K^+\nu\bar{\nu})$ [10 ⁻⁶]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow X_s\gamma)$ [10 ⁻⁶]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \rightarrow X_s\gamma)$		± 0.01	± 0.005	8
$S(B \rightarrow K_S^0\pi^0\gamma)$	$-0.10 \pm 0.31 \pm 0.07$	± 0.11	± 0.035	> 50
$S(B \rightarrow \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	± 0.23	± 0.07	> 50
$C_7/C_9 (B \rightarrow X_s\ell\ell)$	$\sim 20\%$	10%	5%	
$\mathcal{B}(B_s \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 8.7	± 0.3		
$\mathcal{B}(B_s \rightarrow \tau^+\tau^-)$ [10 ⁻³]		< 2		

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
$\mathcal{B}(D_s \rightarrow \mu\nu)$	$5.31 \times 10^{-3} (1 \pm 0.053 \pm 0.038)$	$\pm 2.9\%$	$\pm (0.9\%-1.3\%)$	> 50
$\mathcal{B}(D_s \rightarrow \tau\nu)$	$5.70 \times 10^{-3} (1 \pm 0.037 \pm 0.054)$	$\pm (3.5\%-4.3\%)$	$\pm (2.3\%-3.6\%)$	3-5
y_{CP} [10 ⁻²]	$1.11 \pm 0.22 \pm 0.11$	$\pm (0.11-0.13)$	$\pm (0.05-0.08)$	5-8
A_Γ [10 ⁻²]	$-0.03 \pm 0.20 \pm 0.08$	± 0.10	$\pm (0.03-0.05)$	7 - 9
$A_{CP}^{K^+K^-}$ [10 ⁻²]	$-0.32 \pm 0.21 \pm 0.09$	± 0.11	± 0.06	15
$A_{CP}^{\pi^+\pi^-}$ [10 ⁻²]	$0.55 \pm 0.36 \pm 0.09$	± 0.17	± 0.06	> 50
$A_{CP}^{\phi\gamma}$ [10 ⁻²]	± 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.56 \pm 0.19 \pm_{0.13}^{0.07}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.30 \pm 0.15 \pm_{0.08}^{0.05}$	± 0.08	± 0.05	15
$ q/p ^{K_S\pi^+\pi^-}$	$0.90 \pm_{0.15}^{0.16} \pm_{0.06}^{0.08}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [°]	$-6 \pm 11 \pm_5^4$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10 ⁻²]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K_S^0\pi^0}$ [10 ⁻²]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
	$\tau \rightarrow \mu\gamma$ [10 ⁻⁹]	< 45	< 14.7	< 4.7
	$\tau \rightarrow e\gamma$ [10 ⁻⁹]	< 120	< 39	< 12
	$\tau \rightarrow \mu\mu\mu$ [10 ⁻⁹]	< 21.0	< 3.0	< 0.3

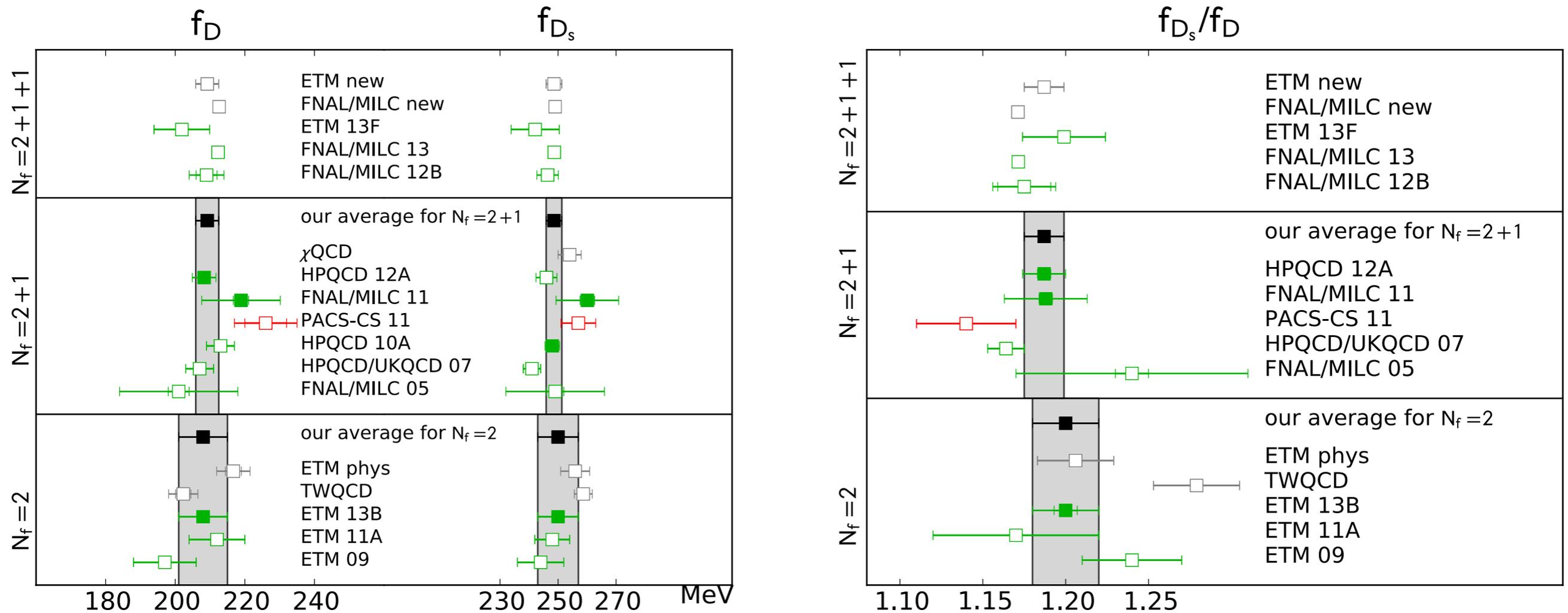
FLAG-2 on charm decay constants



N_f	f_D [MeV]	f_{D_s} [MeV]	f_{D_s}/f_D
2	208(7)	250(7)	1.20(2)
2+1	209.2(3.3)	248.6(2.7)	1.187(12)

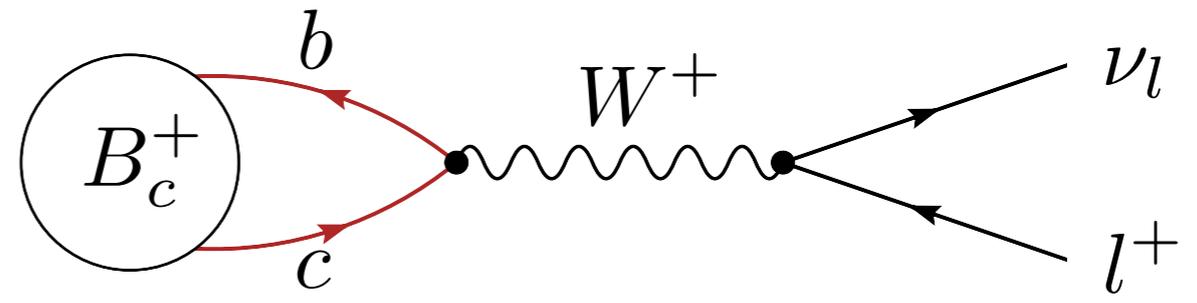
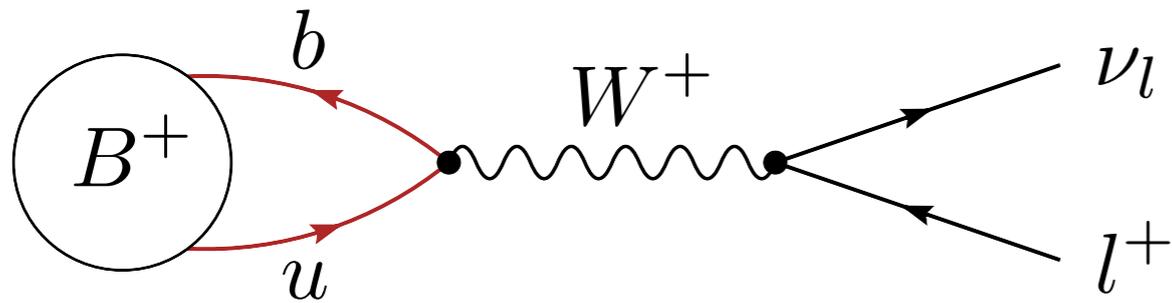
new results for $f_{D(s)}$

new results superimposed on FLAG-2 summary plots:



- close to 1% accuracy: “raw” potential of lattice methods
- already at the level where e.g. electromagnetic effects are relevant

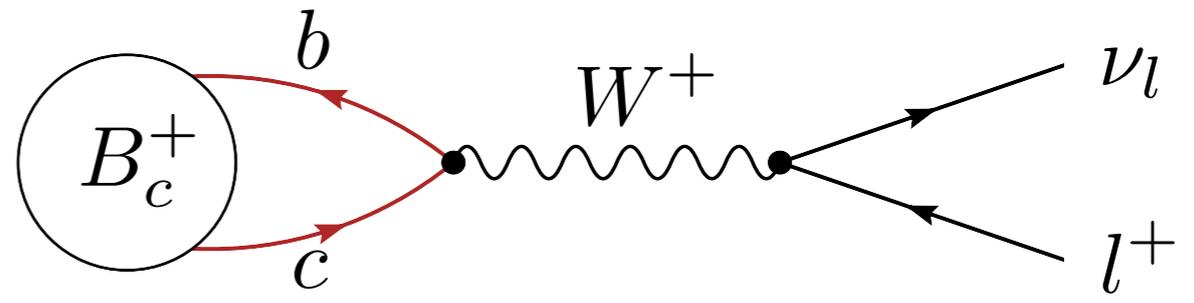
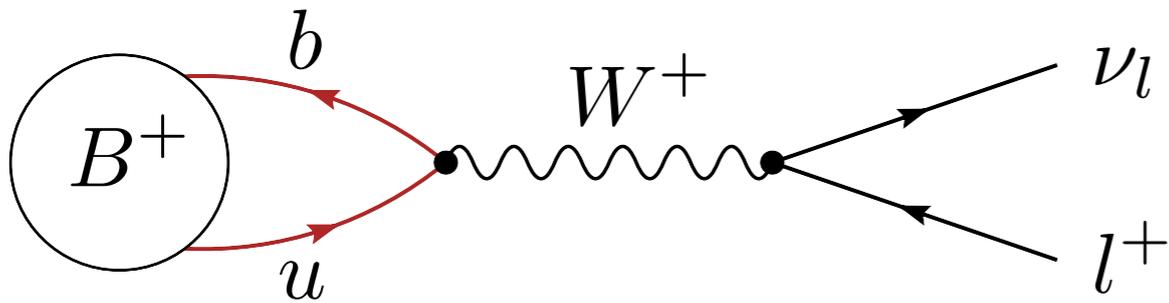
B leptonic decay



$$\frac{\mathcal{B}(B_{(c)} \rightarrow l\nu_l)}{\tau_{B_{(c)}}} = \frac{G_F^2}{8\pi} m_l^2 m_{B_{(c)}} \left(1 - \frac{m_l^2}{m_{B_{(c)}}^2}\right)^2 |V_{qb}|^2 f_{B_{(c)}}^2 [+ \text{h.o. OPE}]$$

$$\langle 0 | \bar{b} \gamma^\mu \gamma^5 q | B_q(p) \rangle = f_{B_q} p^\mu$$

B leptonic decay



$$\frac{\mathcal{B}(B \rightarrow l\nu_l)}{\tau_B} = \frac{G_F^2 m_l^2 m_B}{8\pi} \left(1 - \frac{m_l^2}{m_B^2}\right)^2 |V_{ub}|^2 f_B^2 \quad [+ \text{h.o. OPE}]$$

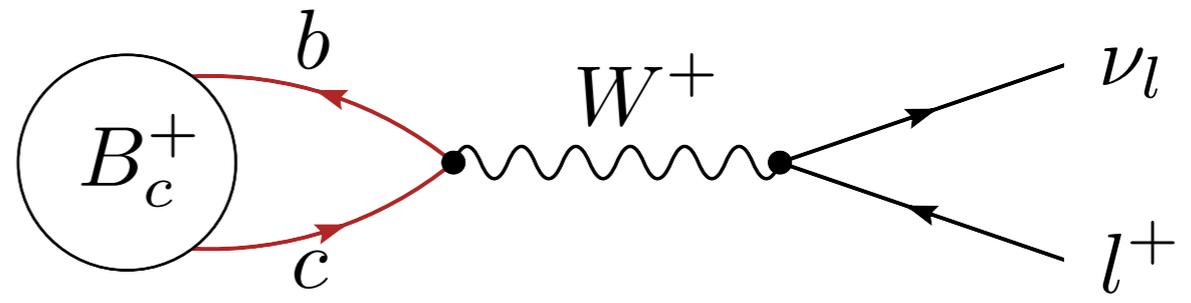
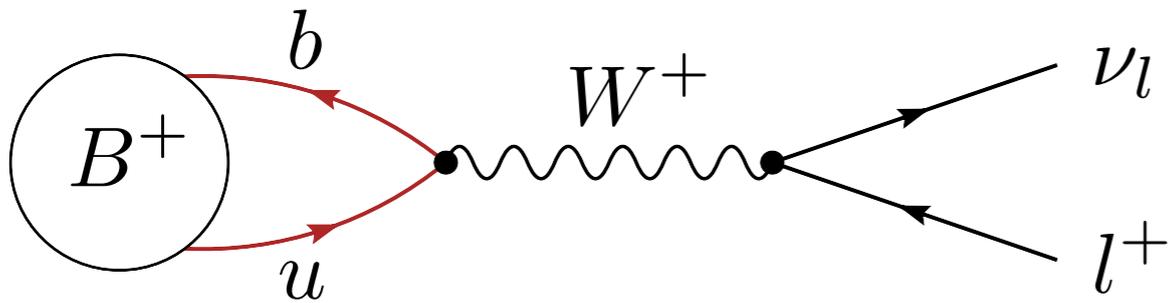
$$\Delta = 0.2\%$$

(negligible)

$$\Delta \sim \mathcal{O}\left(\frac{m_B^2}{M_W^2}\right) \sim 0.4\%$$

(+ e.m. corrections!)

B leptonic decay



$$\frac{\mathcal{B}(B_c \rightarrow l\nu_l)}{\tau_{B_c}} = \frac{G_F^2 m_l^2 m_{B_c}}{8\pi} \left(1 - \frac{m_l^2}{m_{B_c}^2}\right)^2 |V_{qb}|^2 f_{B_c}^2 \quad [+ \text{h.o. OPE}]$$

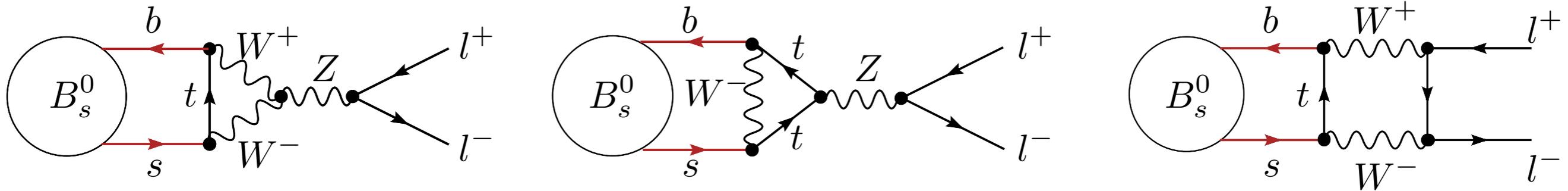
$\Delta = 7.3\%$

(negligible)

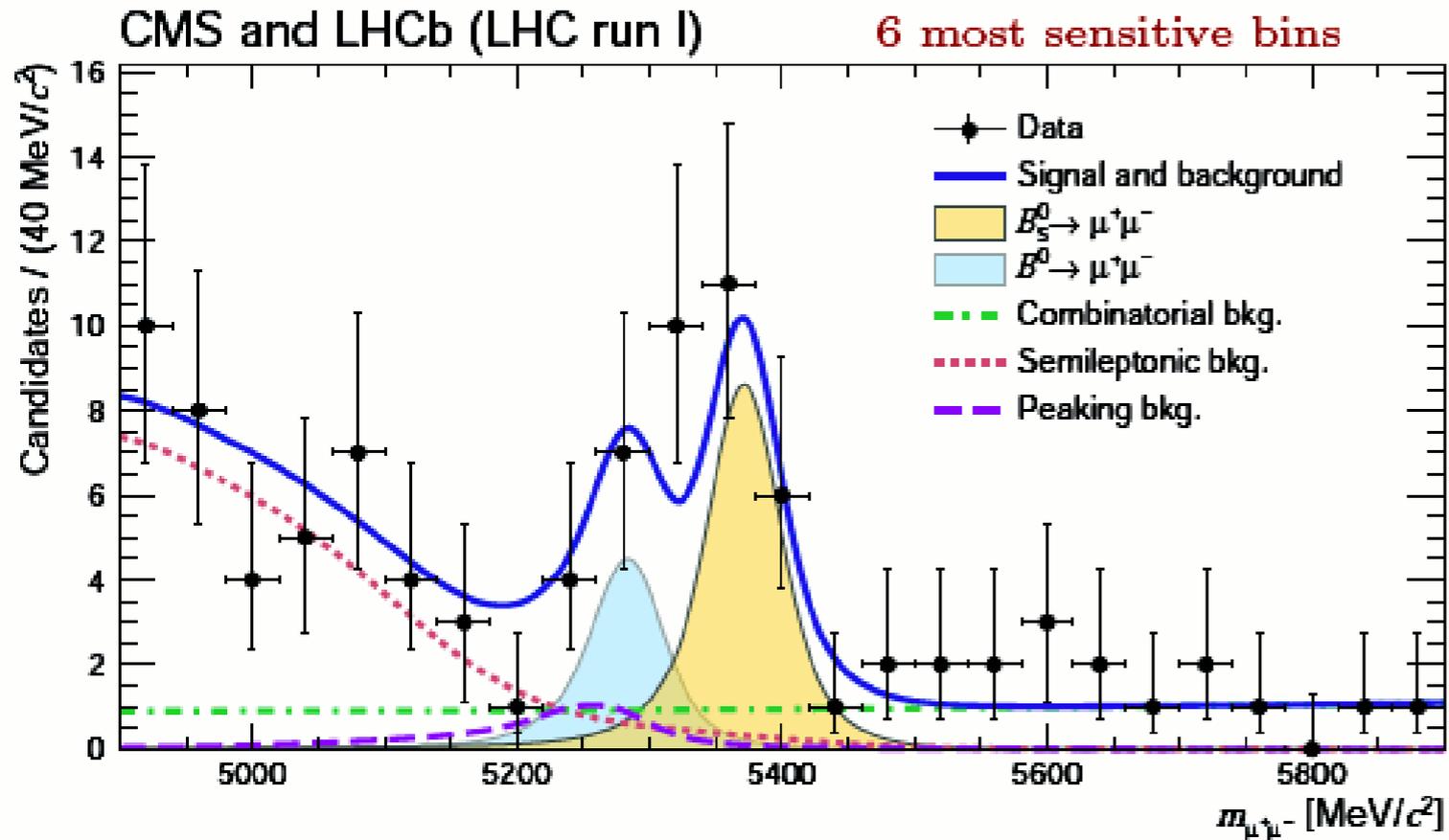
$\Delta \sim \mathcal{O}\left(\frac{m_{B_c}^2}{M_W^2}\right) \sim 0.6\%$

(+ e.m. corrections!)

B leptonic decay



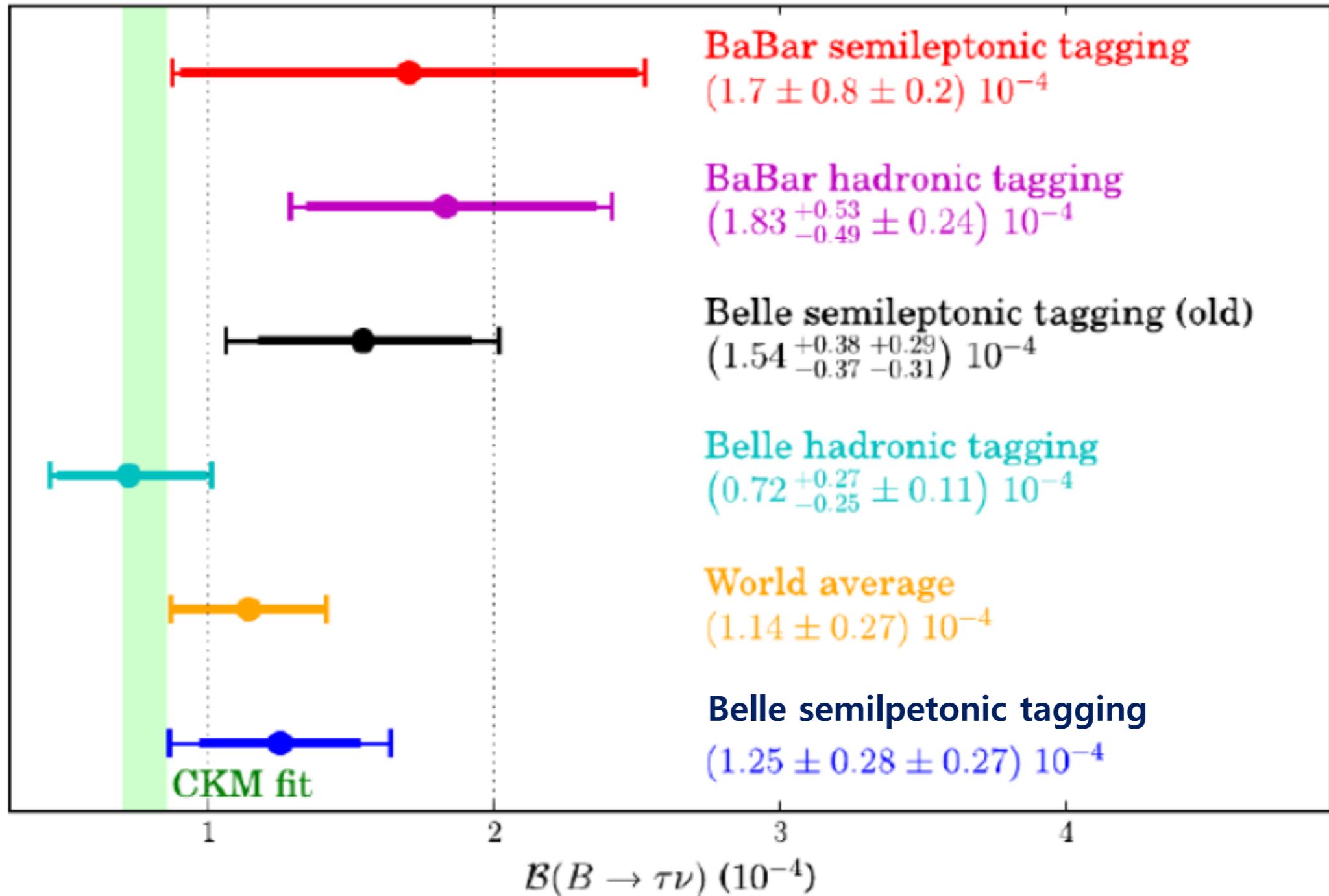
$$\frac{\mathcal{B}(B_q \rightarrow l^+ l^-)}{\tau_{B_q}} = \frac{G_F^2}{\pi} Y \left(\frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 m_{B_q} m_l^2 \sqrt{1 - 4 \frac{m_l^2}{m_{B_q}^2}} |V_{tb}^* V_{tq}|^2 f_{B_q}^2$$



$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) = (2.8_{-0.6}^{+0.7}) \times 10^{-9}$$

[arXiv:1411.4413]

B leptonic decay

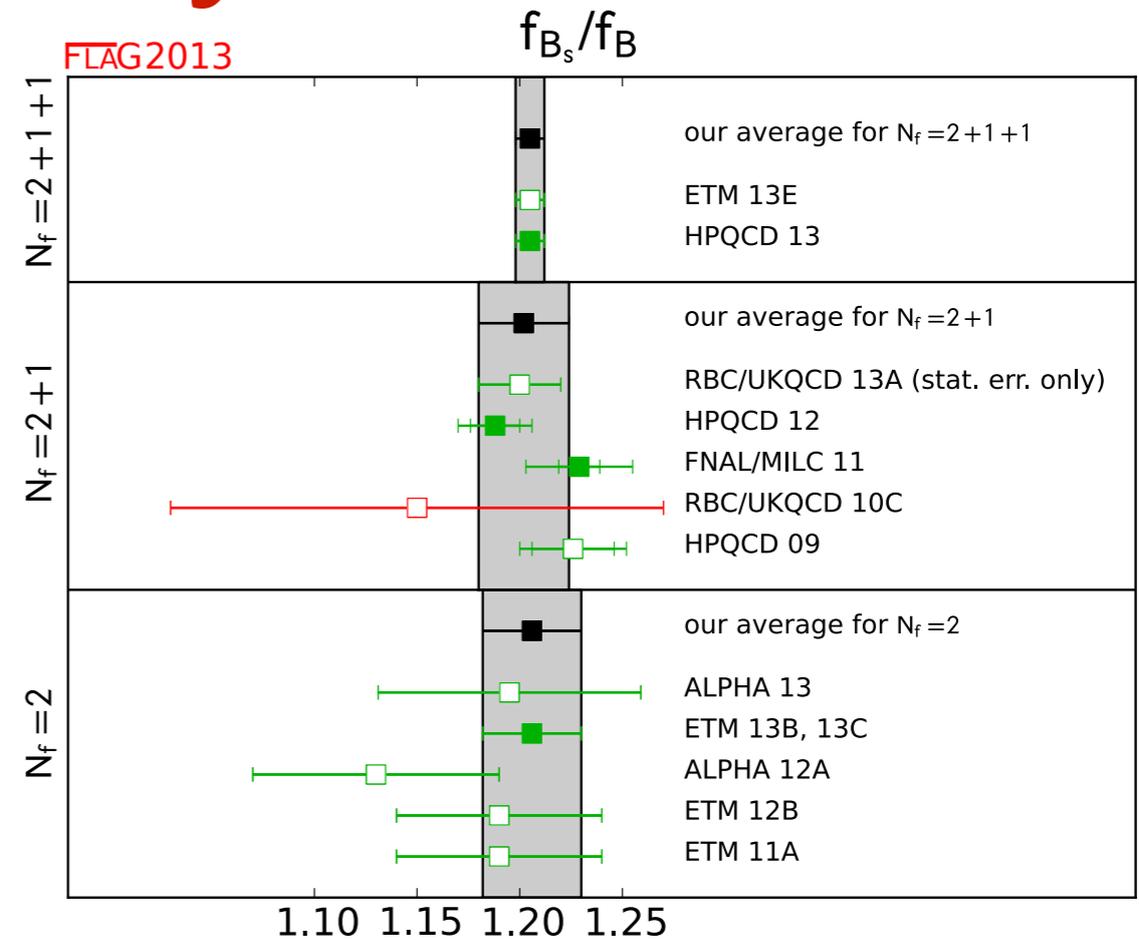
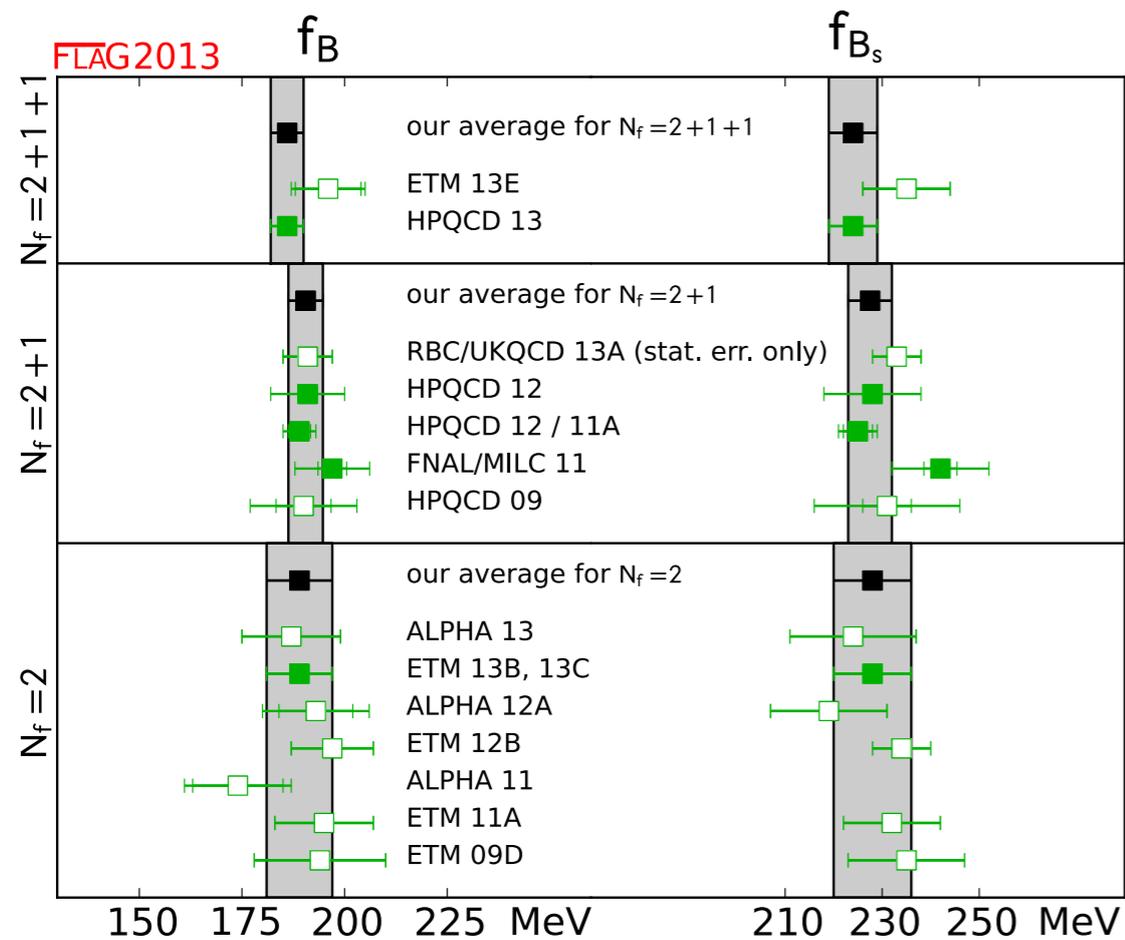


Belle II projections

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
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$S(B \rightarrow K_S^0 K_S^0 K_S^0)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
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$ V_{cb} $ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
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$ V_{ub} $ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \rightarrow \tau\nu)$ [10 ⁻⁶]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \rightarrow \mu\nu)$ [10 ⁻⁶]	< 1.7	5 σ	>> 5 σ	>50
$R(B \rightarrow D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \rightarrow D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
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$\mathcal{B}(B \rightarrow K^+\nu\bar{\nu})$ [10 ⁻⁶]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow X_s\gamma)$ [10 ⁻⁶]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \rightarrow X_s\gamma)$		± 0.01	± 0.005	8
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$C_7/C_9 (B \rightarrow X_s\ell\ell)$	$\sim 20\%$	10%	5%	
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$\mathcal{B}(B_s \rightarrow \tau^+\tau^-)$ [10 ⁻³]		< 2		

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
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$A_{CP}^{\phi\gamma}$ [10 ⁻²]	± 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.56 \pm 0.19 \pm \begin{smallmatrix} 0.07 \\ 0.13 \end{smallmatrix}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.30 \pm 0.15 \pm \begin{smallmatrix} 0.05 \\ 0.08 \end{smallmatrix}$	± 0.08	± 0.05	15
$ q/p ^{K_S\pi^+\pi^-}$	$0.90 \pm \begin{smallmatrix} 0.16 \\ 0.15 \end{smallmatrix} \pm \begin{smallmatrix} 0.08 \\ 0.06 \end{smallmatrix}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [°]	$-6 \pm 11 \pm \begin{smallmatrix} 4 \\ 5 \end{smallmatrix}$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10 ⁻²]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K_S^0\pi^0}$ [10 ⁻²]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
	$\tau \rightarrow \mu\gamma$ [10 ⁻⁹]	< 45	< 14.7	< 4.7
	$\tau \rightarrow e\gamma$ [10 ⁻⁹]	< 120	< 39	< 12
	$\tau \rightarrow \mu\mu\mu$ [10 ⁻⁹]	< 21.0	< 3.0	< 0.3

FLAG-2 on B decay constants



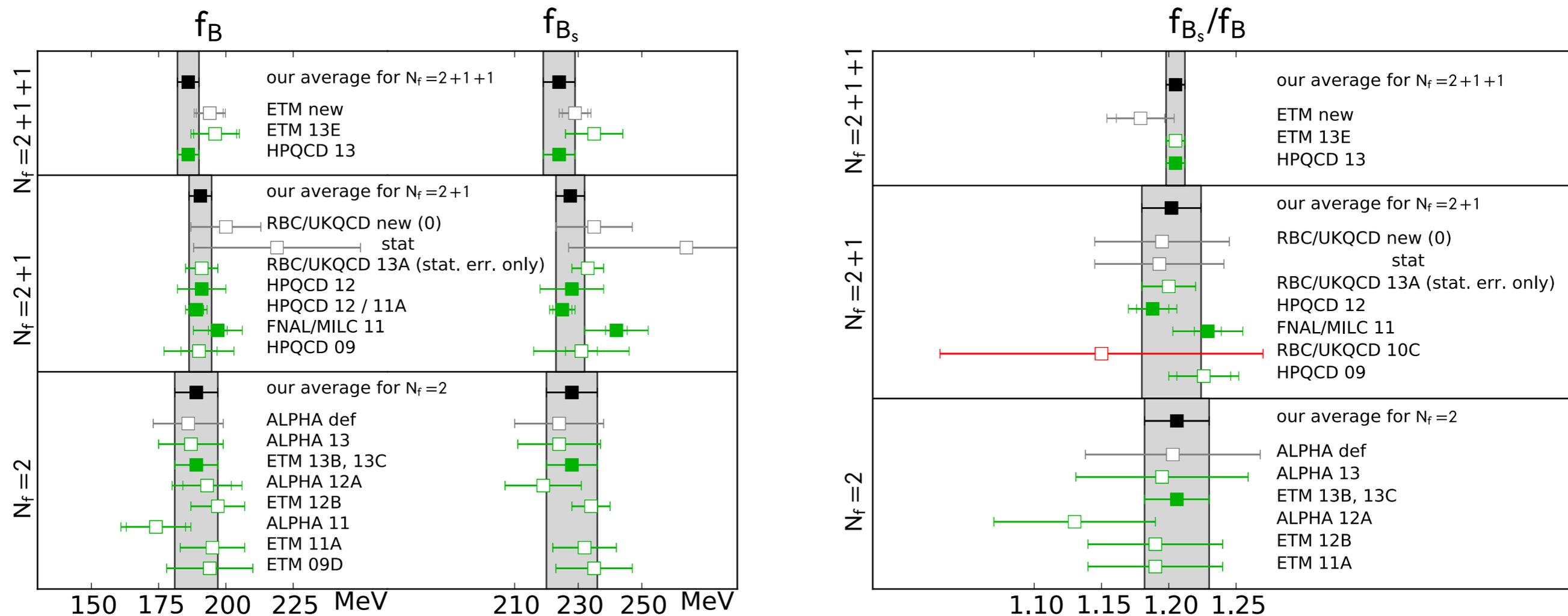
N_f	f_B [MeV]	f_{B_s} [MeV]	f_{B_s}/f_B
2	189(8)	228(8)	1.206(24)
2+1	190.5(4.2)	227.7(4.5)	1.202(22)
2+1+1	186(4)	224(5)	1.205(7)

[FLAG 2013, Eur J Phys C74 (2014) 2890, arXiv:1310.8555v2]

(+ HPQCD results for f_{B_c} , not covered by FLAG) [PRD 86 (2012) 074503]

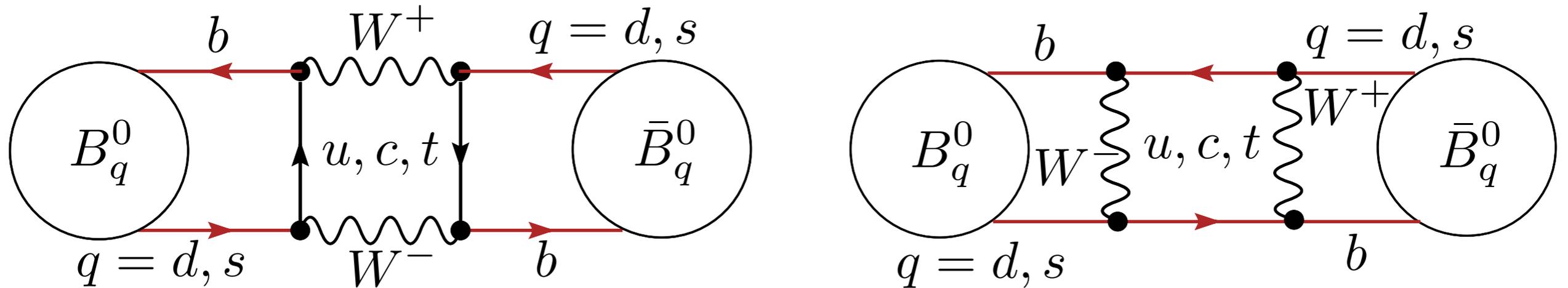
new results for f_{B_q}

new results superimposed on FLAG-2 summary plots:



- still significantly larger error than for charm decay
- few values enter averages, strong need of more results involving different HQ treatments

CP violation: B-mixing



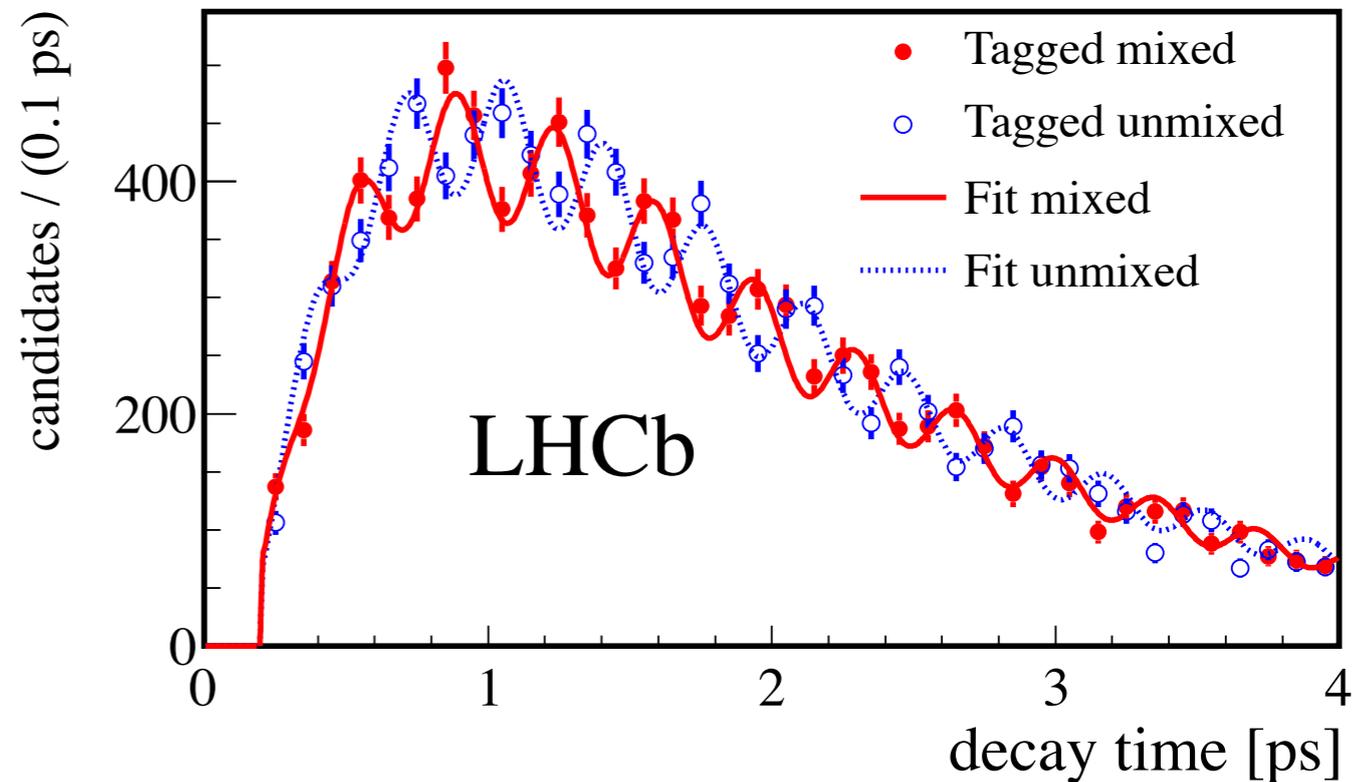
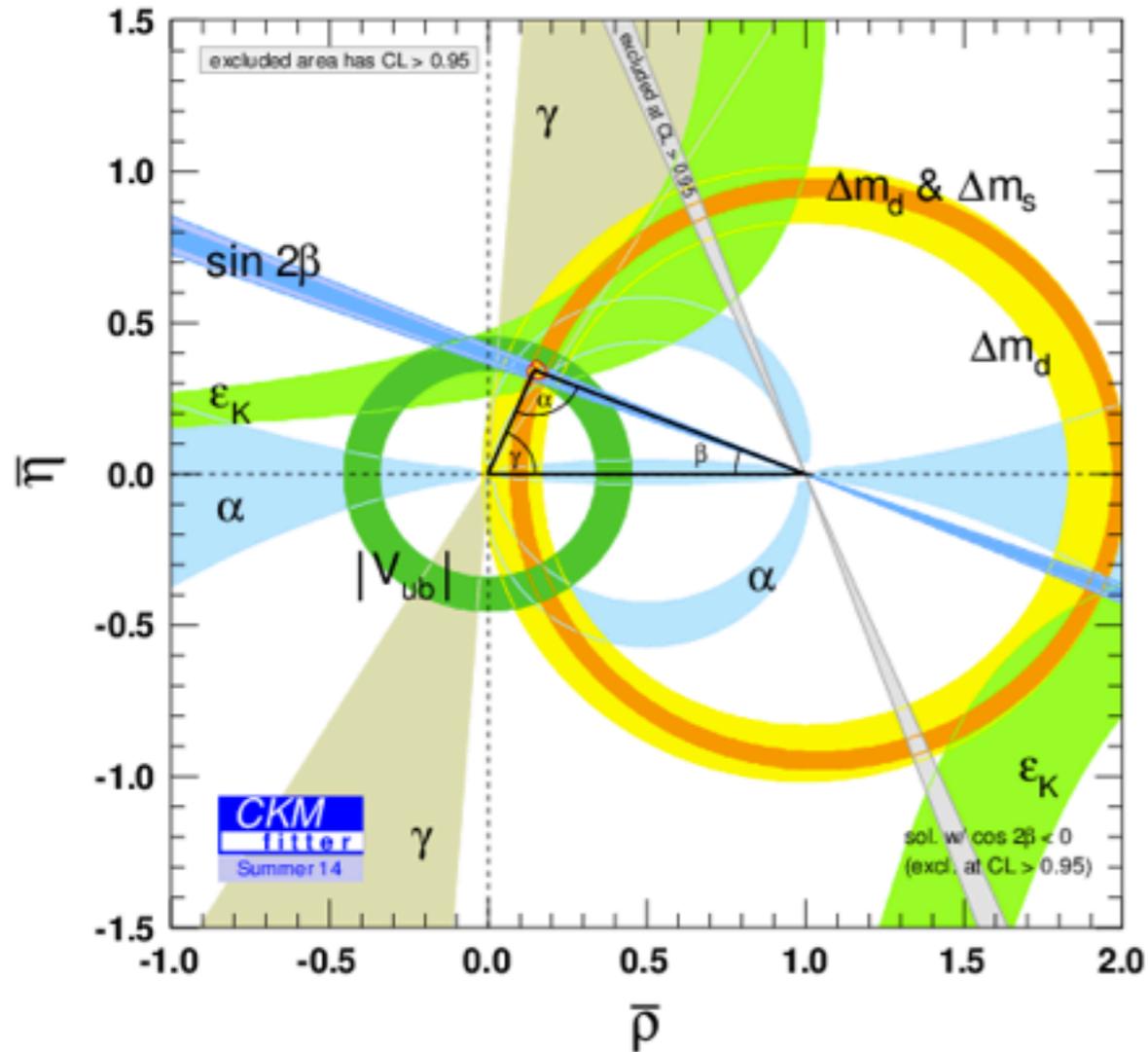
$$H_{\text{eff}}^{\Delta B=2} = \frac{G_F^2 M_W^2}{16\pi^2} (\mathcal{F}_d^0 Q_1^d + \mathcal{F}_s^0 Q_1^s)$$

$$Q_1^q = (\bar{b}\gamma_\mu^L q)(\bar{b}\gamma_\mu^L q) \quad \mathcal{F}_q^0 = \lambda_{tq}^2 S_0(m_t^2/M_W^2), \quad \lambda_{tq} = V_{tq}^* V_{tb}$$

$$\langle B_q^0 | H_{\text{eff}}^{\Delta B=2} | B_q^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} S_0(x_t) \eta_{2B} \lambda_{tq}^2 \langle B_q^0 | Q_{\text{RGI}}^q | B_q^0 \rangle \quad [+ \text{ h.o. OPE}]$$

$$B_{B_q} = \frac{\langle B_q^0 | Q_{\text{RGI}}^q | B_q^0 \rangle}{\frac{8}{3} f_{B_q}^2 m_{B_q}^2}$$

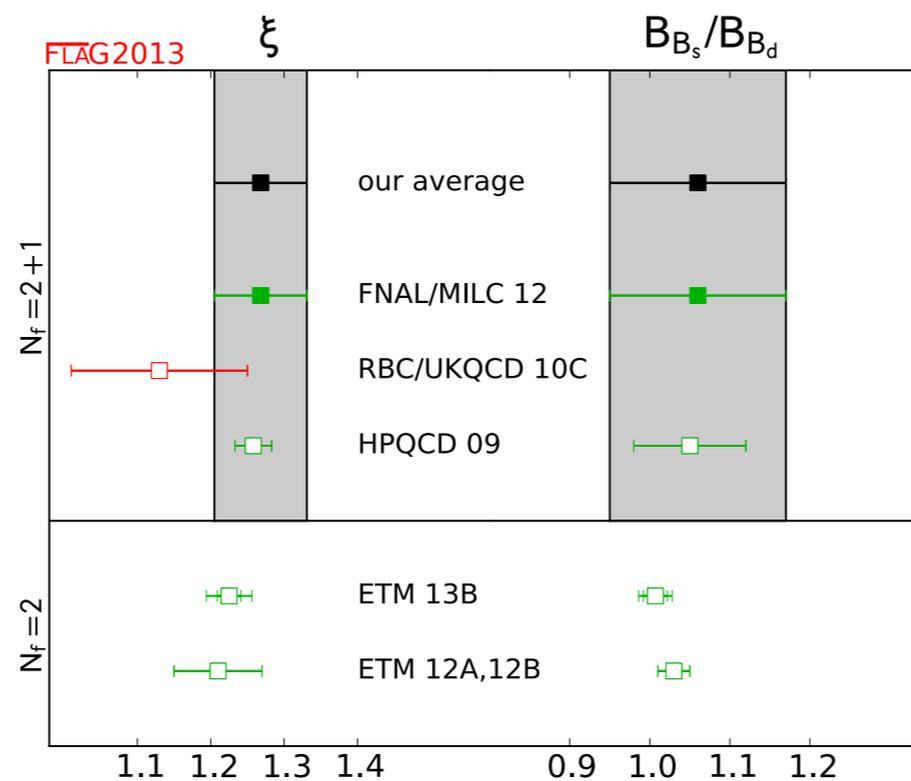
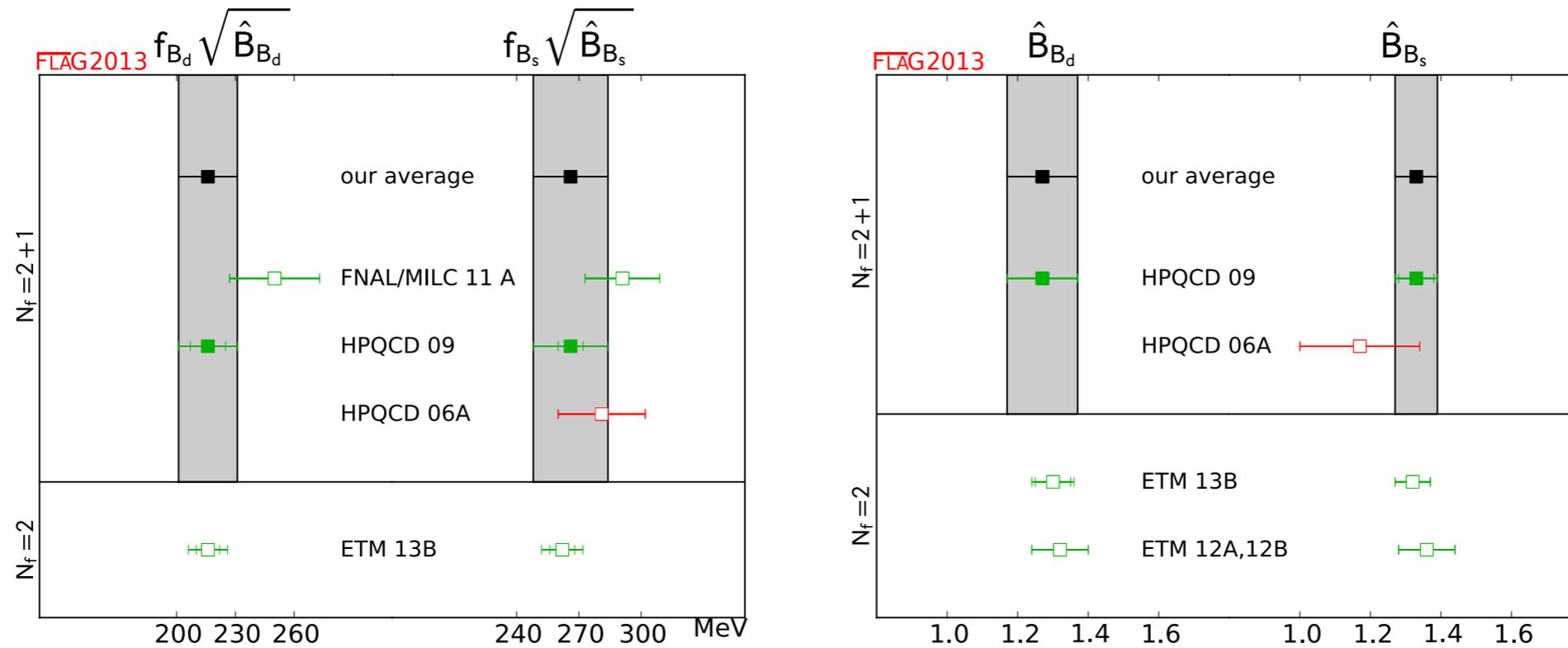
CP violation: B-mixing



$$\langle B_q^0 | H_{\text{eff}}^{\Delta B=2} | B_q^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} S_0(x_t) \eta_{2B} \lambda_{tq}^2 \langle B_q^0 | Q_{\text{RGI}}^q | B_q^0 \rangle \quad [+ \text{ h.o. OPE}]$$

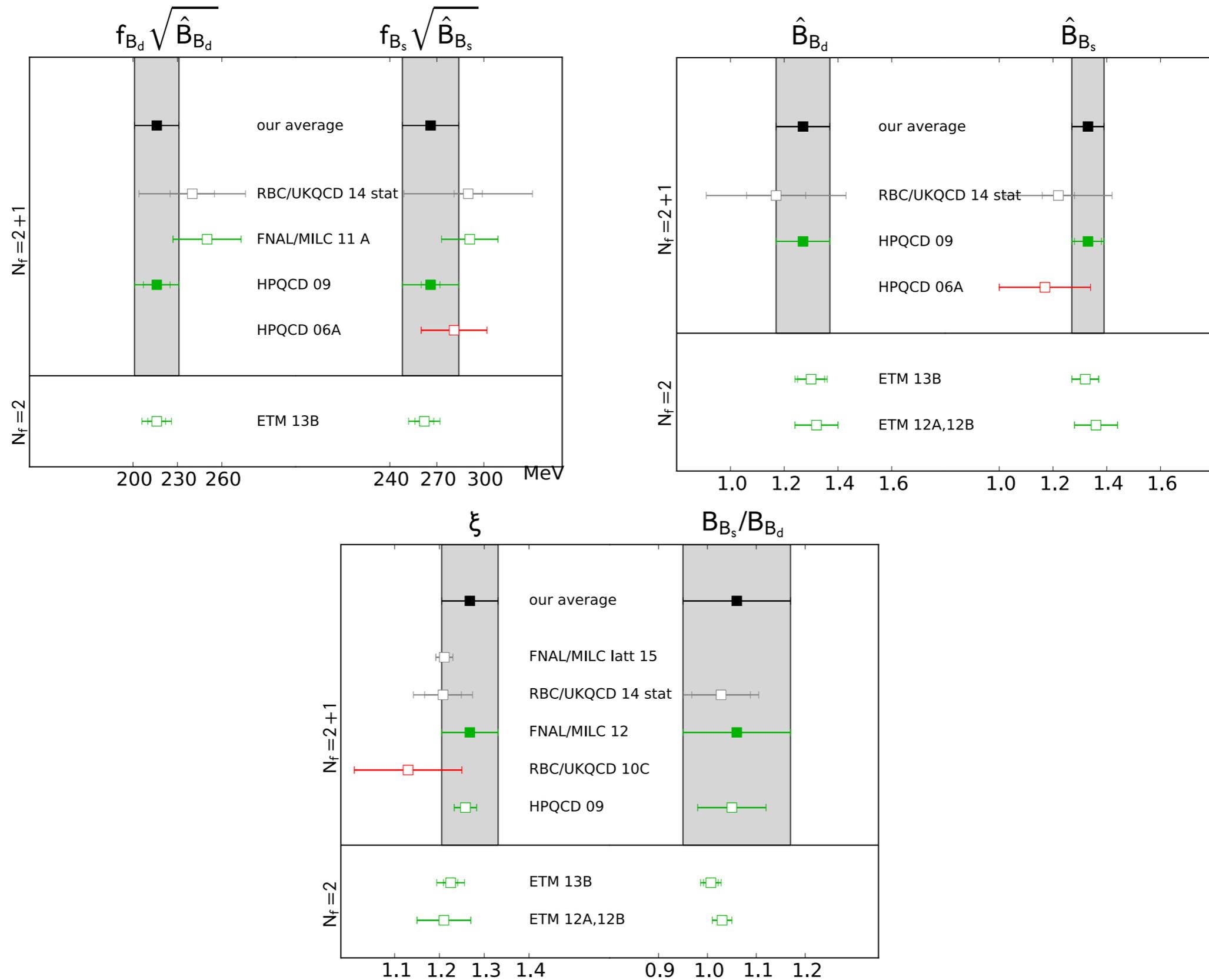
n.b.: in this case one is interested in constraining the apex position, so a priori knowledge of CKM's is needed [$\Delta(\lambda_{tq}^2) \approx 7-8\%$]

FLAG-2 on B-mixing



new results for B-mixing

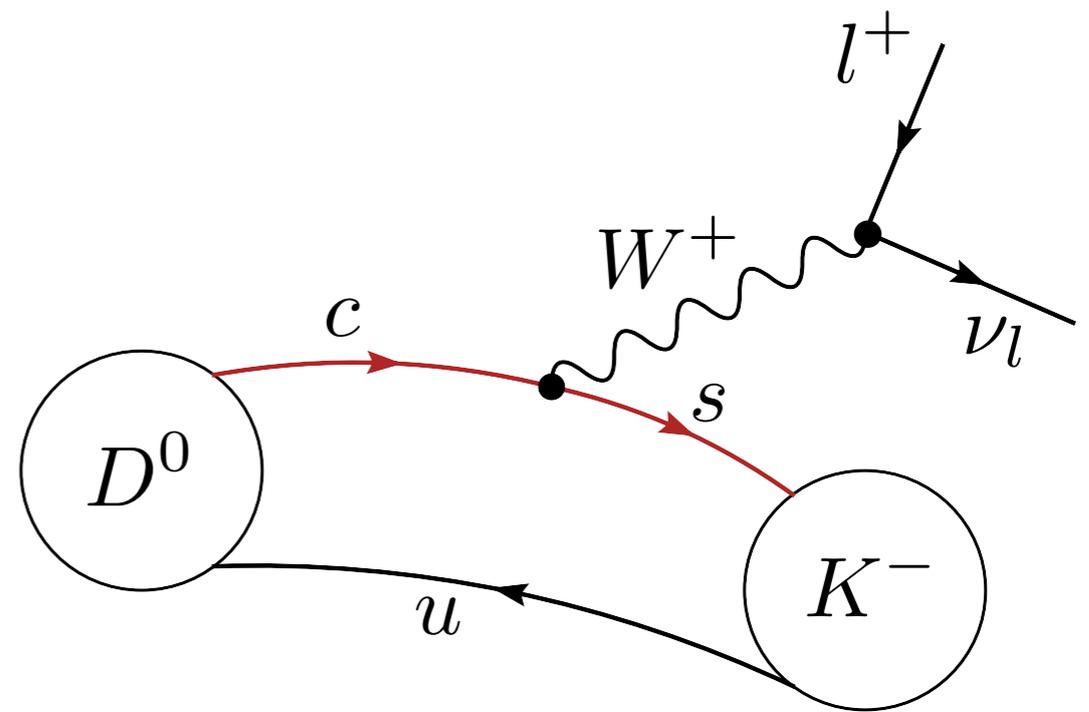
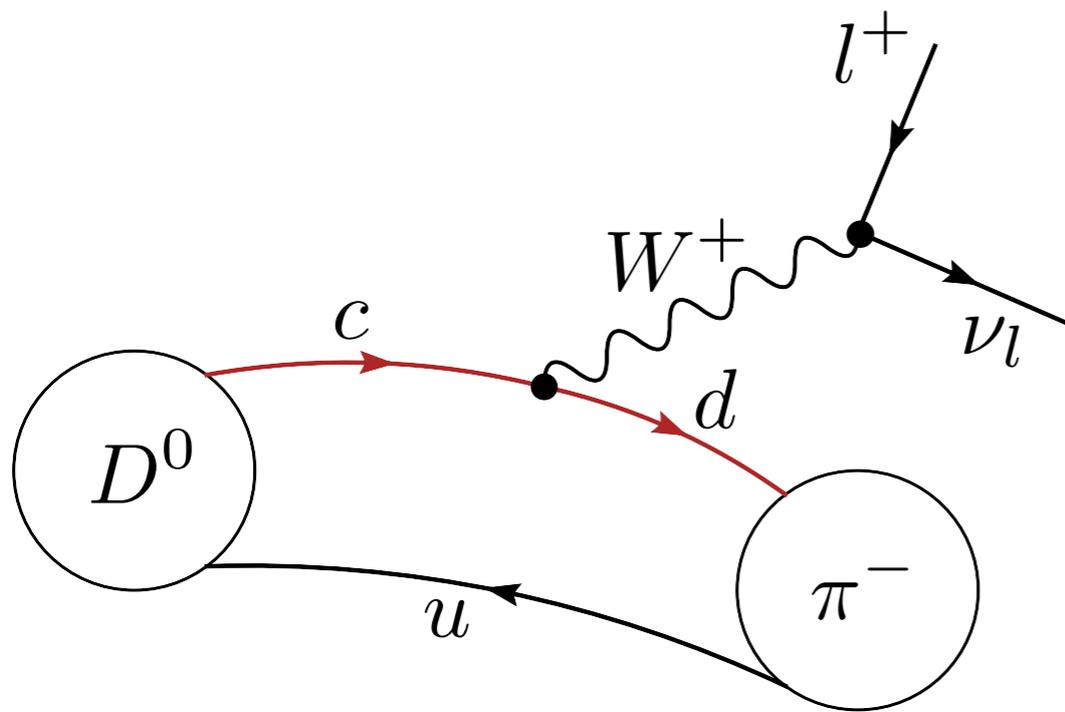
new results superimposed on FLAG-2 summary plots:



plan

- methods: where we stand (brief!)
 - ensembles used in HQ physics, reach
 - HQ approaches
- brief overview of
 - leptonic charm and B decay
 - B mixing
- charm and B semileptonic decays (+ CKM 2nd and 3rd rows)
- the % precision target
- conclusions and outlook

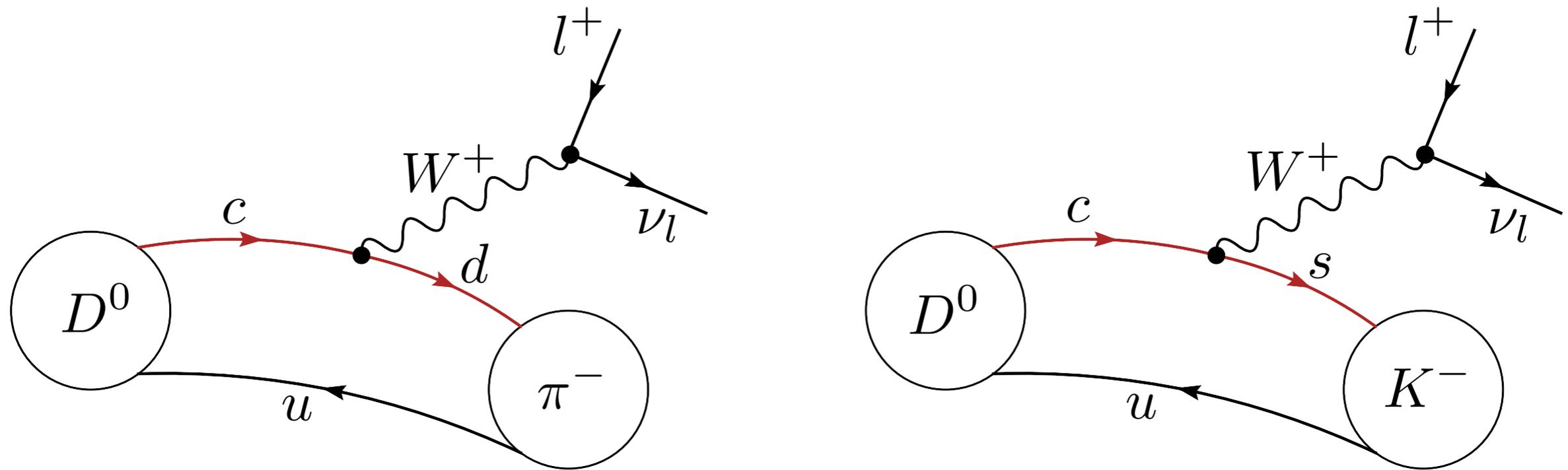
charm semileptonic decay



$$\frac{d\Gamma(D \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2 |V_{cq}|^2 (q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{24\pi^3 q^4 m_D^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} (m_D^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

$$\langle P(p') | \bar{c} \gamma_\mu q | D_q(p) \rangle = f_+(q^2) \left(p_\mu + p'_\mu - \frac{m_{D_q}^2 - m_P^2}{q^2} q_\mu \right) + f_0(q^2) \frac{m_{D_q}^2 - m_P^2}{q^2} q_\mu, \quad q = p - p'$$

charm semileptonic decay

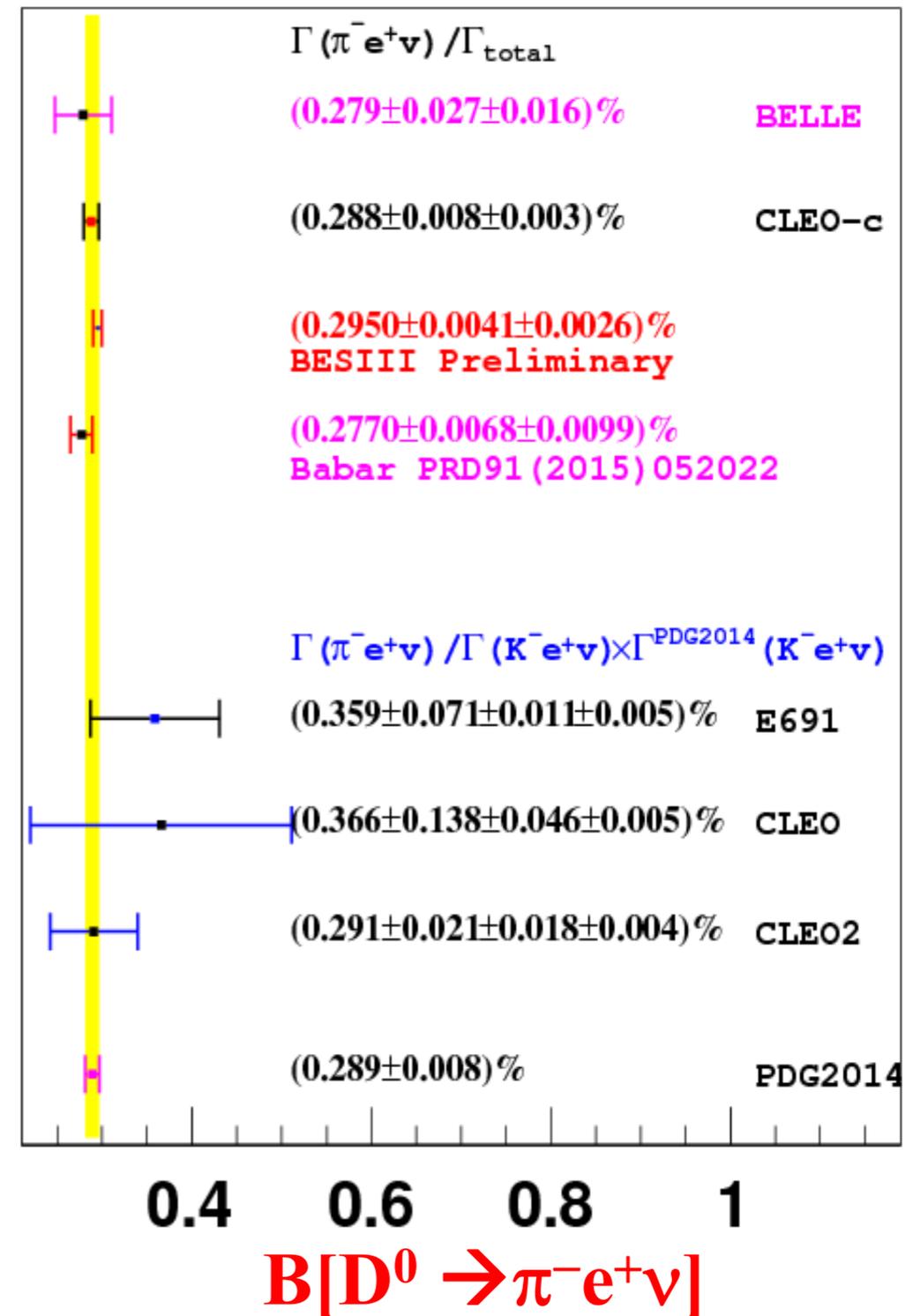
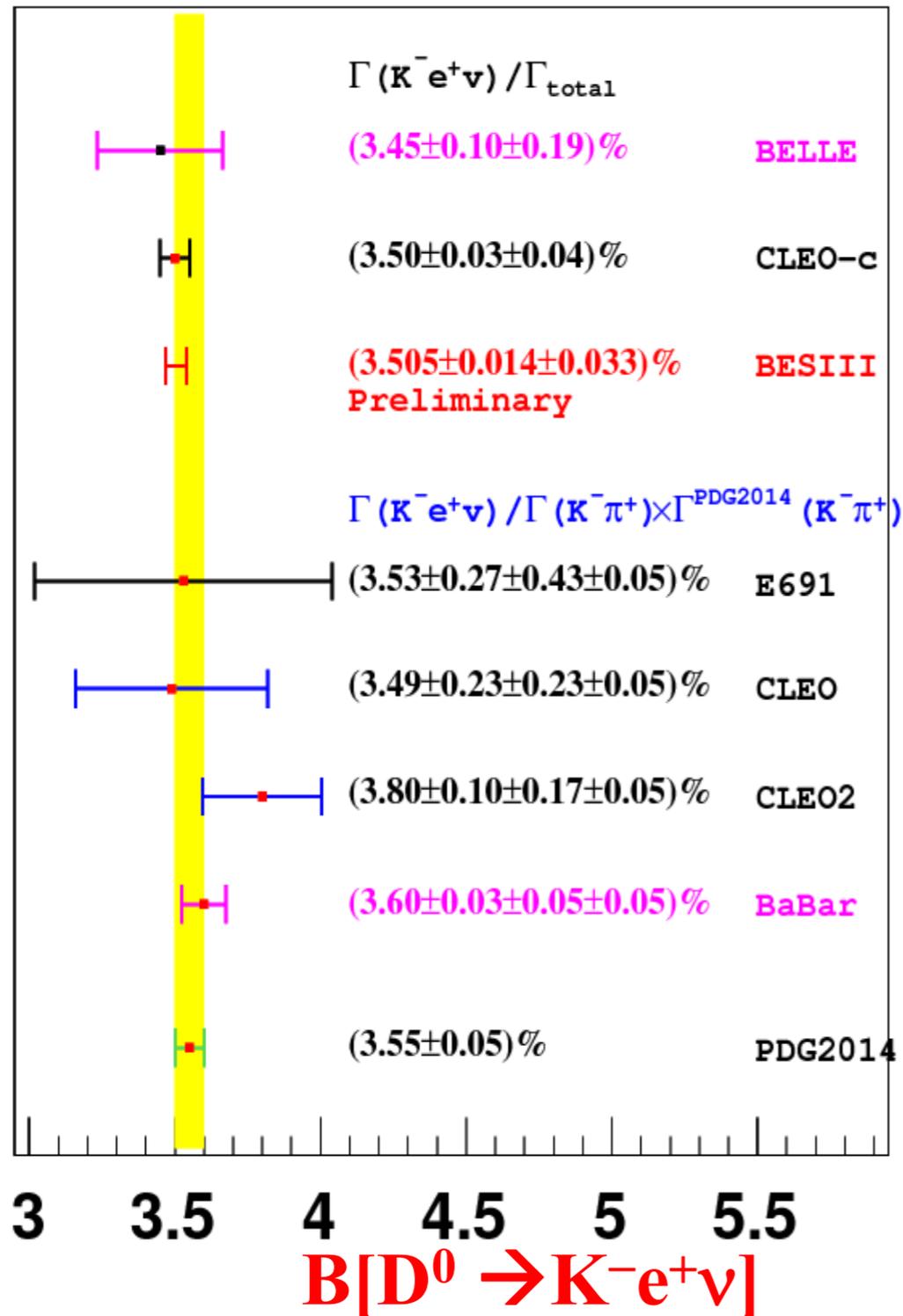


$$\frac{d\Gamma(D \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2 |V_{cq}|^2 (q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{24\pi^3 q^4 m_D^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} (m_D^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

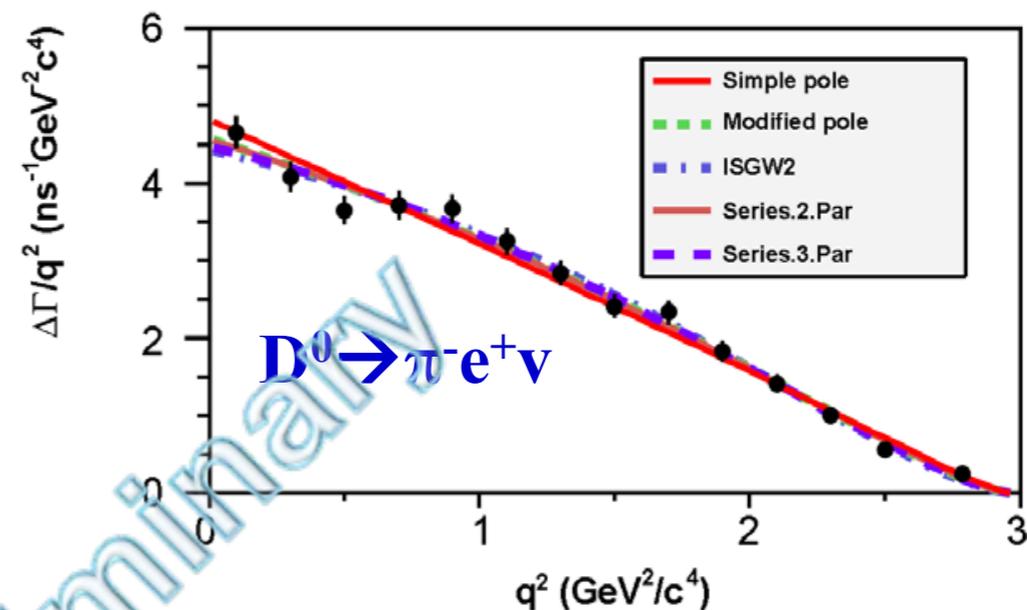
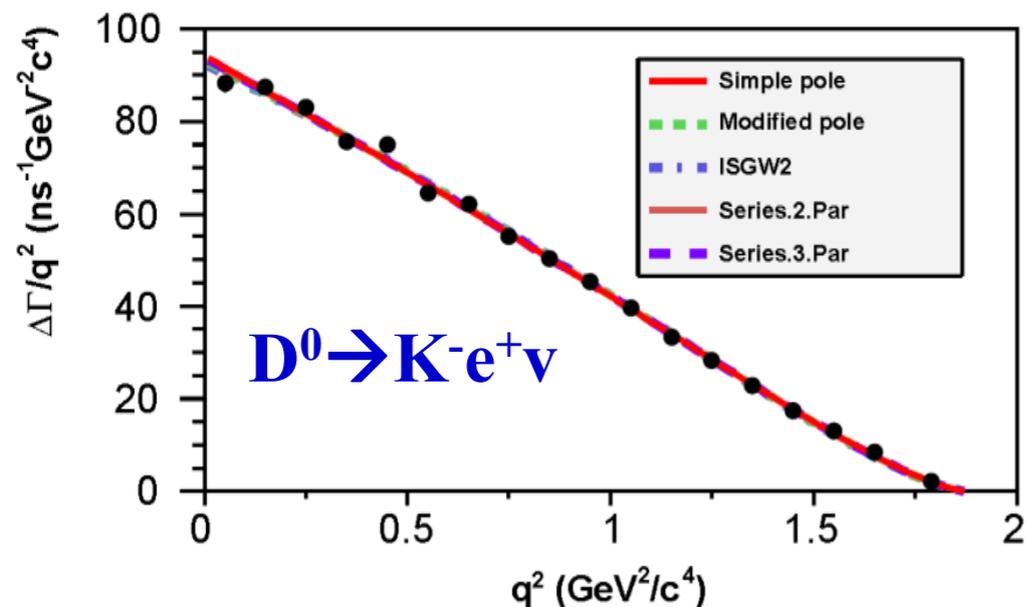
e, μ suppressed

uncertainties from kinematical factors / neglected h.o. OPE at the permille level

charm semileptonic decay

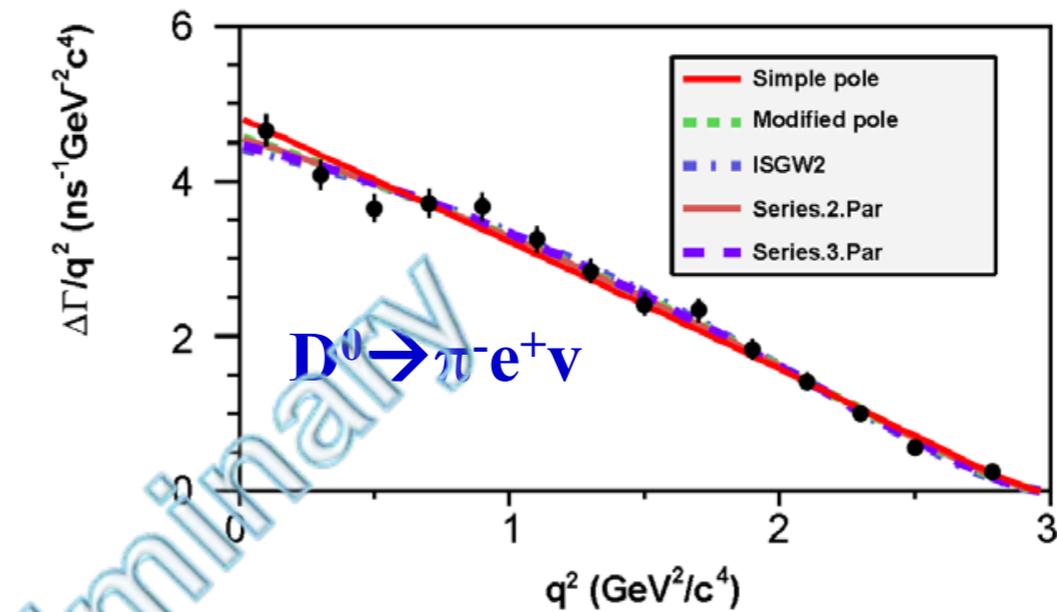
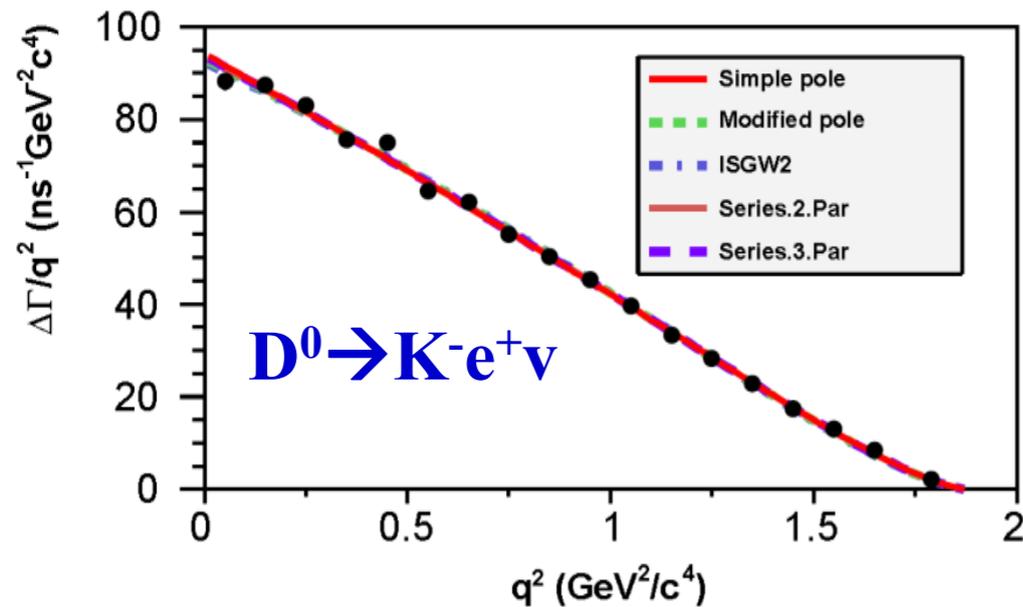


charm semileptonic decay



		$D^0 \rightarrow K^- e^+ \nu$		$D^0 \rightarrow \pi^- e^+ \nu$
Simple Pole	$f_K^+(0) V_{cs} $	$0.7209 \pm 0.0022 \pm 0.0033$	$f_\pi^+(0) V_{cd} $	$0.1475 \pm 0.0014 \pm 0.0005$
	M_{pole}	$1.9207 \pm 0.0103 \pm 0.0069$	M_{pole}	$1.9114 \pm 0.0118 \pm 0.0038$
Mod. Pole	$f_K^+(0) V_{cs} $	$0.7163 \pm 0.0024 \pm 0.0034$	$f_\pi^+(0) V_{cd} $	$0.1437 \pm 0.0017 \pm 0.0008$
	α	$0.3088 \pm 0.0195 \pm 0.0129$	α	$0.2794 \pm 0.0345 \pm 0.0113$
ISGW2	$f_K^+(0) V_{cs} $	$0.7139 \pm 0.0023 \pm 0.0034$	$f_\pi^+(0) V_{cd} $	$0.1415 \pm 0.0016 \pm 0.0006$
	r_{ISGW2}	$1.6000 \pm 0.0141 \pm 0.0091$	r_{ISGW2}	$2.0688 \pm 0.0394 \pm 0.0124$
Series.2.Par	$f_K^+(0) V_{cs} $	$0.7172 \pm 0.0025 \pm 0.0035$	$f_\pi^+(0) V_{cd} $	$0.1435 \pm 0.0018 \pm 0.0009$
	r_1	$-2.2278 \pm 0.0864 \pm 0.0575$	r_1	$-2.0365 \pm 0.0807 \pm 0.0260$
Series.3.Par	$f_K^+(0) V_{cs} $	$0.7196 \pm 0.0035 \pm 0.0041$	$f_\pi^+(0) V_{cd} $	$0.1420 \pm 0.0024 \pm 0.0010$
	r_1	$-2.3331 \pm 0.1587 \pm 0.0804$	r_1	$-1.8434 \pm 0.2212 \pm 0.0690$
	r_2	$3.4223 \pm 3.9090 \pm 2.4092$	r_2	$-1.3871 \pm 1.4615 \pm 0.4677$

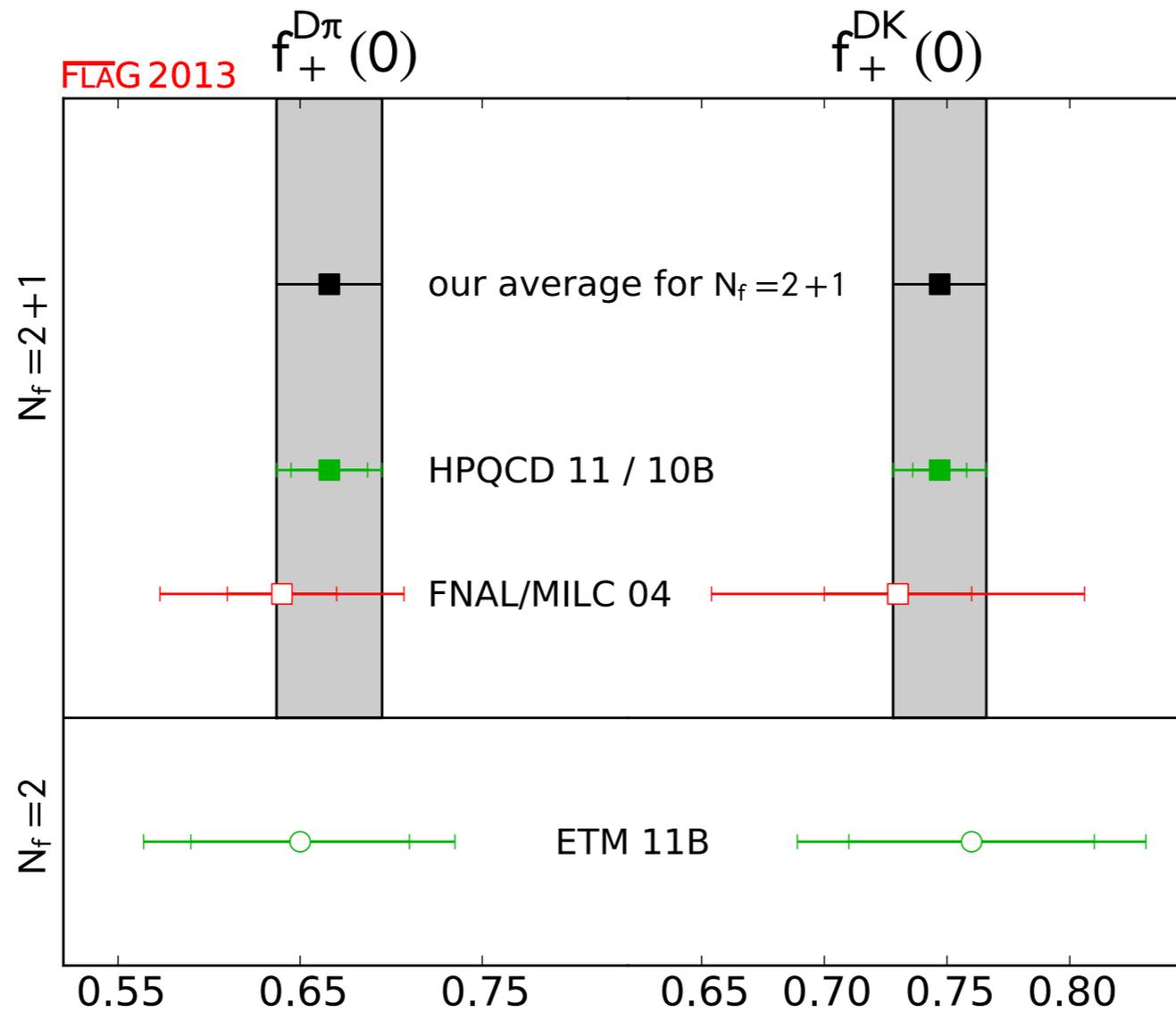
charm semileptonic decay



low q^2 region accessible to lattice computations \Rightarrow CKM can be determined by computing form factors at zero momentum

experimental precision increasing, parametrisation dependence of experimental result for $q^2=0$ relevant: **need to start worrying about q^2 dependence** (which provides a stronger SM test anyway!)

FLAG-2 on charm semileptonic decay



	$f_+^{D\pi}(0)$	$f_+^{DK}(0)$
2+1	0.666(29)	0.747(19)

new results for charm SL form factors

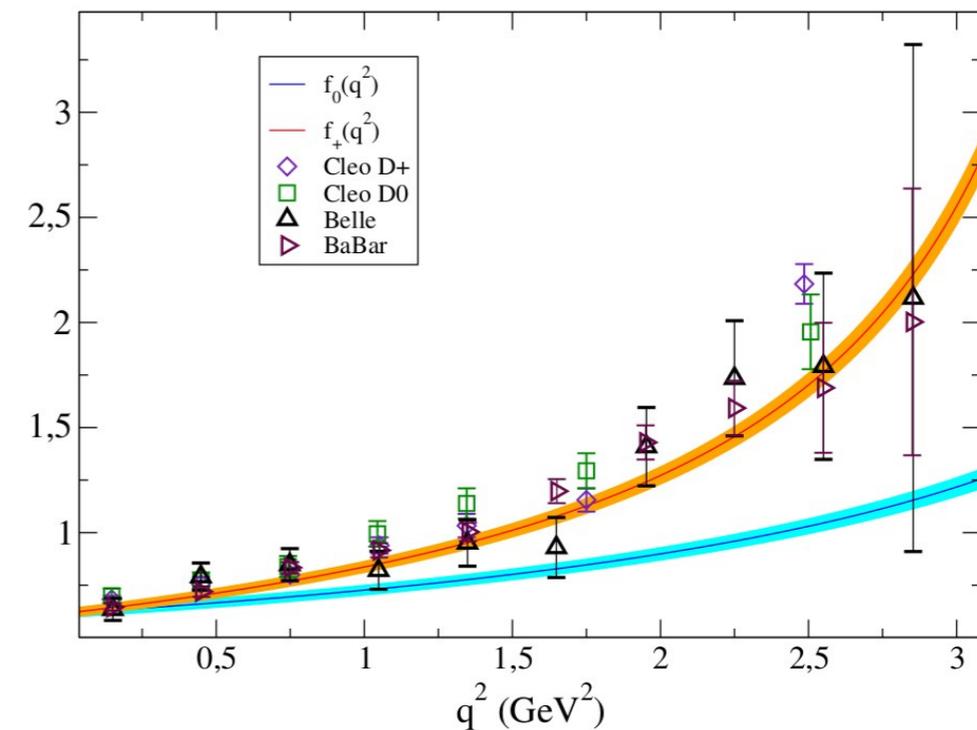
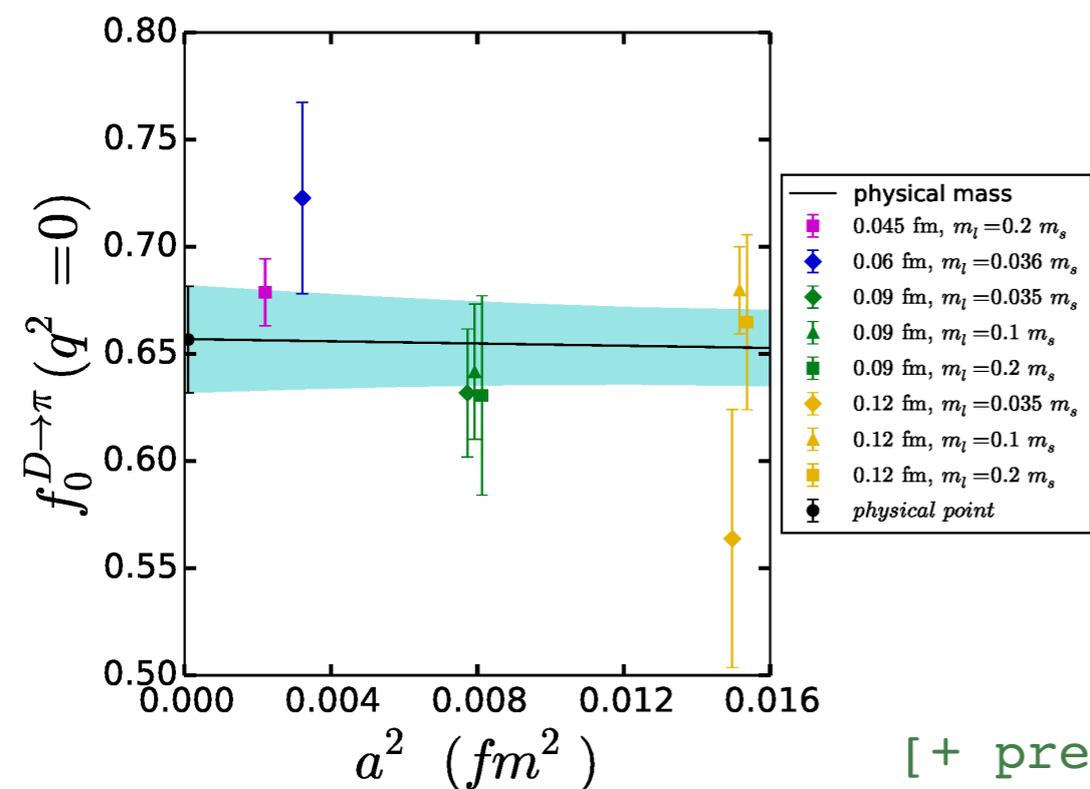
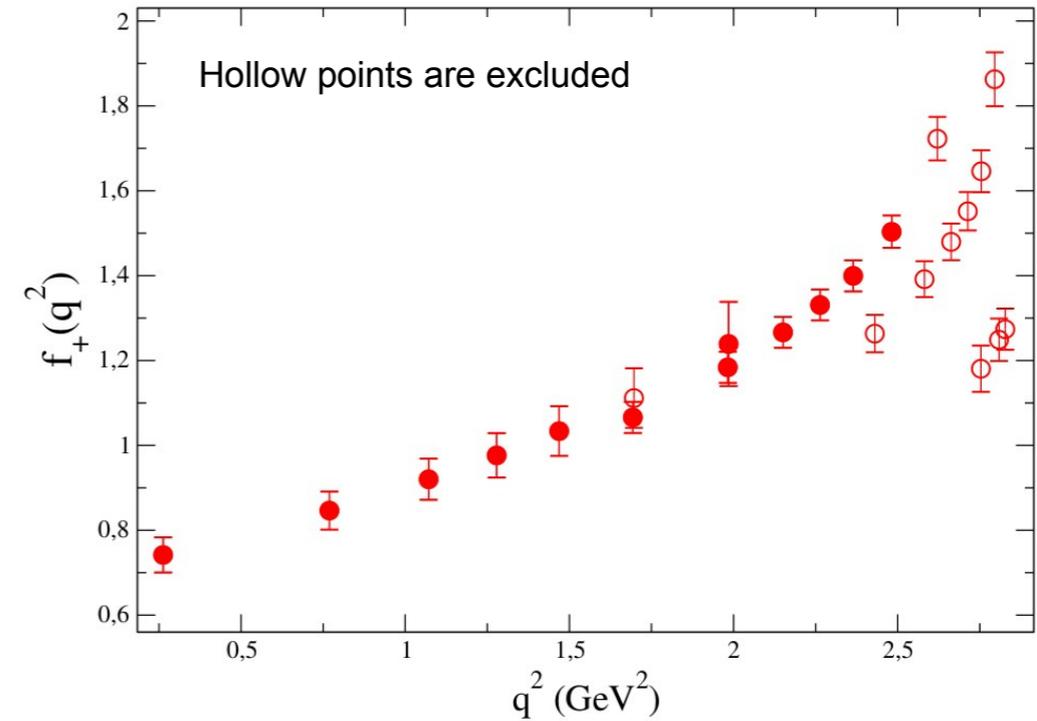
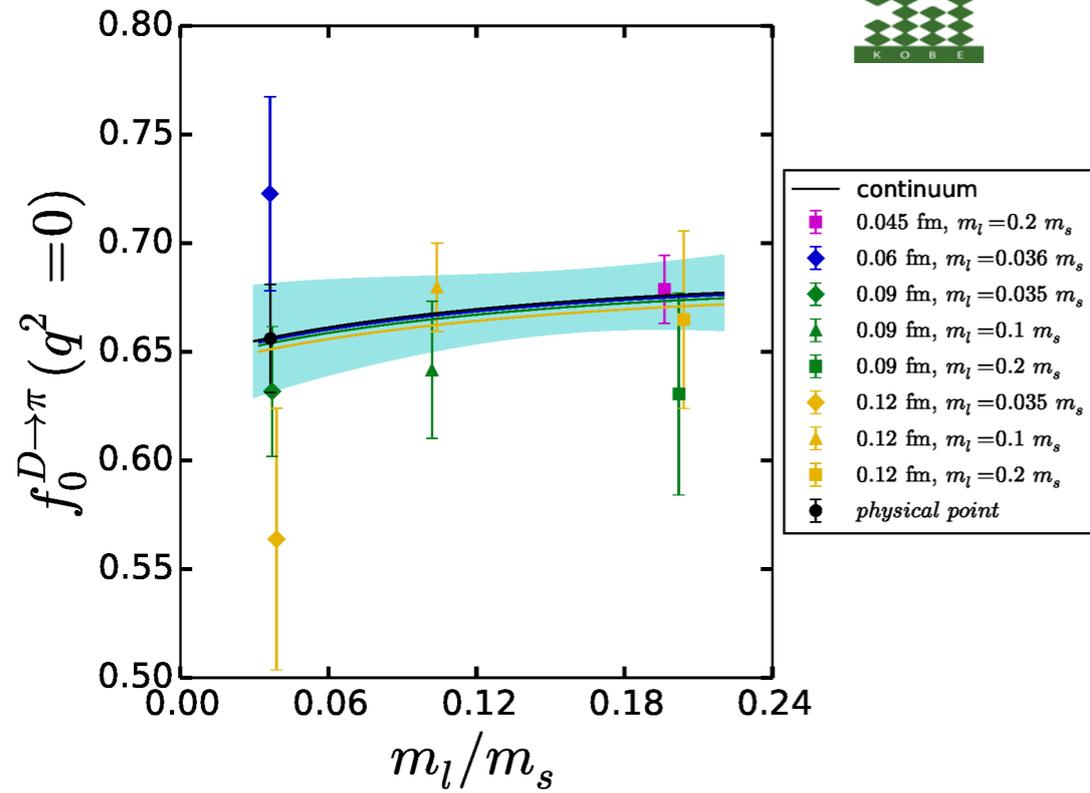
[FNAL/MILC arXiv:1411.1651]

[T Primer, Thu 11:40]



[P Lami, Wed 14:20]

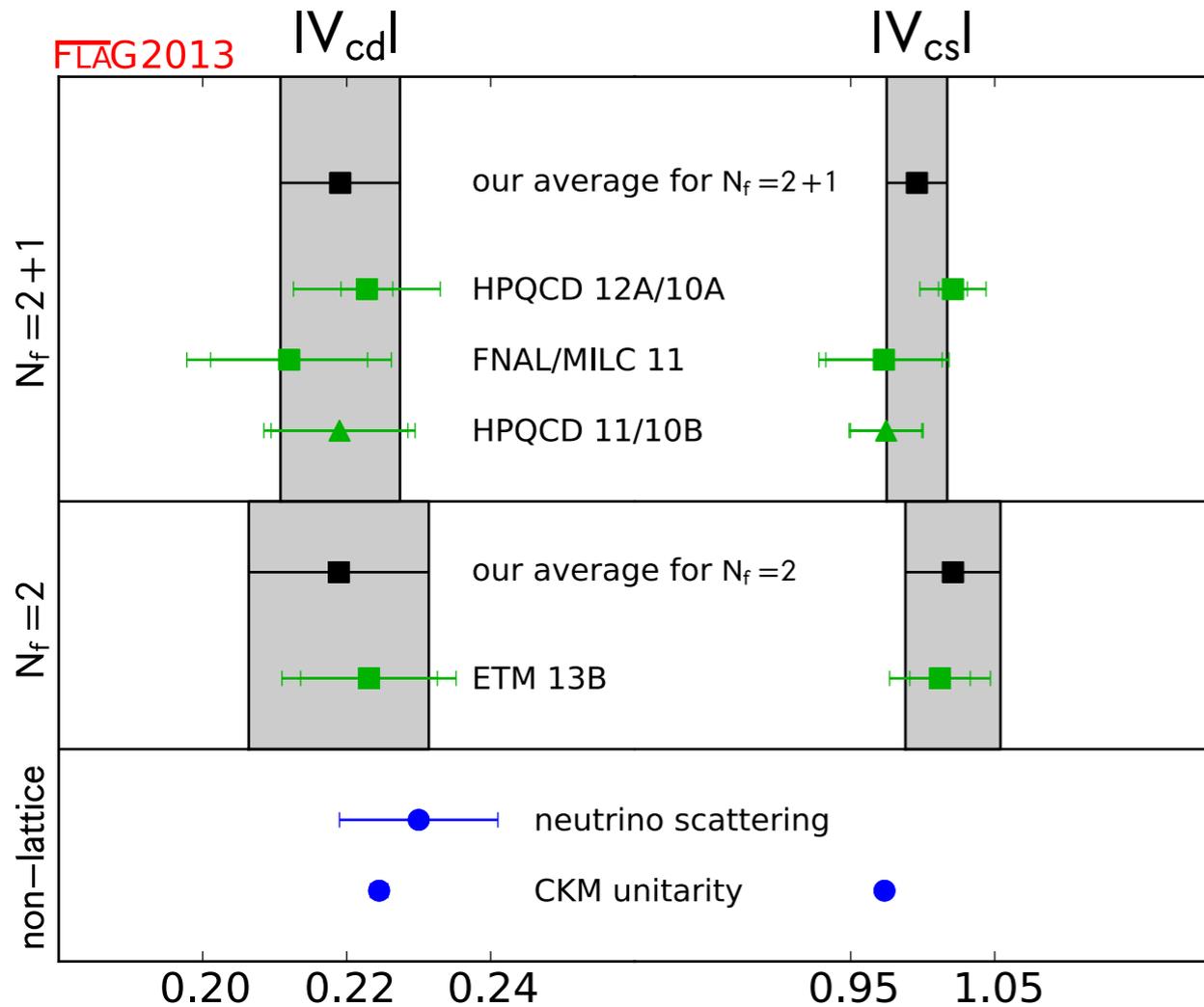
$\beta = 1.90$ $L=24$ $\mu = 0.0060$



[+ preliminary work by JLQCD, cf. T Suzuki's talk]



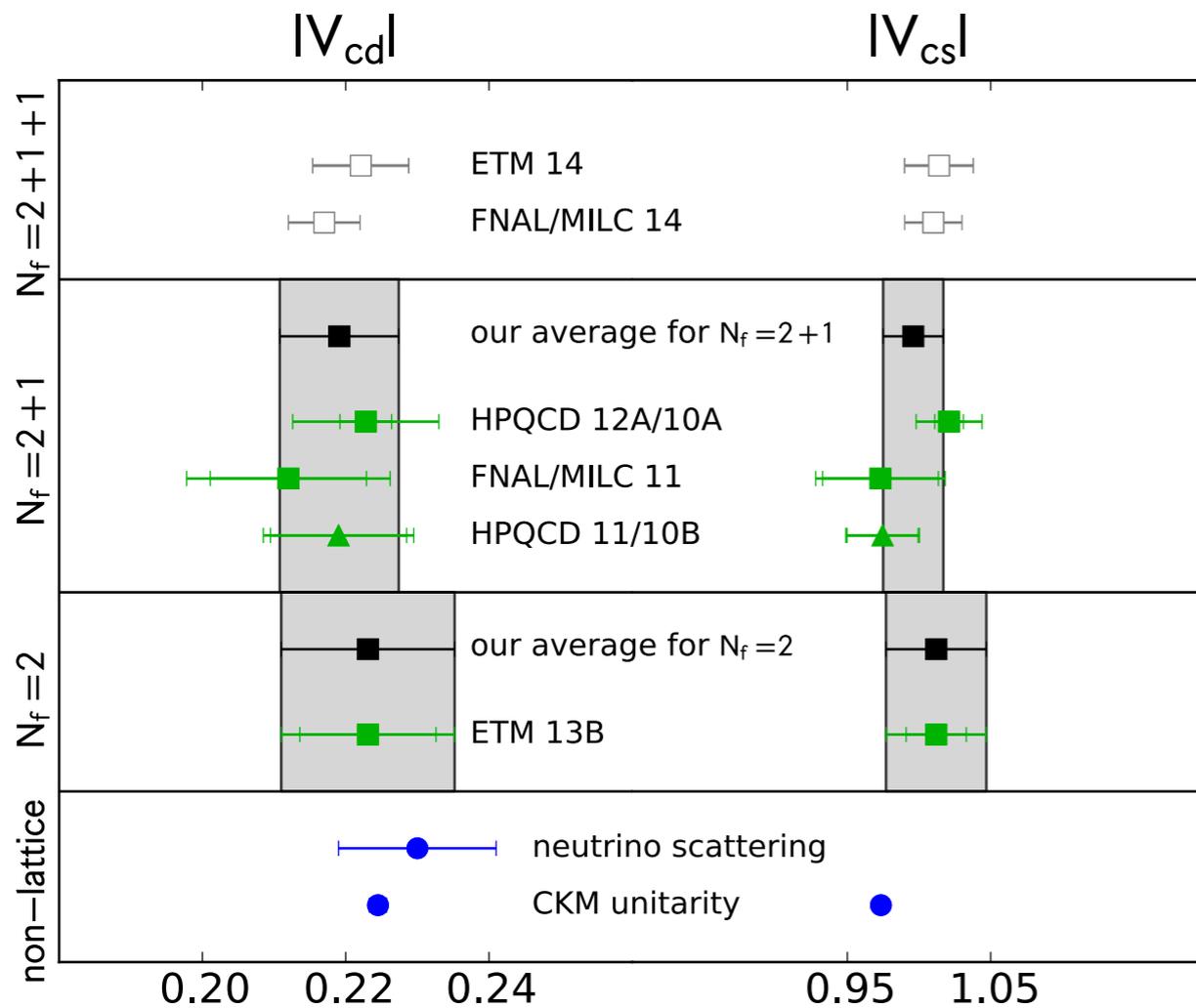
$|V_{cd}|, |V_{cs}|$, 2nd row CKM unitarity



$$|V_{cd}|^2 + |V_{cs}|^2 + |\cancel{V_{cb}}|^2 - 1 = 0.04(6)$$

averaged leptonic (more precise) and semileptonic determinations

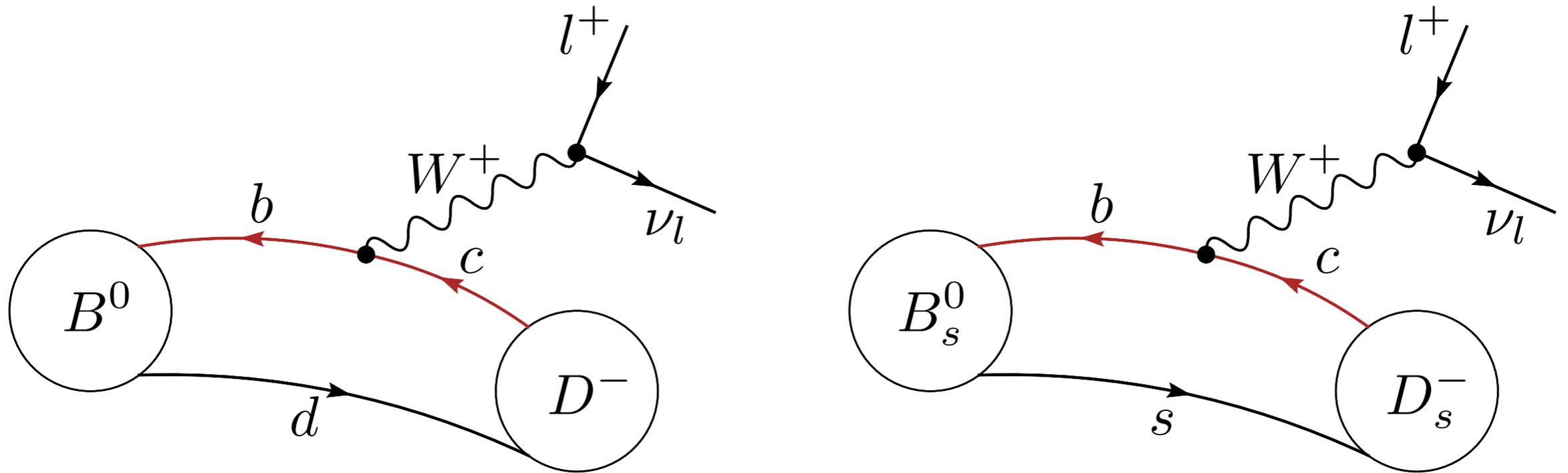
$|V_{cd}|$, $|V_{cs}|$, 2nd row CKM unitarity



$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.04(6)$$

precise semileptonic determination will be interesting, sensitivity to $|V_{cb}|$ around the corner

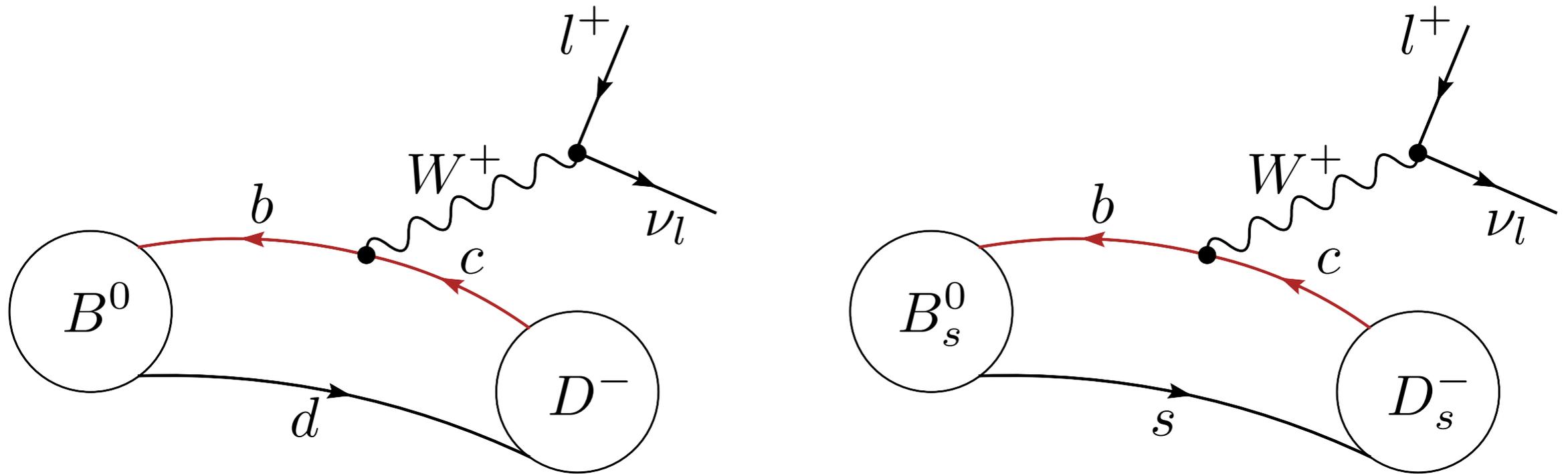
B semileptonic decay: $|V_{cb}|$



$$\frac{d\Gamma(B_{(s)} \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

$$\langle P(p') | \bar{b} \gamma_\mu q | B_q(p) \rangle = f_+(q^2) \left(p_\mu + p'_\mu - \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu \right) + f_0(q^2) \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu, \quad q = p - p'$$

B semileptonic decay: $|V_{cb}|$

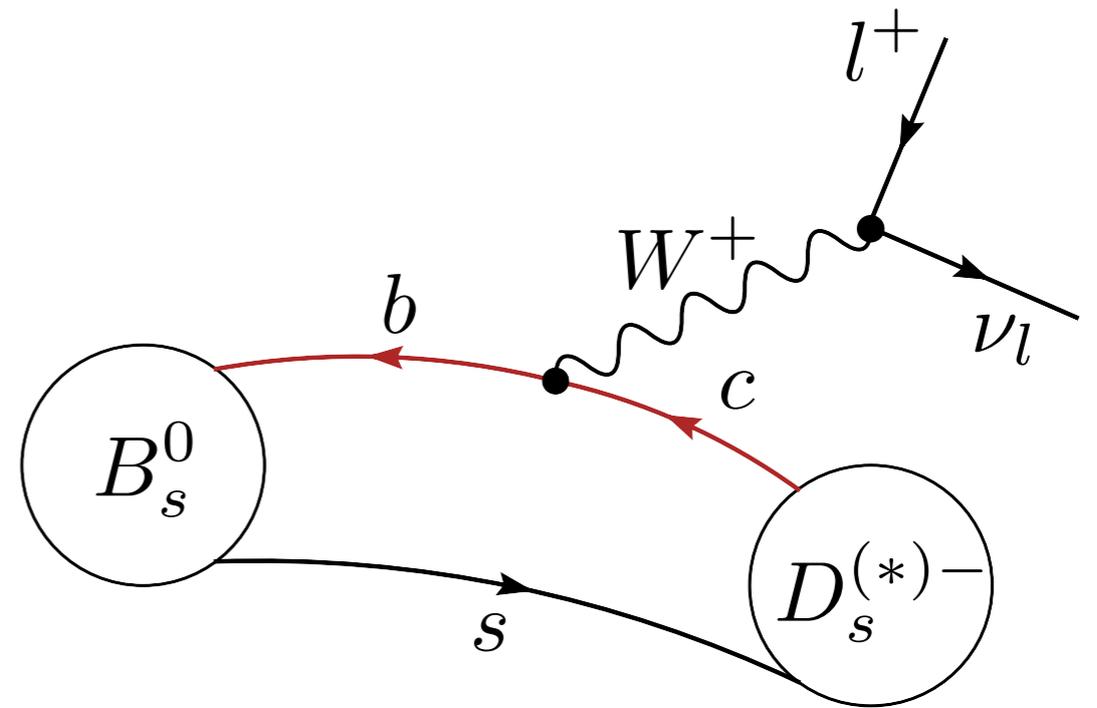
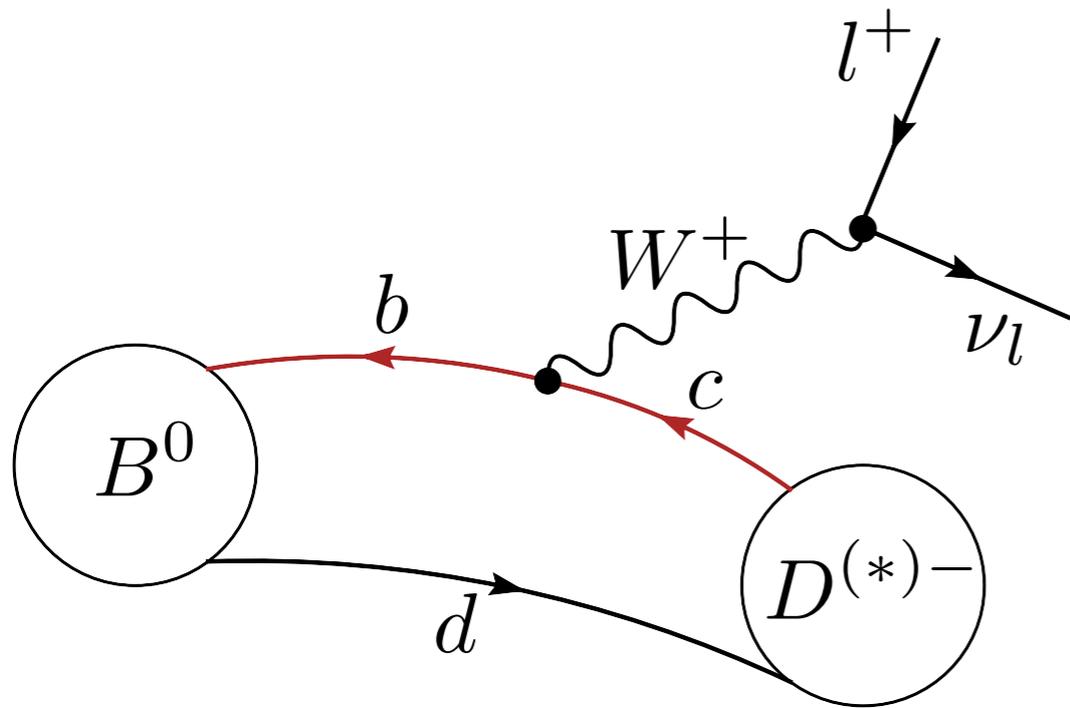


$$\frac{d\Gamma(B_{(s)} \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

e, μ suppressed

uncertainties from kinematical factors / neglected h.o. OPE at the permille level

B semileptonic decay: $|V_{cb}|$



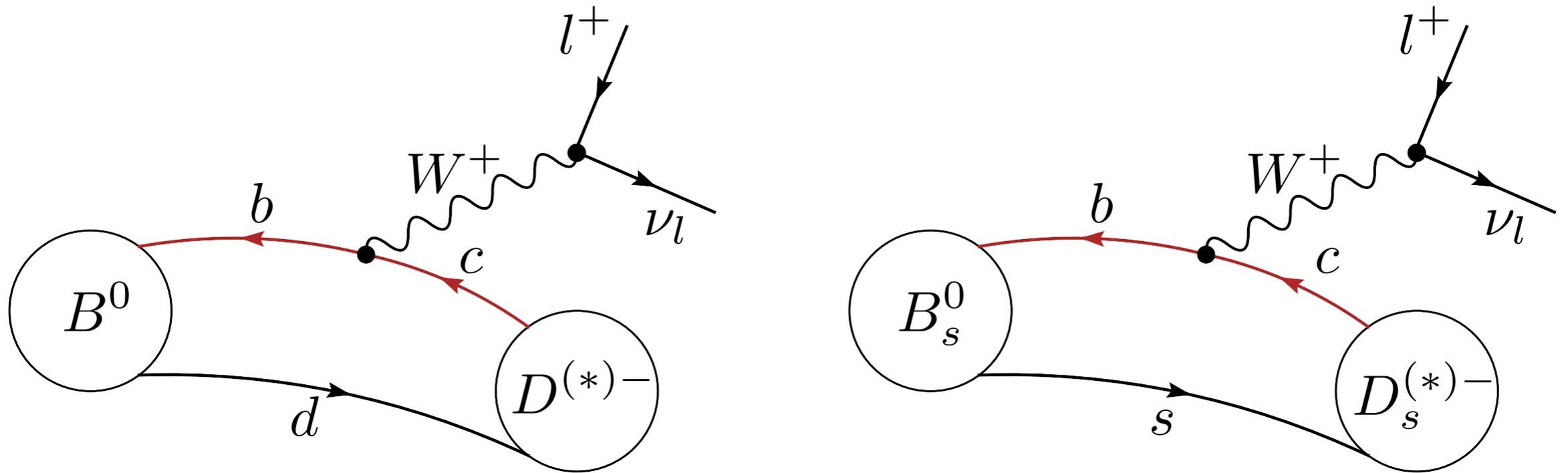
$$\frac{d\Gamma(B \rightarrow D l \nu_l)}{dw} = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 (w^2 - 1)^{3/2} |\eta_{EW}|^2 |V_{cb}|^2 |\mathcal{G}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$\frac{d\Gamma(B \rightarrow D^* l \nu_l)}{dw} = \frac{G_F^2}{4\pi^3} (m_B - m_{D^*})^2 (w^2 - 1)^{1/2} |\eta_{EW}|^2 \chi(w) |V_{cb}|^2 |\mathcal{F}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$w = \frac{p_B \cdot p_{D^{(*)}}}{m_B m_{D^{(*)}}}$$

$$\mathcal{G}(w) = \frac{4 \frac{m_D}{m_B}}{1 + \frac{m_D}{m_B}} f_+(q^2) \quad \text{etc}$$

B semileptonic decay: $|V_{cb}|$



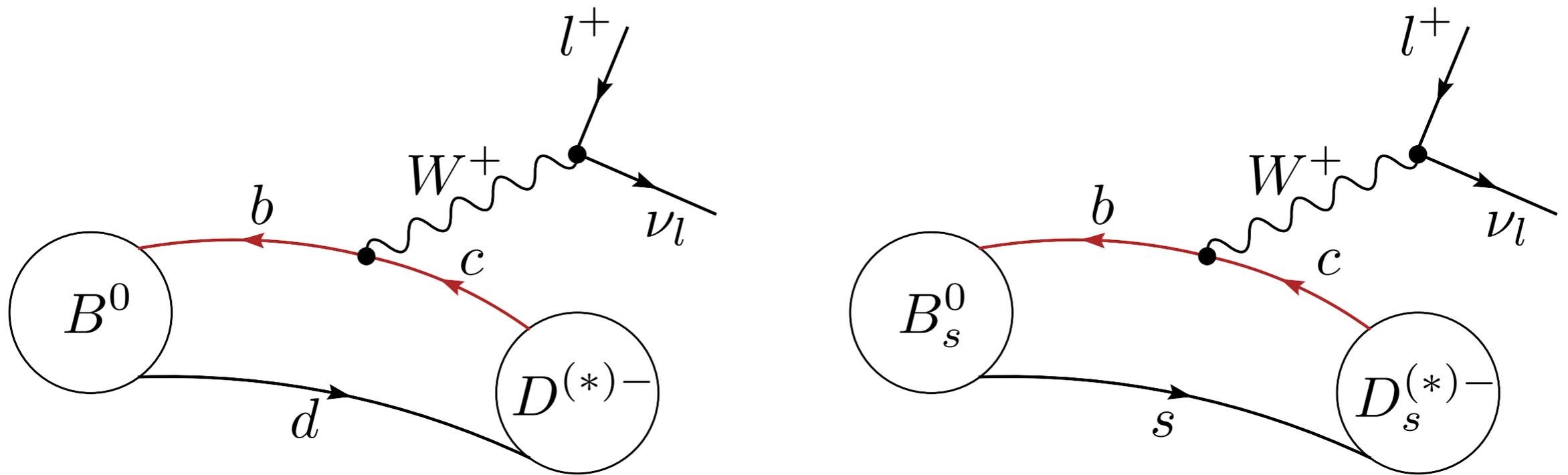
$$\frac{d\Gamma(B \rightarrow D l \nu_l)}{dw} = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 (w^2 - 1)^{3/2} |\eta_{EW}|^2 |V_{cb}|^2 |\mathcal{G}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

$$\frac{d\Gamma(B \rightarrow D^* l \nu_l)}{dw} = \frac{G_F^2}{4\pi^3} (m_B - m_{D^*})^2 (w^2 - 1)^{1/2} |\eta_{EW}|^2 \chi(w) |V_{cb}|^2 |\mathcal{F}(w)|^2 + \mathcal{O}\left(\frac{m_l^2}{q^2}\right)$$

low recoil region accessible to lattice computations \Rightarrow CKM can be determined by computing form factors at $w=1$

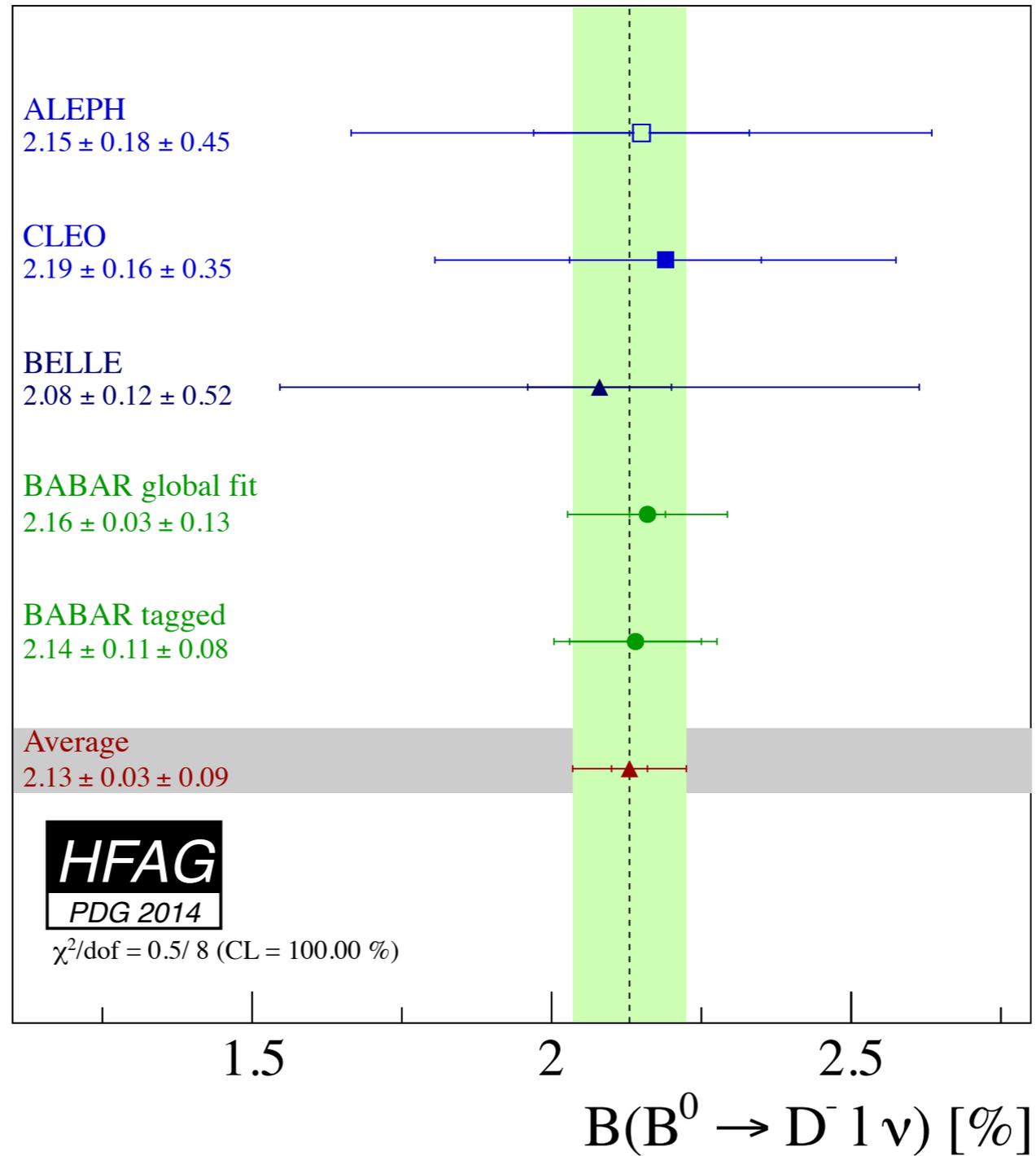
shape information relevant as precision increases

B semileptonic decay: $|V_{cb}|$

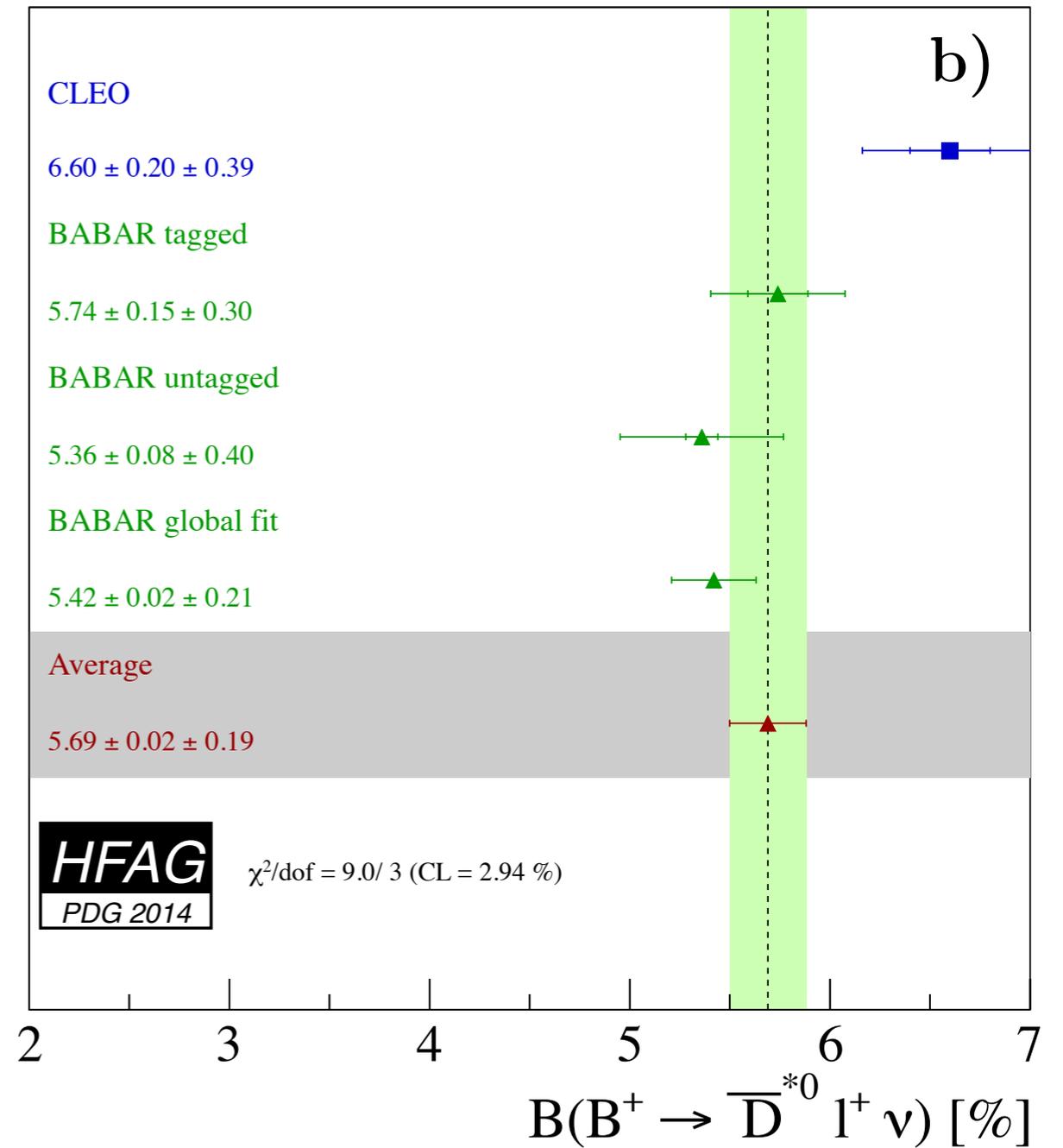
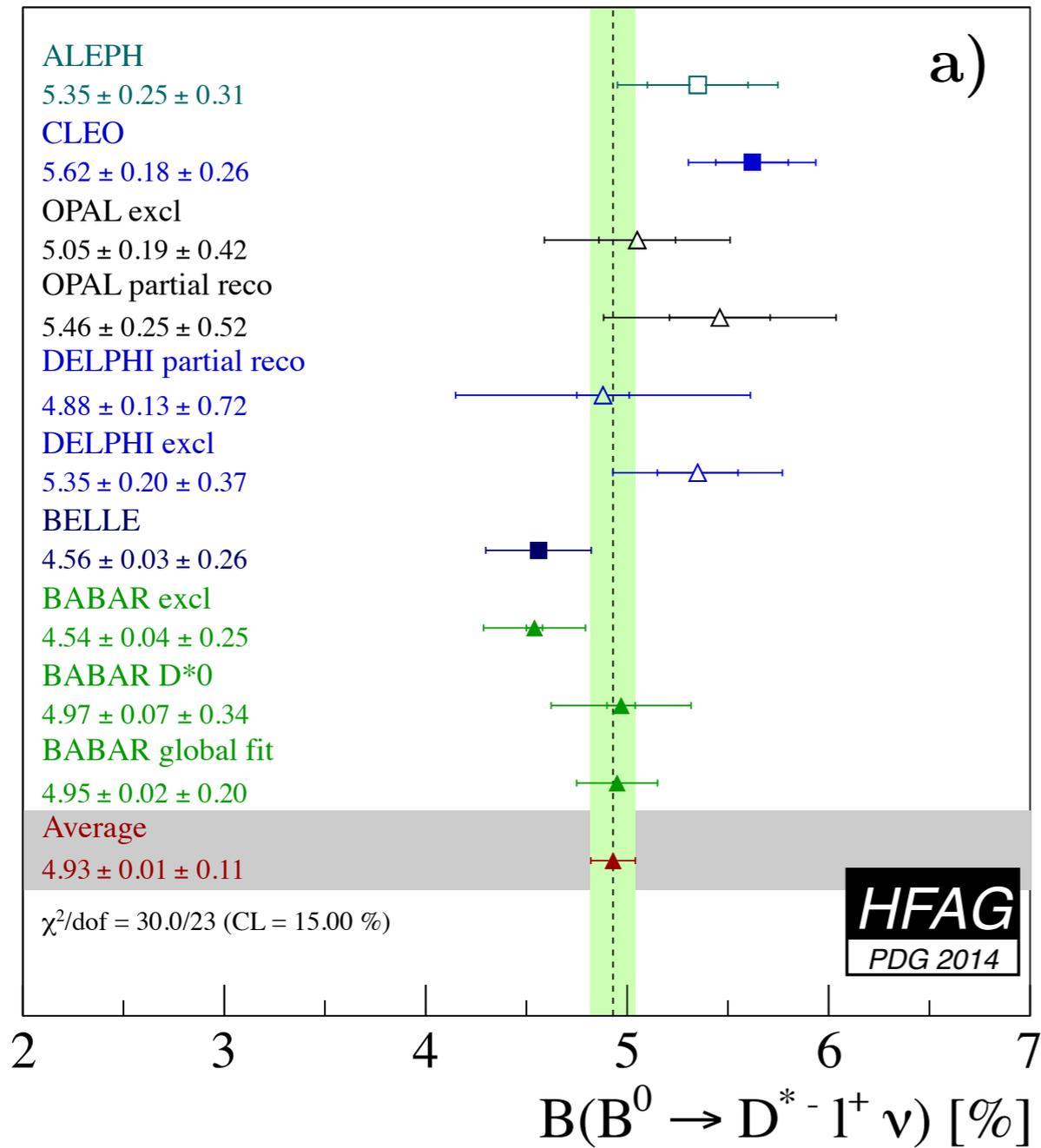


$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu_\ell)} \quad [\longrightarrow f_0(q^2)]$$

B semileptonic decay: $|V_{cb}|$

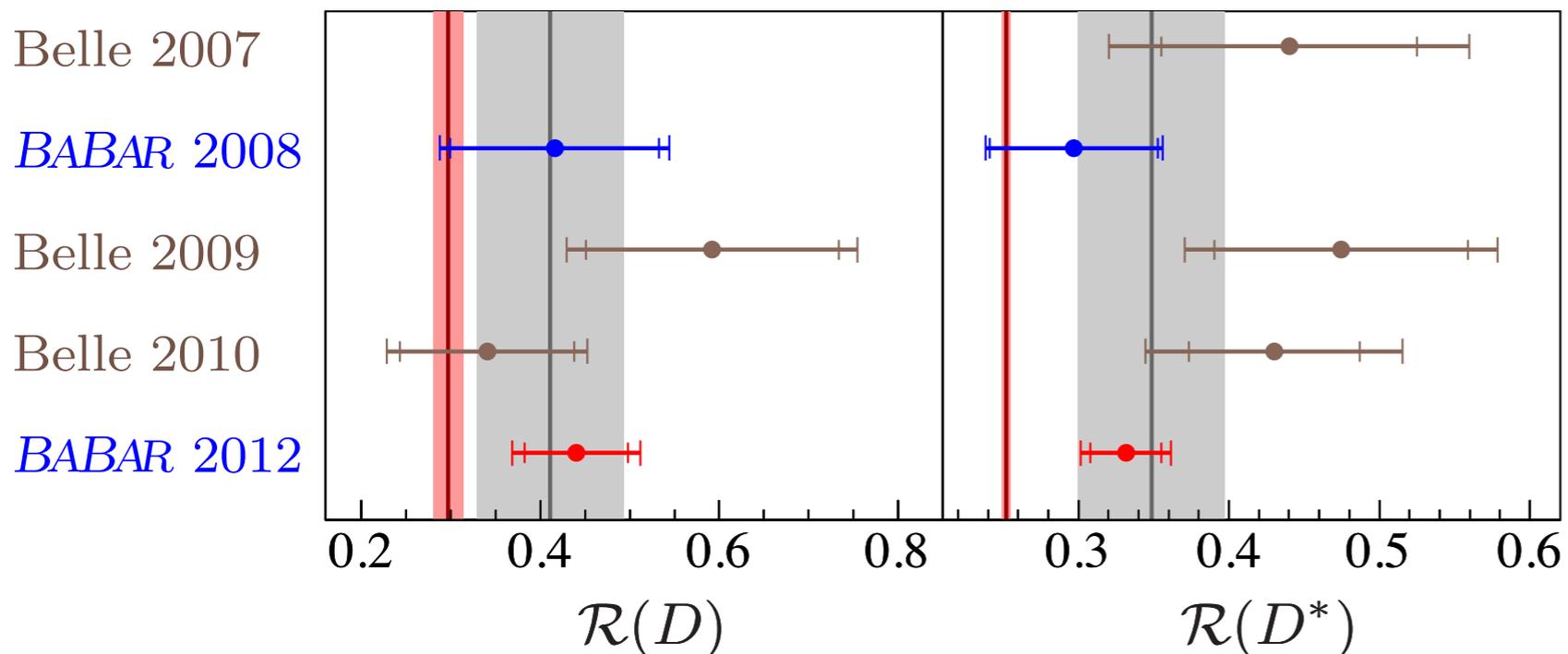


B semileptonic decay: $|V_{cb}|$

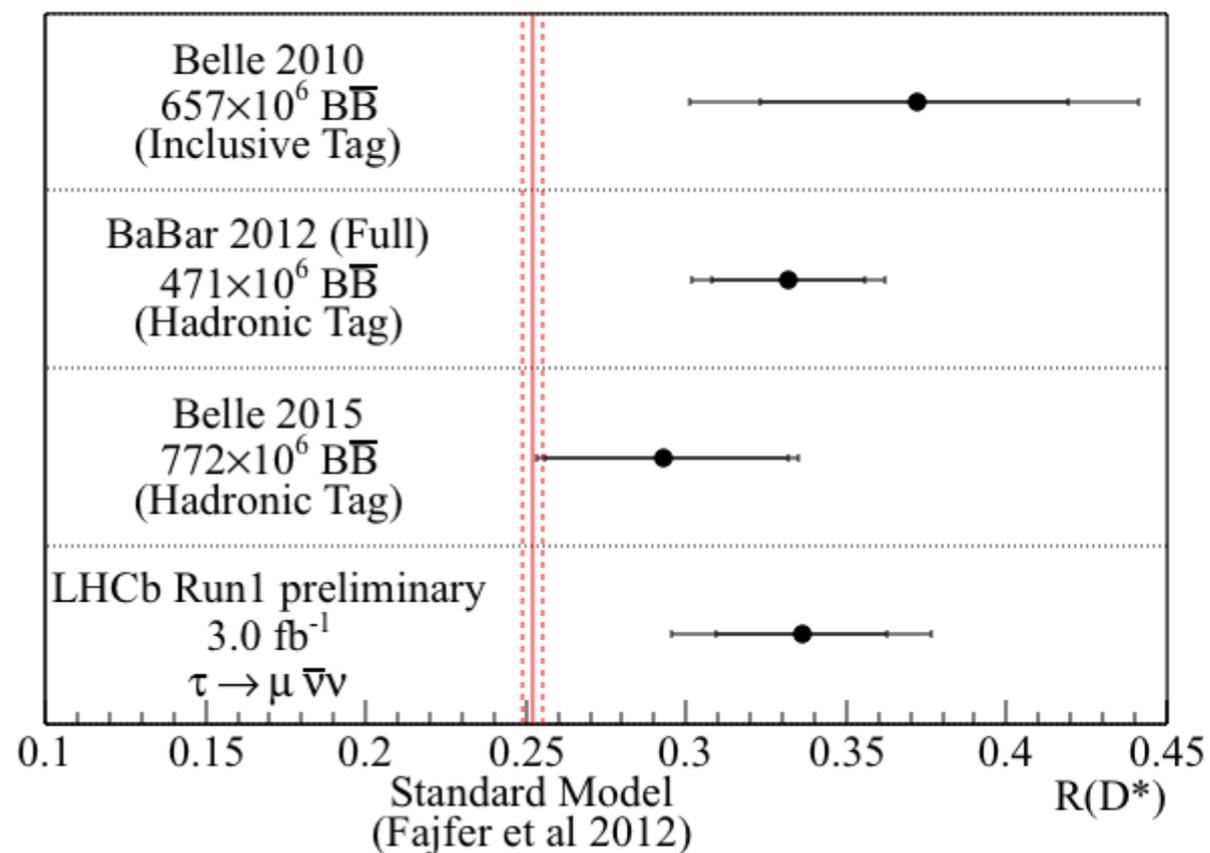


$R(D^{(*)})$, new measurements of $B \rightarrow D^* \tau \nu$

[pre-2015, arXiv:1303.0571]



[Ciezarek/Kuhr, FPCP 2015]
 [LHCb, arXiv:1506.08614]
 [Belle, arXiv:1507.03233]



Belle II projections

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	± 0.012	± 0.008	6
α		$\pm 2^\circ$	$\pm 1^\circ$	
γ	$\pm 14^\circ$	$\pm 6^\circ$	$\pm 1.5^\circ$	
$S(B \rightarrow \phi K^0)$	$0.90^{+0.09}_{-0.19}$	± 0.053	± 0.018	>50
$S(B \rightarrow \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	± 0.028	± 0.011	>50
$S(B \rightarrow K_S^0 K_S^0 K_S^0)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
$ V_{cb} $ incl.	$\pm 2.4\%$	$\pm 1.0\%$		< 1
$ V_{cb} $ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
$ V_{ub} $ incl.	$\pm 6.5\%$	$\pm 3.4\%$	$\pm 3.0\%$	2
$ V_{ub} $ excl. (had. tag.)	$\pm 10.8\%$	$\pm 4.7\%$	$\pm 2.4\%$	20
$ V_{ub} $ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \rightarrow \tau\nu)$ [10 ⁻⁶]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \rightarrow \mu\nu)$ [10 ⁻⁶]	< 1.7	5σ	$\gg 5\sigma$	>50
$R(B \rightarrow D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \rightarrow D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
$\mathcal{B}(B \rightarrow K^{*+}\nu\bar{\nu})$ [10 ⁻⁶]	< 40		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow K^+\nu\bar{\nu})$ [10 ⁻⁶]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow X_s\gamma)$ [10 ⁻⁶]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \rightarrow X_s\gamma)$		± 0.01	± 0.005	8
$S(B \rightarrow K_S^0\pi^0\gamma)$	$-0.10 \pm 0.31 \pm 0.07$	± 0.11	± 0.035	> 50
$S(B \rightarrow \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	± 0.23	± 0.07	> 50
$C_7/C_9 (B \rightarrow X_s\ell\ell)$	$\sim 20\%$	10%	5%	
$\mathcal{B}(B_s \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 8.7	± 0.3		
$\mathcal{B}(B_s \rightarrow \tau^+\tau^-)$ [10 ⁻³]		< 2		

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
$\mathcal{B}(D_s \rightarrow \mu\nu)$	$5.31 \times 10^{-3} (1 \pm 0.053 \pm 0.038)$	$\pm 2.9\%$	$\pm (0.9\%-1.3\%)$	> 50
$\mathcal{B}(D_s \rightarrow \tau\nu)$	$5.70 \times 10^{-3} (1 \pm 0.037 \pm 0.054)$	$\pm (3.5\%-4.3\%)$	$\pm (2.3\%-3.6\%)$	3-5
y_{CP} [10 ⁻²]	$1.11 \pm 0.22 \pm 0.11$	$\pm (0.11-0.13)$	$\pm (0.05-0.08)$	5-8
A_Γ [10 ⁻²]	$-0.03 \pm 0.20 \pm 0.08$	± 0.10	$\pm (0.03-0.05)$	7 - 9
$A_{CP}^{K^+K^-}$ [10 ⁻²]	$-0.32 \pm 0.21 \pm 0.09$	± 0.11	± 0.06	15
$A_{CP}^{\pi^+\pi^-}$ [10 ⁻²]	$0.55 \pm 0.36 \pm 0.09$	± 0.17	± 0.06	> 50
$A_{CP}^{\phi\gamma}$ [10 ⁻²]	± 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.56 \pm 0.19 \pm_{0.13}^{0.07}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.30 \pm 0.15 \pm_{0.08}^{0.05}$	± 0.08	± 0.05	15
$ q/p ^{K_S\pi^+\pi^-}$	$0.90 \pm_{0.15}^{0.16} \pm_{0.06}^{0.08}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [°]	$-6 \pm 11 \pm_5^4$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10 ⁻²]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K_S^0\pi^0}$ [10 ⁻²]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
	$\tau \rightarrow \mu\gamma$ [10 ⁻⁹]	< 45	< 14.7	< 4.7
	$\tau \rightarrow e\gamma$ [10 ⁻⁹]	< 120	< 39	< 12
	$\tau \rightarrow \mu\mu\mu$ [10 ⁻⁹]	< 21.0	< 3.0	< 0.3

FLAG-2 on $B \rightarrow D^{(*)} l \nu$

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	form factor	
FNAL/MILC 13B	[446]	2+1	C [∇]	★	○	★	○	✓	$\mathcal{F}^{B \rightarrow D^*}(1)$	0.906(4)(12)
FNAL/MILC 10	[443]	2+1	C [§]	★	○	★	○	✓	$\mathcal{F}^{B \rightarrow D^*}(1)$	0.9017(51)(87)(83)(89)(30)(33) [‡]
FNAL/MILC 08	[444]	2+1	A	★	○	★	○	✓	$\mathcal{F}^{B \rightarrow D^*}(1)$	0.921(13)(8)(8)(14)(6)(3)(4)
FNAL/MILC 13B	[446]	2+1	C	★	○	★	○	✓	$\mathcal{G}^{B \rightarrow D}(1)$	1.081(25)
FNAL/MILC 04A	[445]	2+1	C	■	■	○*	○ [†]	✓	$\mathcal{G}^{B \rightarrow D}(1)$	1.074(18)(16)
FNAL/MILC 12A	[452]	2+1	A	○	○	★	○	✓	$R(D)$	0.316(12)(7)
Atoui 13	[448]	2	P	★	★	★	—	✓	$\mathcal{G}^{B \rightarrow D}(1)$	1.033(95)
Atoui 13	[448]	2	P	★	★	★	—	✓	$\mathcal{G}^{B_s \rightarrow D_s}(1)$	1.052(46)

$w = 1$

$w \geq 1$

[∇] Update of FNAL/MILC 08 for Lattice 2013.

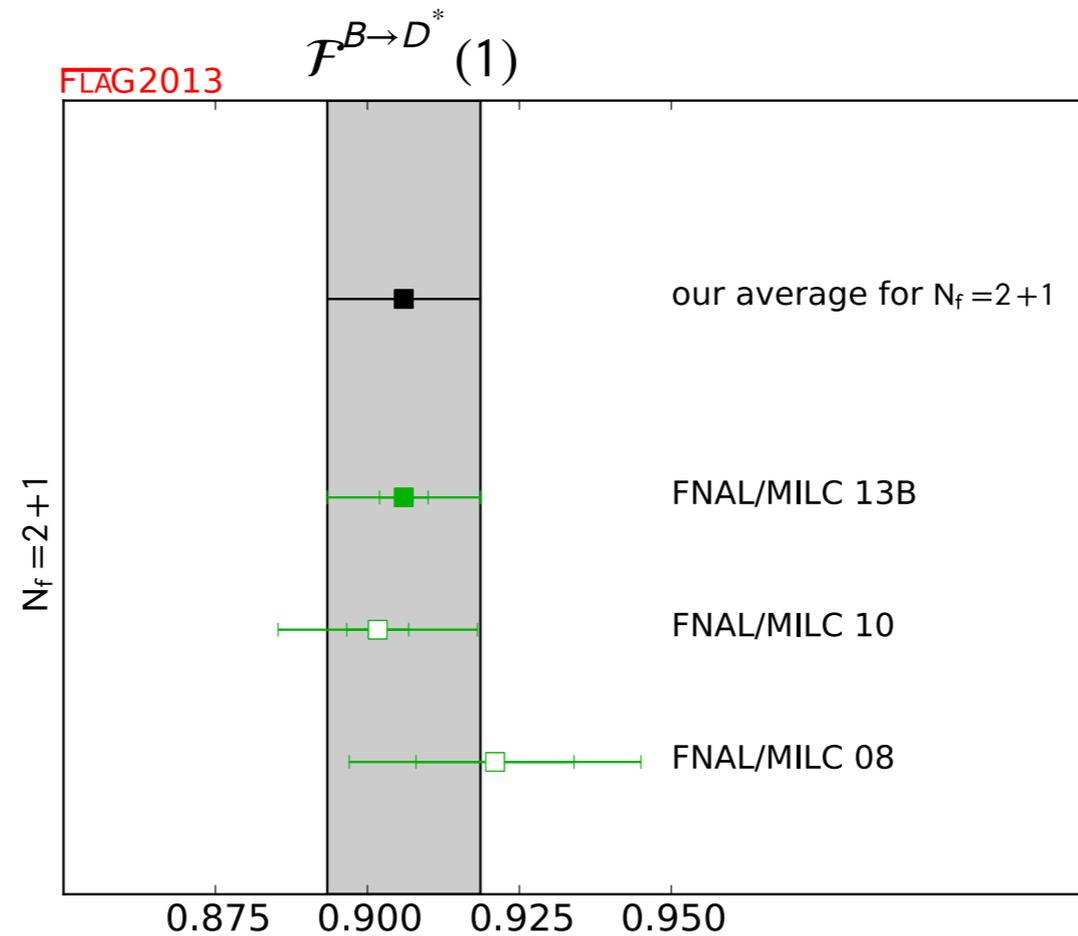
[§] Update of FNAL/MILC 08 for CKM 2010.

[‡] Value of $\mathcal{F}(1)$ presented in Ref. [443] includes 0.7% correction η_{EW} . This correction is unrelated to the lattice calculation and has been removed here.

* No explicit estimate of FV error, but expected to be small.

[†] No explicit estimate of perturbative truncation error in vector current renormalization factor, but expected to be small because of mostly-nonperturbative approach.

FLAG-2 on $B \rightarrow D^{(*)} l \nu$

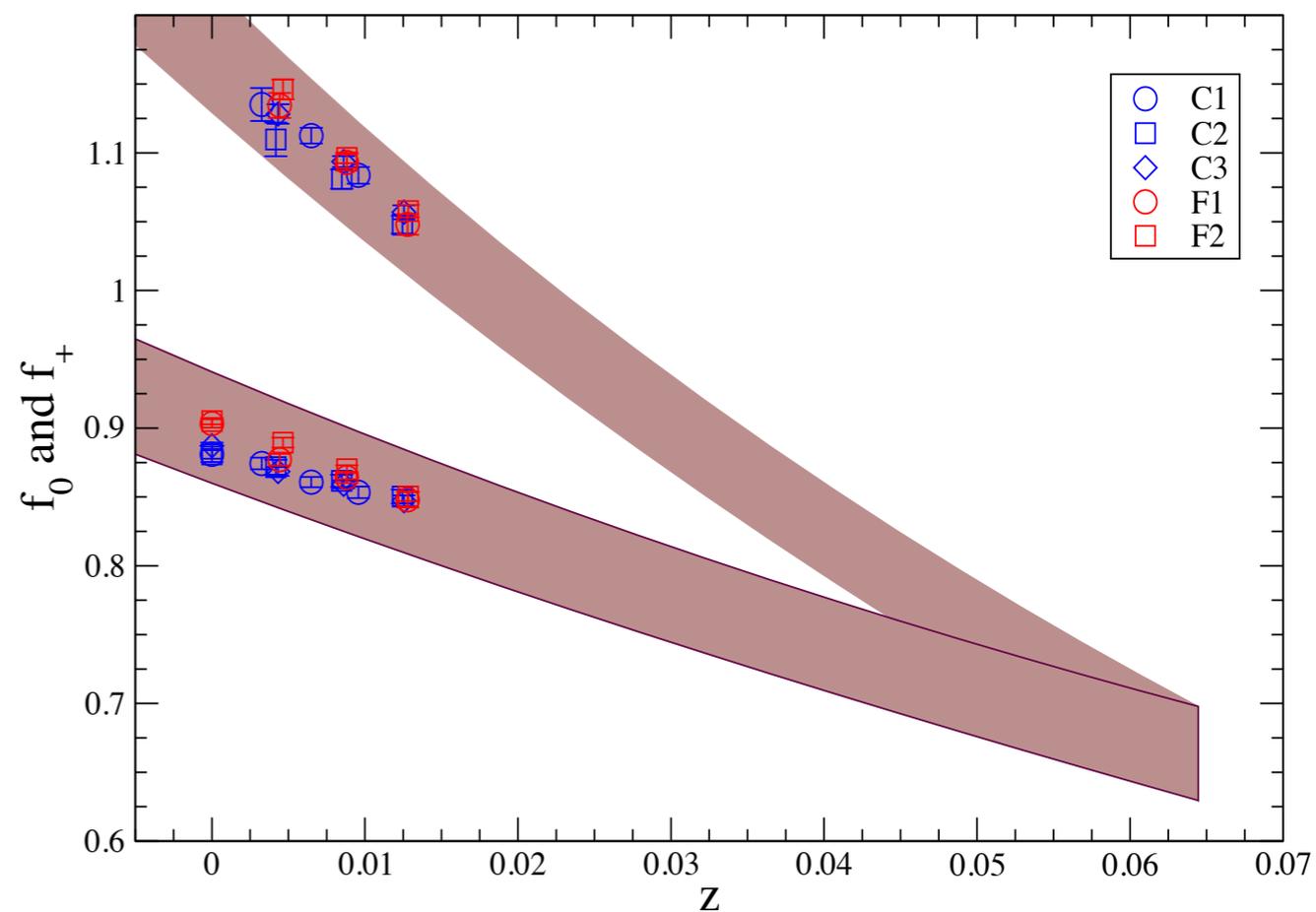
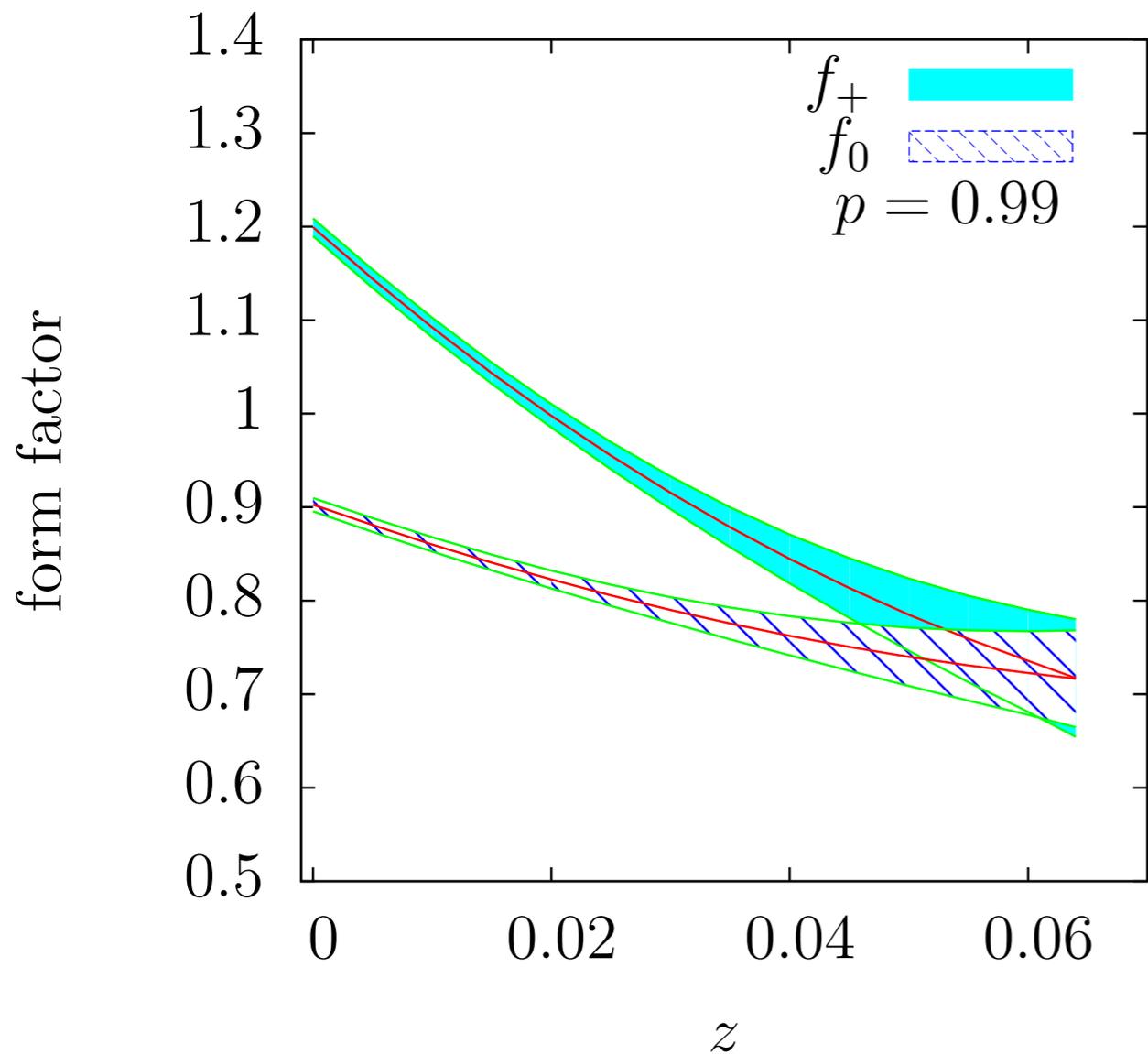


N_f	$\mathcal{G}^{B \rightarrow D}(1)$	$\mathcal{G}^{B_s \rightarrow D_s}(1)$	$\mathcal{F}^{B \rightarrow D^*}(1)$	$R(D)$
2	1.033(95)	1.052(46)	—	—
2+1	1.081(25)	—	0.906(4)(12)	0.316(12)(7)

new results for $B \rightarrow D l \nu$

[FNAL/MILC]

[HPQCD]



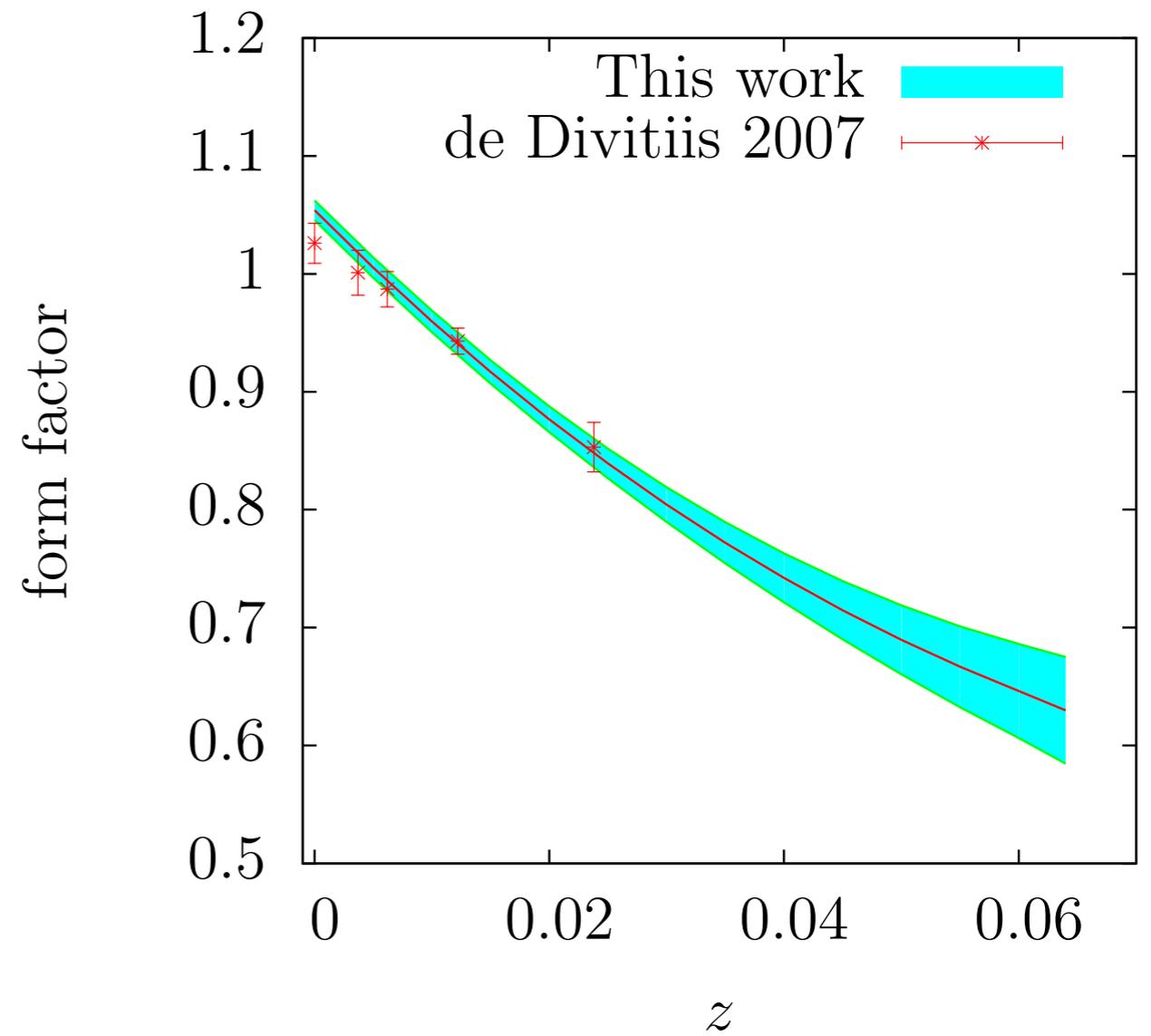
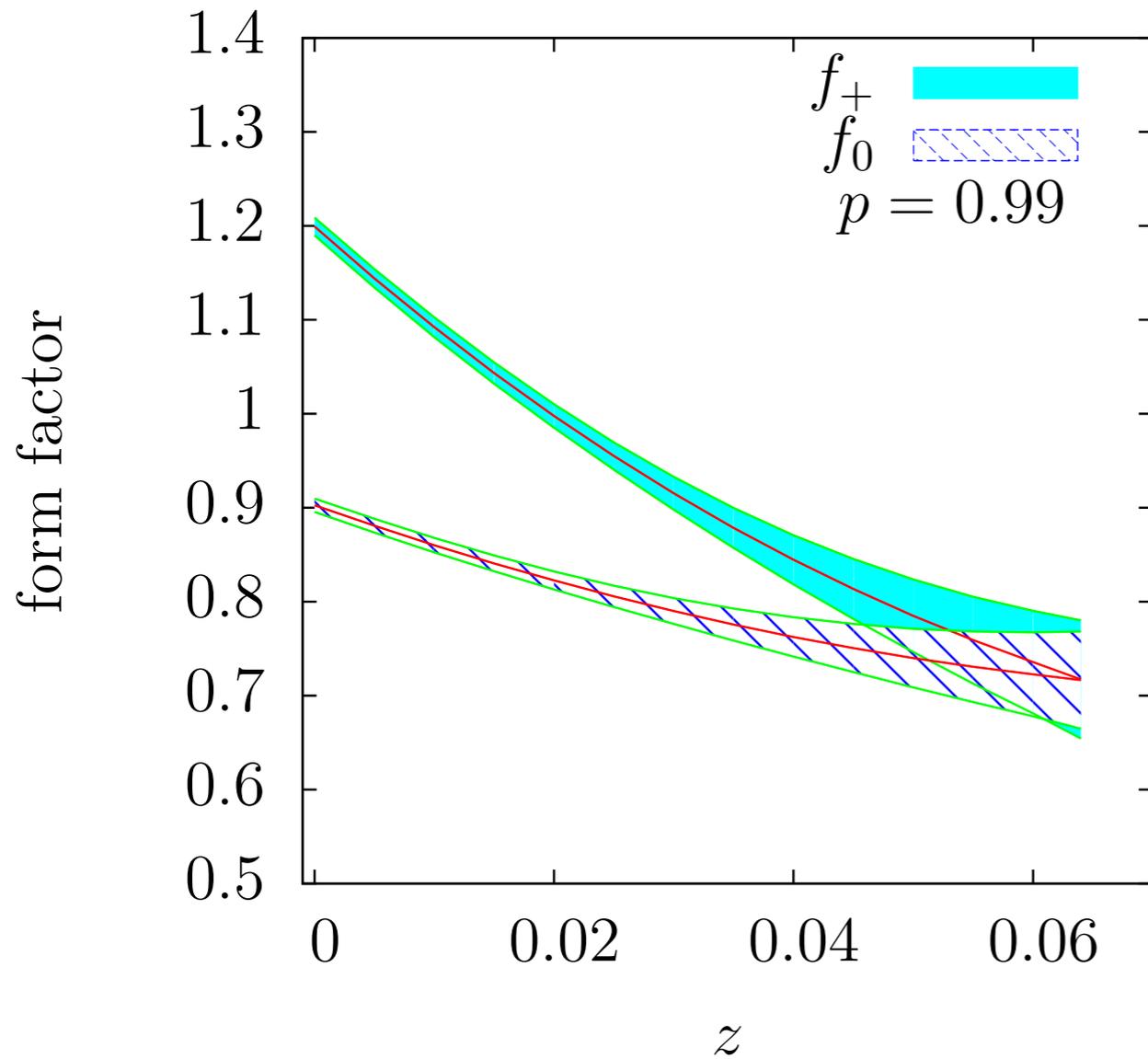
$$z(w) = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}$$

$$w_{\text{latt}} \in [1.00, 1.16] \quad w_{\text{exp}} \in [1.00, 1.58]$$

$$z_{\text{latt}} \in [0.00, 0.02] \quad z_{\text{exp}} \in [0.00, 0.06]$$

new results for $B \rightarrow Dl\nu$

[FNAL/MILC]



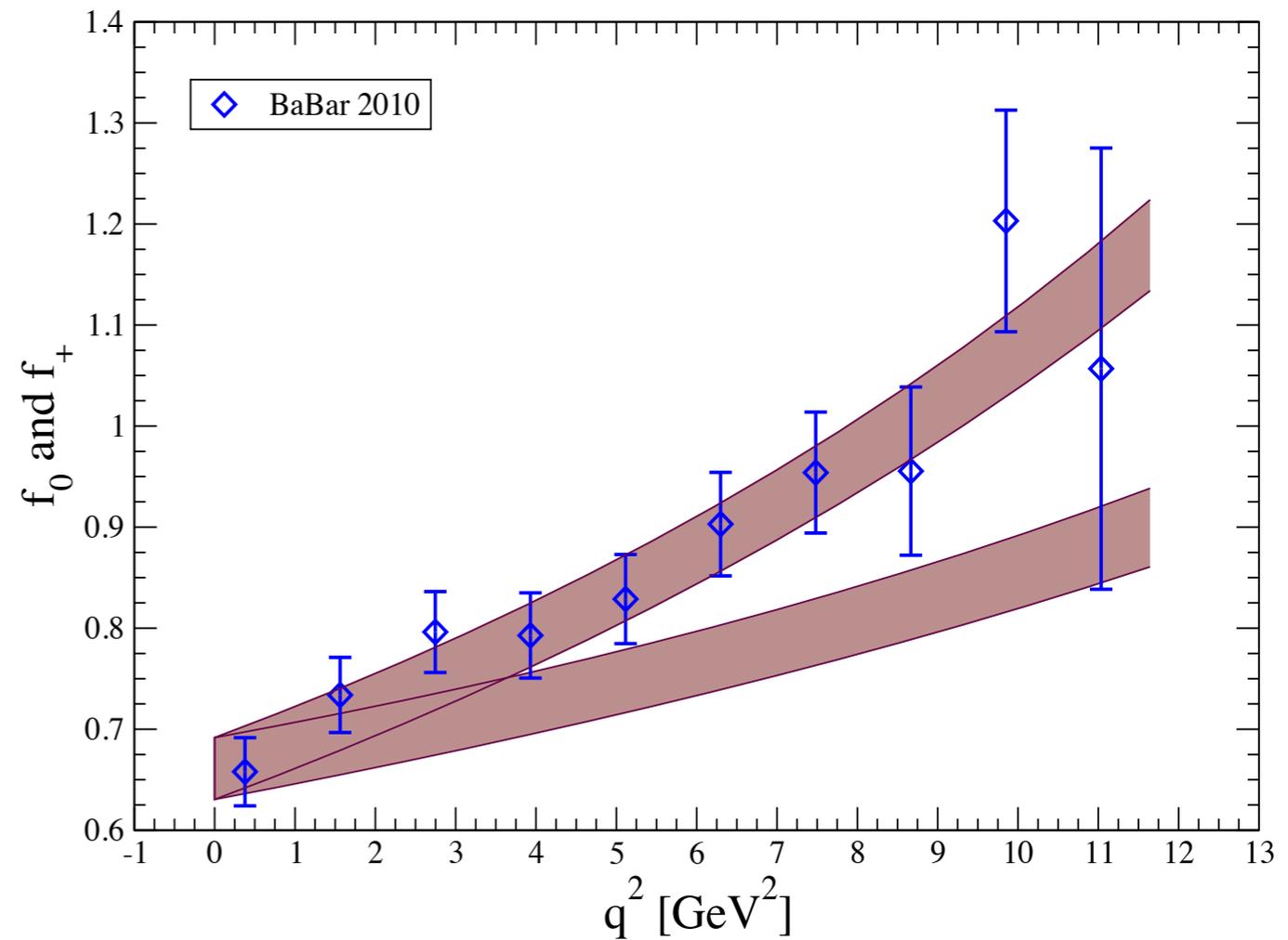
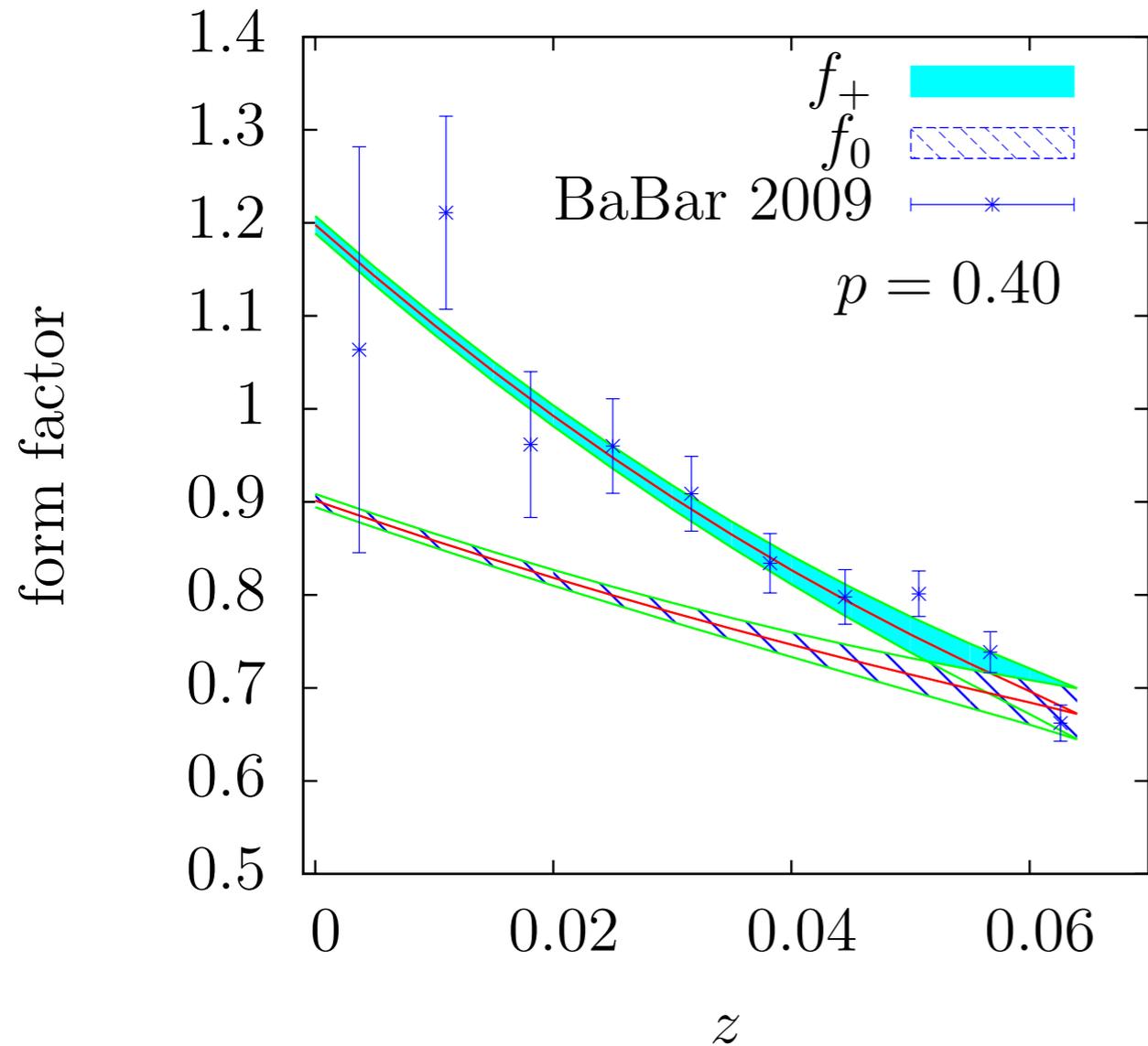
comparison with quenched results

[De Divitiis, Molinaro, Petronzio, Tantalò PLB 655 (2007) 45]

new results for $B \rightarrow D l \nu$

[FNAL/MILC]

[HPQCD]



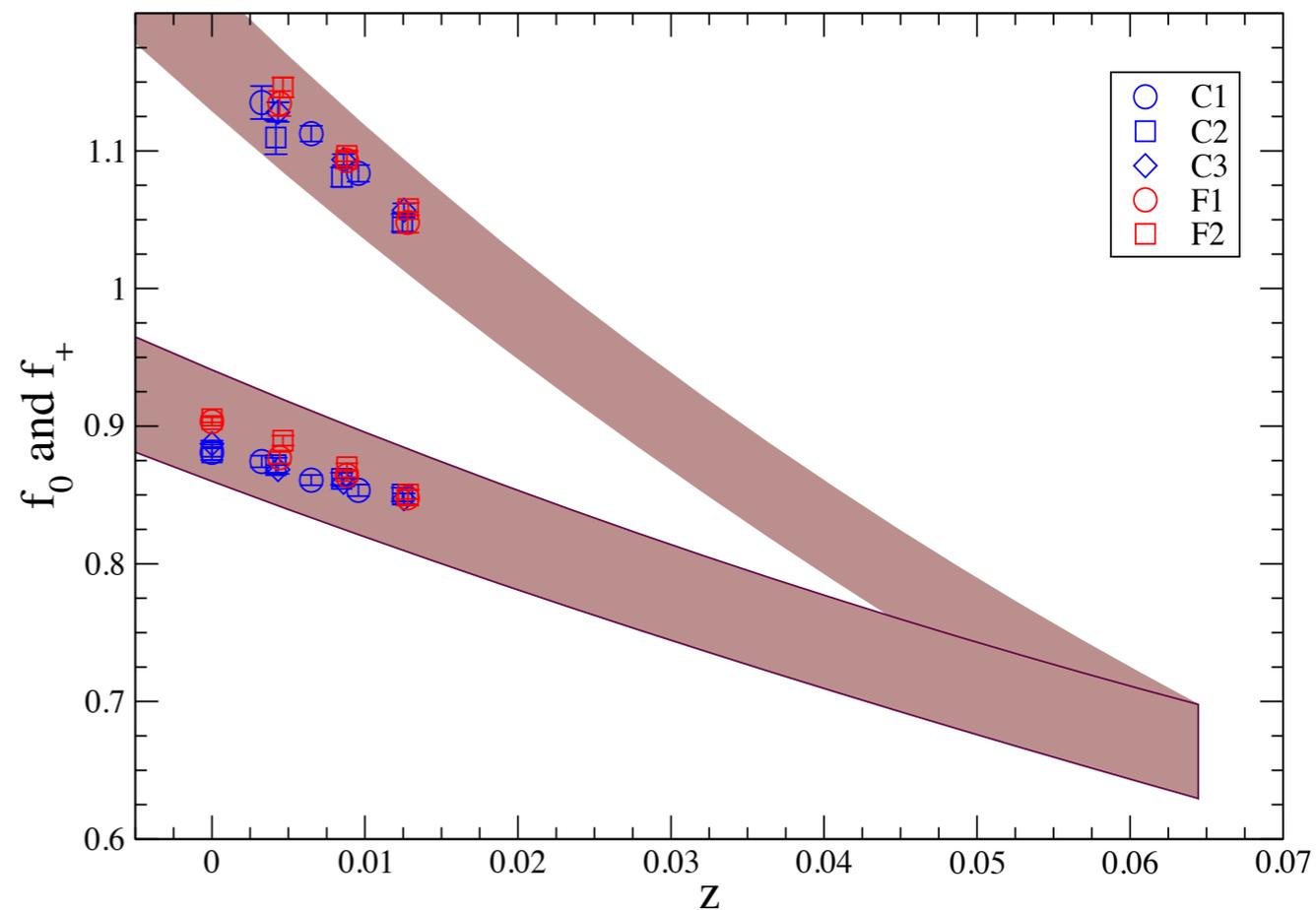
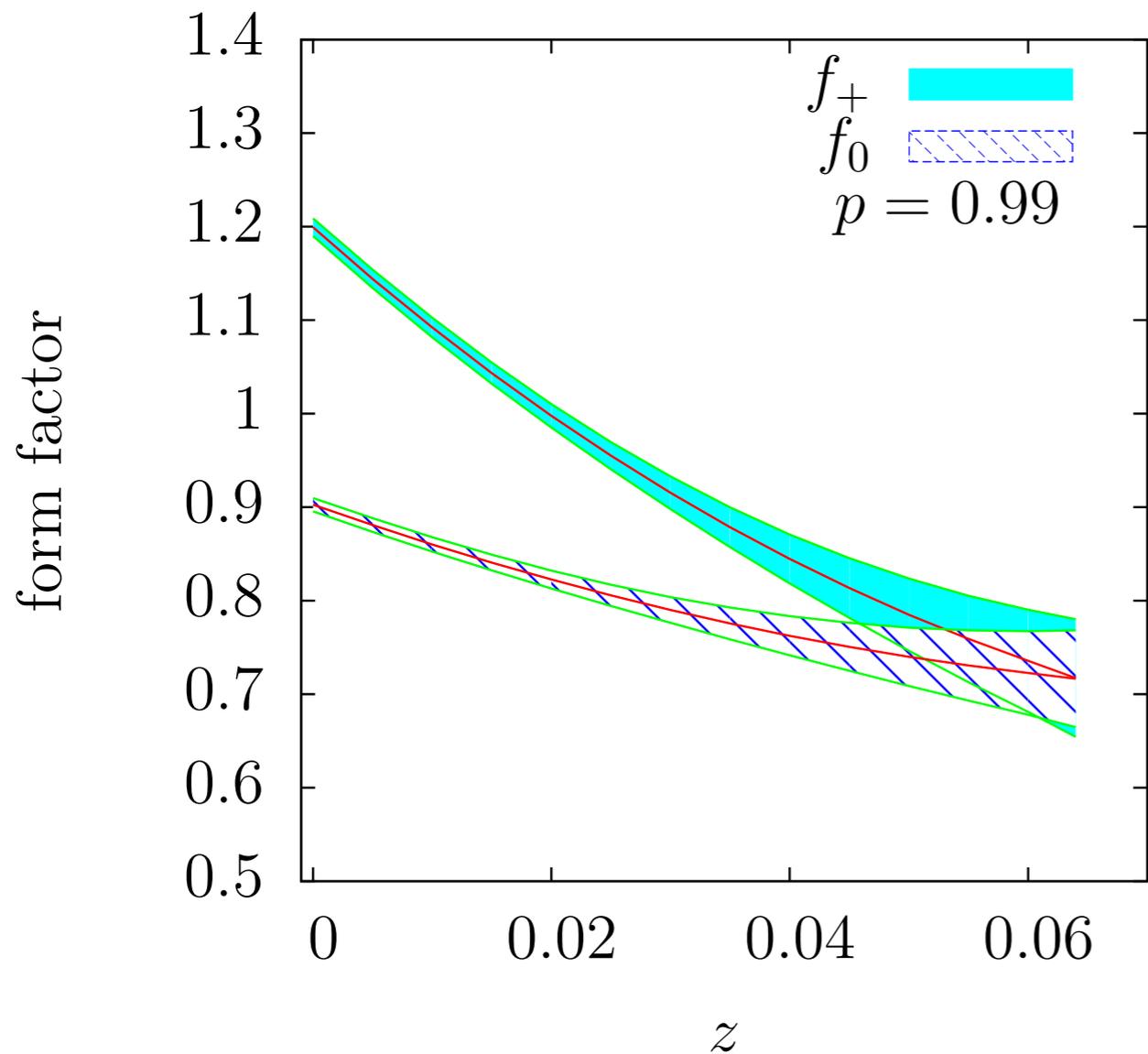
$$|V_{cb}| = 39.6(1.7)_{\text{QCD}+\text{exp}}(0.2)_{\text{QED}} \times 10^{-3}$$

$$|V_{cb}| = 40.2(17)(13) \times 10^{-3}$$

new results for $B \rightarrow D l \nu$

[FNAL/MILC]

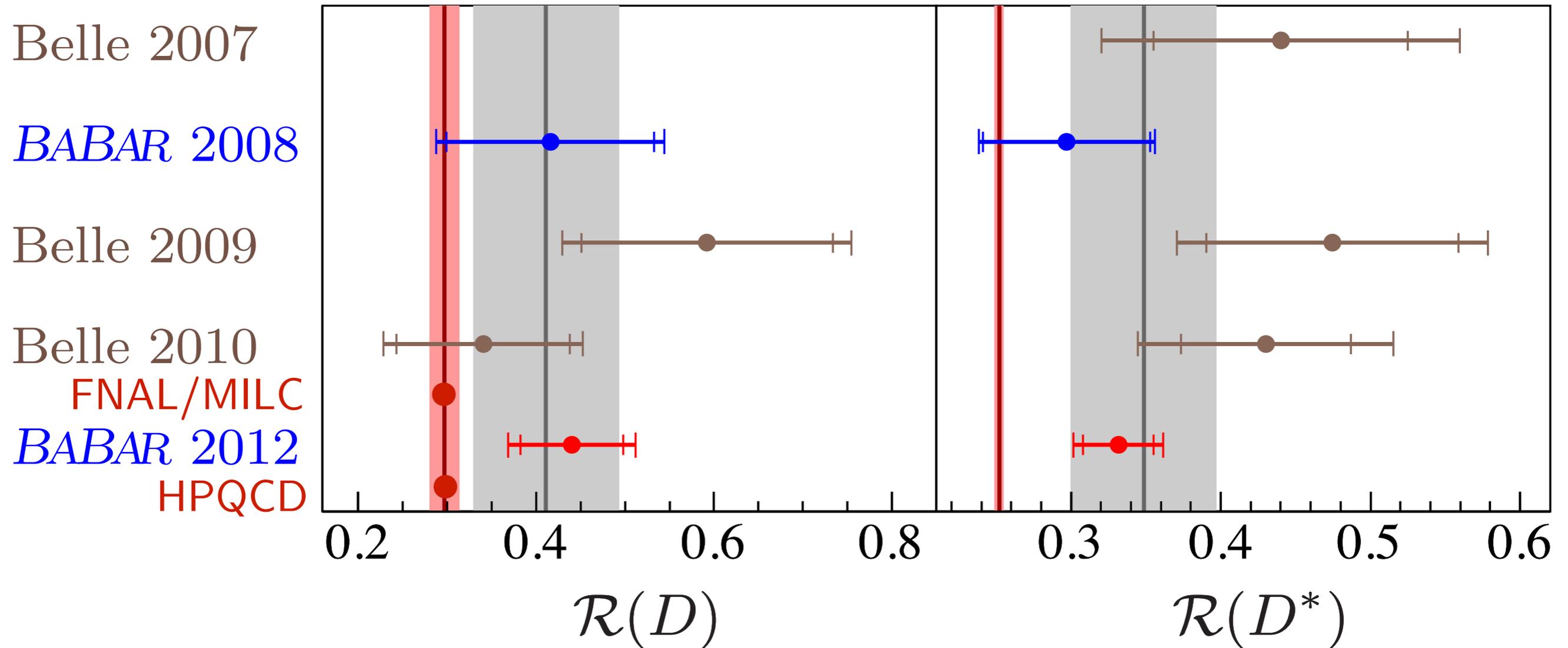
[HPQCD]



$$R(D) = \frac{\mathcal{B}(B \rightarrow D\tau\nu)}{\mathcal{B}(B \rightarrow D l \nu)} = 0.299(11)$$

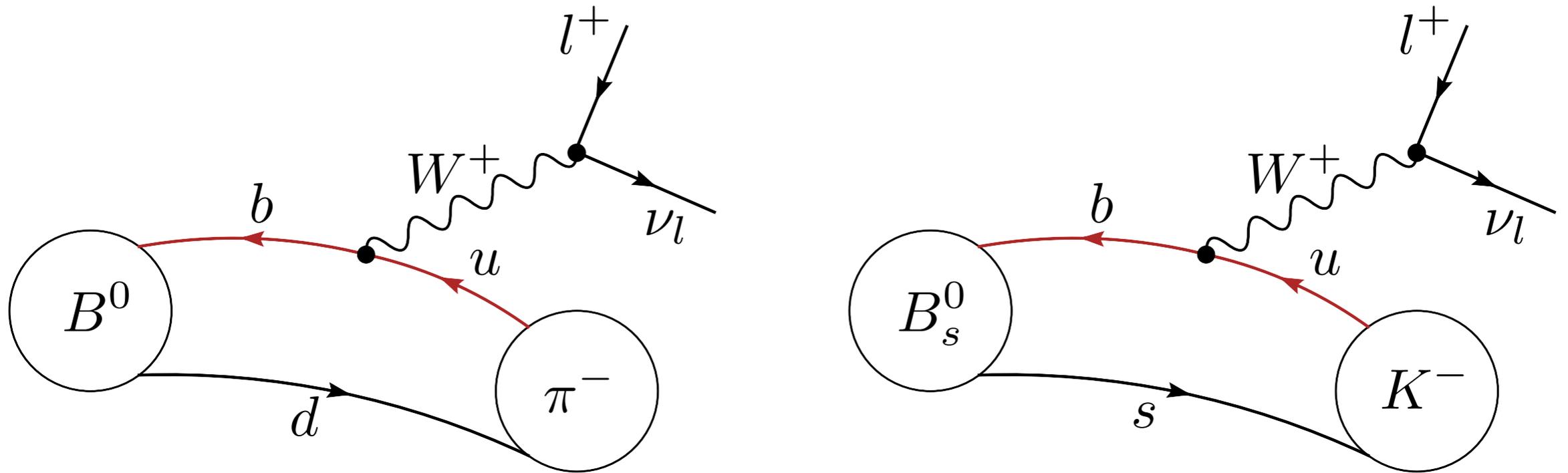
$$0.300(8)$$

status of $R(D)$



n.b.: Belle II expected to decrease error by factor of 3 at 5/ab, and a factor 5 at 50/ab

B semileptonic decay: $|V_{ub}|$



$$\frac{d\Gamma(B_{(s)} \rightarrow Pl\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_l^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_{B_{(s)}}^2} \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B_{(s)}}^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} (m_{B_{(s)}}^2 - m_P^2)^2 |f_0(q^2)|^2 \right]$$

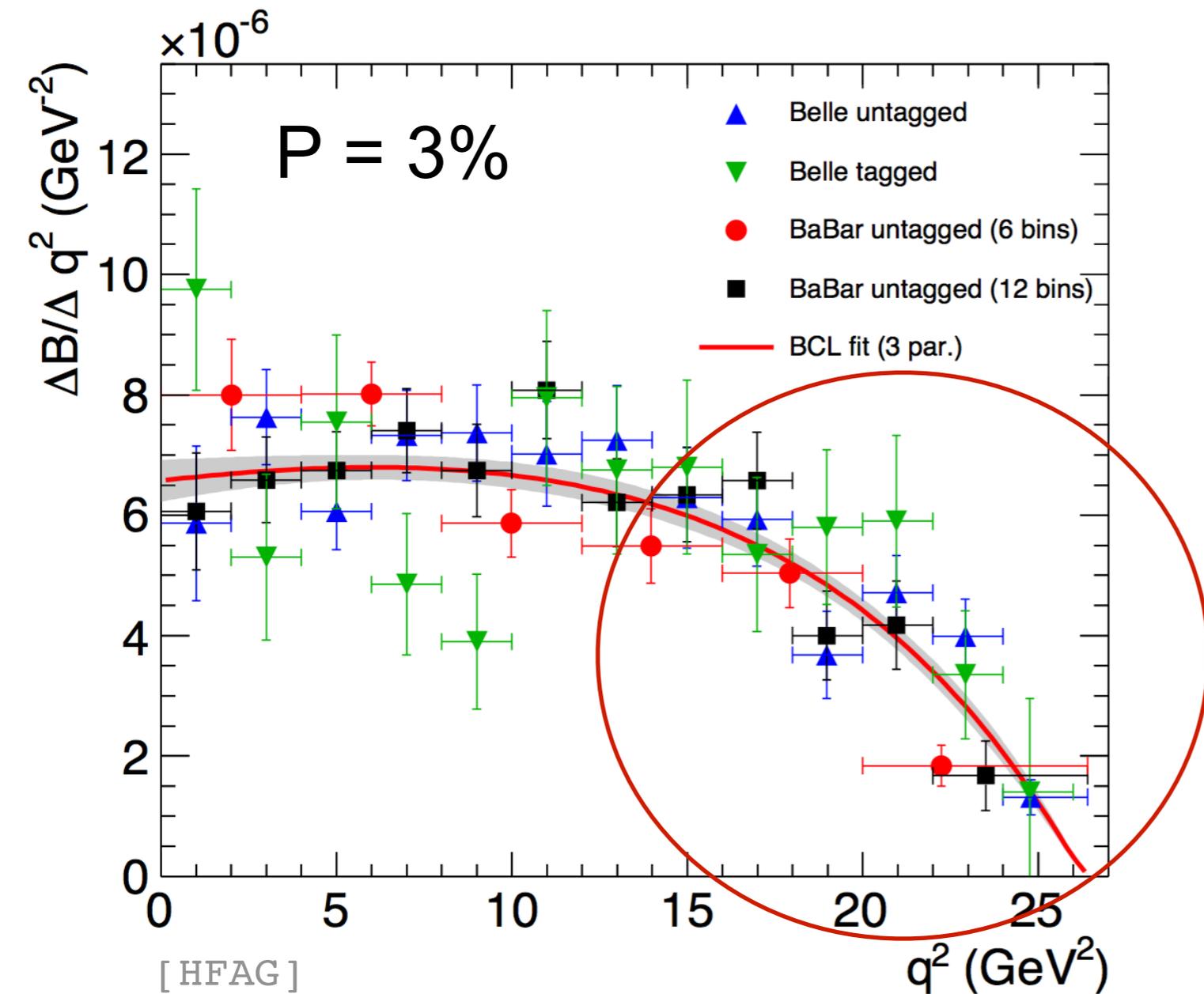
$$\langle P(p') | \bar{b} \gamma_\mu q | B_q(p) \rangle = f_+(q^2) \left(p_\mu + p'_\mu - \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu \right) + f_0(q^2) \frac{m_{B_q}^2 - m_P^2}{q^2} q_\mu, \quad q = p - p'$$

Belle II projections

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
$\sin 2\beta$	$0.667 \pm 0.023 \pm 0.012$	± 0.012	± 0.008	6
α		$\pm 2^\circ$	$\pm 1^\circ$	
γ	$\pm 14^\circ$	$\pm 6^\circ$	$\pm 1.5^\circ$	
$S(B \rightarrow \phi K^0)$	$0.90^{+0.09}_{-0.19}$	± 0.053	± 0.018	>50
$S(B \rightarrow \eta' K^0)$	$0.68 \pm 0.07 \pm 0.03$	± 0.028	± 0.011	>50
$S(B \rightarrow K_S^0 K_S^0 K_S^0)$	$0.30 \pm 0.32 \pm 0.08$	± 0.100	± 0.033	44
$ V_{cb} $ incl.	$\pm 2.4\%$	$\pm 1.0\%$		< 1
$ V_{cb} $ excl.	$\pm 3.6\%$	$\pm 1.8\%$	$\pm 1.4\%$	< 1
$ V_{ub} $ incl.	$\pm 6.5\%$	$\pm 3.4\%$	$\pm 3.0\%$	2
$ V_{ub} $ excl. (had. tag.)	$\pm 10.8\%$	$\pm 4.7\%$	$\pm 2.4\%$	20
$ V_{ub} $ excl. (untag.)	$\pm 9.4\%$	$\pm 4.2\%$	$\pm 2.2\%$	3
$\mathcal{B}(B \rightarrow \tau\nu)$ [10 ⁻⁶]	96 ± 26	$\pm 10\%$	$\pm 5\%$	46
$\mathcal{B}(B \rightarrow \mu\nu)$ [10 ⁻⁶]	< 1.7	5σ	$\gg 5\sigma$	>50
$R(B \rightarrow D\tau\nu)$	$\pm 16.5\%$	$\pm 5.6\%$	$\pm 3.4\%$	4
$R(B \rightarrow D^*\tau\nu)$	$\pm 9.0\%$	$\pm 3.2\%$	$\pm 2.1\%$	3
$\mathcal{B}(B \rightarrow K^{*+}\nu\bar{\nu})$ [10 ⁻⁶]	< 40		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow K^+\nu\bar{\nu})$ [10 ⁻⁶]	< 55		$\pm 30\%$	>50
$\mathcal{B}(B \rightarrow X_s\gamma)$ [10 ⁻⁶]	$\pm 13\%$	$\pm 7\%$	$\pm 6\%$	< 1
$A_{CP}(B \rightarrow X_s\gamma)$		± 0.01	± 0.005	8
$S(B \rightarrow K_S^0\pi^0\gamma)$	$-0.10 \pm 0.31 \pm 0.07$	± 0.11	± 0.035	> 50
$S(B \rightarrow \rho\gamma)$	$-0.83 \pm 0.65 \pm 0.18$	± 0.23	± 0.07	> 50
$C_7/C_9 (B \rightarrow X_s\ell\ell)$	$\sim 20\%$	10%	5%	
$\mathcal{B}(B_s \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 8.7	± 0.3		
$\mathcal{B}(B_s \rightarrow \tau^+\tau^-)$ [10 ⁻³]		< 2		

Observables	Belle	Belle II		\mathcal{L}_s [ab ⁻¹]
	(2014)	5 ab ⁻¹	50 ab ⁻¹	
$\mathcal{B}(D_s \rightarrow \mu\nu)$	$5.31 \times 10^{-3}(1 \pm 0.053 \pm 0.038)$	$\pm 2.9\%$	$\pm(0.9\%-1.3\%)$	> 50
$\mathcal{B}(D_s \rightarrow \tau\nu)$	$5.70 \times 10^{-3}(1 \pm 0.037 \pm 0.054)$	$\pm(3.5\%-4.3\%)$	$\pm(2.3\%-3.6\%)$	3-5
y_{CP} [10 ⁻²]	$1.11 \pm 0.22 \pm 0.11$	$\pm(0.11-0.13)$	$\pm(0.05-0.08)$	5-8
A_Γ [10 ⁻²]	$-0.03 \pm 0.20 \pm 0.08$	± 0.10	$\pm(0.03-0.05)$	7 - 9
$A_{CP}^{K^+K^-}$ [10 ⁻²]	$-0.32 \pm 0.21 \pm 0.09$	± 0.11	± 0.06	15
$A_{CP}^{\pi^+\pi^-}$ [10 ⁻²]	$0.55 \pm 0.36 \pm 0.09$	± 0.17	± 0.06	> 50
$A_{CP}^{\phi\gamma}$ [10 ⁻²]	± 5.6	± 2.5	± 0.8	> 50
$x^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.56 \pm 0.19 \pm^{0.07}_{0.13}$	± 0.14	± 0.11	3
$y^{K_S\pi^+\pi^-}$ [10 ⁻²]	$0.30 \pm 0.15 \pm^{0.05}_{0.08}$	± 0.08	± 0.05	15
$ q/p ^{K_S\pi^+\pi^-}$	$0.90 \pm^{0.16}_{0.15} \pm^{0.08}_{0.06}$	± 0.10	± 0.07	5-6
$\phi^{K_S\pi^+\pi^-}$ [°]	$-6 \pm 11 \pm^4_5$	± 6	± 4	10
$A_{CP}^{\pi^0\pi^0}$ [10 ⁻²]	$-0.03 \pm 0.64 \pm 0.10$	± 0.29	± 0.09	> 50
$A_{CP}^{K_S^0\pi^0}$ [10 ⁻²]	$-0.10 \pm 0.16 \pm 0.09$	± 0.08	± 0.03	> 50
$Br(D^0 \rightarrow \gamma\gamma)$ [10 ⁻⁶]	< 1.5	$\pm 30\%$	$\pm 25\%$	2
	$\tau \rightarrow \mu\gamma$ [10 ⁻⁹]	< 45	< 14.7	< 4.7
	$\tau \rightarrow e\gamma$ [10 ⁻⁹]	< 120	< 39	< 12
	$\tau \rightarrow \mu\mu\mu$ [10 ⁻⁹]	< 21.0	< 3.0	< 0.3

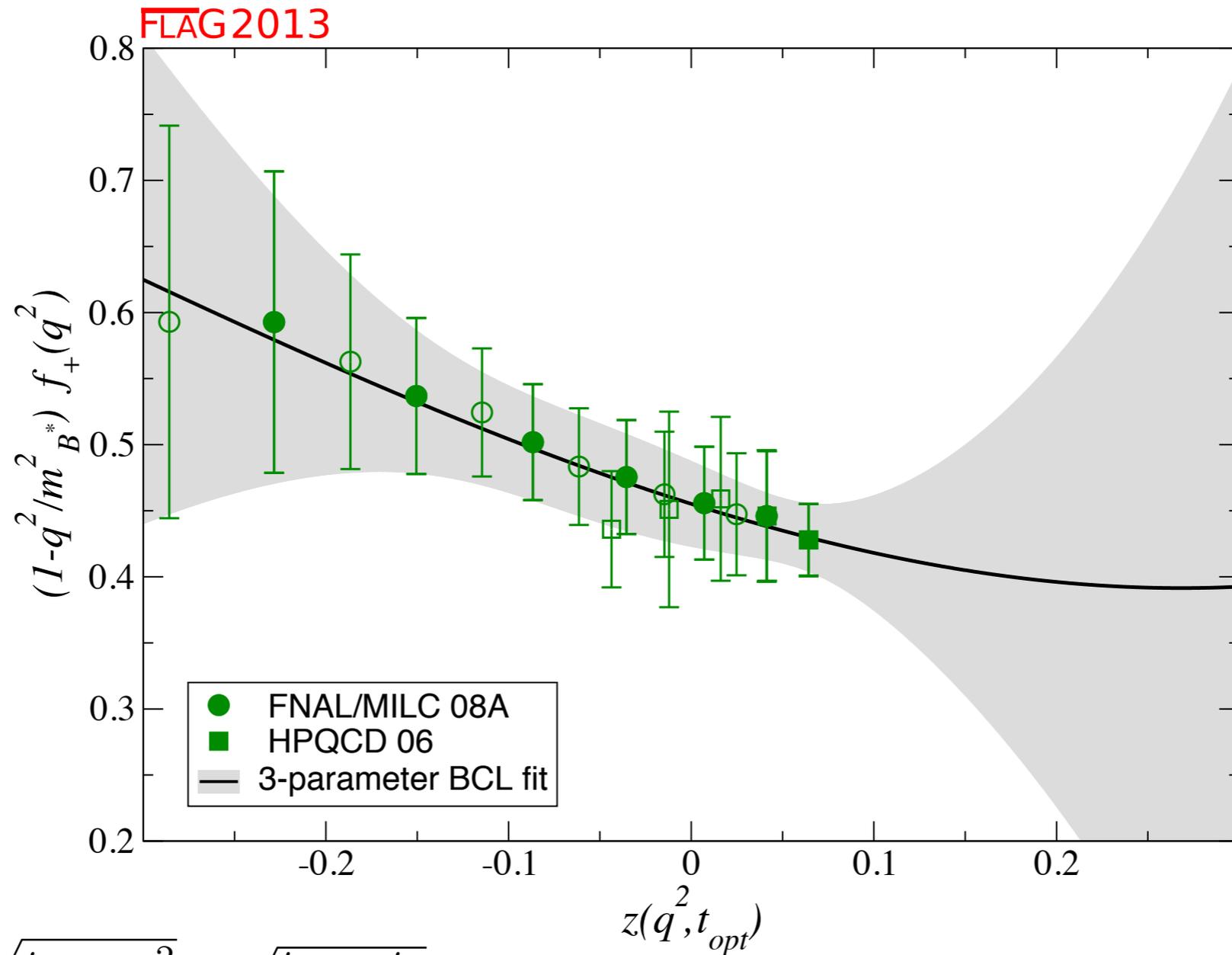
B semileptonic decay: $|V_{ub}|$



easily accessible kinematics on the lattice (not-too-fast pions)

large phase space \Rightarrow accurate description of q^2 dependence over a significant region crucial for a precise CKM determination

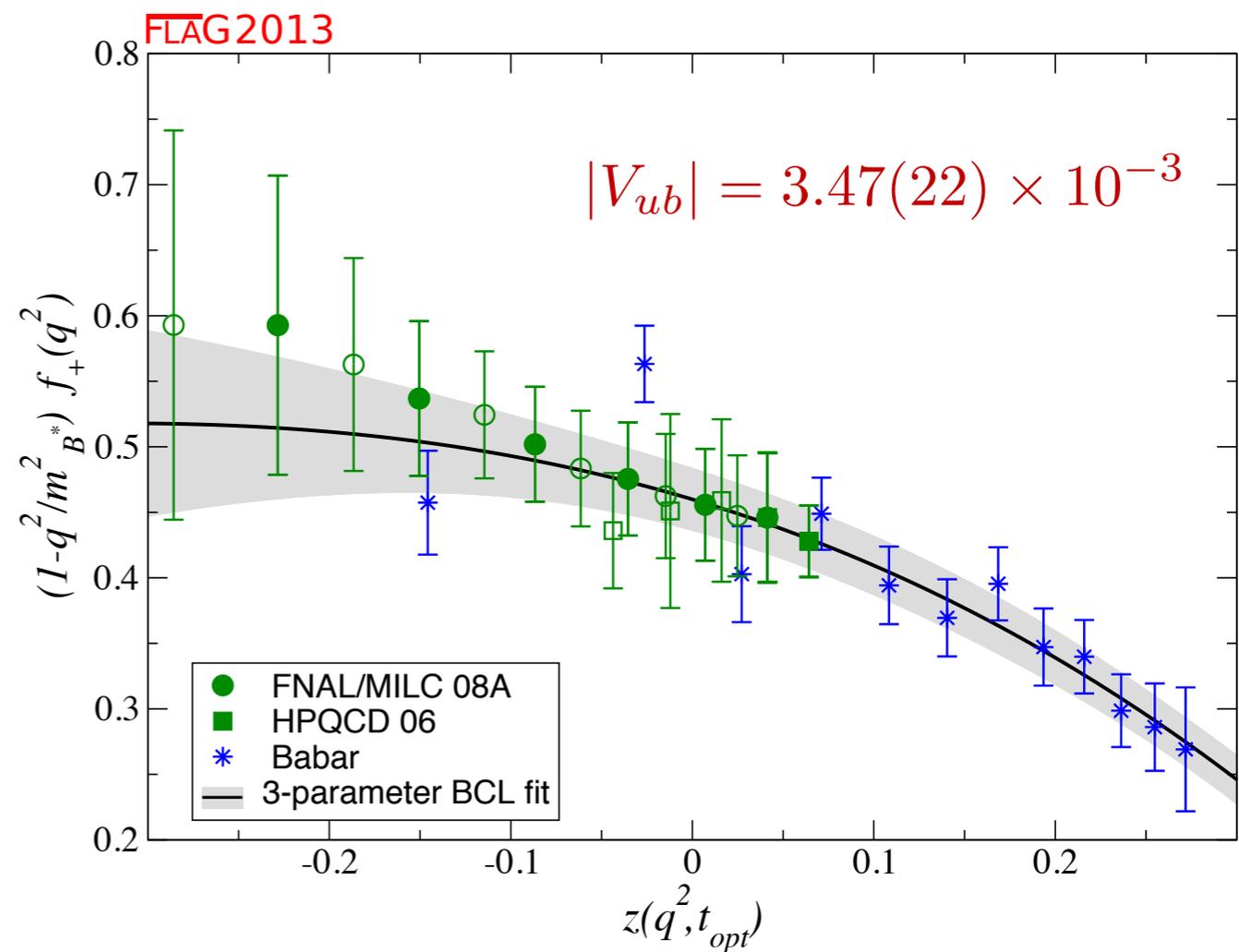
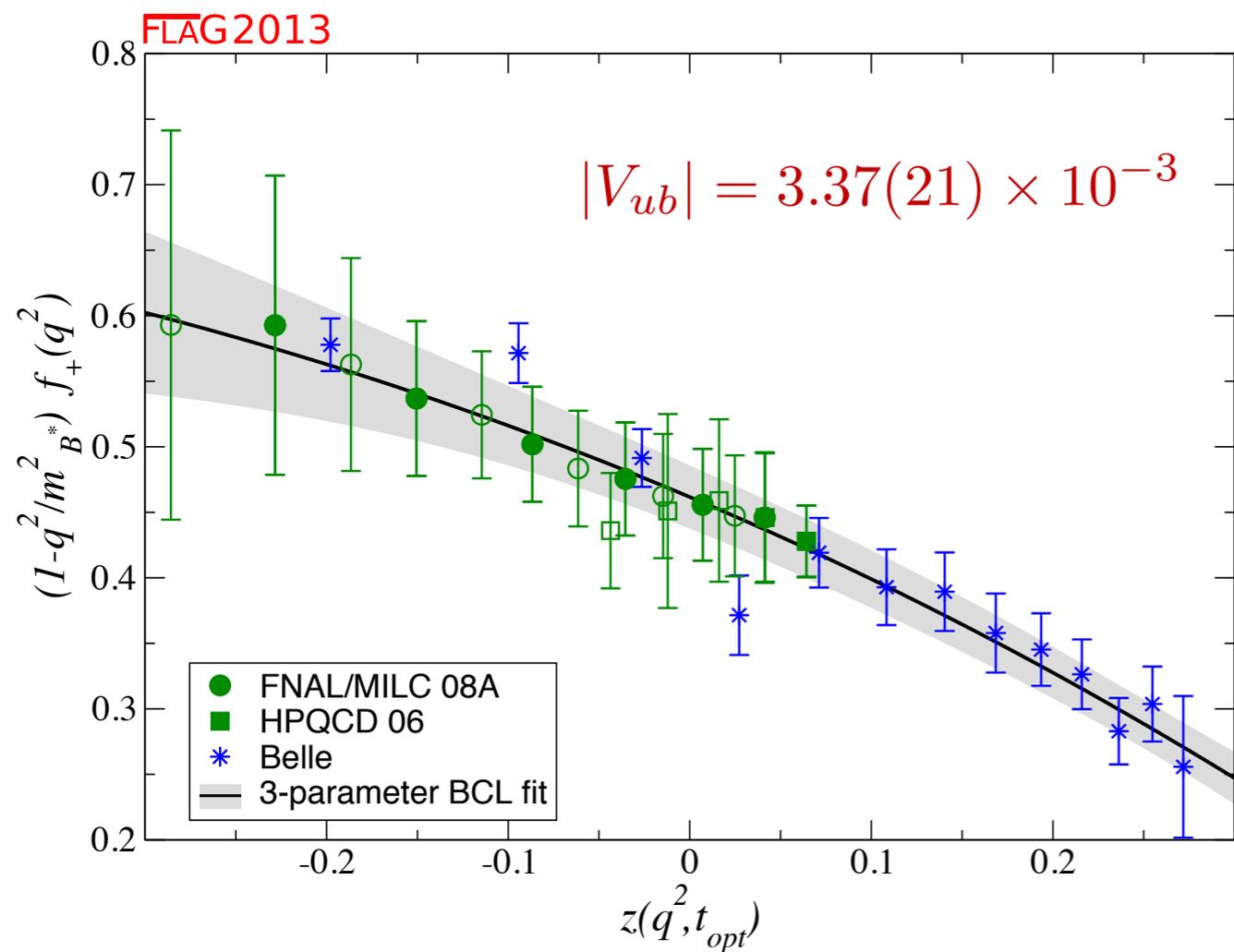
FLAG-2 on $B \rightarrow \pi l \nu$



$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

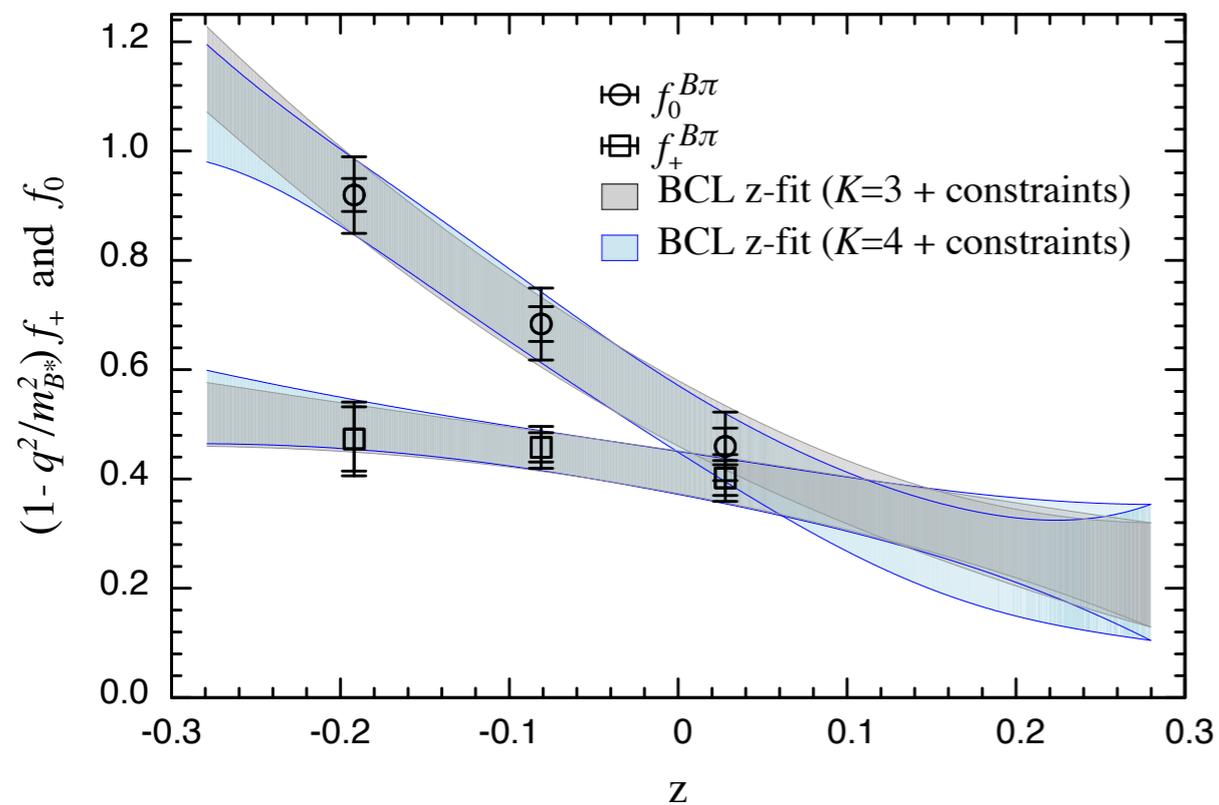
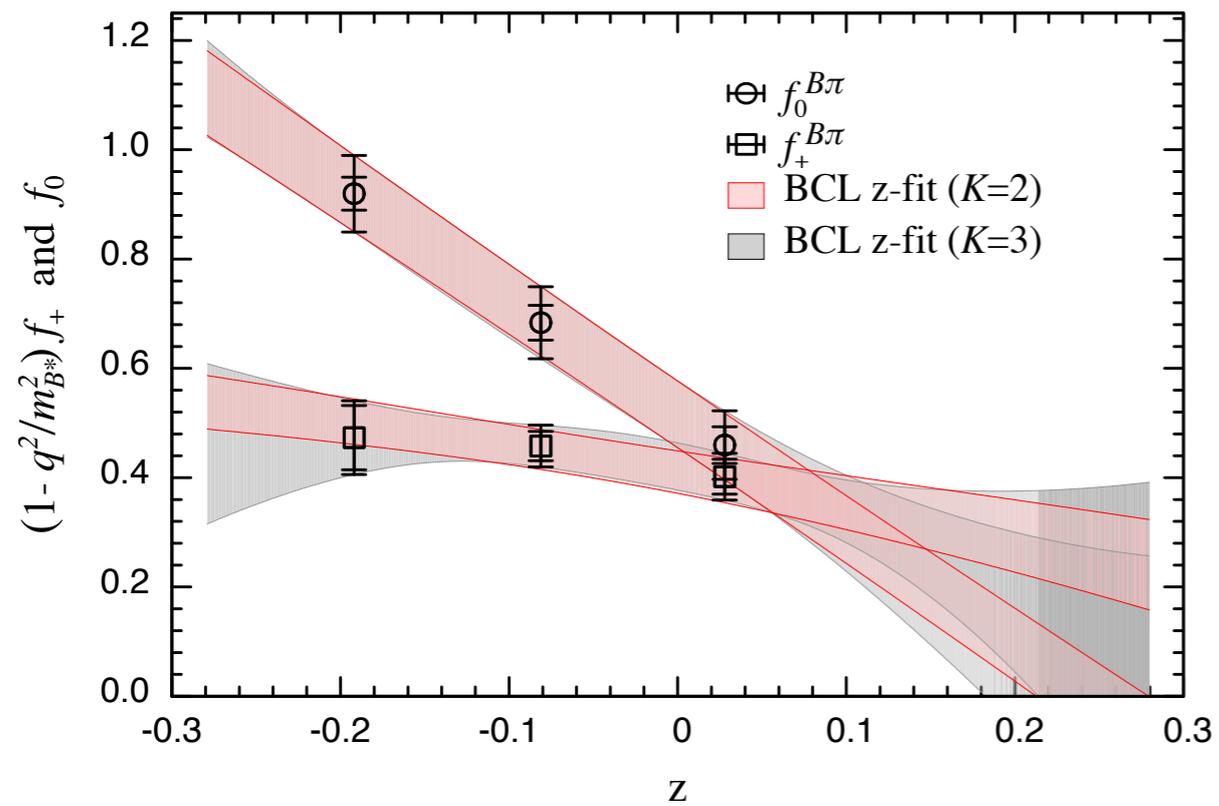
$$t_+ = (m_B + m_\pi)^2, \quad t_0 < t_+$$

FLAG-2 on $B \rightarrow \pi l \nu$

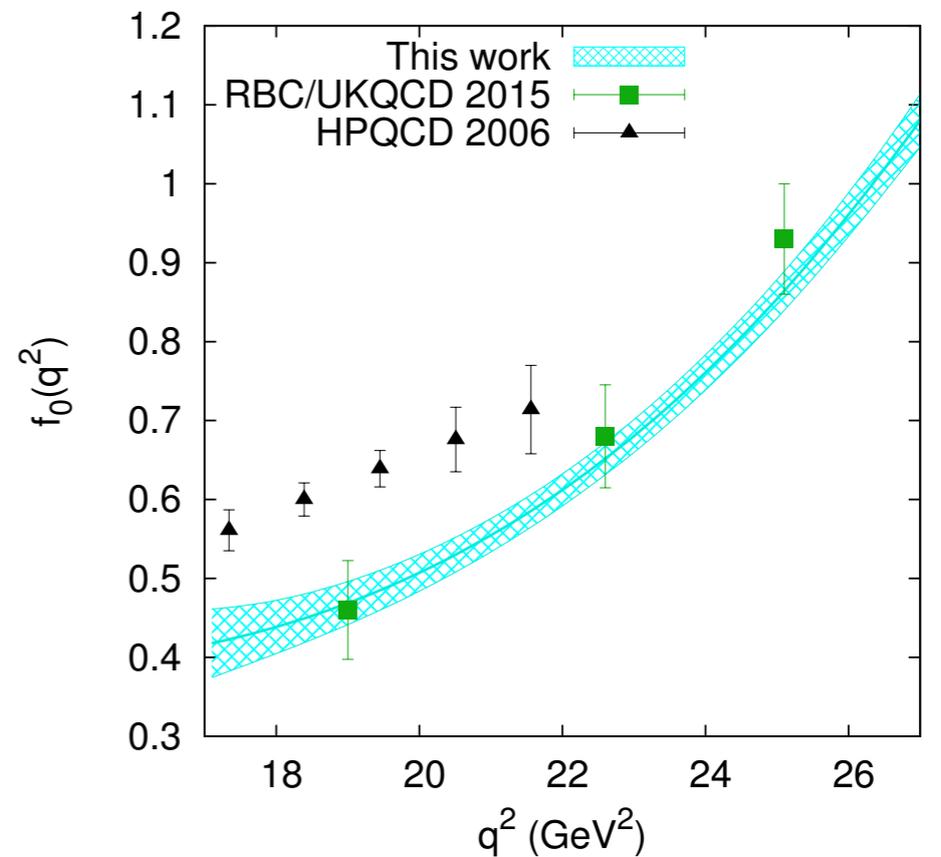
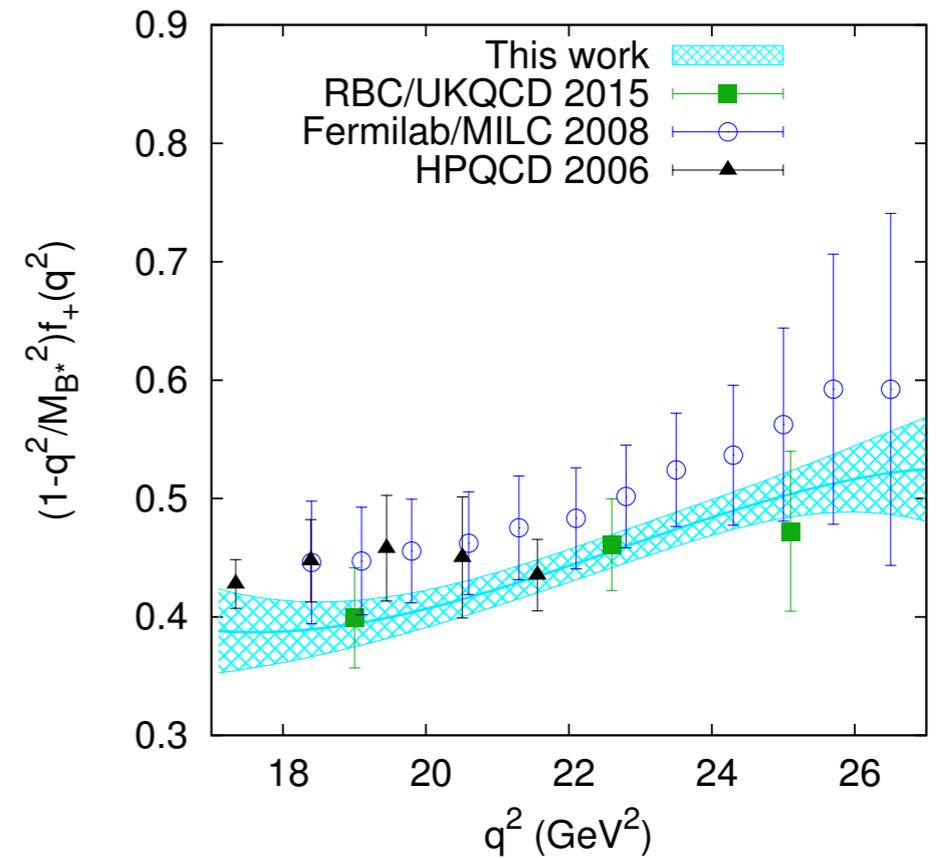


new results for $B \rightarrow \pi l \nu$

[RBC/UKQCD]



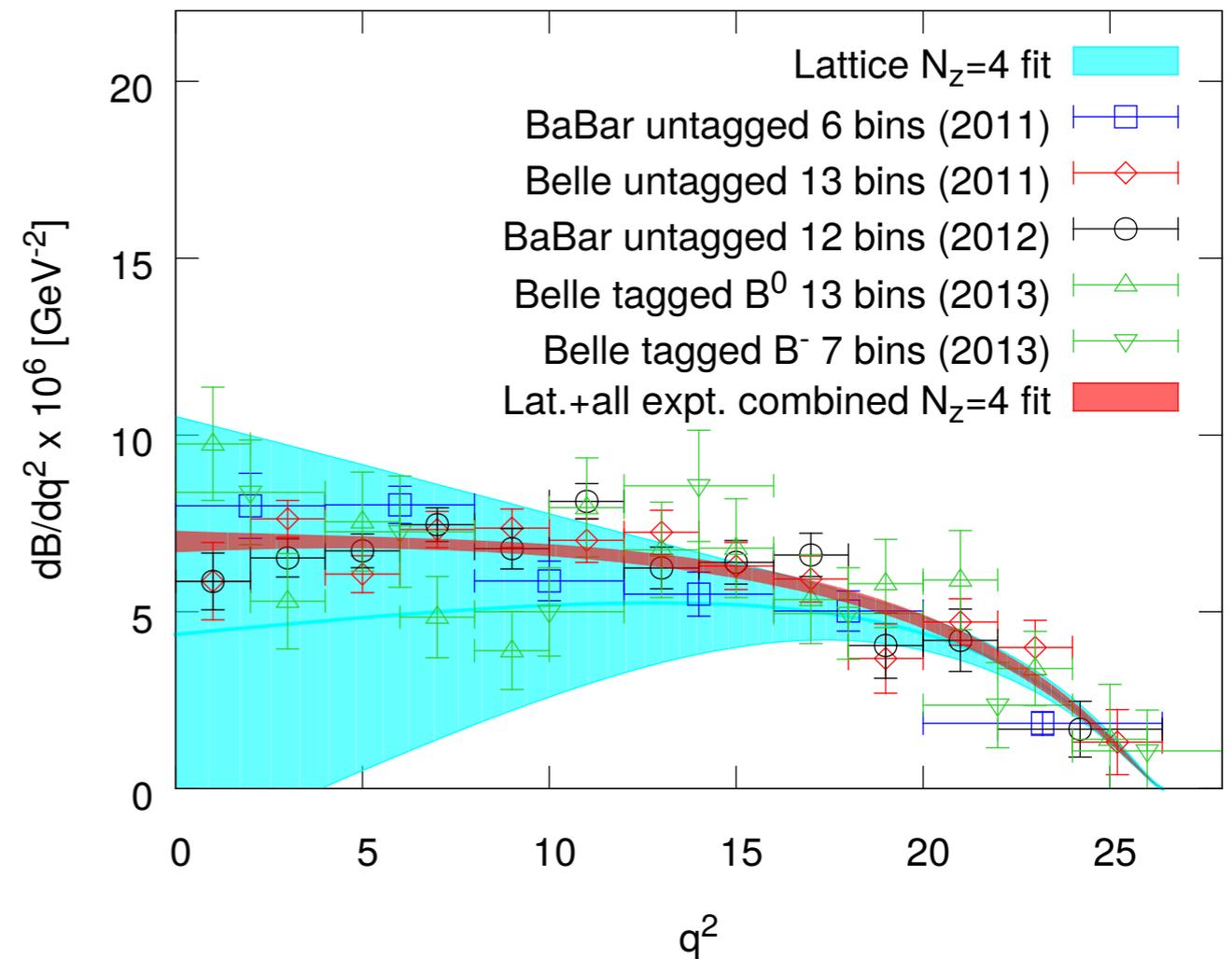
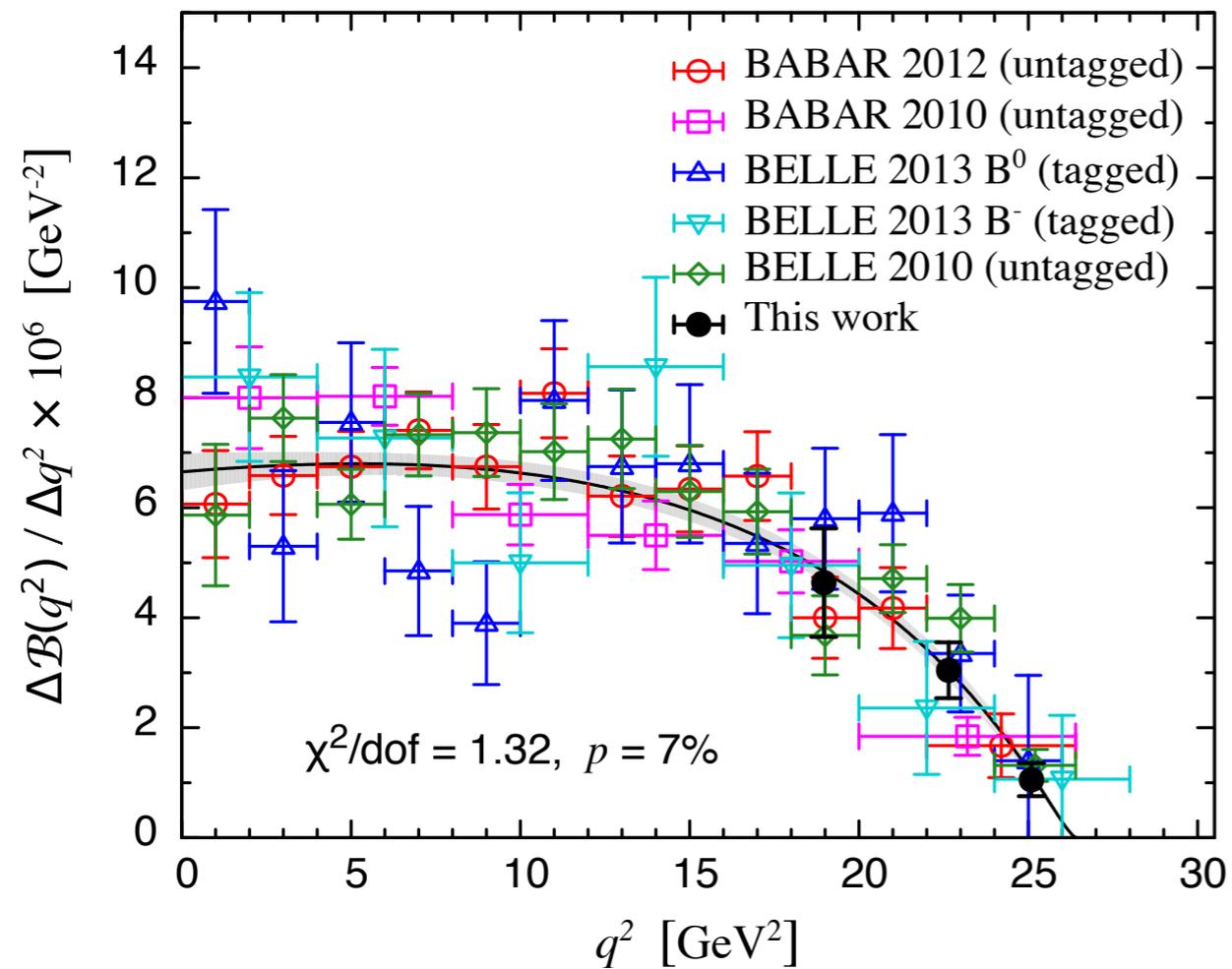
[FNAL/MILC]



new results for $B \rightarrow \pi l \nu$

[RBC/UKQCD]

[FNAL/MILC]

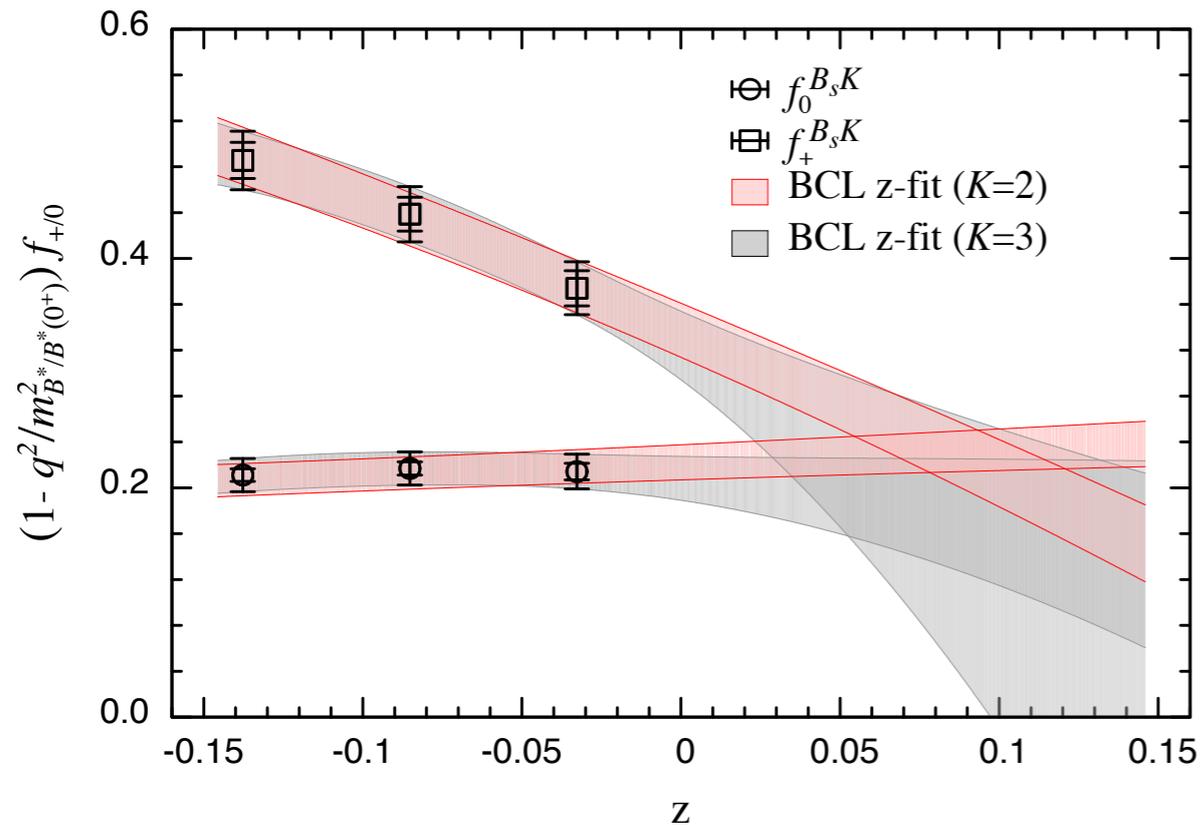


$$|V_{ub}| = 3.61(32) \times 10^{-3}$$

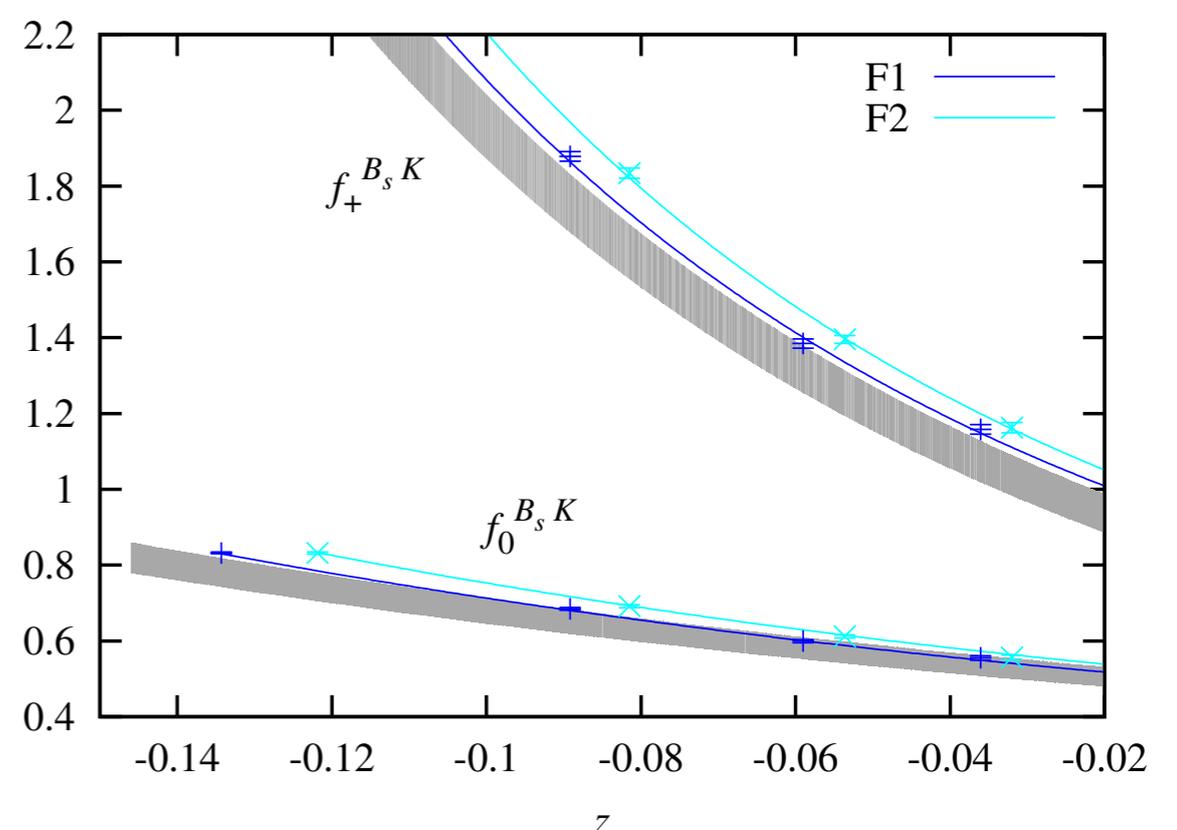
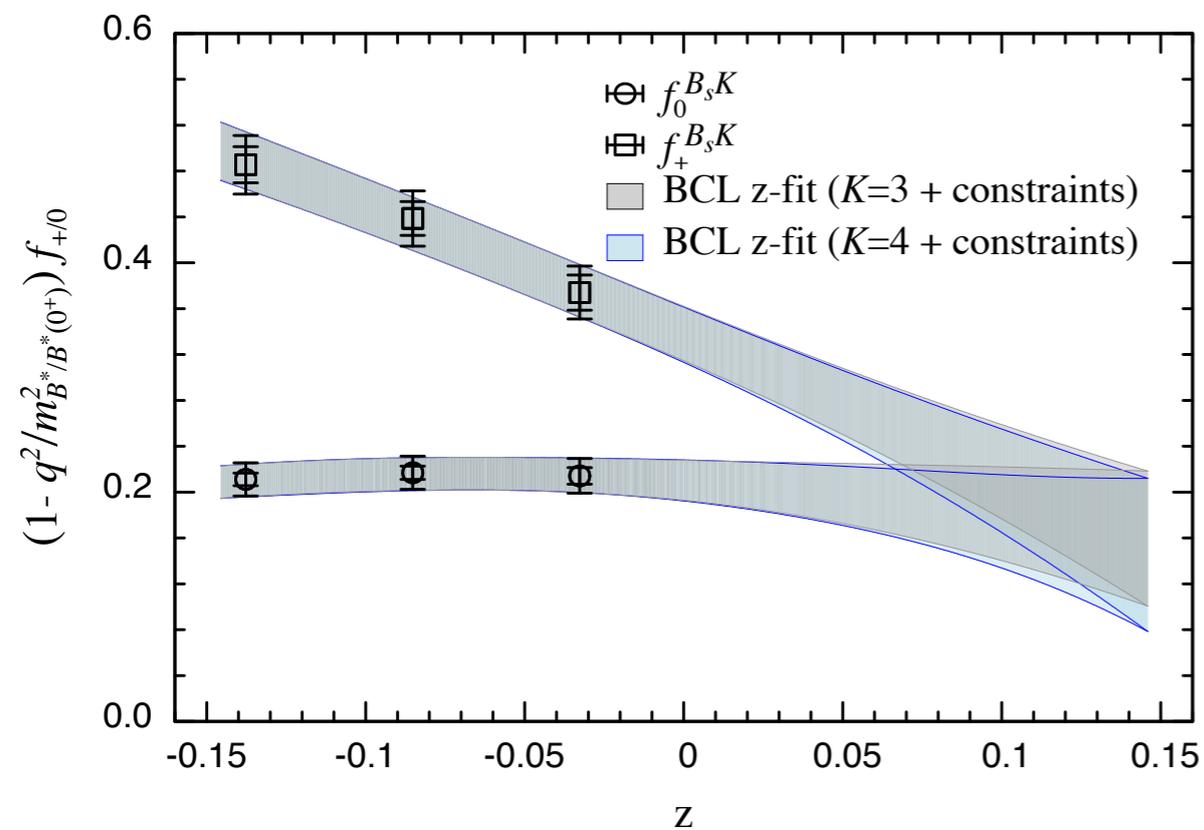
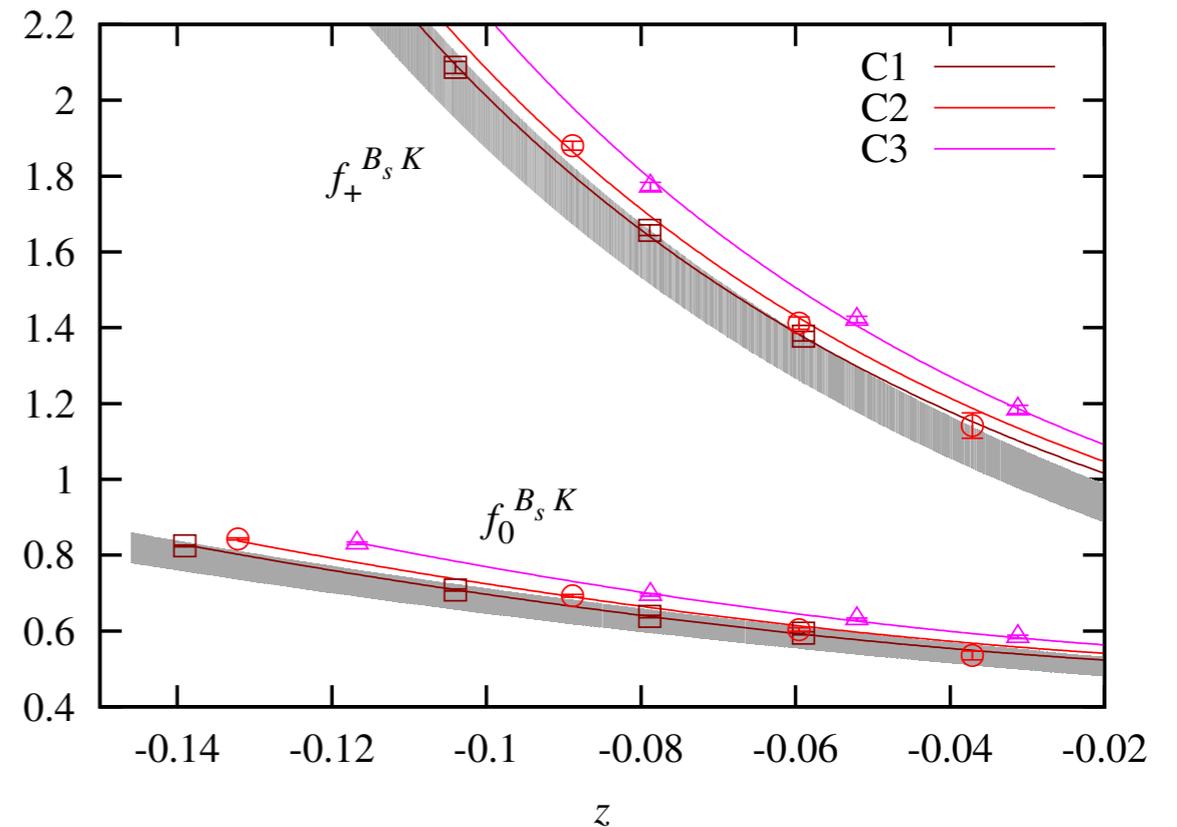
$$|V_{ub}| = 3.72(16) \times 10^{-3}$$

$B_s \rightarrow Kl\nu$

[RBC/UKQCD]



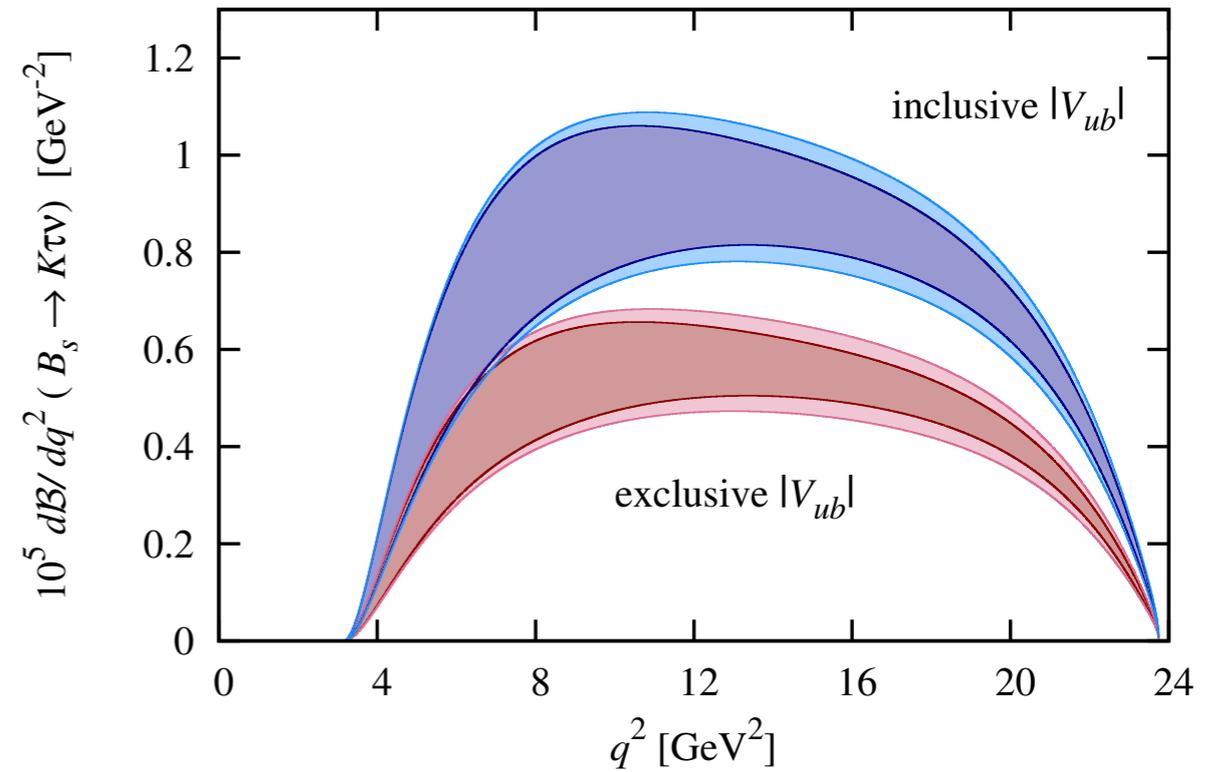
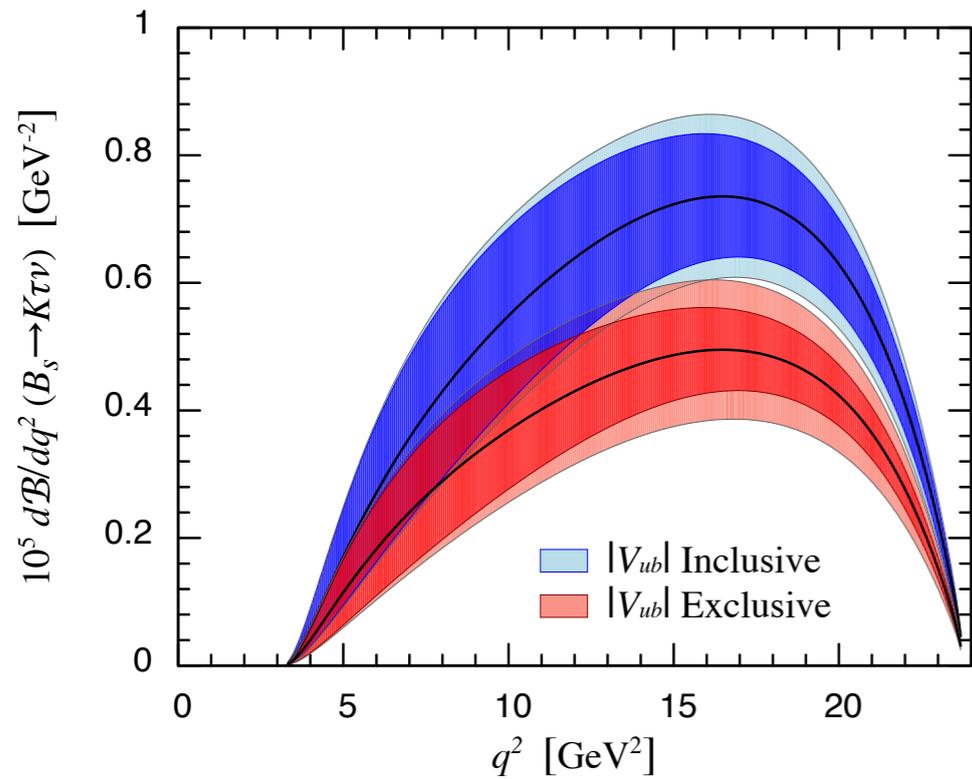
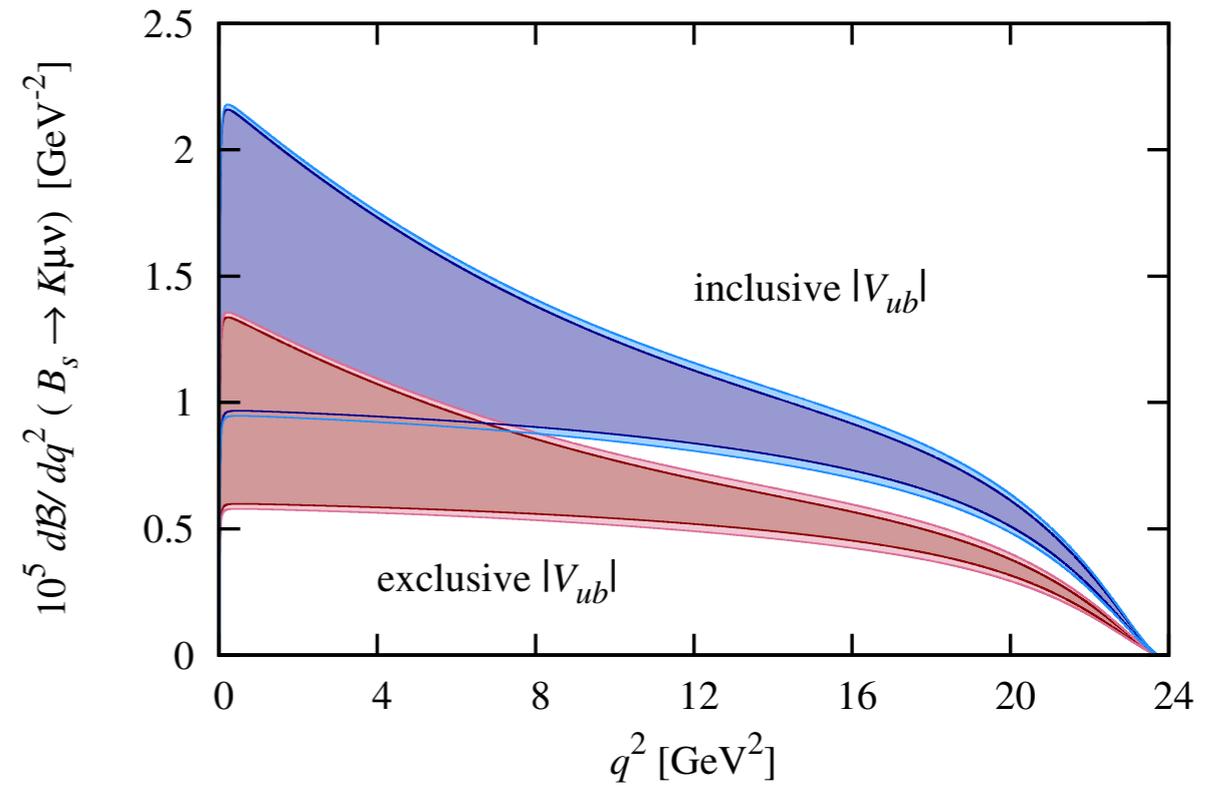
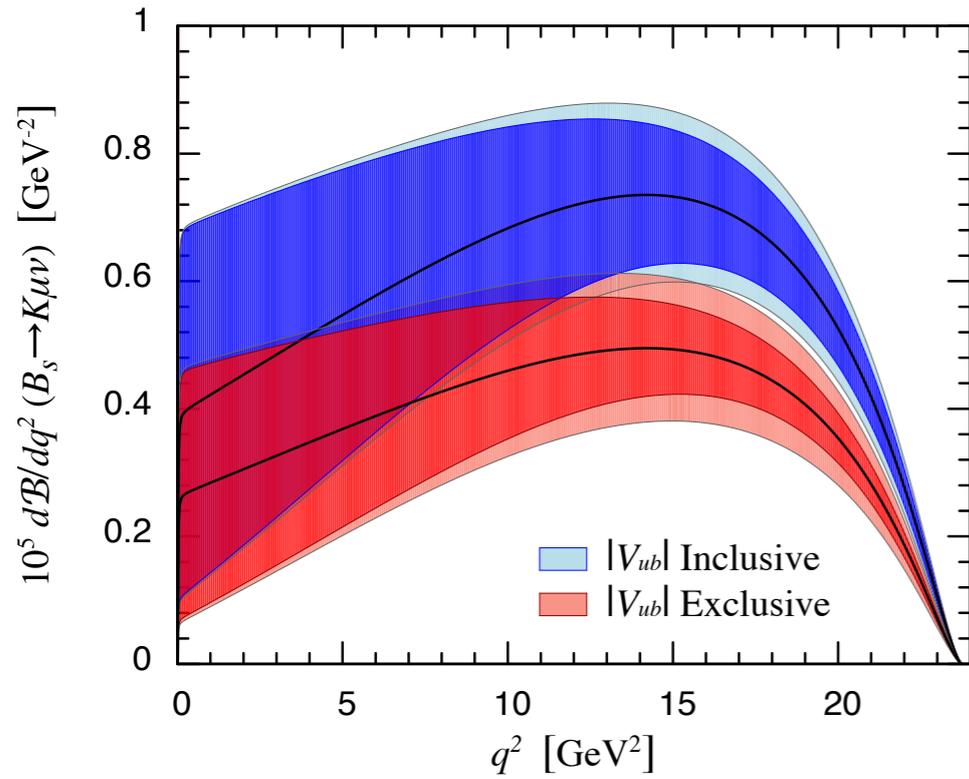
[HPQCD]



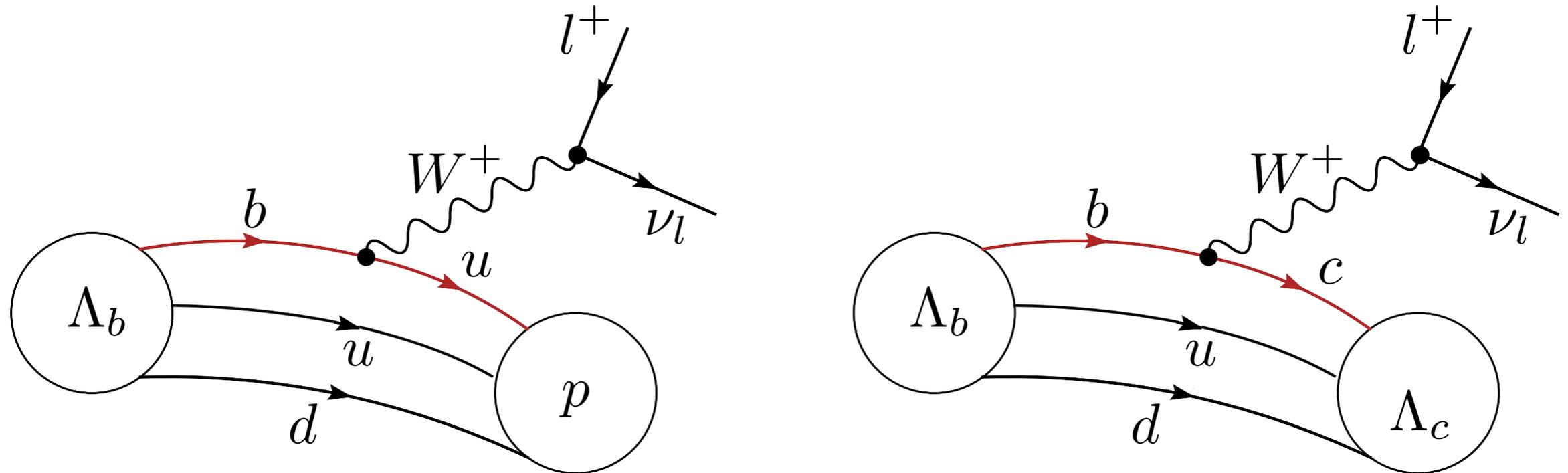
$B_s \rightarrow Kl\nu$

[RBC / UKQCD]

[HPQCD]



$|V_{ub}|$ and $|V_{cb}|$ from SL baryon decay



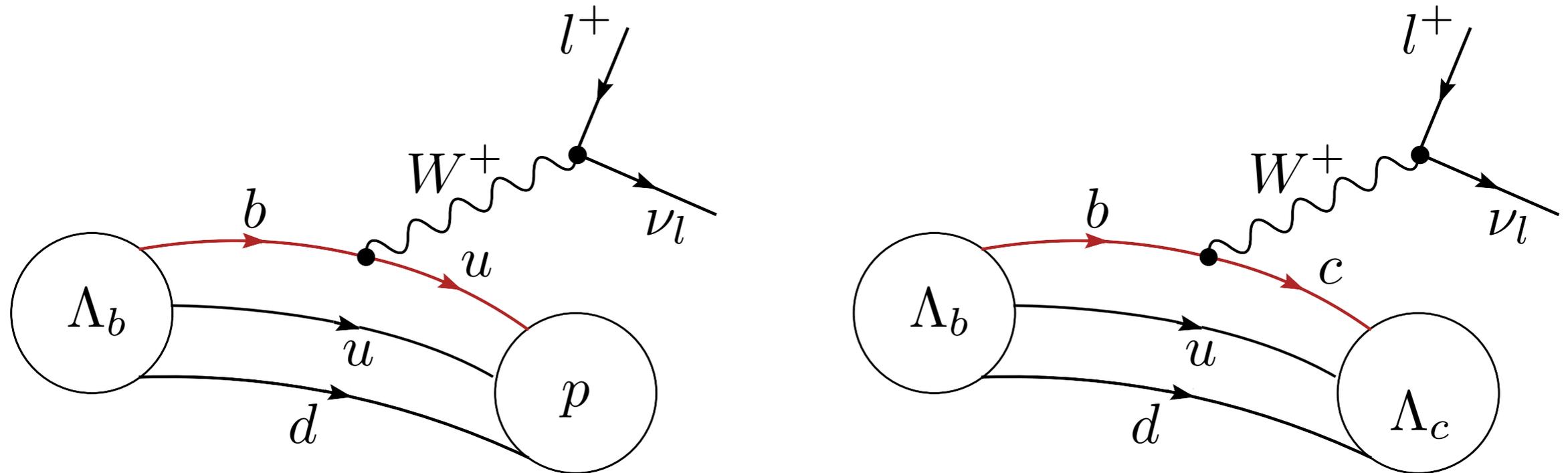
new exclusive determination of $|V_{cb}|/|V_{ub}|$ from LHCb measurement + LQCD computation of form factors



Kenneth G. Wilson Award for Excellence in Lattice
Field Theory 2015: **S Meinel**



$|V_{ub}|$ and $|V_{cb}|$ from SL baryon decay



helicity-based parametrisation: [Feldmann, Yip PRD 85 (2012) 014035]

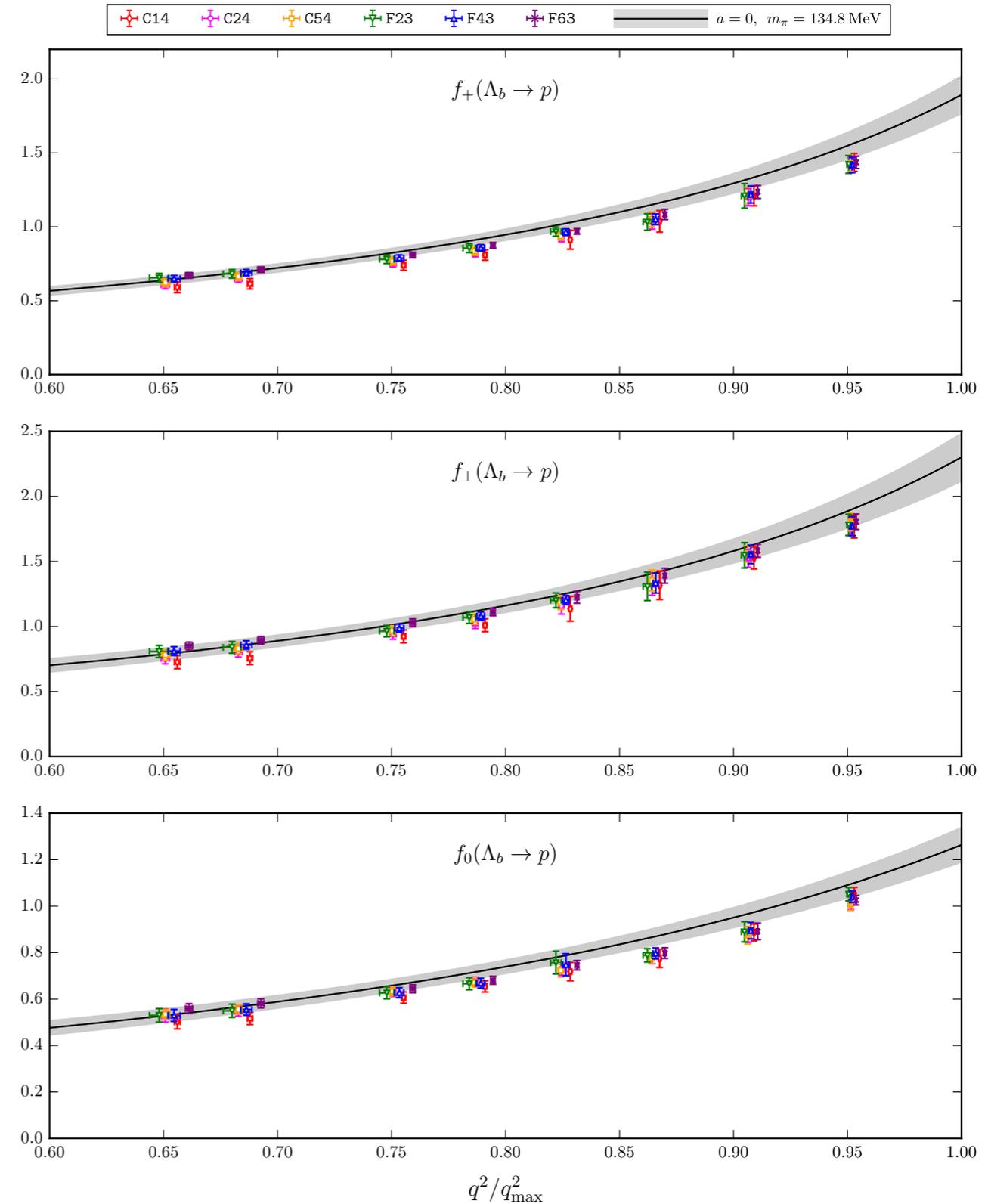
$$\begin{aligned}
 \langle X(p', s') | \bar{q} \gamma^\mu b | \Lambda_b(p, s) \rangle &= \bar{u}_X(p', s') \left[f_0(q^2) (m_{\Lambda_b} - m_X) \frac{q^\mu}{q^2} \right. \\
 &\quad + f_+(q^2) \frac{m_{\Lambda_b} + m_X}{s_+} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_X^2) \frac{q^\mu}{q^2} \right) \\
 &\quad \left. + f_\perp(q^2) \left(\gamma^\mu - \frac{2m_X}{s_+} p^\mu - \frac{2m_{\Lambda_b}}{s_+} p'^\mu \right) \right] u_{\Lambda_b}(p, s), \\
 \langle X(p', s') | \bar{q} \gamma^\mu \gamma_5 b | \Lambda_b(p, s) \rangle &= -\bar{u}_X(p', s') \gamma_5 \left[g_0(q^2) (m_{\Lambda_b} + m_X) \frac{q^\mu}{q^2} \right. \\
 &\quad + g_+(q^2) \frac{m_{\Lambda_b} - m_X}{s_-} \left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_X^2) \frac{q^\mu}{q^2} \right) \\
 &\quad \left. + g_\perp(q^2) \left(\gamma^\mu + \frac{2m_X}{s_-} p^\mu - \frac{2m_{\Lambda_b}}{s_-} p'^\mu \right) \right] u_{\Lambda_b}(p, s).
 \end{aligned}$$

$|V_{ub}|$ and $|V_{cb}|$ from SL baryon decay

[Detmold, Lehner, Meitner arXiv:1503.01421]

[cf. Detmold, Lin, Meitner, Wingate PRD 88 (2013) 014512]

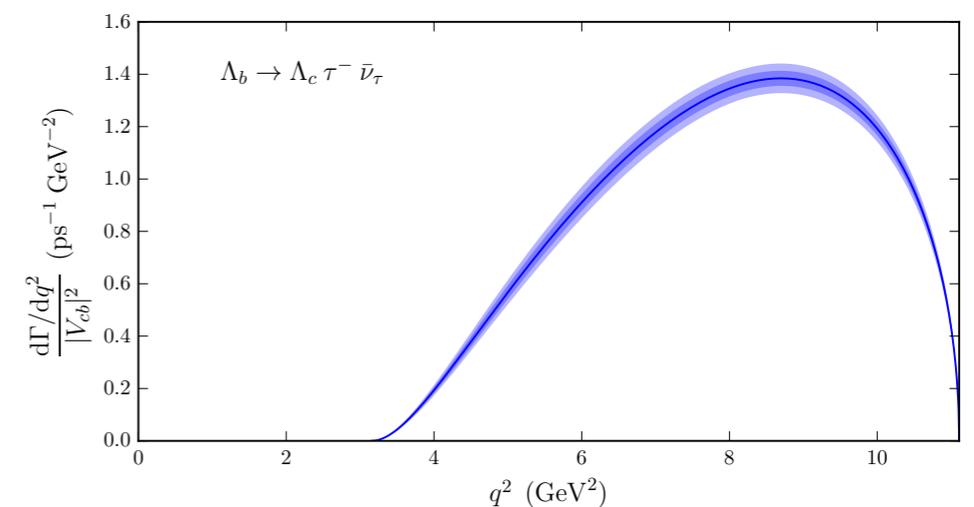
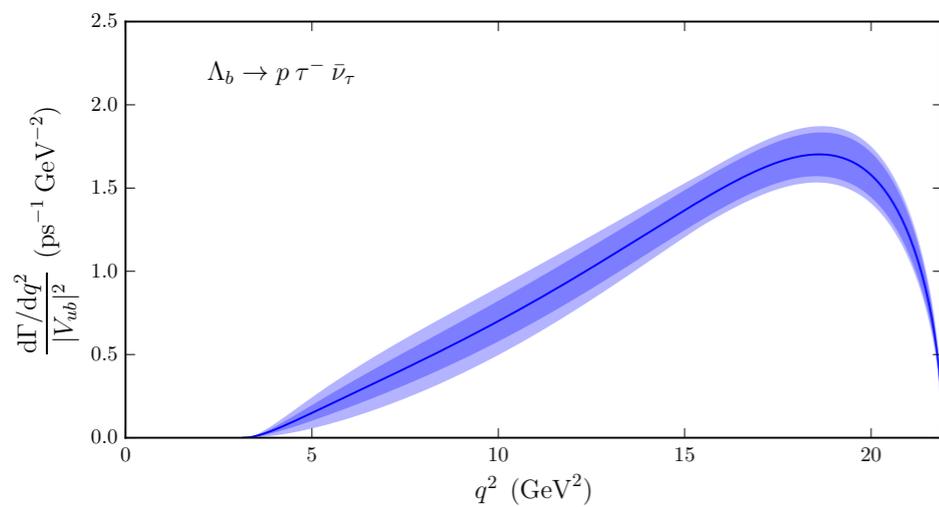
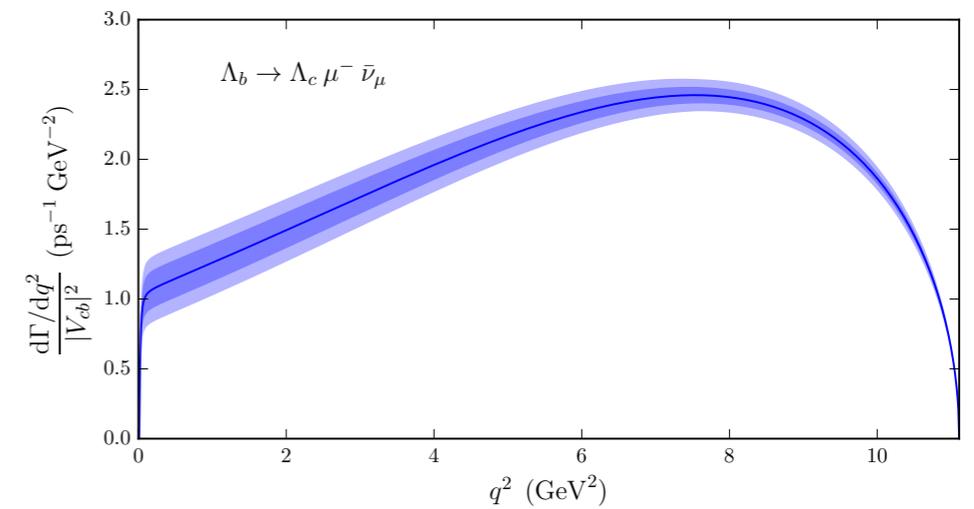
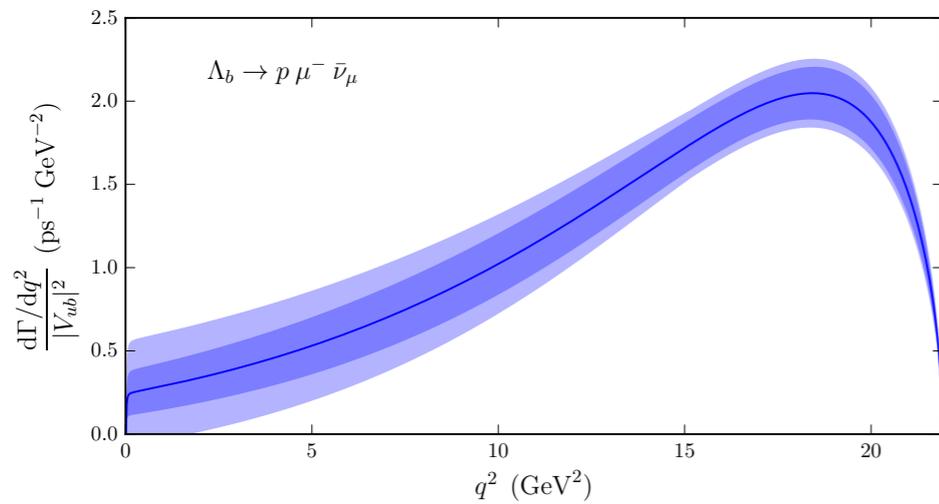
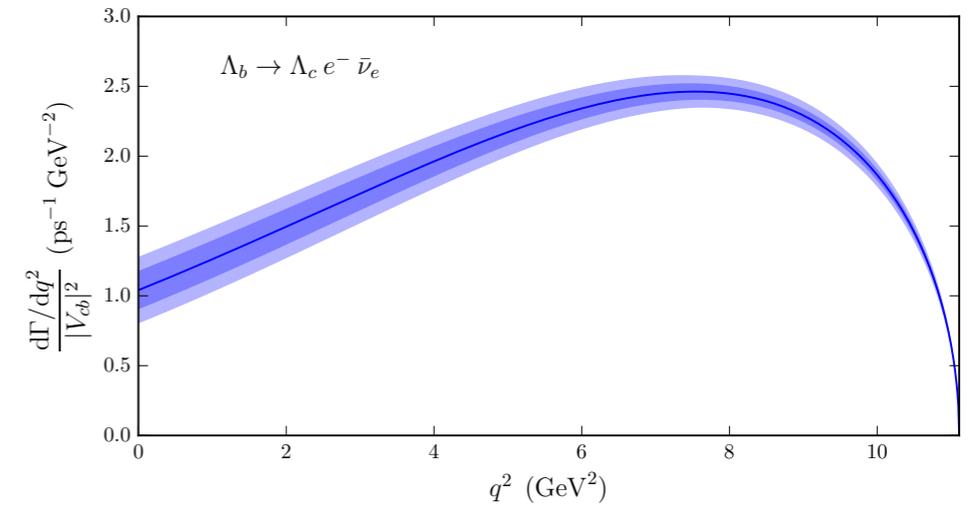
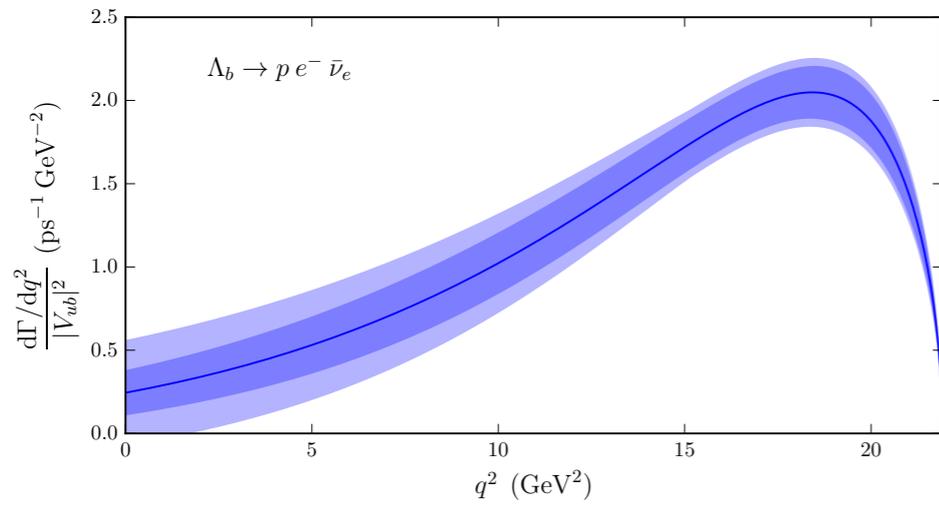
ensembles	RBC/UKQCD
N_f	2+1
a (fm)	2/0.085, 0.111
M_π^{\min} [MeV]	320 (sea) / 230 (val)
$M_\pi^{\min} L$	4.3 (sea) / 3.1 (val)
l quarks	DWF
b quark	RHQ



$|V_{ub}|$ and $|V_{cb}|$ from SL baryon decay

[Detmold, Lehner, Meitnel arXiv:1503.01421]

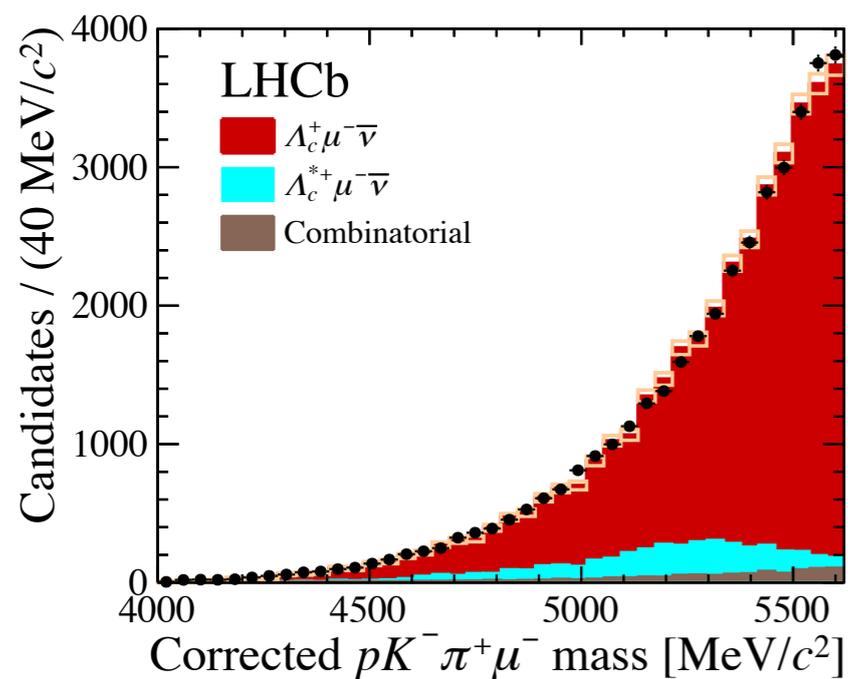
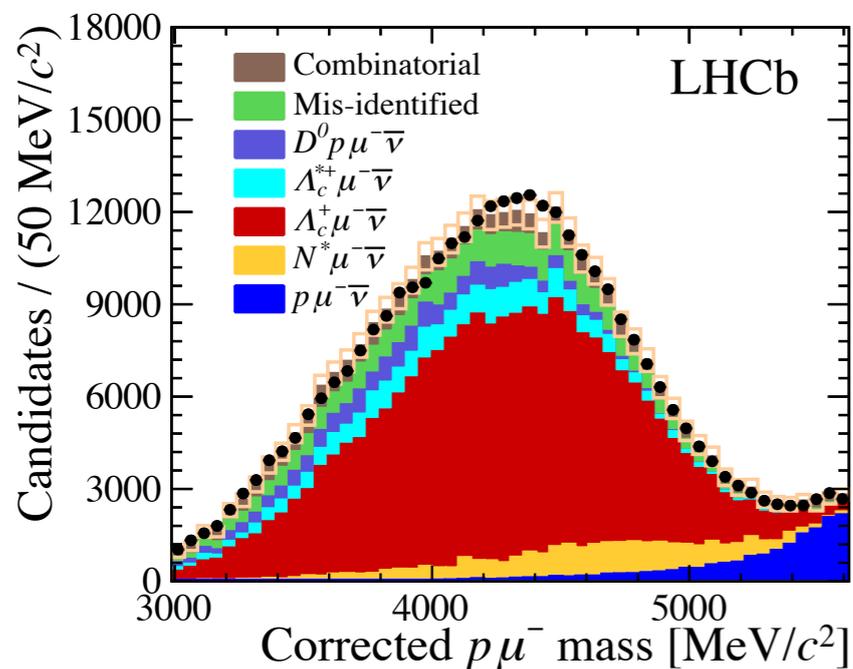
[cf. Detmold, Lin, Meitnel, Wingate PRD 88 (2013) 014512]



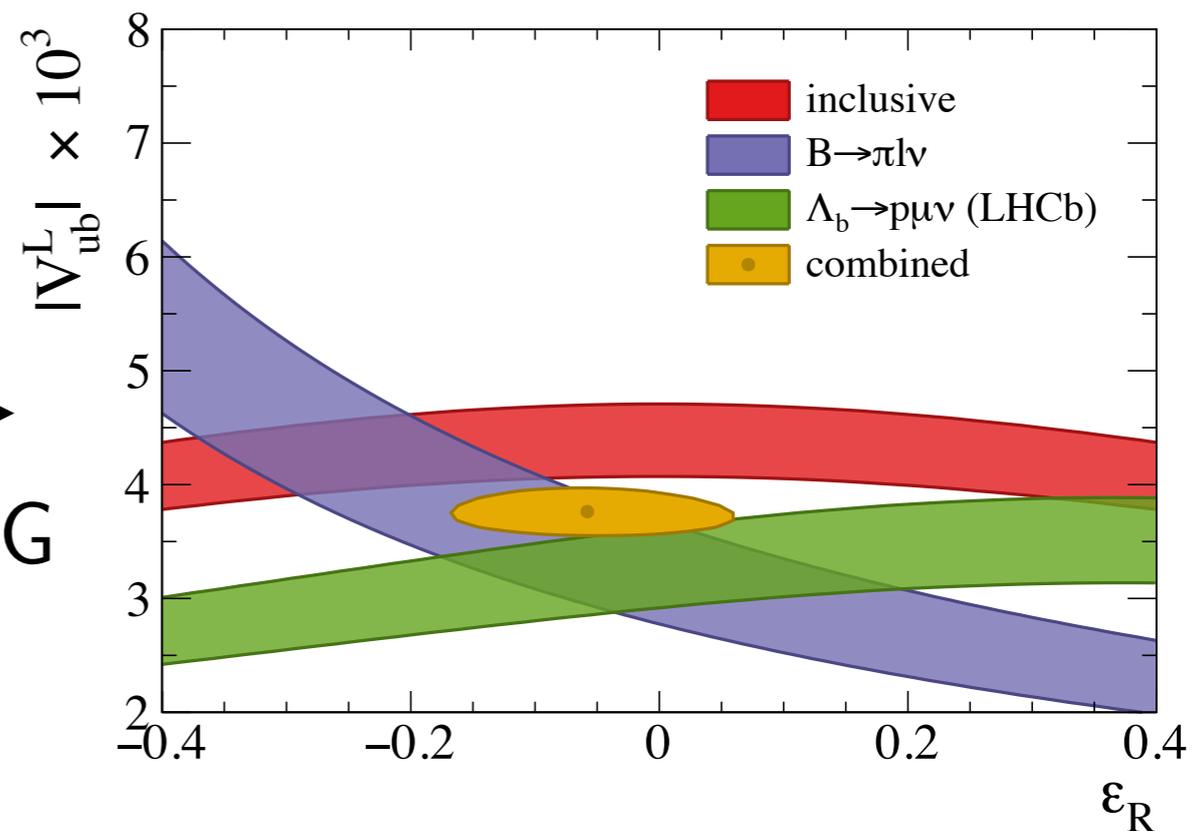
$|V_{ub}|$ and $|V_{cb}|$ from SL baryon decay

[LHCb arXiv:1504.01568]

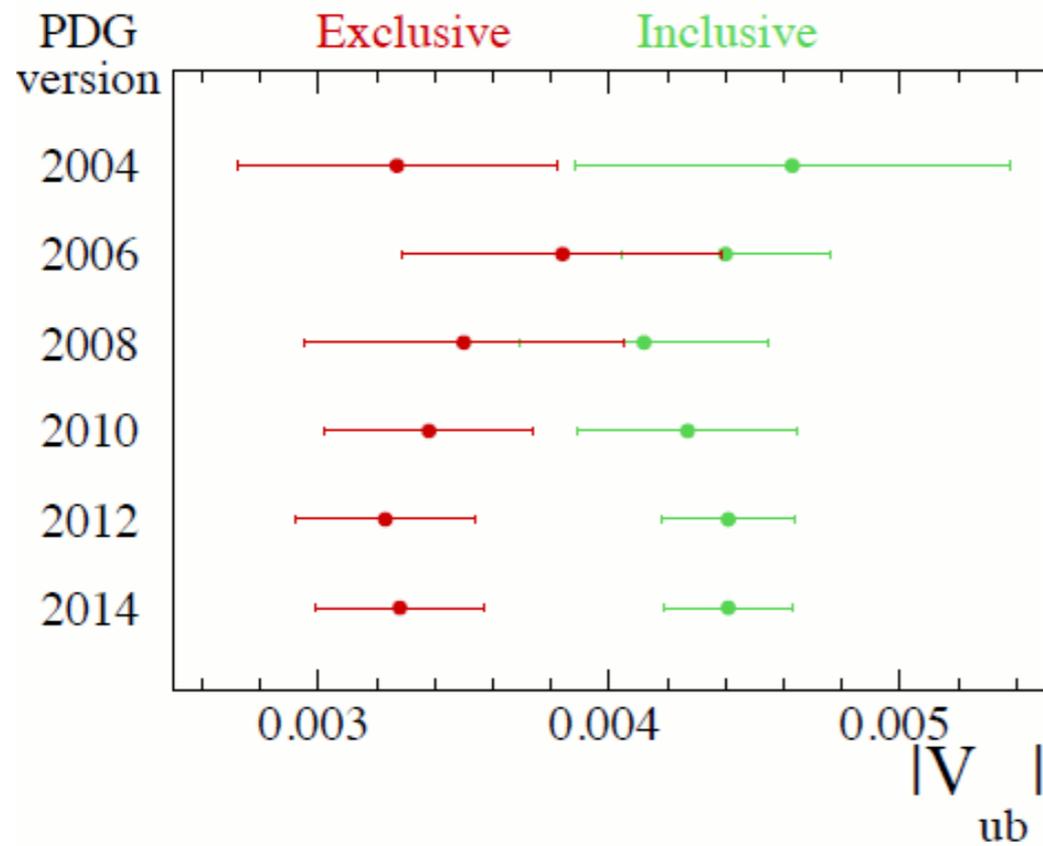
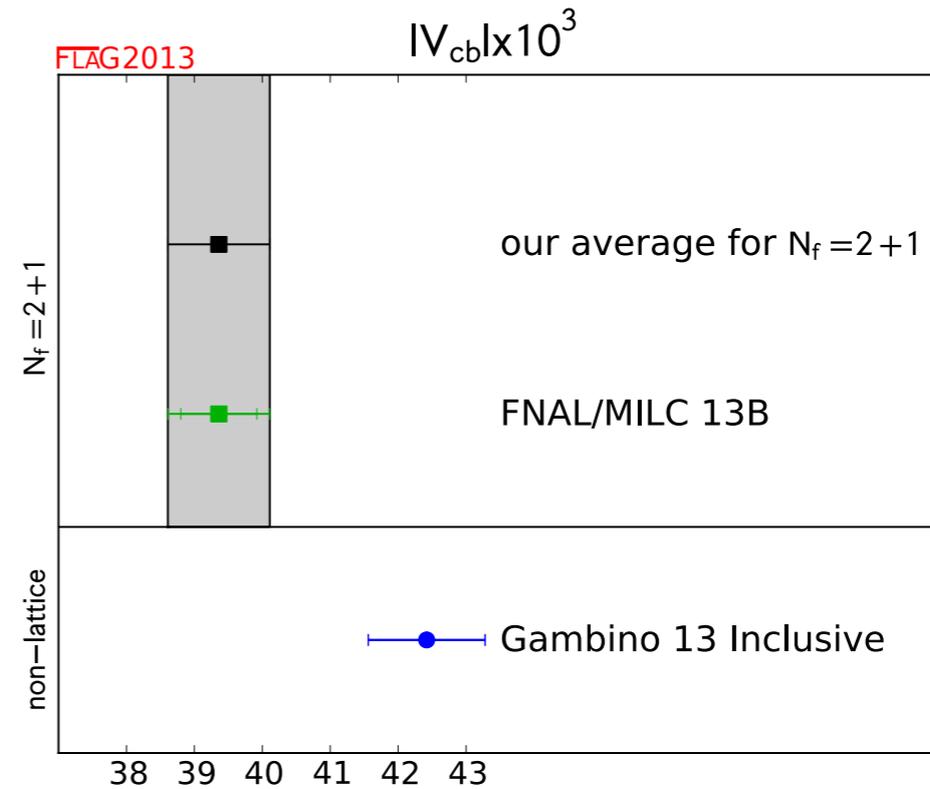
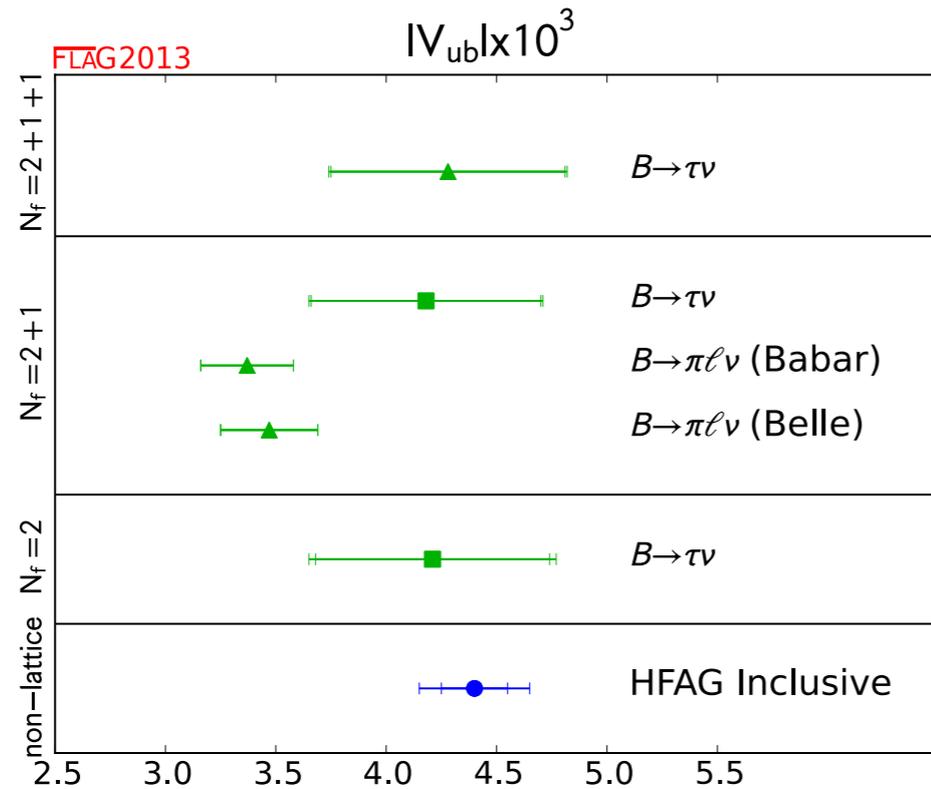
LHCb measurement of $\frac{\mathcal{B}(\Lambda_b^0 \rightarrow p\mu^-\bar{\nu}_\mu)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\bar{\nu}_\mu)} = 1.00(4)(8) \times 10^{-2}$



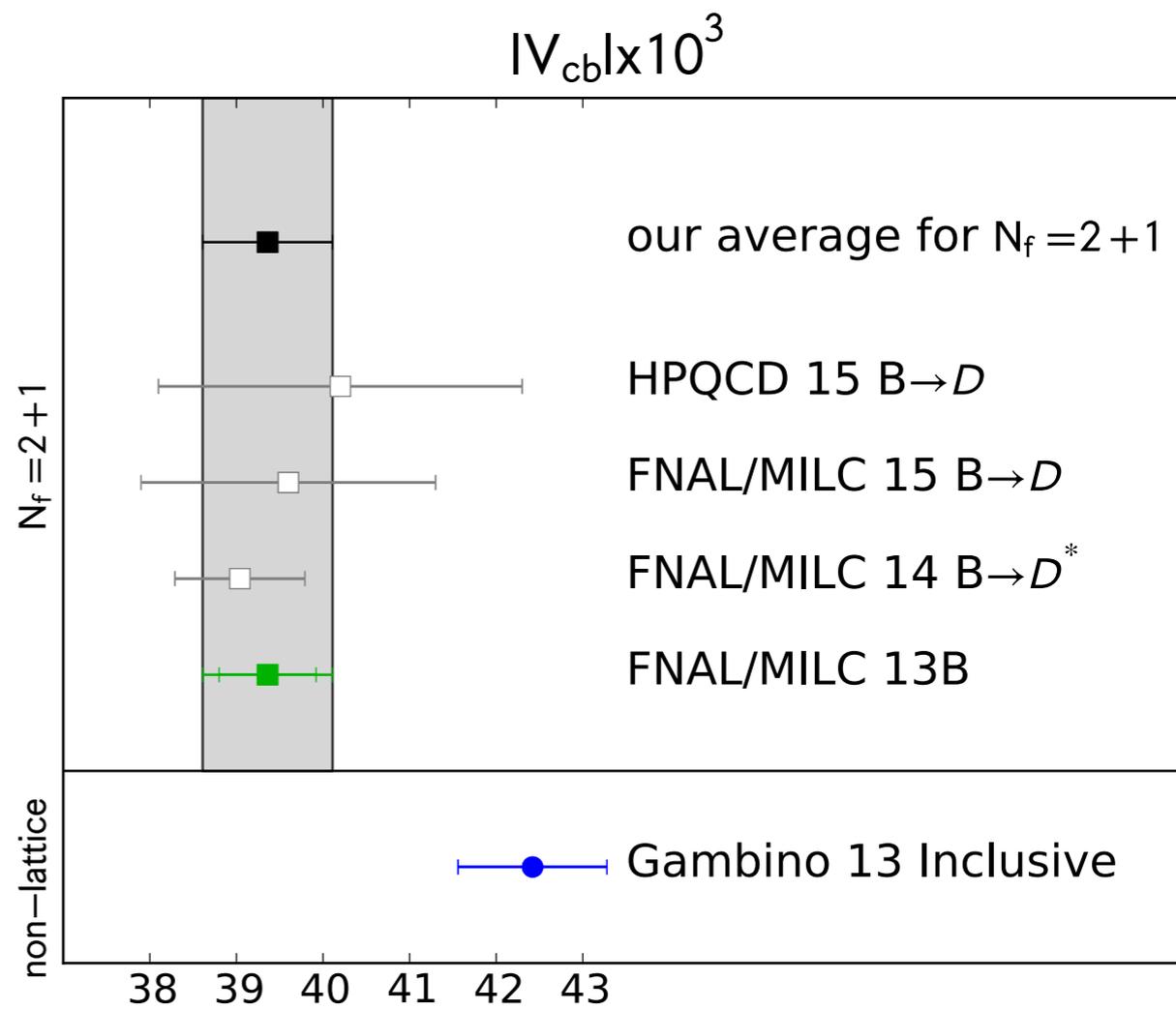
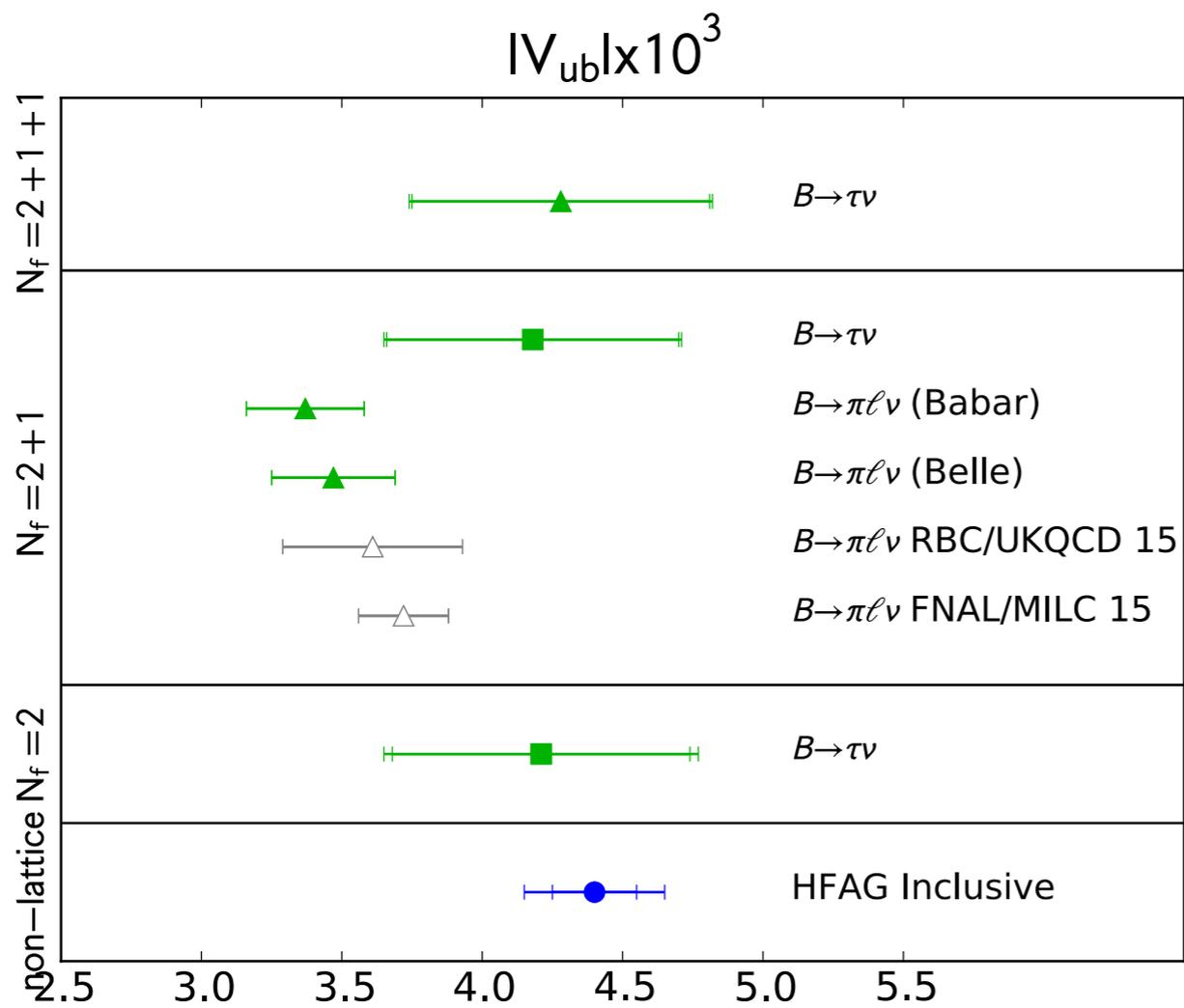
$|V_{cb}|$ from PDG
+ LQCD FFs



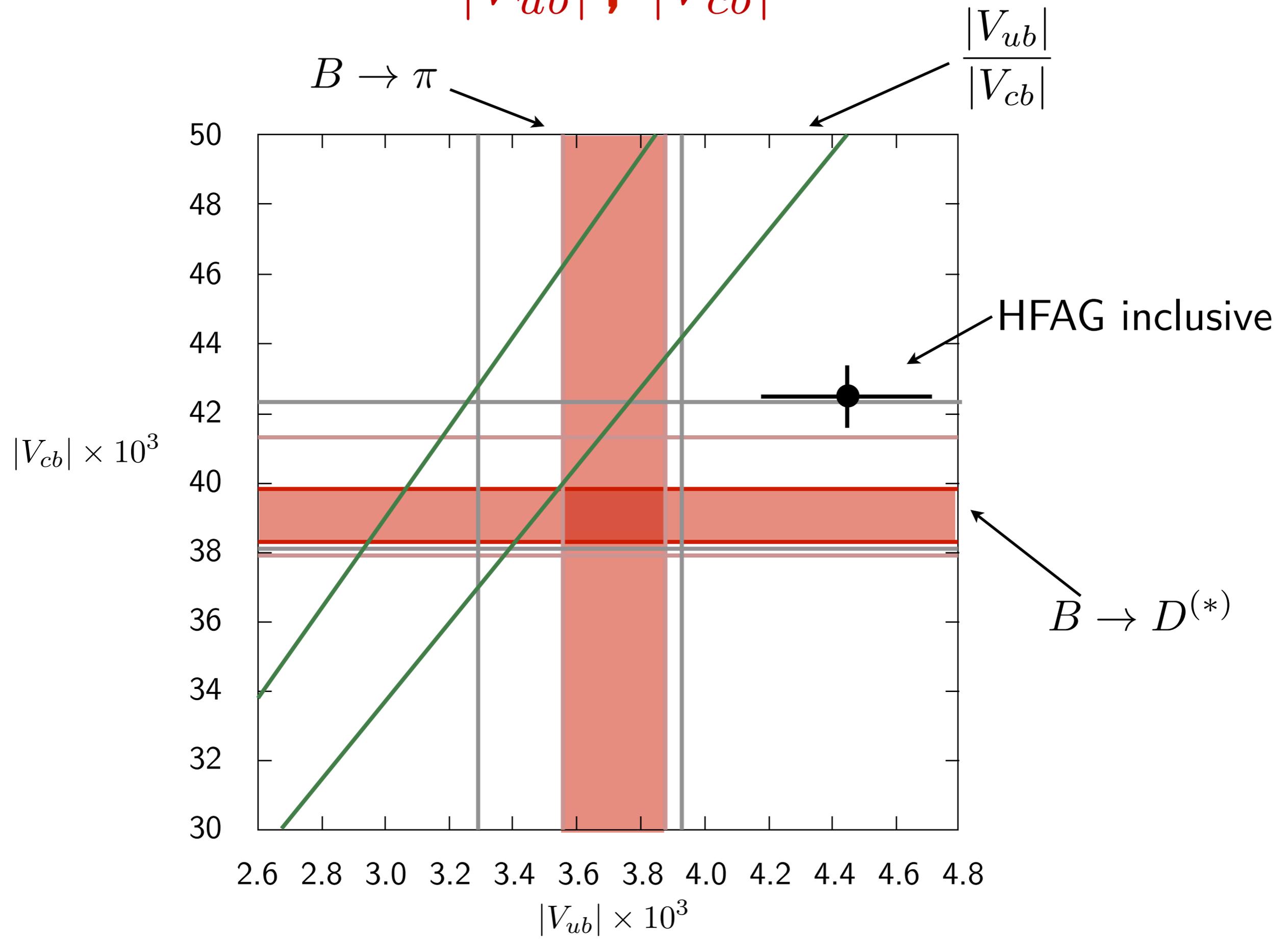
$|V_{ub}|$, $|V_{cb}|$



$|V_{ub}|, |V_{cb}|$



$|V_{ub}|, |V_{cb}|$



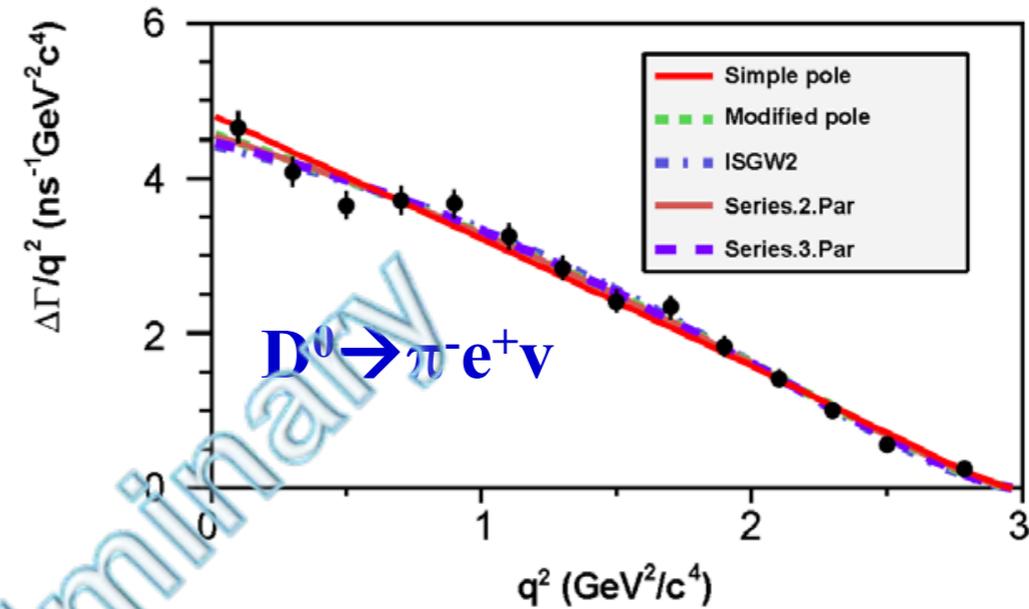
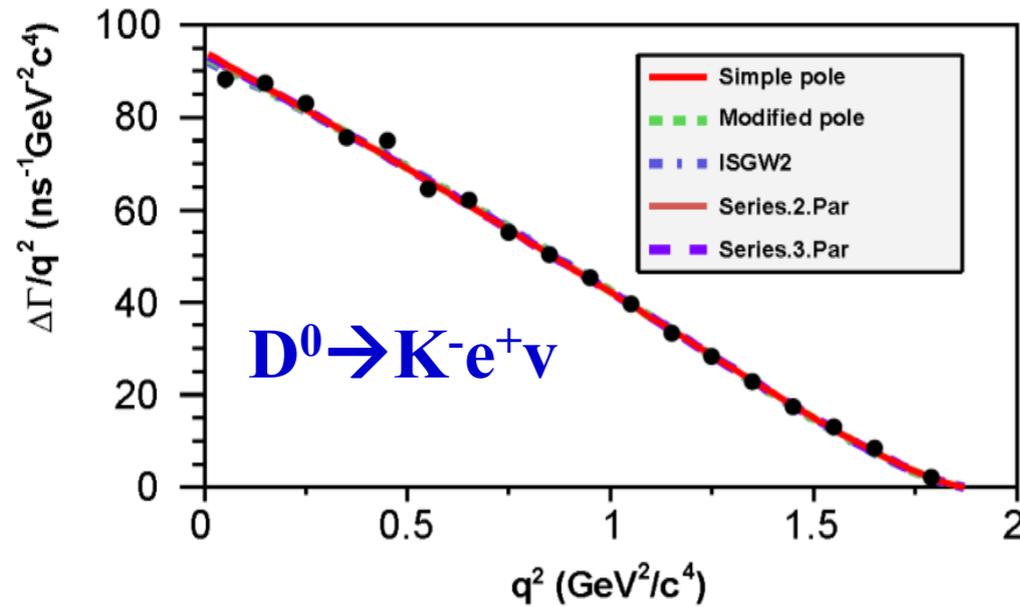
plan

- methods: where we stand (brief!)
 - ensembles used in HQ physics, reach
 - HQ approaches
- brief overview of
 - leptonic charm and B decay
 - B mixing
- charm and B semileptonic decays (+ CKM 2nd and 3rd rows)
- the % precision target
- conclusions and outlook

the % semileptonic precision target

- in order to achieve few % precision there are common issues with other quantities ...
 - limitations of HQ approaches
 - (related:) reliance on perturbation theory
 - mixed-action aspects
 - correct treatment of resonances
 - QED (also inclusive determinations), isospin, OPE corrections
- ... and some specific questions
 - contamination from excited states
 - matching of currents / four-quark operators
 - chiPT for form factors
 - **momentum dependence of form factors**

q^2 dependence of form factors



		$D^0 \rightarrow K^- e^+ \nu$		$D^0 \rightarrow \pi^- e^+ \nu$	
Simple Pole	$f_K^+(0) V_{cs} $	$0.7209 \pm 0.0022 \pm 0.0033$	$f_\pi^+(0) V_{cd} $	$0.1475 \pm 0.0014 \pm 0.0005$	
	M_{pole}	$1.9207 \pm 0.0103 \pm 0.0069$	M_{pole}	$1.9114 \pm 0.0118 \pm 0.0038$	
Mod. Pole	$f_K^+(0) V_{cs} $	$0.7163 \pm 0.0024 \pm 0.0034$	$f_\pi^+(0) V_{cd} $	$0.1437 \pm 0.0017 \pm 0.0008$	
	α	$0.3088 \pm 0.0195 \pm 0.0129$	α	$0.2794 \pm 0.0345 \pm 0.0113$	
ISGW2	$f_K^+(0) V_{cs} $	$0.7139 \pm 0.0023 \pm 0.0034$	$f_\pi^+(0) V_{cd} $	$0.1415 \pm 0.0016 \pm 0.0006$	
	r_{ISGW2}	$1.6000 \pm 0.0141 \pm 0.0091$	r_{ISGW2}	$2.0688 \pm 0.0394 \pm 0.0124$	
Series.2.Par	$f_K^+(0) V_{cs} $	$0.7172 \pm 0.0025 \pm 0.0035$	$f_\pi^+(0) V_{cd} $	$0.1435 \pm 0.0018 \pm 0.0009$	
	r_1	$-2.2278 \pm 0.0864 \pm 0.0575$	r_1	$-2.0365 \pm 0.0807 \pm 0.0260$	
Series.3.Par	$f_K^+(0) V_{cs} $	$0.7196 \pm 0.0035 \pm 0.0041$	$f_\pi^+(0) V_{cd} $	$0.1420 \pm 0.0024 \pm 0.0010$	
	r_1	$-2.3331 \pm 0.1587 \pm 0.0804$	r_1	$-1.8434 \pm 0.2212 \pm 0.0690$	
	r_2	$3.4223 \pm 3.9090 \pm 2.4092$	r_2	$-1.3871 \pm 1.4615 \pm 0.4677$	

a benchmark case: $f_+(B \rightarrow \pi l \nu)$

various parametrisations based on pole dominance: Bećirević-Kaidalov, Ball-Zwicky, Hill, ... difficult to systematically improve precision

[Bećirević, Kaidalov PLB 478 (2000) 417]

[Ball, Zwicky PRD 71 (2005) 014015]

[Hill PRD 73 (2006) 014012]

z-parametrisations proposed to solve this issue (almost) rigourously by exploiting unitarity and crossing symmetry

[Okubo PRD 3 (1971) 2807, 4 (1971) 725]

[Bourrely, Machet, de Rafael NPB 189 (1981) 157]

[Boyd, Grinstein, Lebed PRL 74 (1995) 4603]

[Lellouch NPB 479 (1996) 353]

[Bourrely, Caprini, Micu EJPC 27 (2003) 439]

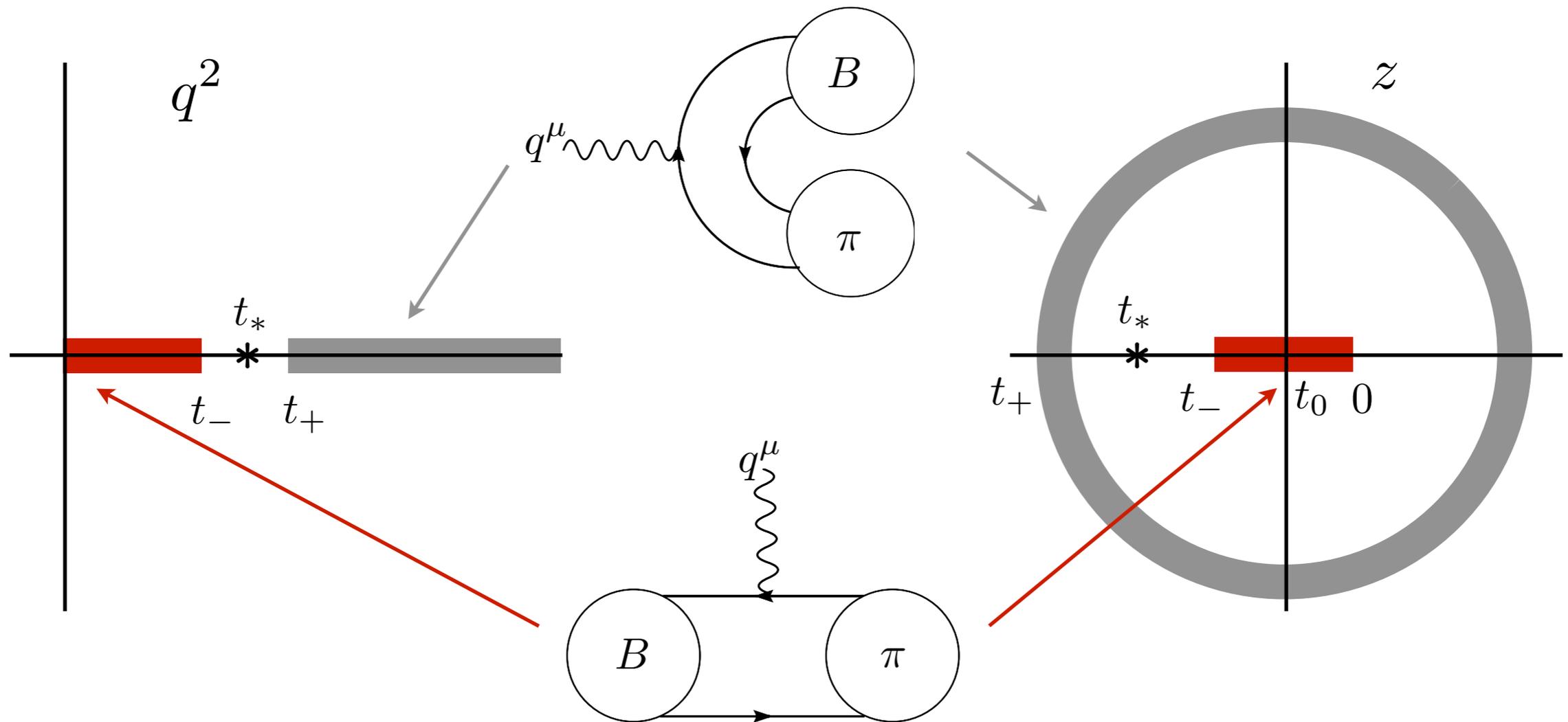
[Arnesen, Grinstein, Rothstein, Stewart PRL 95 (2005) 071802]

[Becher, Hill PLB 633 (2006) 61]

[Flynn, Nieves PRD 75 (2007) 013008]

[Bourrely, Caprini, Lellouch PRD 79 (2009) 013008]

a benchmark case: $f_+(B \rightarrow \pi l \nu)$



$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$



$$f_+(q^2) = \frac{1}{B(q^2)\phi(q^2, t_0)} \sum_{n \geq 0} a_n z(q^2, t_0)^n$$

$$t_+ = (m_B + m_\pi)^2, \quad t_0 < t_+$$

$$\text{unitarity bound: } \sum_{m, n} B_{mn}^{(\phi)} a_m a_n \leq 1$$

a benchmark case: $f_+(B \rightarrow \pi l \nu)$

$$f_+(q^2) = \frac{1}{B(q^2)\phi(q^2, t_0)} \sum_{n \geq 0} a_n z(q^2, t_0)^n \quad B(q^2) = z(q^2, m_{B^*}^2)$$

BGL: complicated outer function $\phi \longrightarrow \sum_{n \geq 0} |a_n|^2 \lesssim 1$

[Boyd, Grinstein, Lebed PRL 74 (1995) 4603]

$$\text{BCL: } f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{n \geq 0} a_n z^n \longrightarrow \sum_{m, n \geq 0} B_{mn} a_m a_n \lesssim 1$$

(recommended by FLAG)

[Bourenly, Caprini, Lellouch PRD 79 (2009) 013008]

crucial for optimal use:

- all sub-threshold poles included in Blaschke factor
- fixed kinematics (coefficients implicitly depend on quark masses)

does the unitarity bound apply?

- using a z -parametrisation as part of a global fit including a , m_q , ... (modified z -expansion) tricky
 - poles can cross threshold as quark masses change
 - complicated entanglement of (m_q, a) dependence (complete form factor vs. z -parametrisation coefficient)
- pole structure not always well-known (scalar channels, D decay), or complicated (Λ_b decay)
- missing sub-threshold poles may imply convergence breakdown (proton charge radius analysis by Hill, Paz et al, D semileptonic decay data by Bećirević et al)

[Hill, Paz PRD 82 (2010) 113005]

[Bhattacharya, Hill, Paz PRD 84 (2011) 073006]

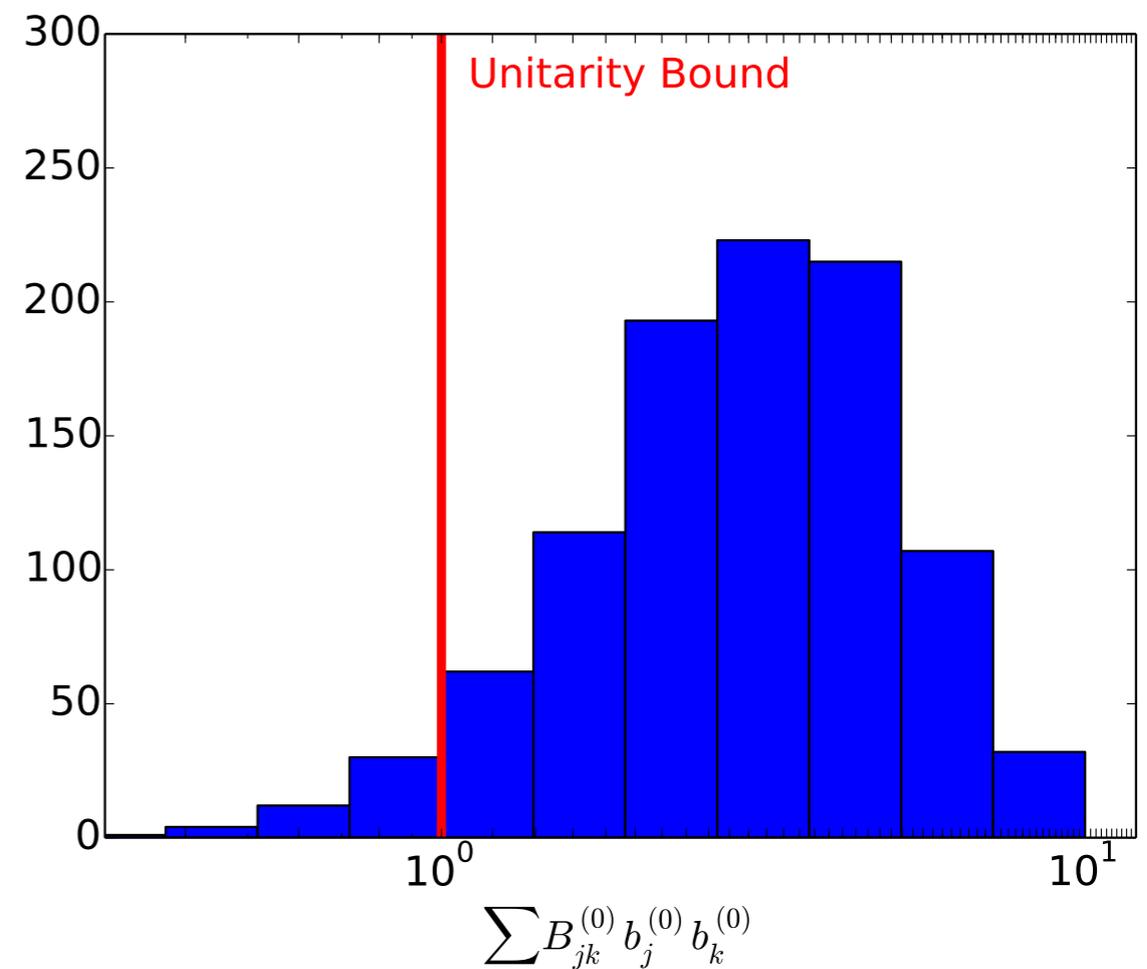
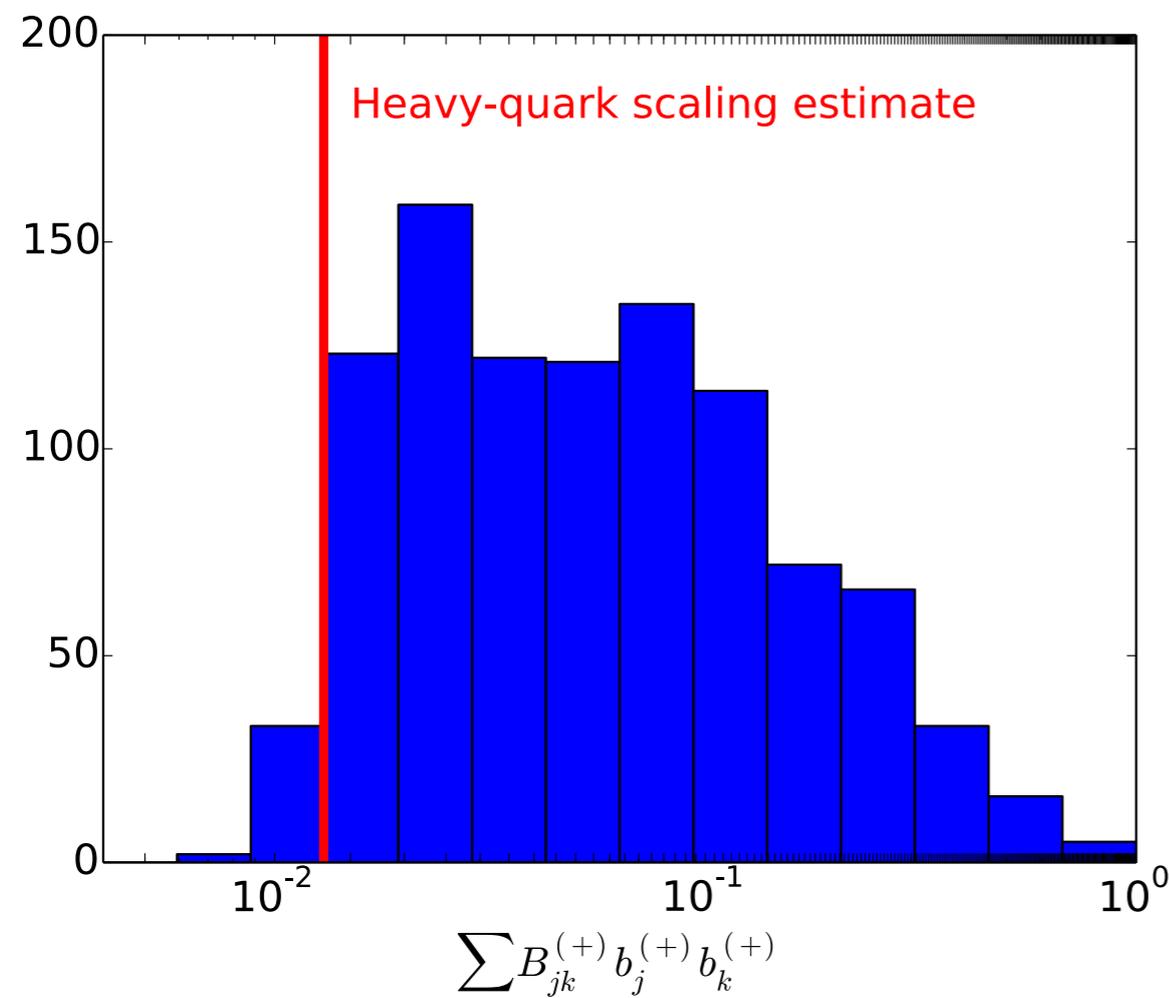
[Epstein, Paz, Roy PRD 90 (2014) 074027]

[cf talk by J Zanotti]

[Bećirević et al arXiv:1407.1019]

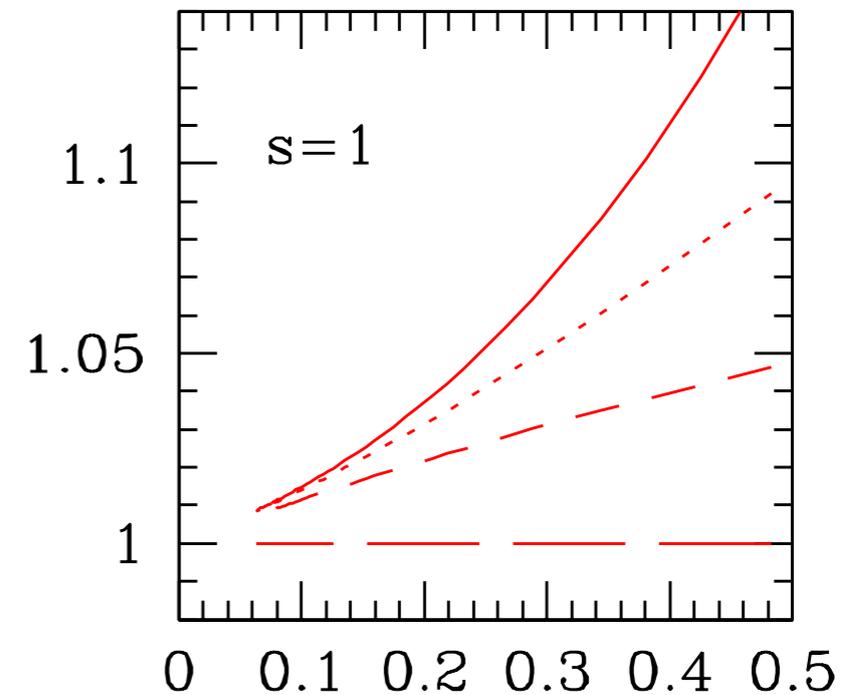
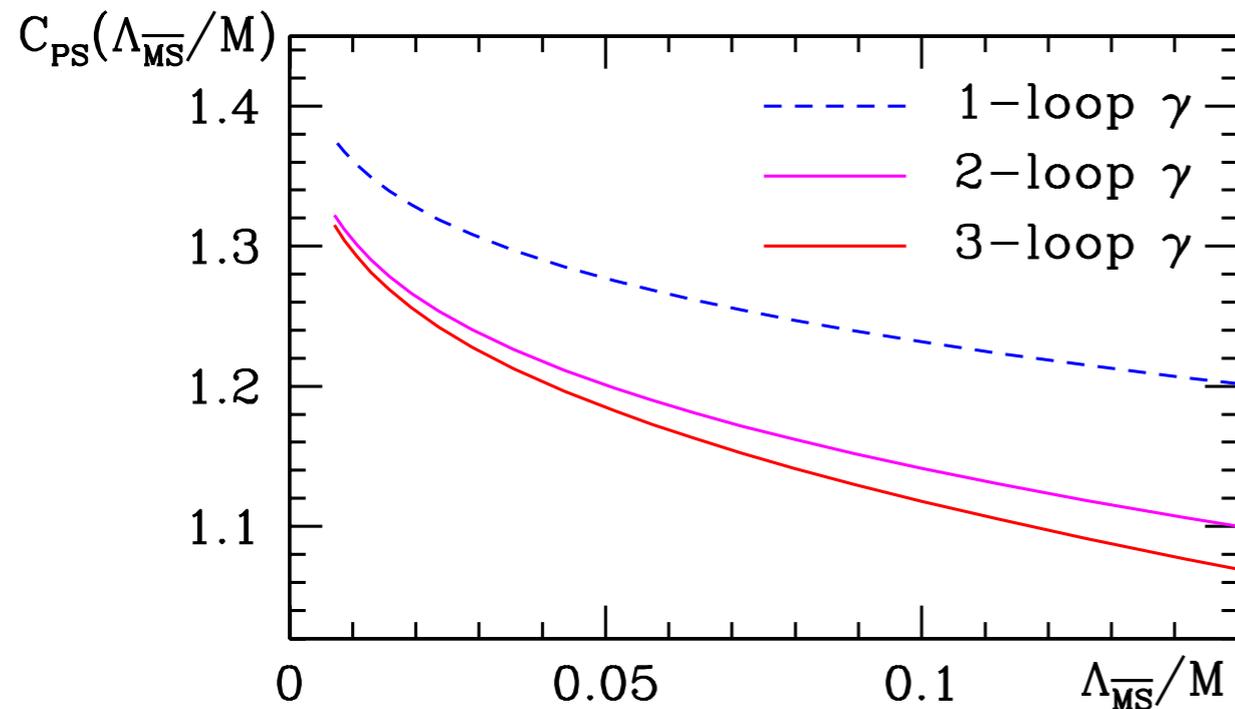
is your z-parametrisation well-behaved?

convergence properties can actually be tested



matching/renormalisation

perturbative convergence known to be poor at b scale



[R Sommer, Les Houches Lectures arXiv:1008.0710]

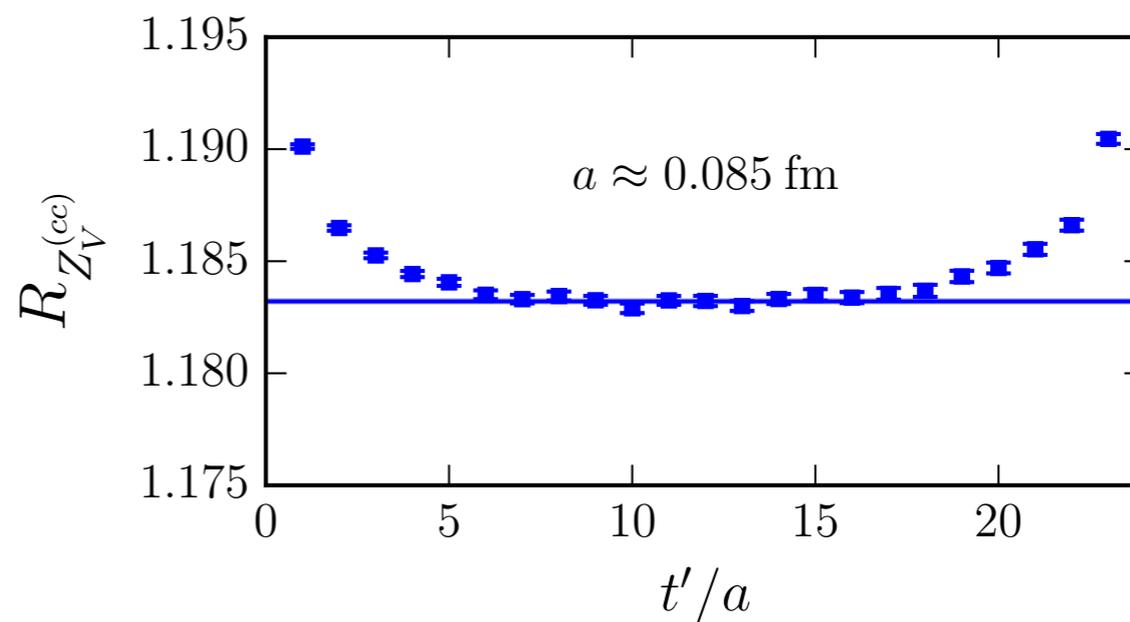
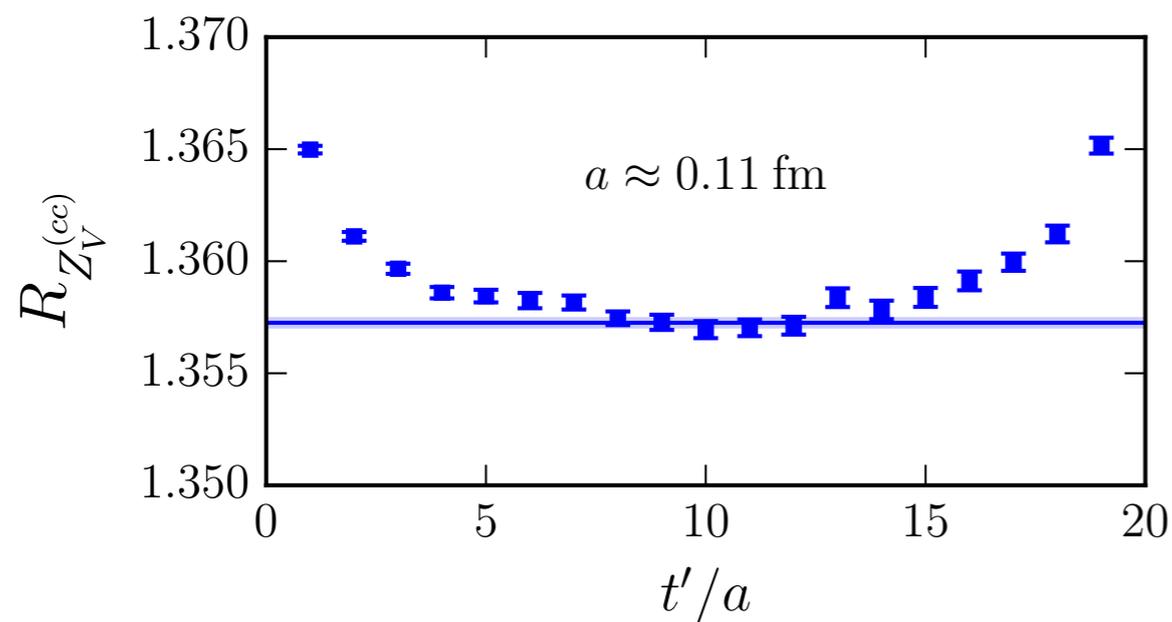
e.g. systematics due to using perturbative running in HQET may well be $O(4\%)$ for B decay constant

[cf. P Fritzsche's talk]

extensive one-loop input for matchings/renormalisation needed in RHQ actions and operators (even tree-level, e.g. $B \rightarrow D$ transition amplitudes at non-zero recoil)

matching/renormalisation

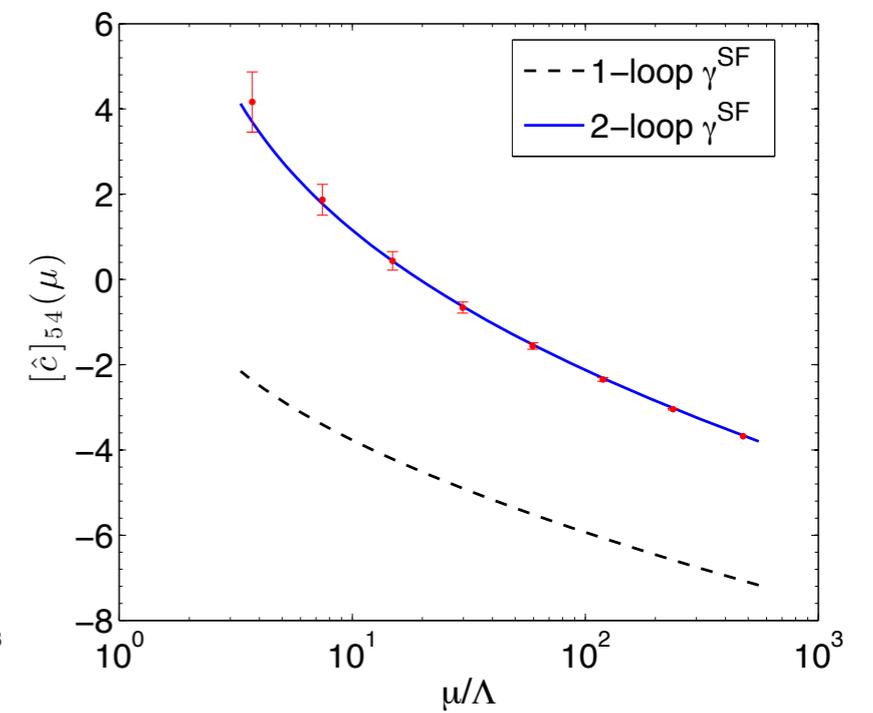
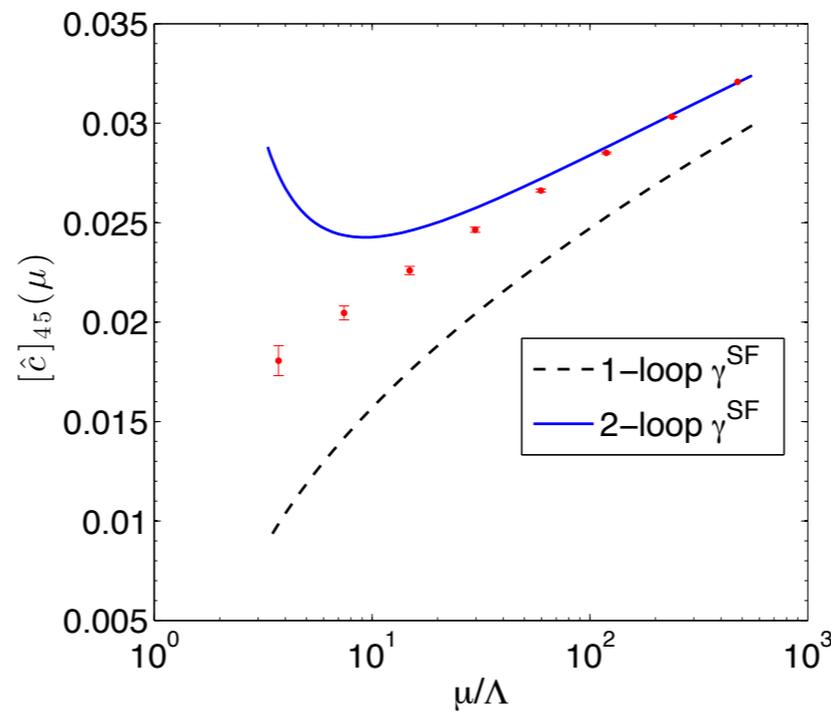
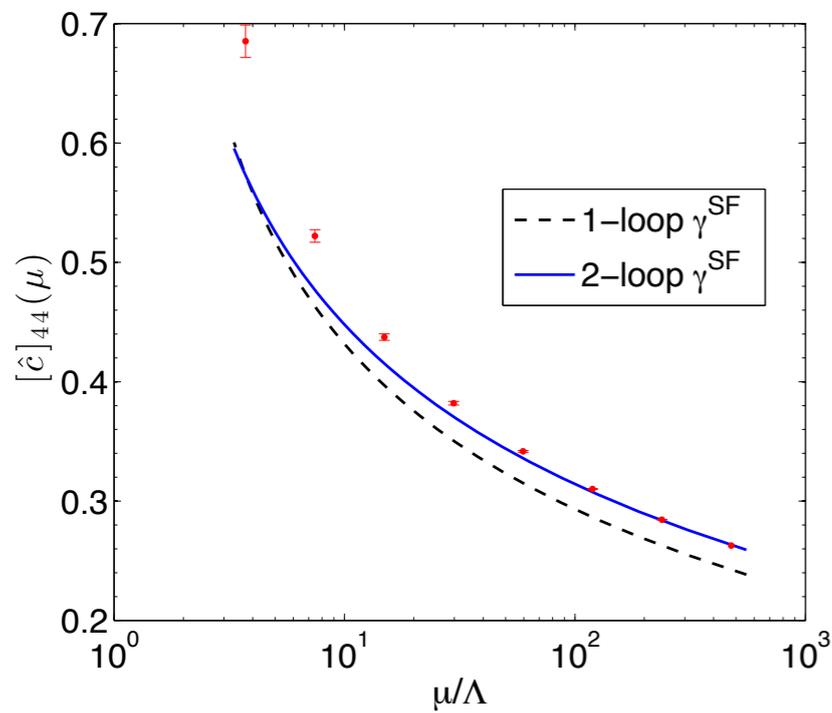
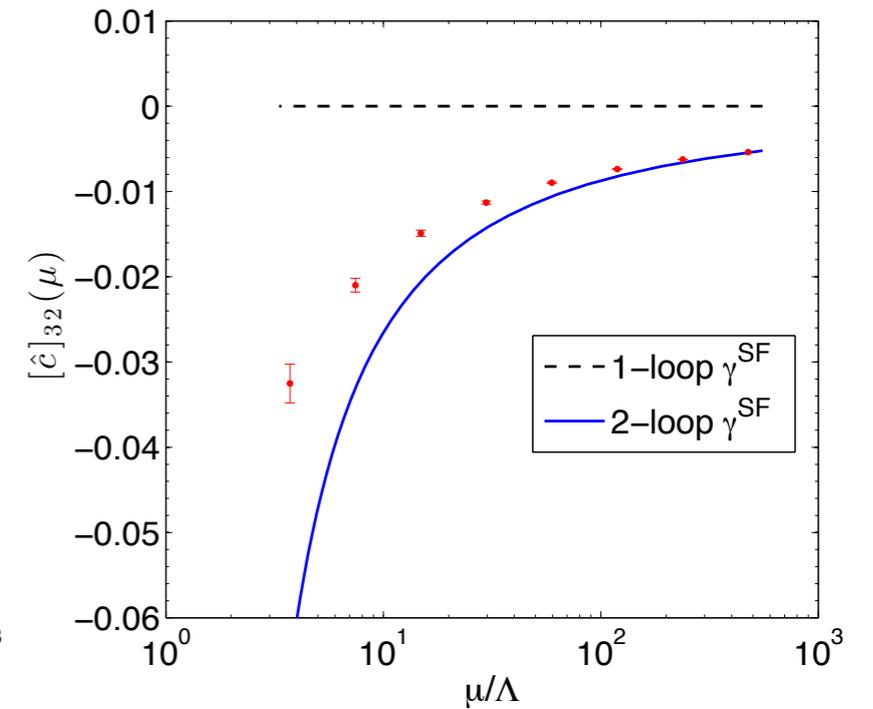
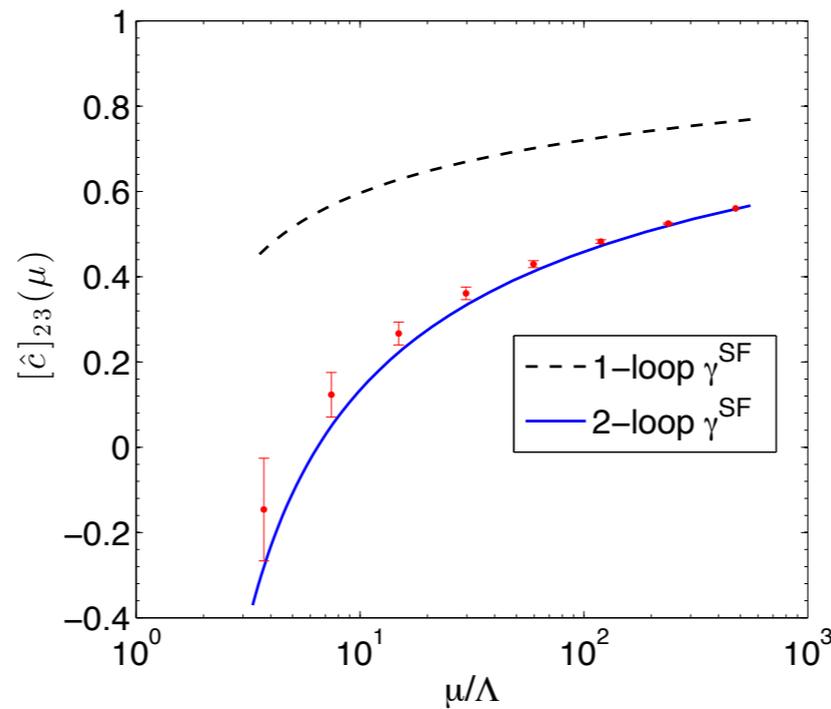
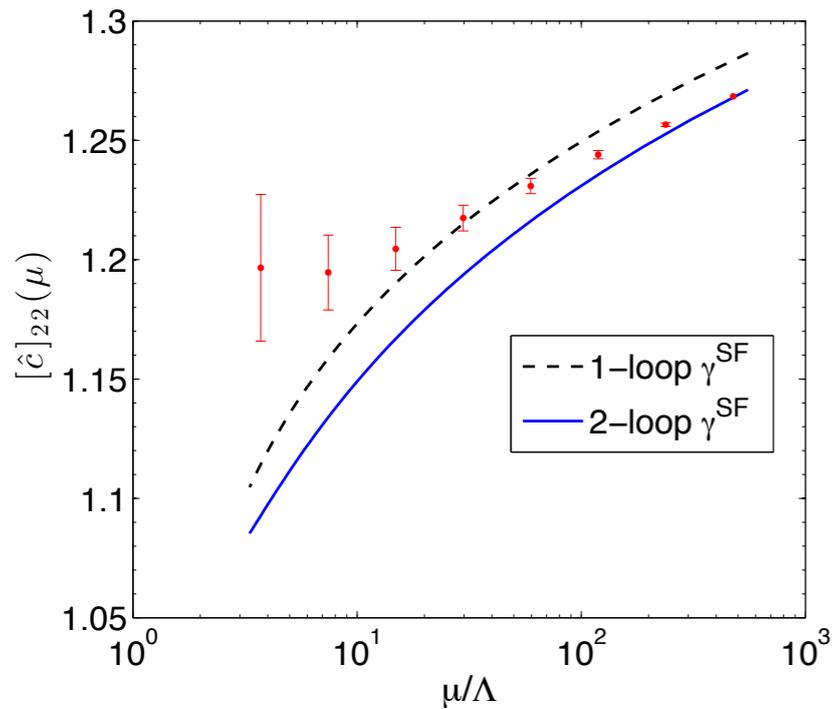
non-perturbative current normalisations for RHQ actions significantly large, huge cutoff dependence; expected effect of RG trajectory tuning?



Parameter	coarse	fine
$Z_V^{(bb)}$	10.037(34)	5.270(13)
$Z_V^{(cc)}$	1.35725(23)	1.18321(14)
$Z_V^{(uu)}$	0.71651(46)	0.74475(12)

[Detmold, Lehner, Meinel arXiv:1503.01421]

matching/renormalisation



chiPT for SL form factors

- issues in fitting FF light quark mass dependence:
 - how reliable HQchiPT is?
 - uncertainties from g_{H^*HP}
 - explored range in momentum transfer goes well beyond slow pion kinematics

- extension of chiPT to hard pions being rapidly incorporated into analyses

[Bijnens, Jemos NPB 840 (2010) 54, 844(2011) 182]

- recent efforts aimed at improving precision on g_{H^*HP}

[RBC/UKQCD arXiv:1311.1251]

[ALPHA PLB 740 (2015) 278]

[A Gérardin, Fri 17:30]



- (HQchiPT systematic dependence should become less relevant as physical pion points enter analyses — H Leutwyler allowing)

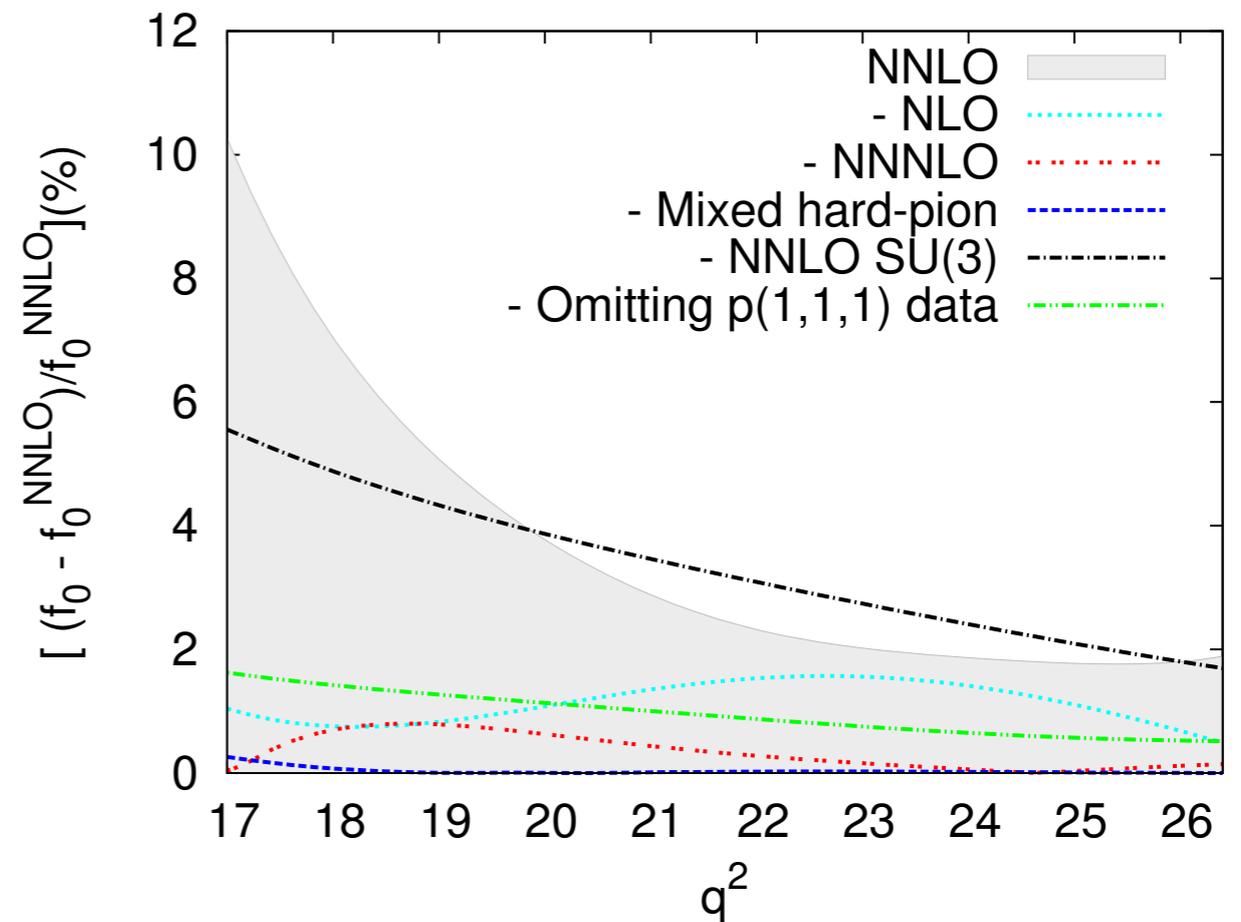
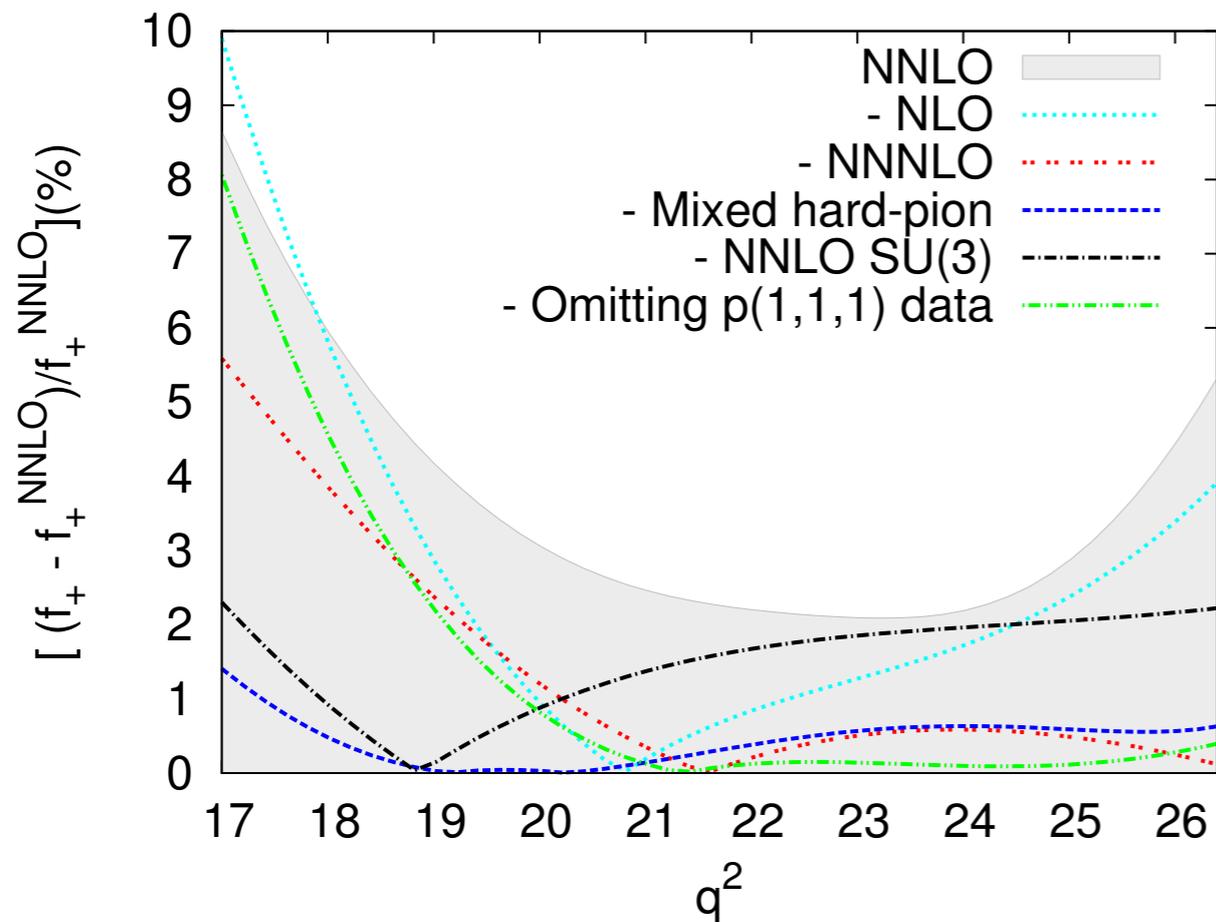
[cf. A Jüttner's talk]



chiPT for SL form factors

FNAL/MILC analysis of hard-pion chiPT impact in $B \rightarrow \pi$

[arXiv:1503.07839]



conclusions and outlook

- HQ physics making great progress, remarkably in semileptonics
- still much way to go to meet the new era precision requirements
 - crosscheck HQ approaches as much as possible
 - full incorporation of available ensembles to HQ physics
 - many systematics to be improved: use of perturbation theory, momentum dependence of FFs, incorporation of QED effects, resonances ...
- decrease the lattice spacing and get direct access to the b region
- FLAG-3 review foreseen for early 2016: keep tuned

backup

Flavour Lattice Averaging Group

advisory board: S Aoki, C Bernard, C Sachrajda

editorial board: G Colangelo, H Leutwyler, A Vladikas, U Wenger

working groups:

quark masses

V_{ud} , V_{us}

LECs

B_K

α_s

f_{D_q} , f_{B_q} , B_{B_q}

SL, rare

T Blum, L Lellouch, V Lubicz

A Jüttner, T Kaneko, S Simula

S Dürr, H Fukaya, S Necco

J Laiho, S Sharpe, H Wittig

R Horsley, T Onogi, R Sommer

Y Aoki, M Della Morte, A El-Khadra

E Lunghi, CP, R Van de Water

FLAG-2 review published in 2014, includes results up to \approx end 2013

FLAG-3

AB: S Aoki, C Bernard, H Leutwyler, C Sachrajda

EB: G Colangelo, S Hashimoto, A Jüttner, S Sharpe, A Vladikas, U Wenger

WGs:

quark masses (+HQ)

V_{ud} , V_{us}

LECs

B_K (+BSM)

α_s

f_{D_q} , f_{B_q} , B_{B_q}

SL, rare

T Blum, L Lellouch, V Lubicz

P Boyle, T Kaneko, S Simula

S Dürr, H Fukaya, U Heller

P Dimopoulos, R Mawhinney, H Wittig

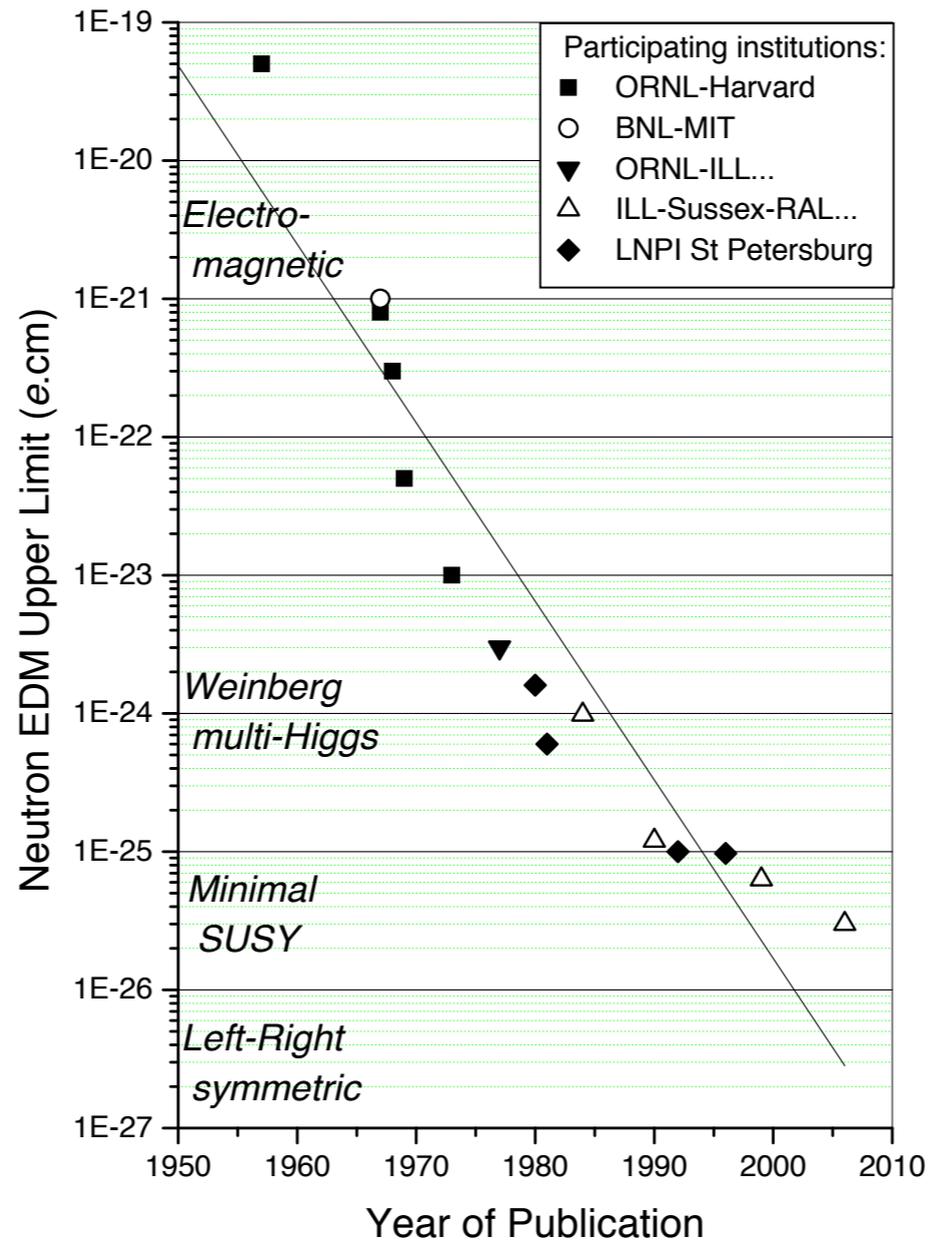
R Horsley, T Onogi, R Sommer

Y Aoki, D Lin, M Della Morte

D Bećirević, S Gottlieb, E Lunghi, CP

expected publication early 2016, **no preliminary averages yet :-)**

bounds on neutron EDM



[P Harris, arXiv:0709.3100]

confined ultracold atoms $\Rightarrow |d_n| \sim 10^{-27} \text{---} 10^{-28} \text{ e} \cdot \text{cm}$

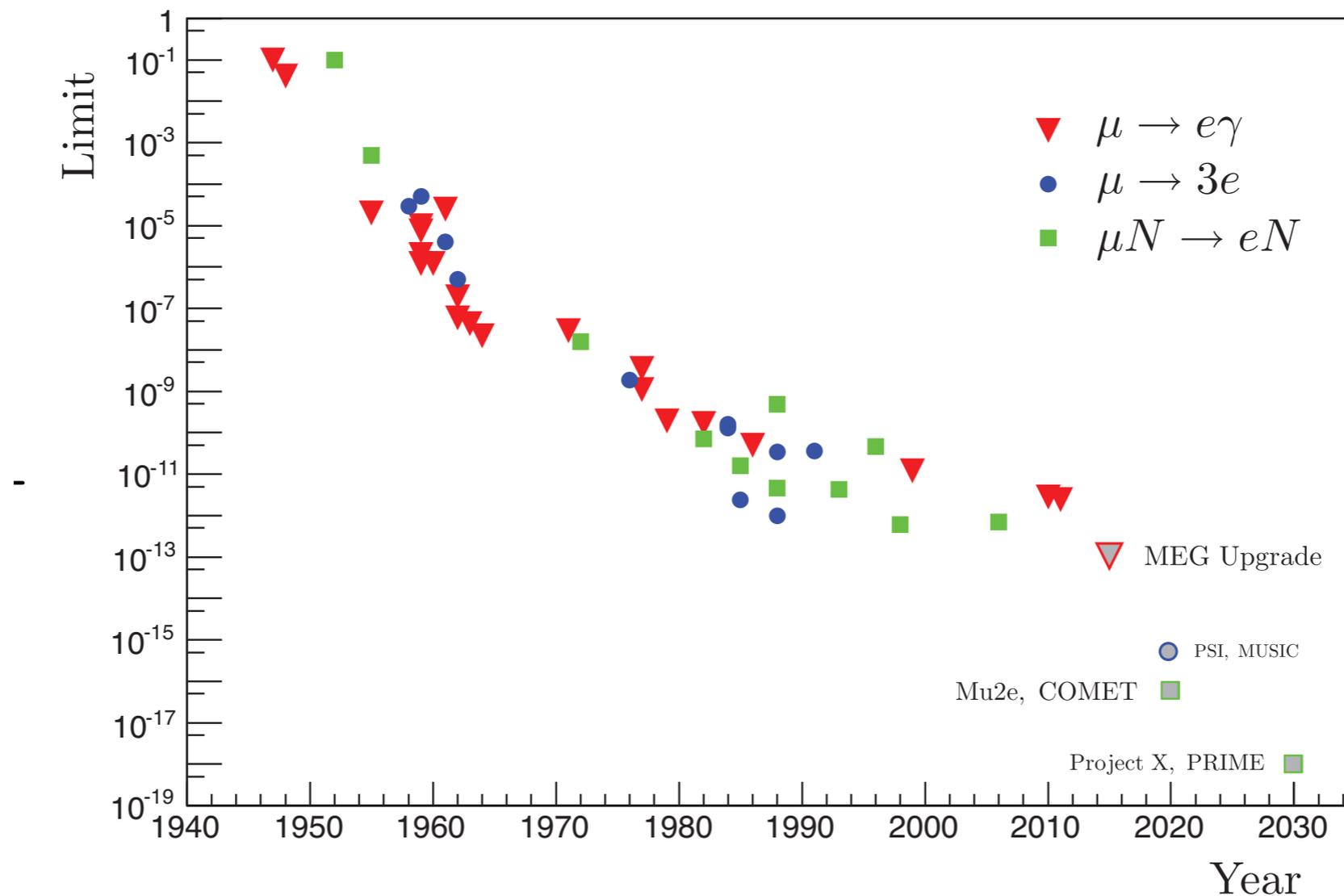
(slow neutron sources at Oak Ridge, J-PARC, PSI, TRIUMF, Lund)

[H Shimizu, FPCP15]

bounds on charged lepton flavour violation

LFV not present in the SM for $m_\nu \neq 0$, no reason to impose it

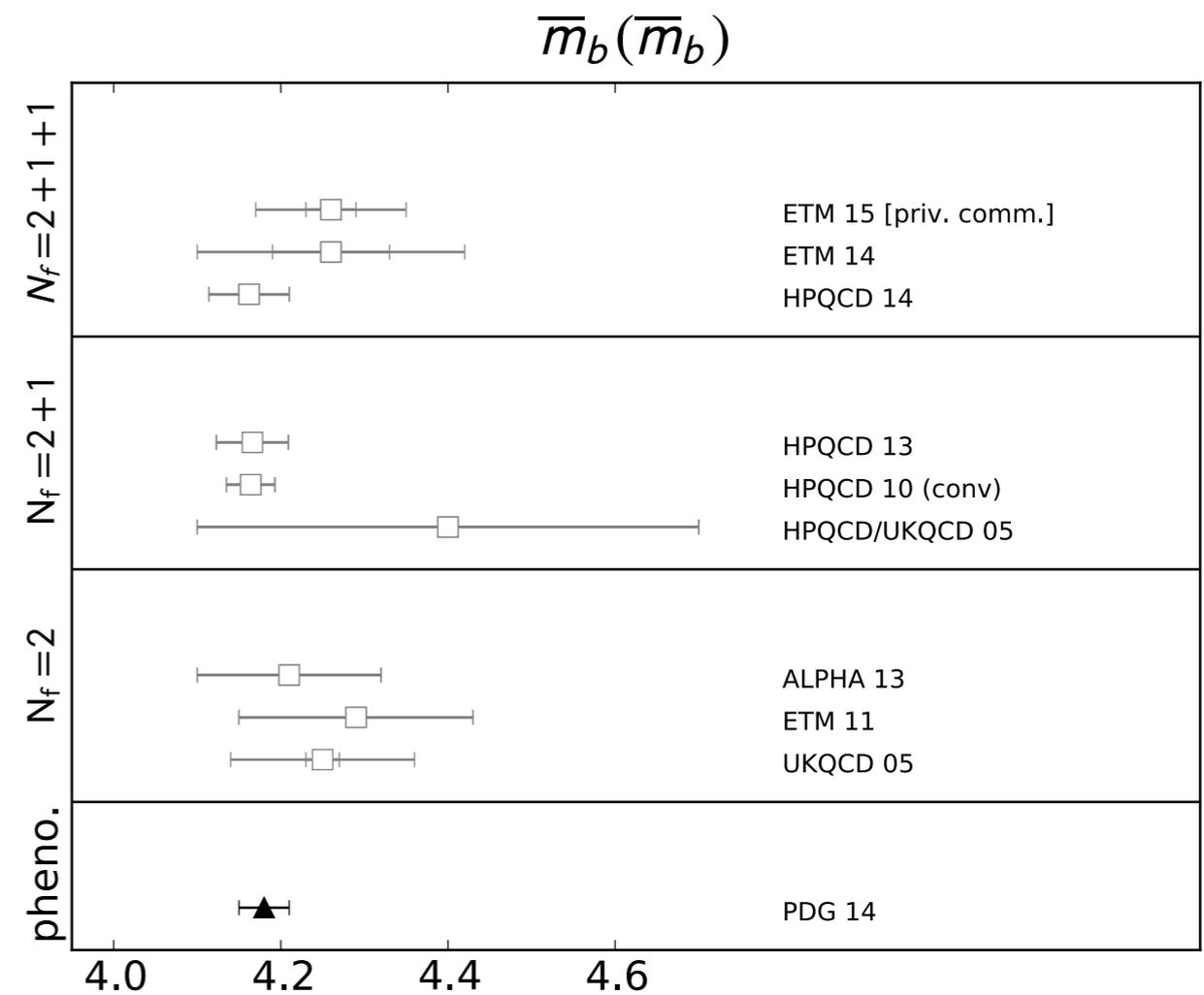
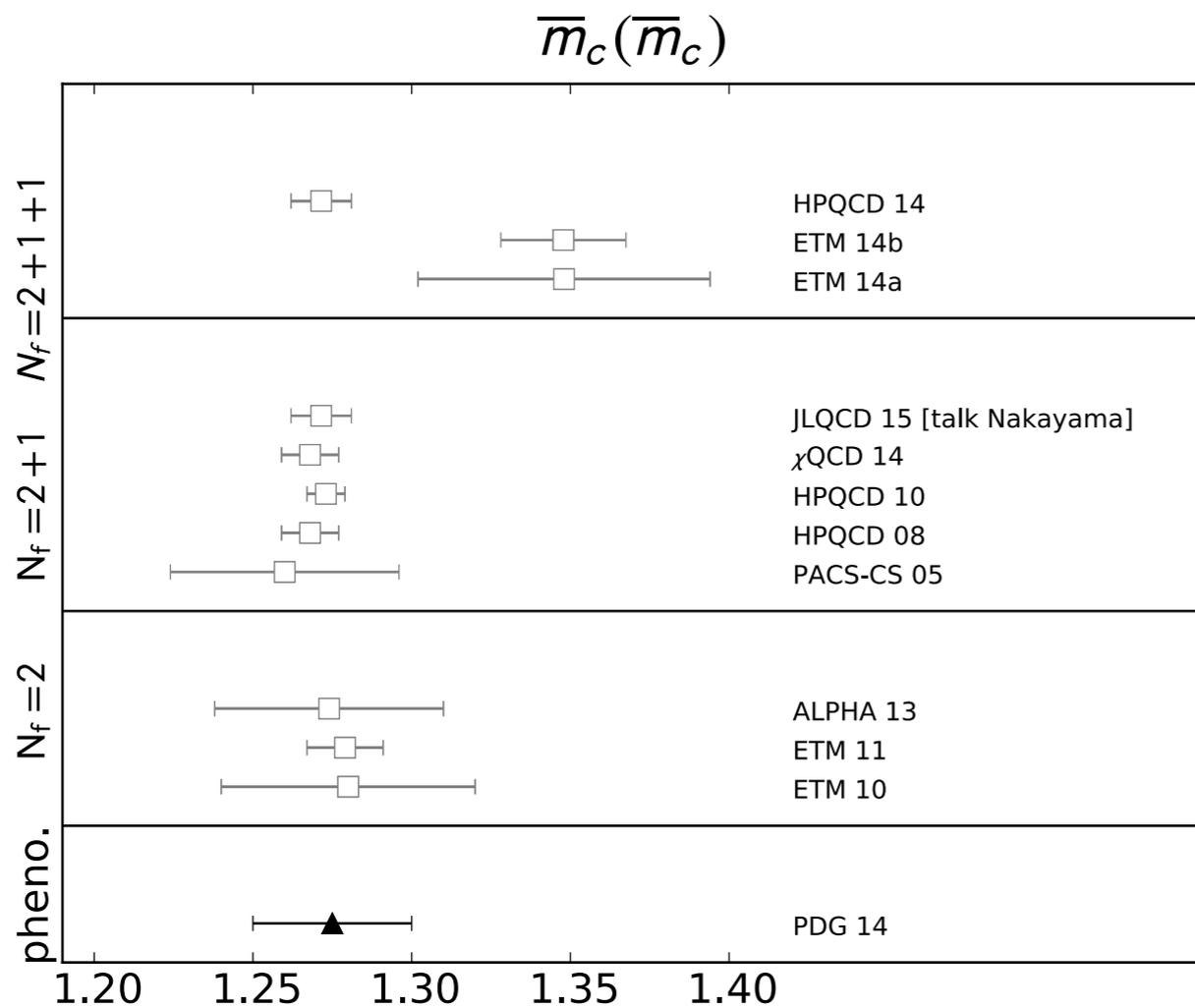
History of $\mu \rightarrow e\gamma$, $\mu N \rightarrow eN$, and $\mu \rightarrow 3e$



[plot from Z Ligeti's review talk at FPCP 2015]

c and *b* quark masses

almost no new results after F Sanfilippo's review at Lattice 2014



see also talk on JLQCD results from moments method

[K Nakayama, Wed 16:50]



FLAG-2 on charm decay constants

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	f_D	f_{D_s}	f_{D_s}/f_D
ETM 13F	[154]	2+1+1	C	○	○	○	★	✓	202(8)	242(8)	1.199(25)
FNAL/MILC 13 [∇]	[328]	2+1+1	C	★	★	★	★	✓	212.3(0.3)(1.0)	248.7(0.2)(1.0)	1.1714(10)(25)
FNAL/MILC 12B	[329]	2+1+1	C	★	★	★	★	✓	209.2(3.0)(3.6)	246.4(0.5)(3.6)	1.175(16)(11)
HPQCD 12A	[330]	2+1	A	○	○	★	★	✓	208.3(1.0)(3.3)	246.0(0.7)(3.5)	1.187(4)(12)
FNAL/MILC 11	[331]	2+1	A	○	○	★	○	✓	218.9(11.3)	260.1(10.8)	1.188(25)
PACS-CS 11	[332]	2+1	A	■	★	■	○	✓	226(6)(1)(5)	257(2)(1)(5)	1.14(3)
HPQCD 10A	[94]	2+1	A	★	○	★	★	✓	213(4)*	248.0(2.5)	
HPQCD/UKQCD 07	[164]	2+1	A	★	○	★	★	✓	207(4)	241 (3)	1.164(11)
FNAL/MILC 05	[333]	2+1	A	○	○	★	○	✓	201(3)(17)	249(3)(16)	1.24(1)(7)
ETM 13B [□]	[334]	2	P	★	○	★	★	✓	208(7)	250(7)	1.20(2)
ETM 11A	[335]	2	A	★	○	★	★	✓	212(8)	248(6)	1.17(5)
ETM 09	[168]	2	A	○	○	★	★	✓	197(9)	244(8)	1.24(3)

[∇] Update of FNAL/MILC 12B.

* This result is obtained by using the central value for f_{D_s}/f_D from HPQCD/UKQCD 07 and increasing the error to account for the effects from the change in the physical value of r_1 .

[□] Update of ETM 11A and ETM 09.

new results for $f_{D(s)}$

	TWQCD	ETMC phys	χ QCD	FNAL/MILC	ETMC
f_D [MeV]	202.3(2.2)(2.6)	216.7(2.3)(4.2)	—	212.6(0.4)($^{+1.2}_{-1.0}$)	209.2(3.3)
f_{D_s} [MeV]	258.7(1.1)(2.9)	255.9(0.5)(5.0)	254(2)(4)	249.0(0.3)($^{+1.1}_{-1.5}$)	248.6(2.7)
f_{D_s}/f_D	1.279(26)	1.206(23)	—	1.1712(10)($^{+29}_{-32}$)	1.187(12)
ensembles	TWQCD	ETMC	RBC/UKQCD	MILC	ETMC
N_f	2	2	2+1	2+1+1	2+1+1
a (fm)	1/0.06	1/0.09	2/0.085, 0.111	4/0.06 – 0.15	3/0.062 – 0.089
M_π^{\min} [MeV]	259	132	320	130	210
$M_\pi^{\min} L$	2	3.0	4.3	3.2	3.1
sea	DW	tmQCD	DW	HISQ	tmQCD
valence	DW	tmQCD/OS	overlap	HISQ	tmQCD/OS
reference	[1404.3648]	[priv. comm.]	[1410.3343]	[1407.3772]	[1411.7908]

+ report of work in progress by RBC/UKQCD

[T Tsang, Thu 10:40]



FLAG-2 on B decay constants

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	f_{B^+}	f_{B^0}	f_B	f_{B_s}
ETM 13E	[398]	2+1+1	C	○	○	○	○	✓	—	—	196(9)	235(9)
HPQCD 13	[399]	2+1+1	A	★	★	★	○	✓	184(4)	188(4)	186(4)	224(5)
RBC/UKQCD 13A	[400]	2+1	C	○	○	★	○	✓	—	—	191(6) [◇] _{stat}	233(5) [◇] _{stat}
HPQCD 12	[401]	2+1	A	○	○	★	○	✓	—	—	191(9)	228(10)
HPQCD 12	[401]	2+1	A	○	○	★	○	✓	—	—	189(4) [△]	—
HPQCD 11A	[365]	2+1	A	★	○	★	★	✓	—	—	—	225(4) [▽]
FNAL/MILC 11	[331]	2+1	A	○	○	★	○	✓	197(9)	—	—	242(10)
HPQCD 09	[402]	2+1	A	○	○	★	○	✓	—	—	190(13) [•]	231(15) [•]
ALPHA 13	[403]	2	C	★	★	★	★	✓	—	—	187(12)(2)	224(13)
ETM 13B, 13C	[334, 404]	2	P [†]	★	○	★	○	✓	—	—	189(8)	228(8)
ALPHA 12A	[369]	2	C	★	★	★	★	✓	—	—	193(9)(4)	219(12)
ETM 12B	[392]	2	C	★	○	★	○	✓	—	—	197(10)	234(6)
ALPHA 11	[364]	2	C	★	○	★	★	✓	—	—	174(11)(2)	—
ETM 11A	[335]	2	A	○	○	★	○	✓	—	—	195(12)	232(10)
ETM 09D	[391]	2	A	○	○	○	○	✓	—	—	194(16)	235(12)

[◇]Statistical errors only.

[△]Obtained by combining f_{B_s} from HPQCD 11A with f_{B_s}/f_B calculated in this work.

[▽]This result uses one ensemble per lattice spacing with light to strange sea-quark mass ratio $m_l/m_s \approx 0.2$.

[•]This result uses an old determination of $r_1 = 0.321(5)$ fm from Ref. [379] that has since been superseded.

[†]Update of ETM 11A and 12B.

new results for f_{B_q}

	ALPHA (def)	RBC/UKQCD	RBC/UKQCD _{stat}	ETMC
f_B [MeV]	186(13)	196(15)/200(13)*	219(31)	194(5)(3)
f_{B_s} [MeV]	224(14)	235(12)	264(37)	229(4)(3)
f_{B_s}/f_B	1.203(65)	1.223(71)/1.197(50)*	1.193(48)	1.179(18)(18)
ensembles	CLS	RBC/UKQCD	RBC/UKQCD	ETMC
N_f	2	2+1	2+1	2+1+1
a (fm)	3/0.0483 – 0.0749	2/0.085, 0.111	2/0.085, 0.111	3/0.062 – 0.089
M_π^{\min} [MeV]	190	289	289	210
$M_\pi^{\min} L$	4.0	4.0	4.0	3.1
l quarks	NP O(a) improved	DW	DW	tmQCD
b quark	npHQET	RHQ (Columbia)	static	tmQCD/OS
reference	[1404.3590]	[1404.4670]	[1406.6192]	[priv. comm.]

* f_{B^+}/f_{B^0}

+ report of work in progress by RBC/UKQCD, FNAL/MILC

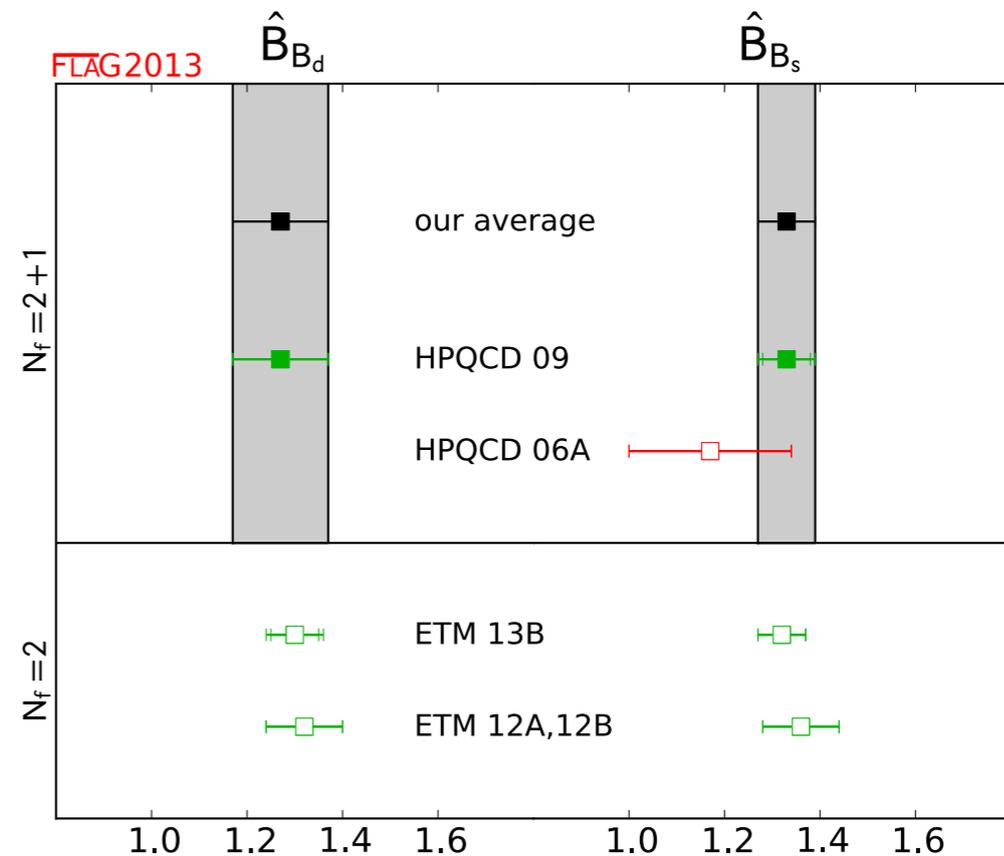
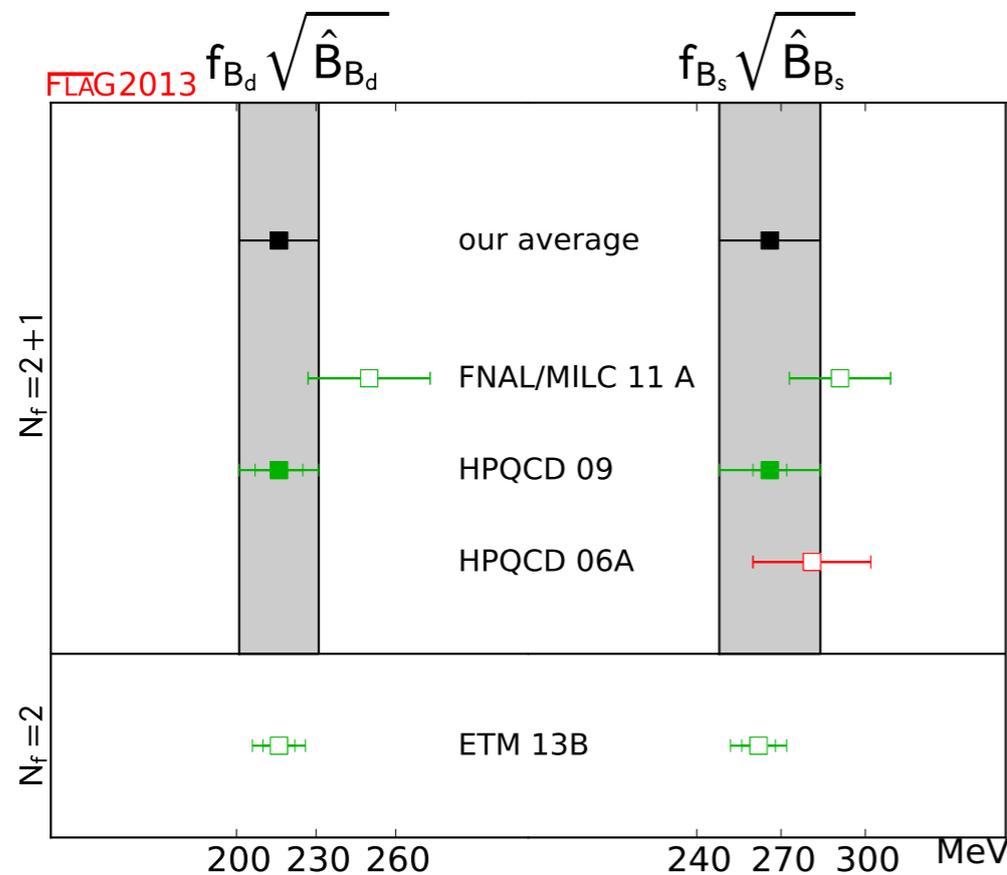
[T Ishikawa, Thu 8:30]
[C DeTar, Thu 8:50]



FLAG-2 on B-mixing

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	$f_{B_d} \sqrt{\hat{B}_{B_d}}$	$f_{B_d} \sqrt{\hat{B}_{B_s}}$	\hat{B}_{B_d}	\hat{B}_{B_s}
FNAL/MILC 11A	[411]	2+1	C	★	○	★	○	✓	250(23) [†]	291(18) [†]	–	–
HPQCD 09	[402]	2+1	A	○	○ [▽]	★	○	✓	216(15) [*]	266(18) [*]	1.27(10) [*]	1.33(6) [*]
HPQCD 06A	[412]	2+1	A	■	■	★	○	✓	–	281(21)	–	1.17(17)
ETM 13B	[334]	2	P	★	○	★	★	✓	216(6)(8)	262(6)(8)	1.30(5)(3)	1.32(5)(2)
ETM 12A, 12B	[392, 413]	2	C	★	○	★	★	✓	–	–	1.32(8) [◇]	1.36(8) [◇]

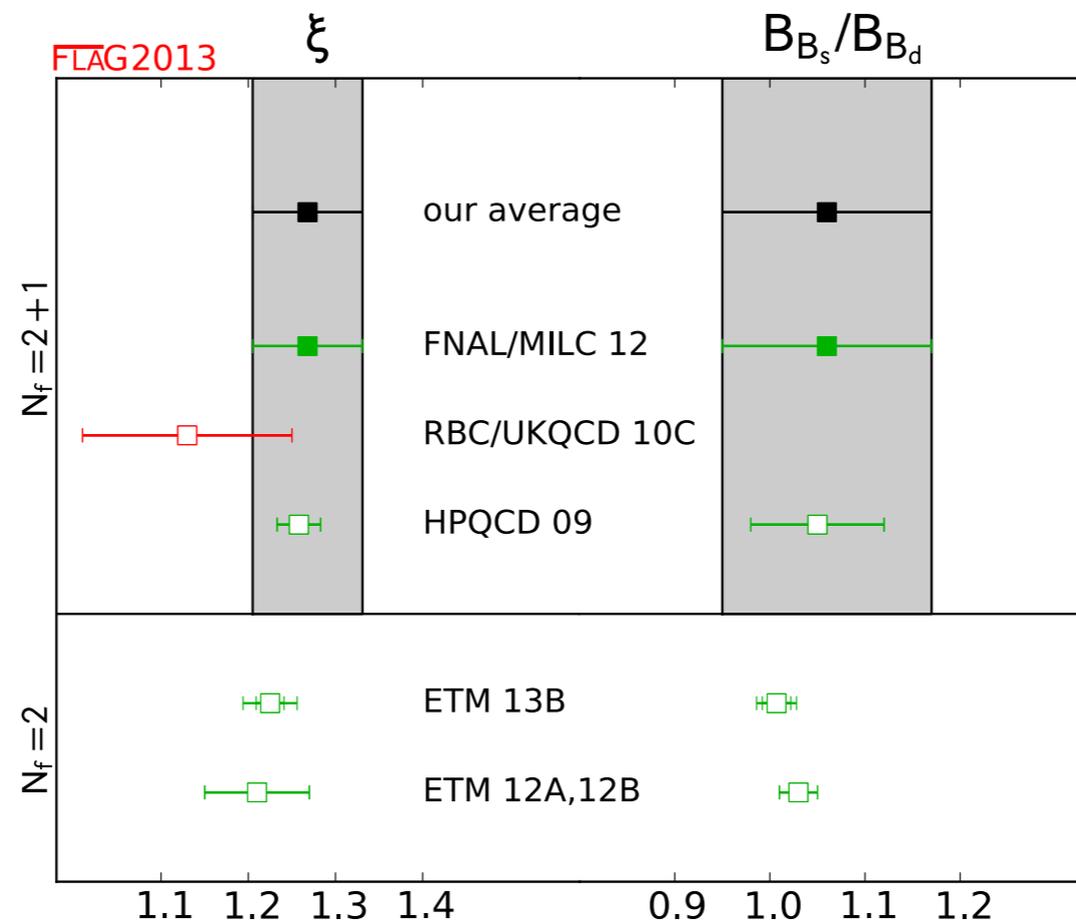
now published



FLAG-2 on B-mixing

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization/matching	heavy quark treatment	ξ	B_{B_s}/B_{B_d}
FNAL/MILC 12	[414]	2+1	A	○	○	★	○	✓	1.268(63)	1.06(11)
RBC/UKQCD 10C	[405]	2+1	A	■	■	★	○	✓	1.13(12)	—
HPQCD 09	[402]	2+1	A	○	○ [▽]	★	○	✓	1.258(33)	1.05(7)
ETM 13B	[334]	2	P	★	○	★	★	✓	1.225(16)(14)(22)	1.007(15)(14)
ETM 12A, 12B	[392, 413]	2	C	★	○	★	★	✓	1.21(6)	1.03(2)

now published



new results for B-mixing

RBC/UKQCD static limit

[Y Aoki et al, PRD 91 (2014) 114505, arXiv:1406.6192]

ensembles	RBC/UKQCD	
N_f	2+1	$f_{B_d} \sqrt{\hat{B}_{B_d}} = 240(15)(33)$
a (fm)	2/0.085, 0.111	$f_{B_s} \sqrt{\hat{B}_{B_s}} = 290(9)(40)$
M_π^{\min} [MeV]	320	$\hat{B}_{B_d} = 1.17(11)(24)$
$M_\pi^{\min} L$	4.3	$\hat{B}_{B_s} = 1.22(6)(19)$
l quarks	DWF	$B_{B_s}/B_{B_d} = 1.028(60)(49)$
b quark	static	$\xi = 1.208(41)(52)$

preliminary FNAL/MILC $N_f=2+1$: $\xi = 1.211(19)$

+ work in progress by FNAL/MILC, RBC/UKQCD



[J Simone, Thu 9:10]
[A Khamseh, Thu 11:00]
[T Ishikawa, Thu 8:30]
[T Kawanai, Fri 16:30]
[O Witzel, Fri 17:10]
[P Korcyl, Tue 16:30]

FLAG-2 on charm semileptonic decay

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$f_+^{D\pi}(0)$	$f_+^{DK}(0)$
HPQCD 11	[337]	2+1	A	○	○	★	★	✓	0.666(29)	
HPQCD 10B	[341]	2+1	A	○	○	★	★	✓		0.747(19)
FNAL/MILC 04	[356]	2+1	A	■	■	★	○	✓	0.64(3)(6)	0.73(3)(7)
ETM 11B	[344]	2	C	○	○	★	★	✓	0.65(6)(6)	0.76(5)(5)

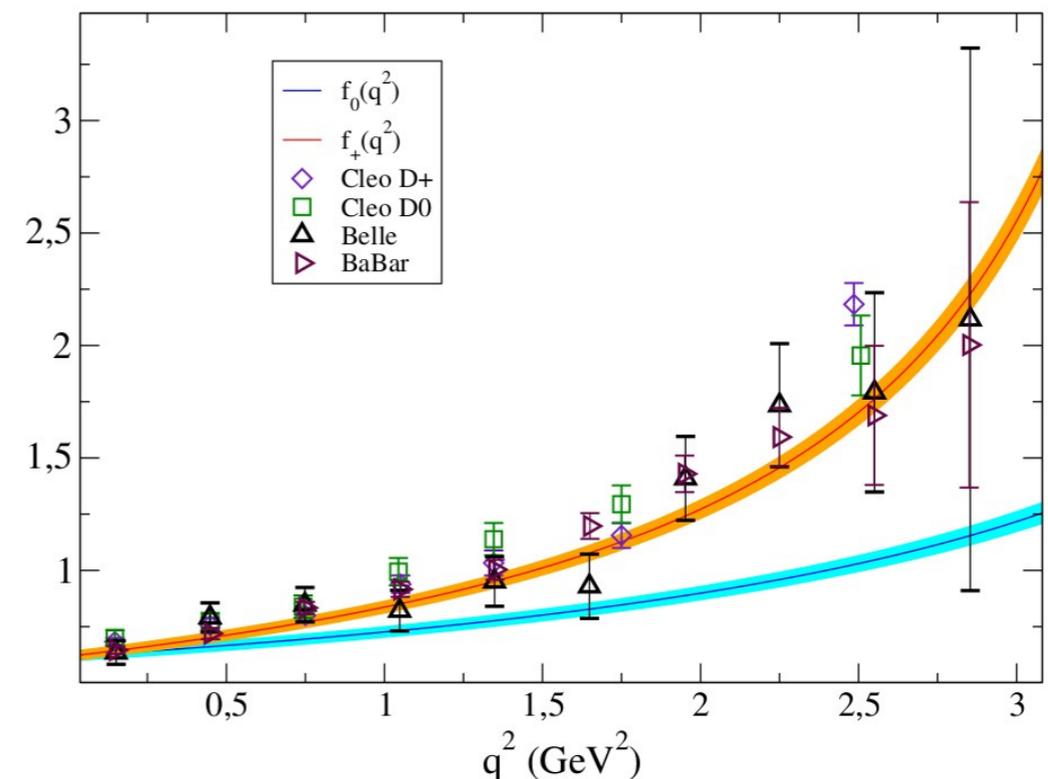
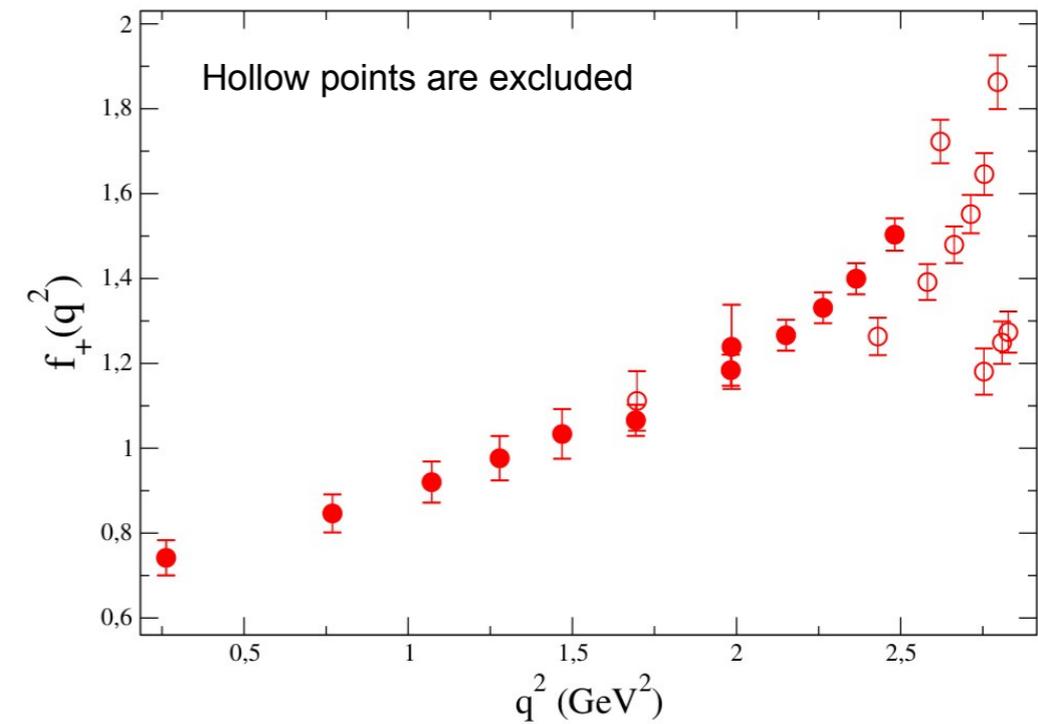
new results for charm SL form factors

[P Lami, Wed 14:20]



$\beta = 1.90$ $L=24$ $\mu = 0.0060$

ensembles	ETM
N_f	2+1+1
a (fm)	3/0.062 - 0.089
M_π^{\min} [MeV]	210
$M_\pi^{\min} L$	3.1
sea	tmQCD
valence	OS



- accurate analysis of $(aq)^2$ cutoff effects
- tbc allow for fine momentum spacing
- preliminary results with stat error only

new results for charm SL form factors

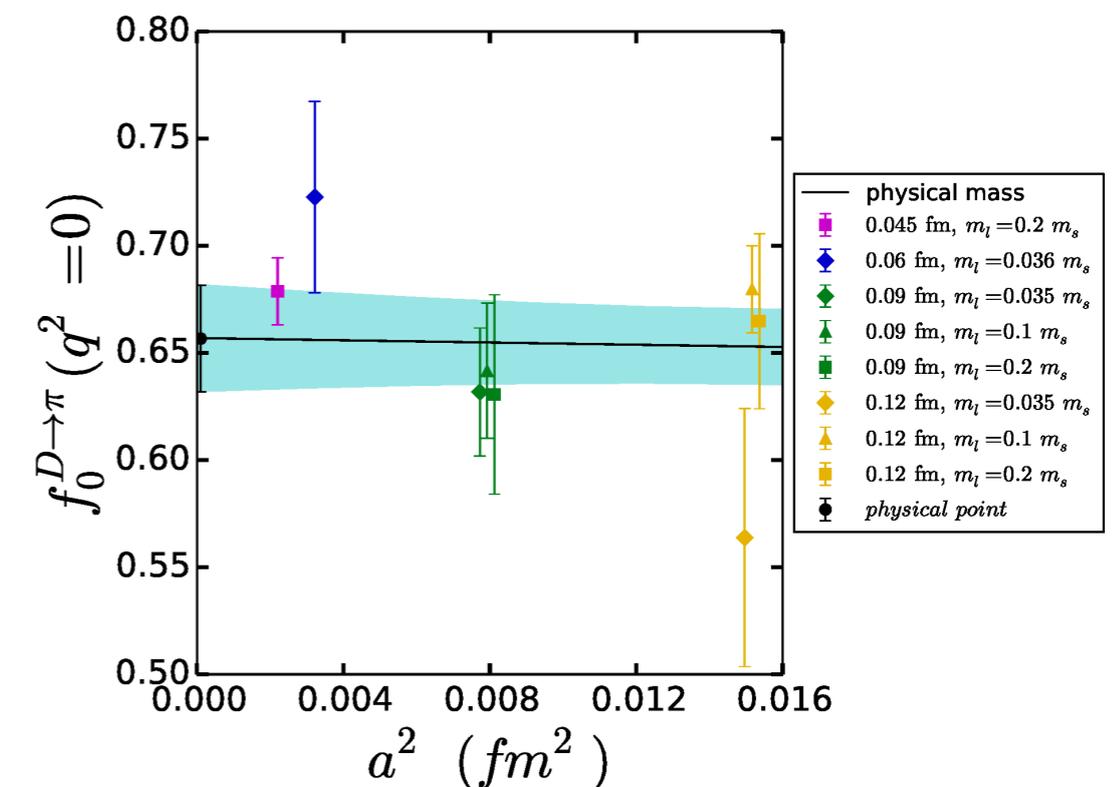
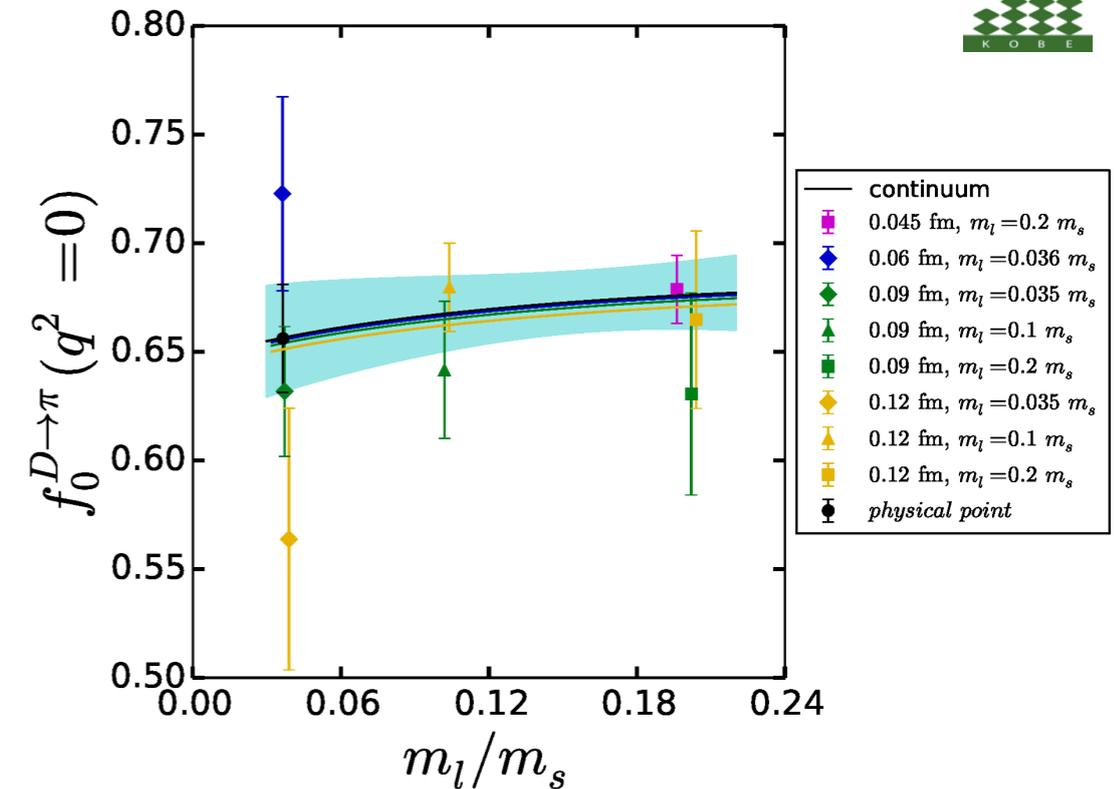
[FNAL/MILC arXiv:1411.1651]

[T Primer, Thu 11:40]



ensembles	MILC
N_f	2+1+1
a (fm)	4/0.042 - 0.12
M_π^{\min} [MeV]	130
$M_\pi^{\min} L$	3.2
sea	HISQ
valence	HISQ

- tbc to get precise form factor at $q^2=0$
- tbc allow for fine momentum spacing
- no quotable numbers outside plots

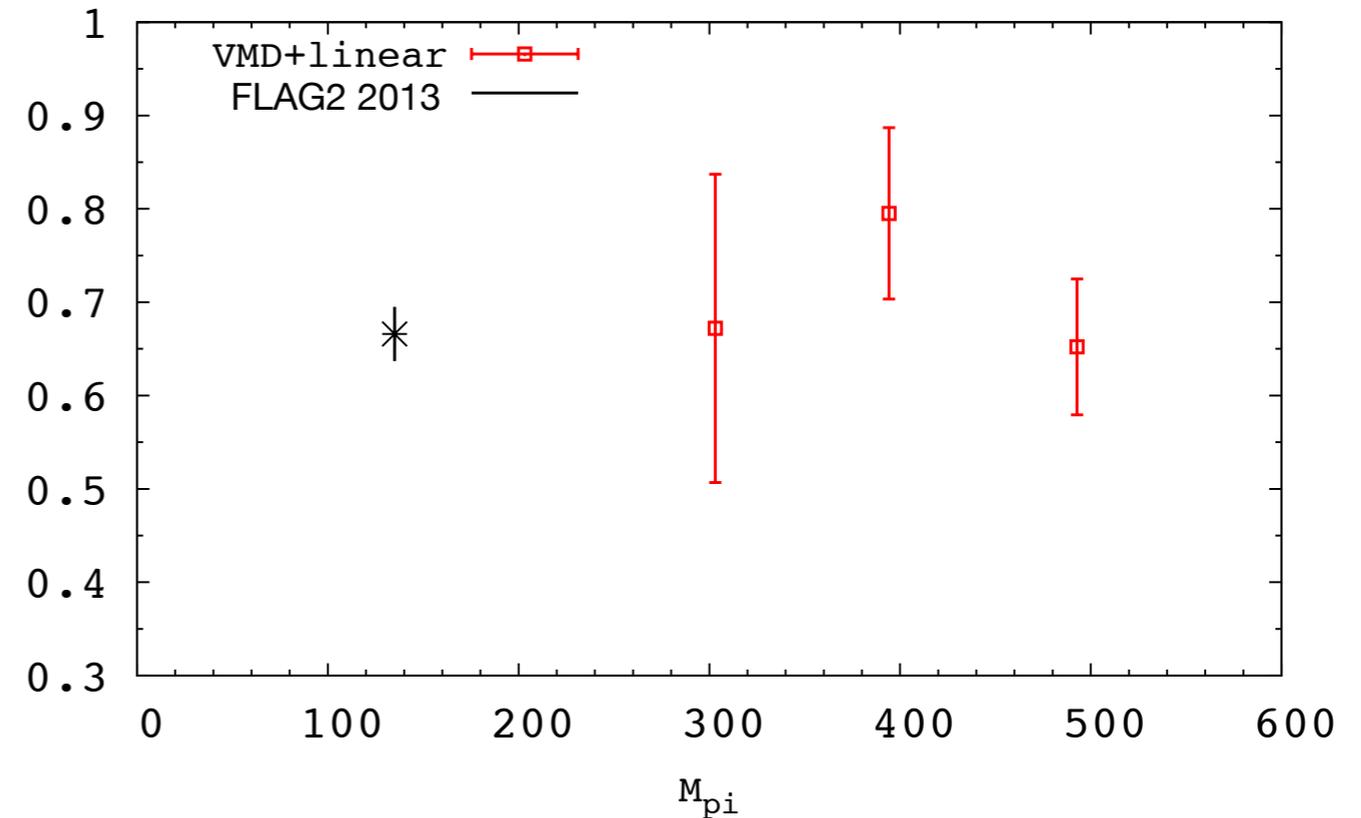


new results for charm SL form factors

[T Suzuki, Thu 11:20]



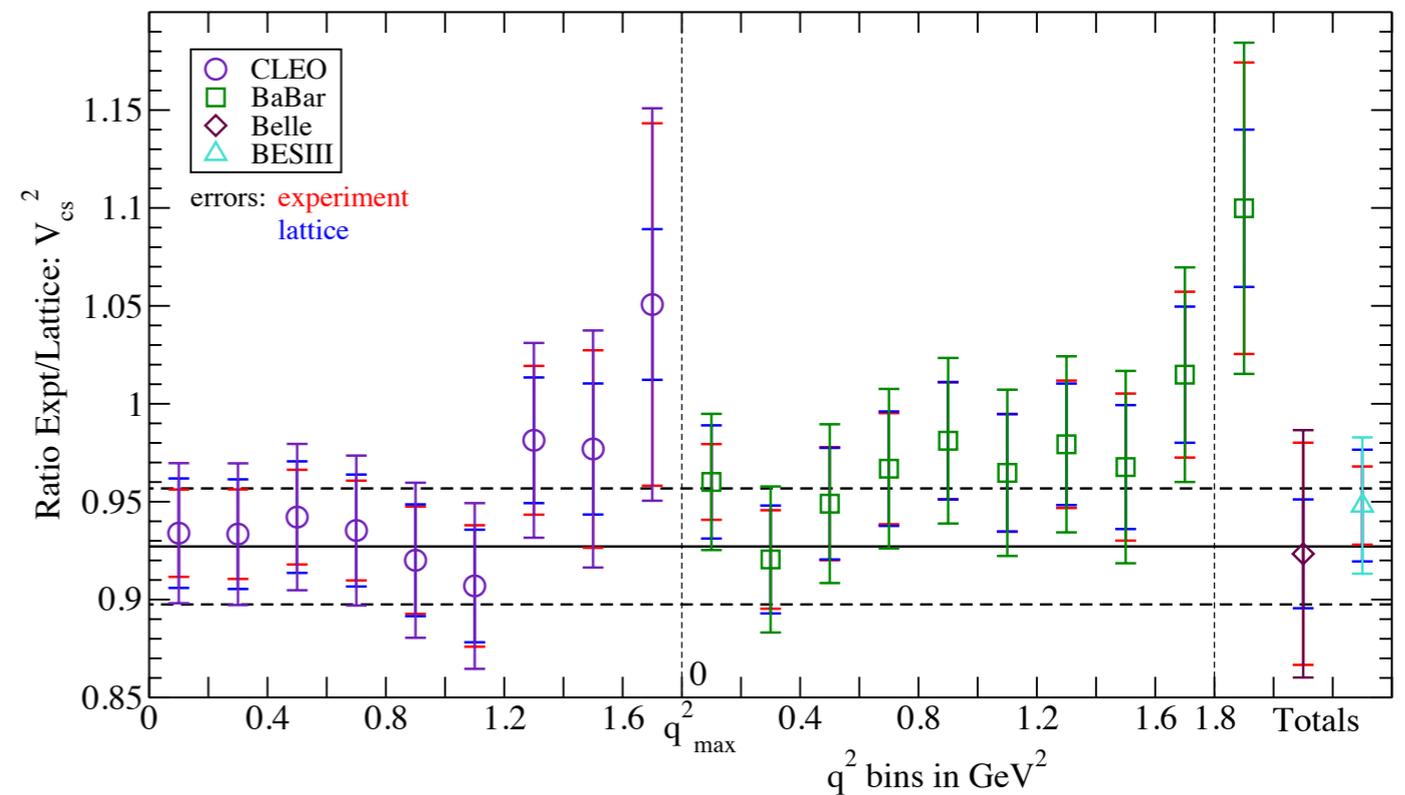
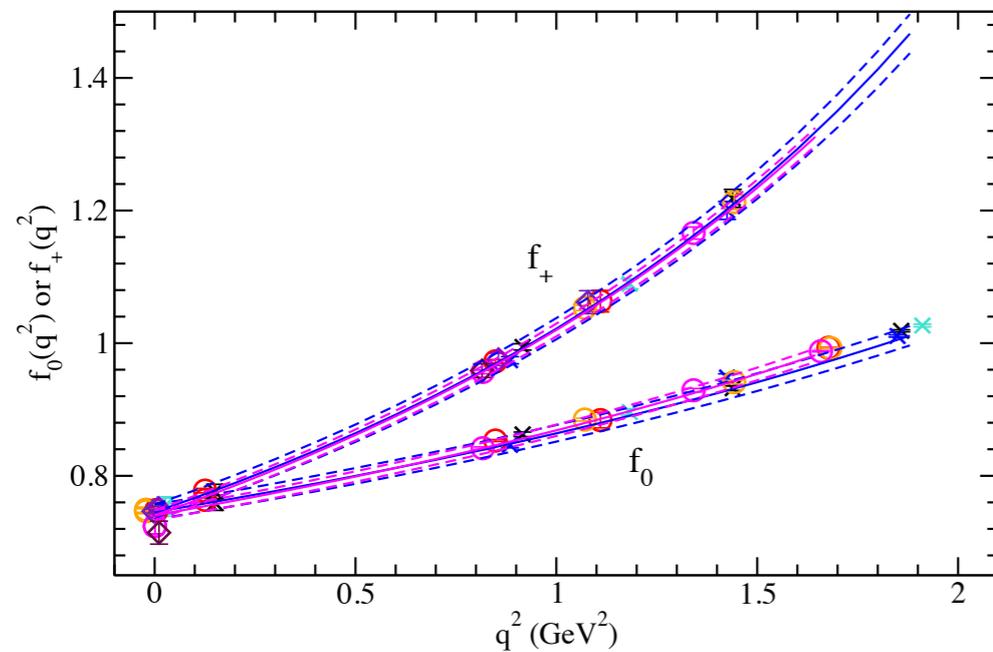
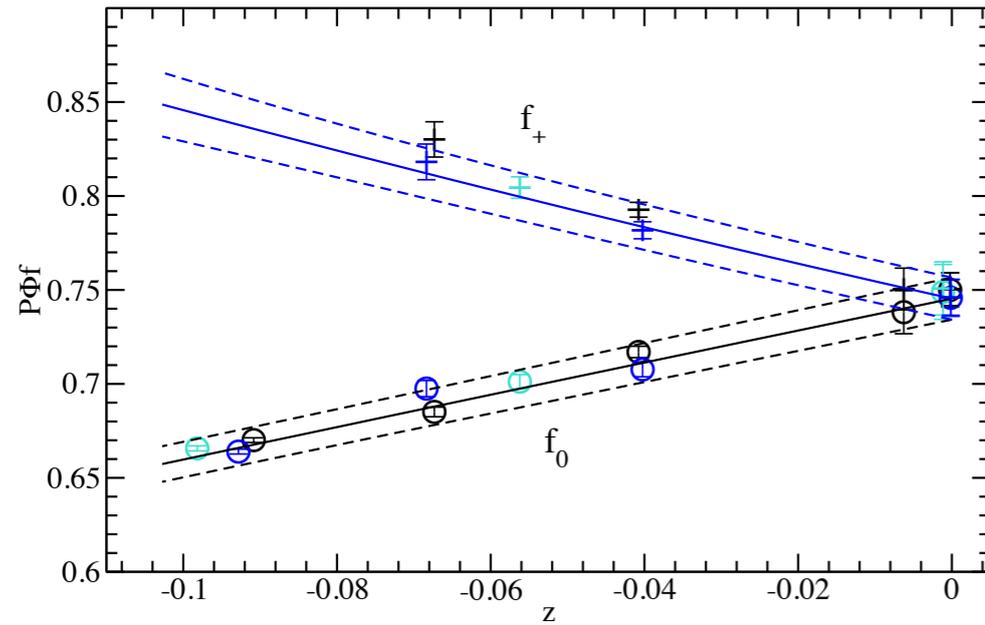
ensembles	JLQCD
N_f	2+1
a (fm)	1/0.08 fm
M_π^{\min} [MeV]	300
$M_\pi^{\min} L$	3.9
sea	mDWF
valence	mDWF



$$f^{D\pi}(q^2 = 0)$$

- very preliminary
- results consistent with FLAG “average” and CLEO-c shape within large errors

HPQCD arXiv:1305.1465 ($D \rightarrow K$)



discussed in FLAG-2, unable to rate/average it due to incomplete information on systematics

new results for $B \rightarrow D^{(*)} l \nu$

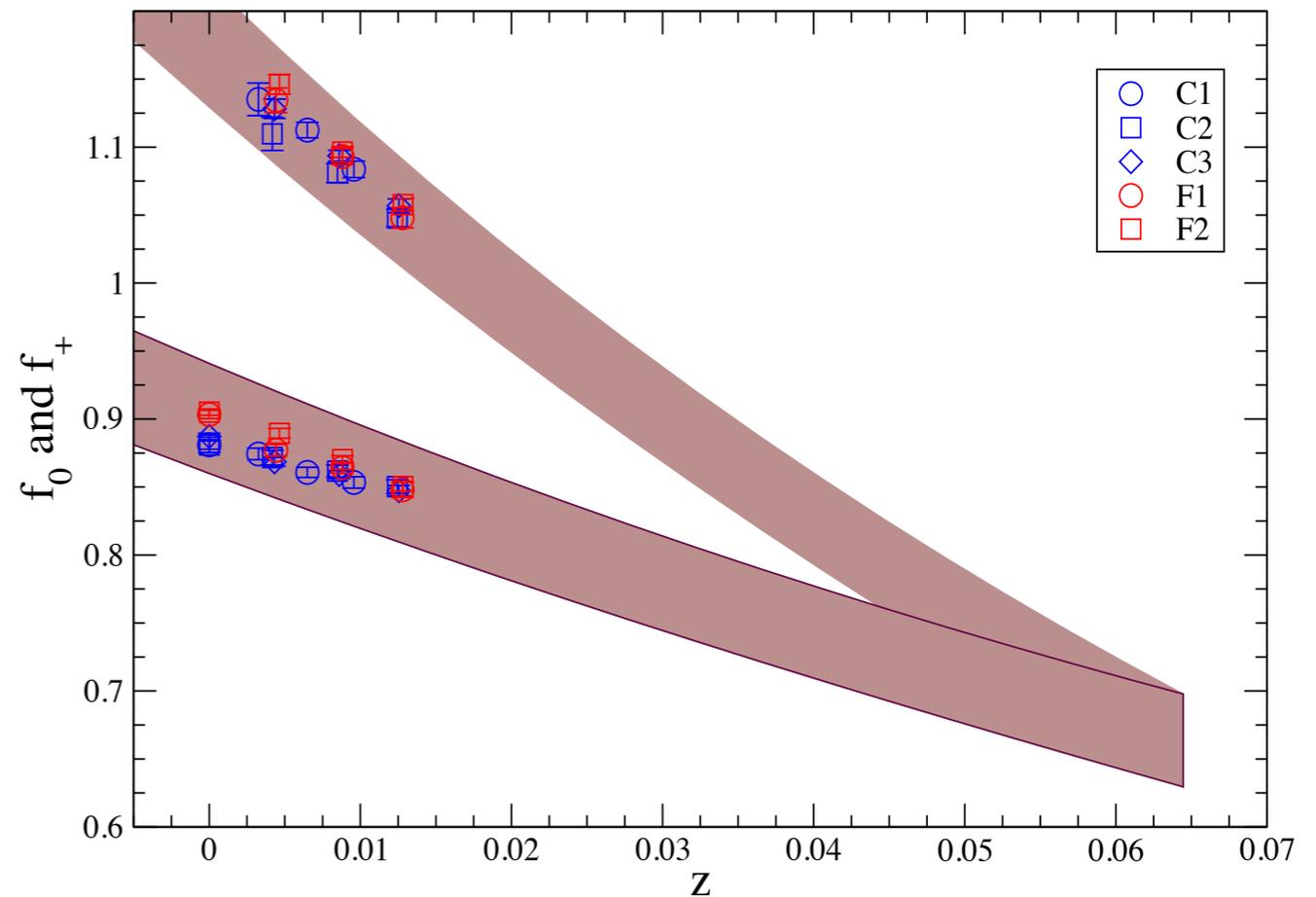
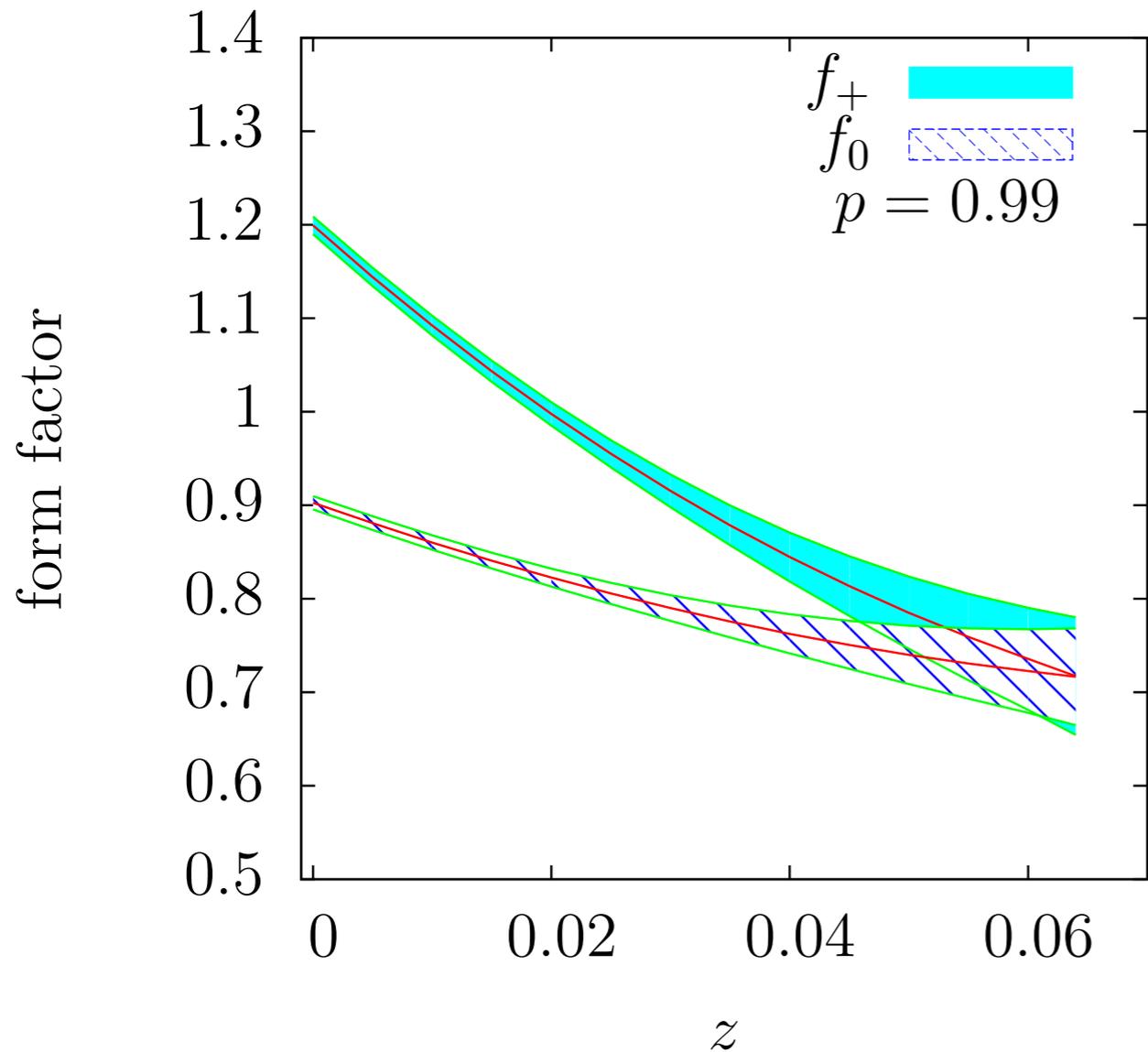
	FNAL/MILC*	FNAL/MILC	HPQCD
process	$B \rightarrow D^* l \nu$	$B \rightarrow D l \nu$	$B \rightarrow D l \nu$
kinematics	$w = 1$	$w \geq 1$	$w \geq 1$
ensembles	MILC	MILC	MILC
N_f	2+1	2+1	2+1
a (fm)	5/0.045 – 0.15	4/0.045 – 0.12	2/0.09, 0.12
M_π^{\min} [MeV]	260	220	260
$M_\pi^{\min} L$	3.8	3.8	3.8
l quarks	asqtad	asqtad	asqtad
c quark	RHQ (Fermilab)	RHQ (Fermilab)	HISQ
b quark	RHQ (Fermilab)	RHQ (Fermilab)	NRQCD
reference	[1403.0635]	[1503.07237]	[1505.03925]

(* full publication of $B \rightarrow D^*$ results, no changes wrt proceedings value quoted in FLAG)

new results for $B \rightarrow D l \nu$

[FNAL/MILC]

[HPQCD]



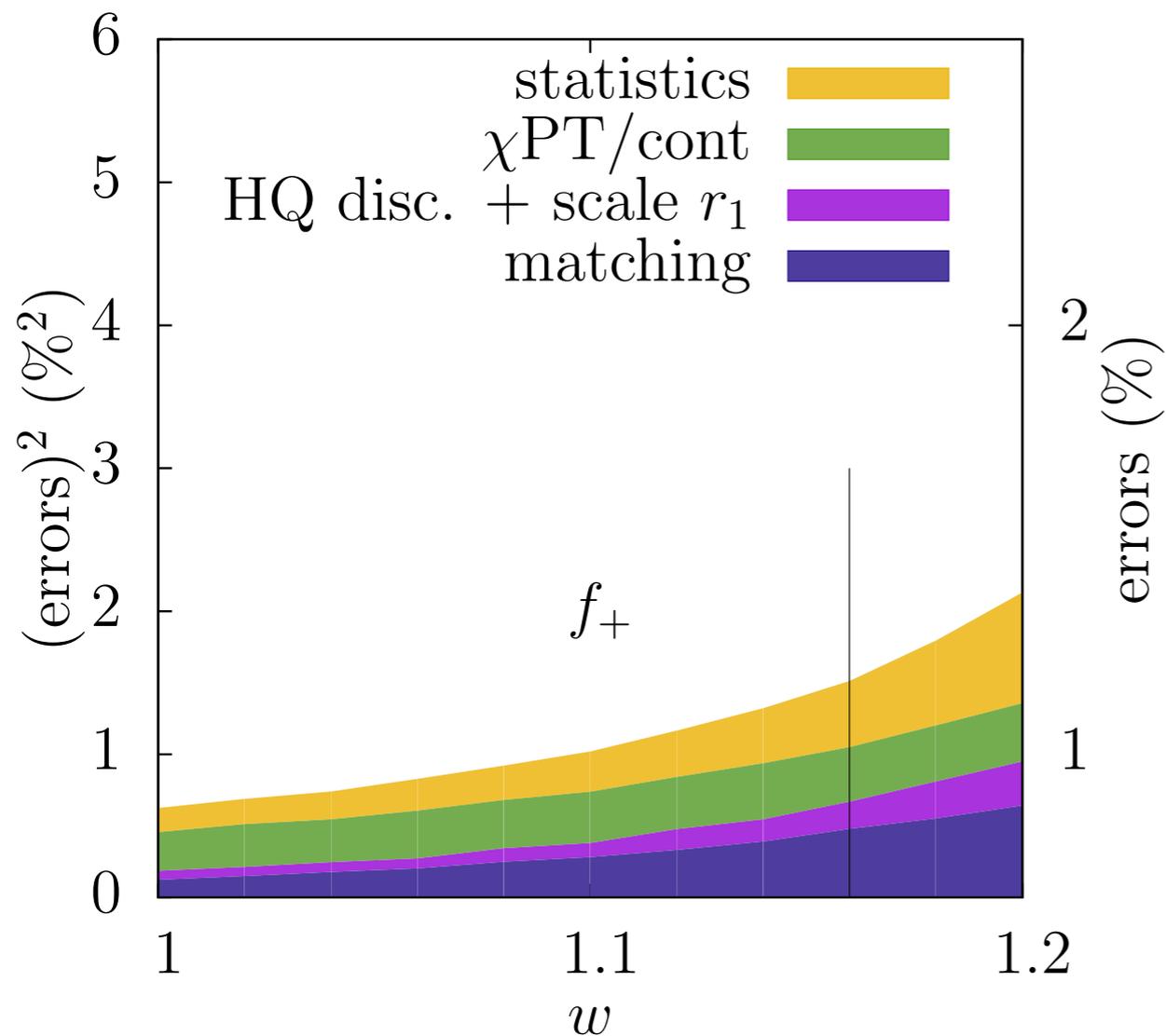
FFs from combination of double and single ratios of current matrix elements

FFs from direct fit to three-point functions

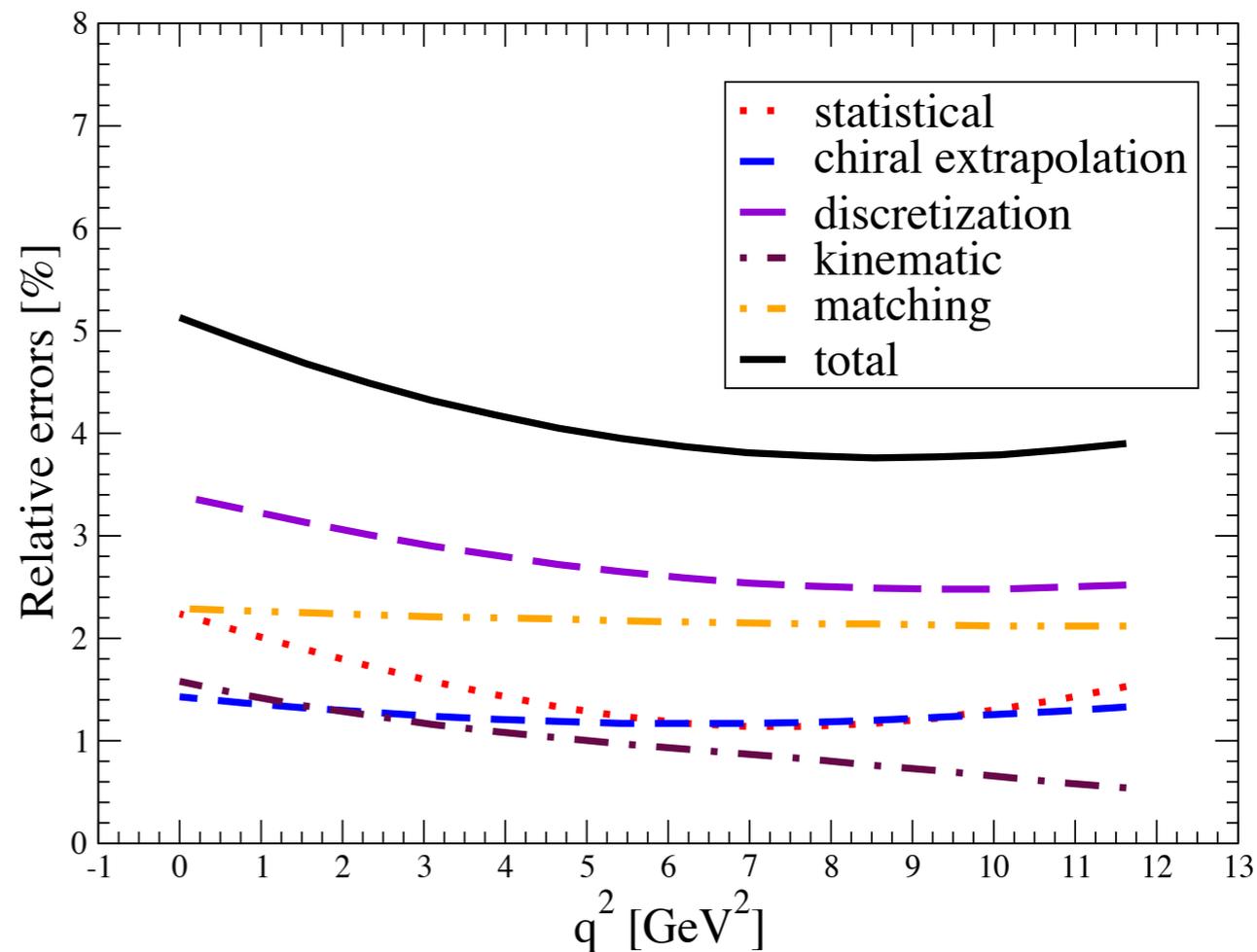
new results for $B \rightarrow Dl\nu$

[FNAL/MILC]

[HPQCD]



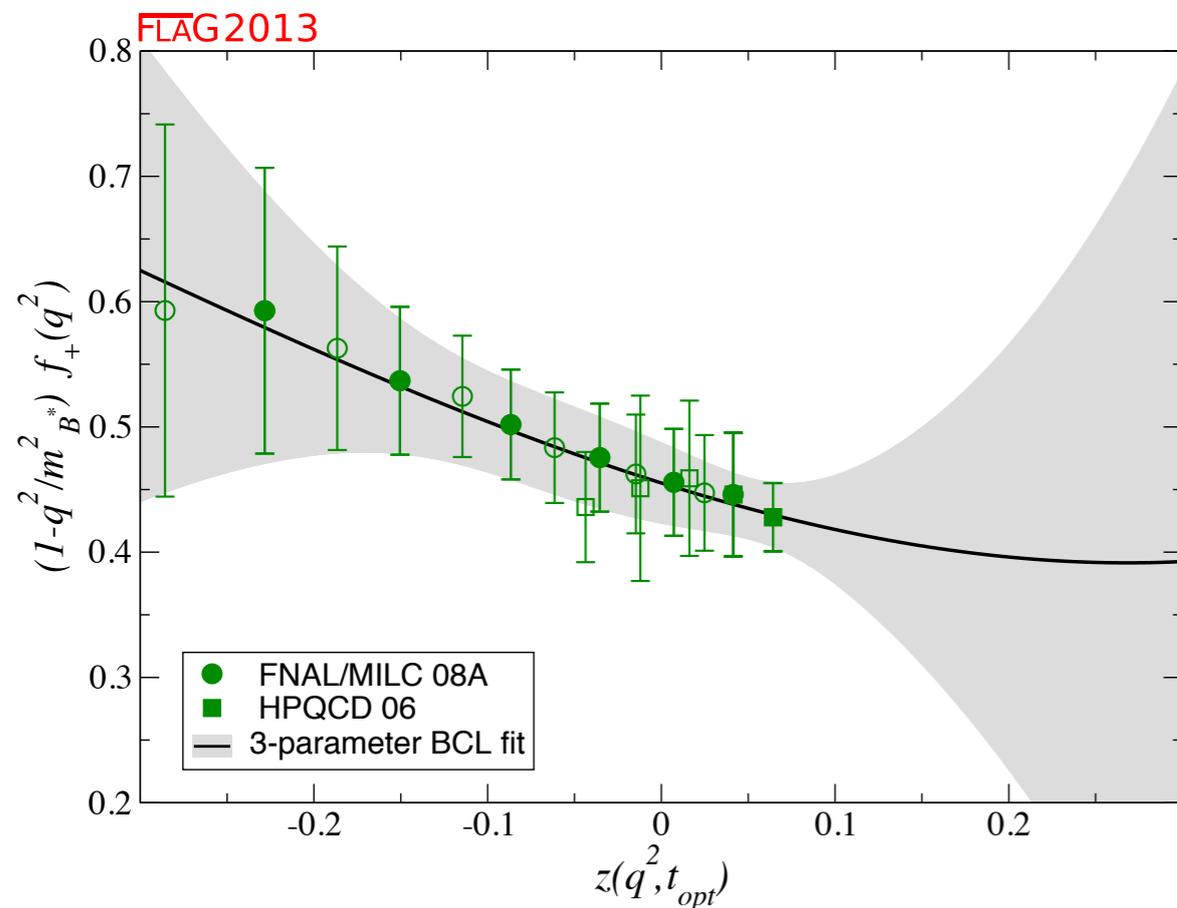
FFs from combination of double and single ratios of current matrix elements



FFs from direct fit to three-point functions

FLAG-2 on $B \rightarrow \pi l \nu$

Collaboration	Ref.	N_f	Publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$\Delta\zeta^{B\pi}$	z -parameterization type	$\{a_0, a_1, a_2\}$	cov. matrix
FNAL/MILC 08A [350]		2+1	A	○	○	★	○	✓	$2.21^{+0.47}_{-0.42}$	BGL [‡]	$\{0.0216(27), -0.038(19), -0.113(27)\}$	yes [§]
HPQCD 06 [426]		2+1	A	○	○	★	○	✓	2.07(41)(39)	–	–	no

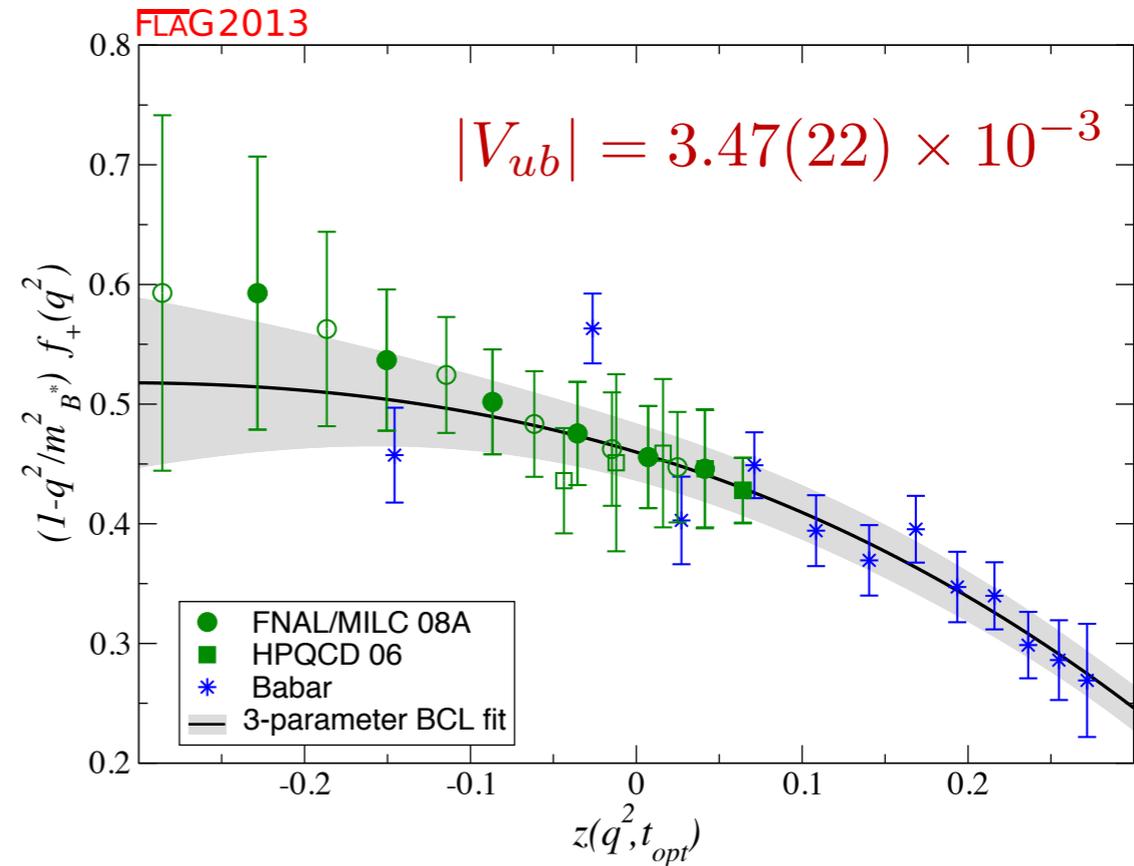
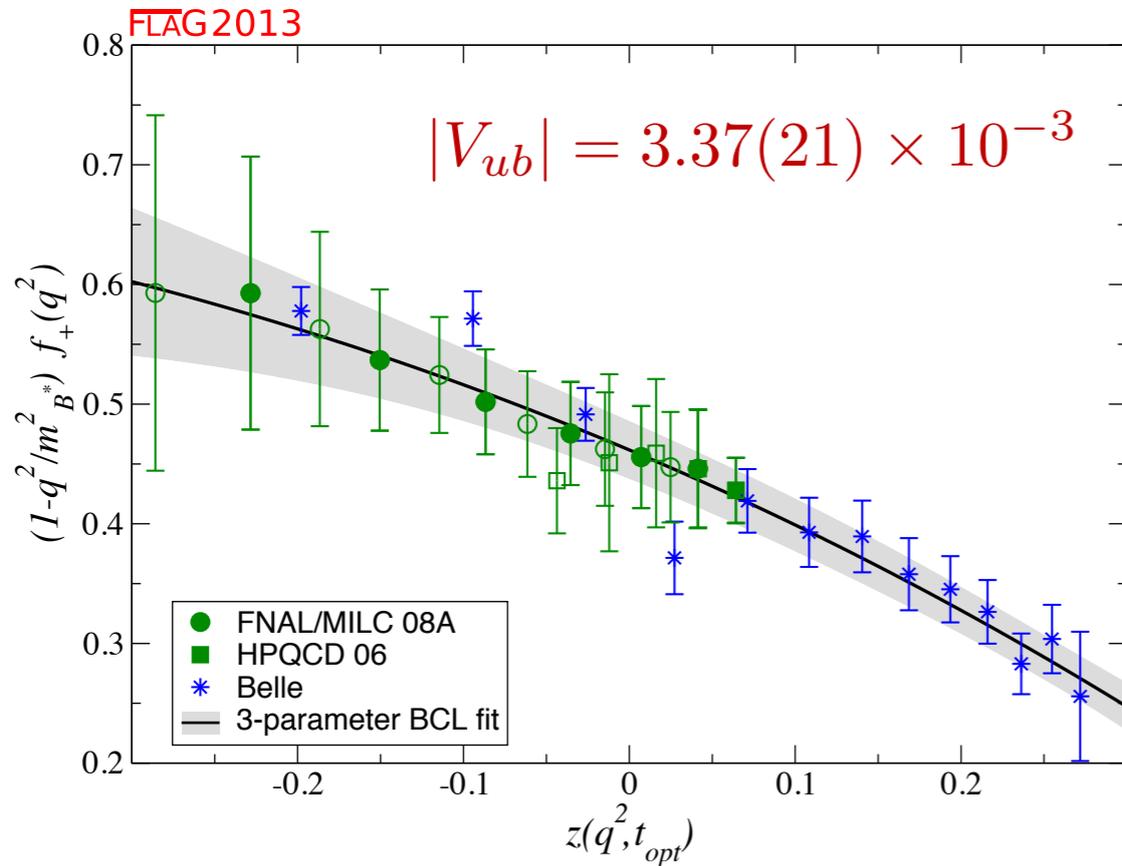


$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = (m_B + m_\pi)^2, \quad t_0 < t_+$$

FLAG-2 on $B \rightarrow \pi l \nu$

Collaboration	Ref.	N_f	Publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$\Delta\zeta^{B\pi}$	z -parameterization type	$\{a_0, a_1, a_2\}$	cov. matrix
FNAL/MILC 08A [350]		2+1	A	○	○	★	○	✓	$2.21^{+0.47}_{-0.42}$	BGL [‡]	$\{0.0216(27), -0.038(19), -0.113(27)\}$	yes [§]
HPQCD 06 [426]		2+1	A	○	○	★	○	✓	2.07(41)(39)	–	–	no

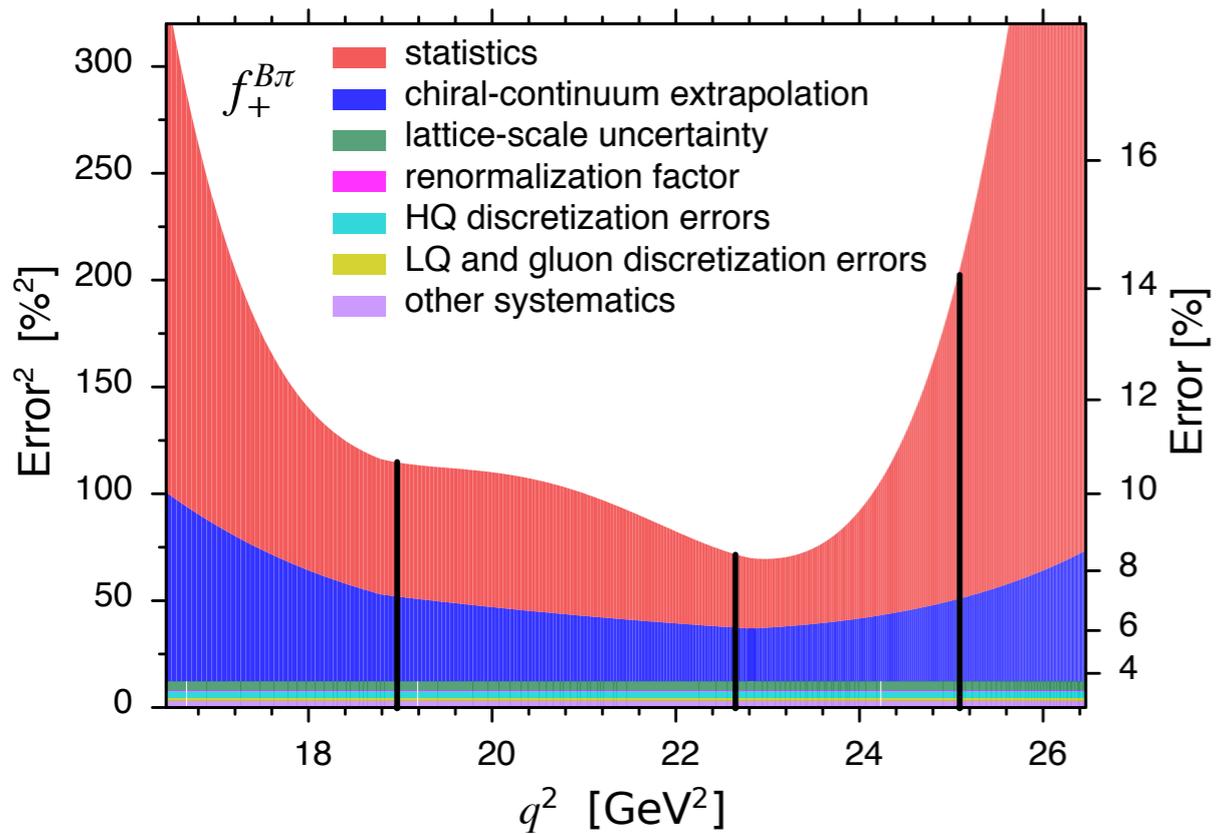


new results for $B \rightarrow \pi l \nu$

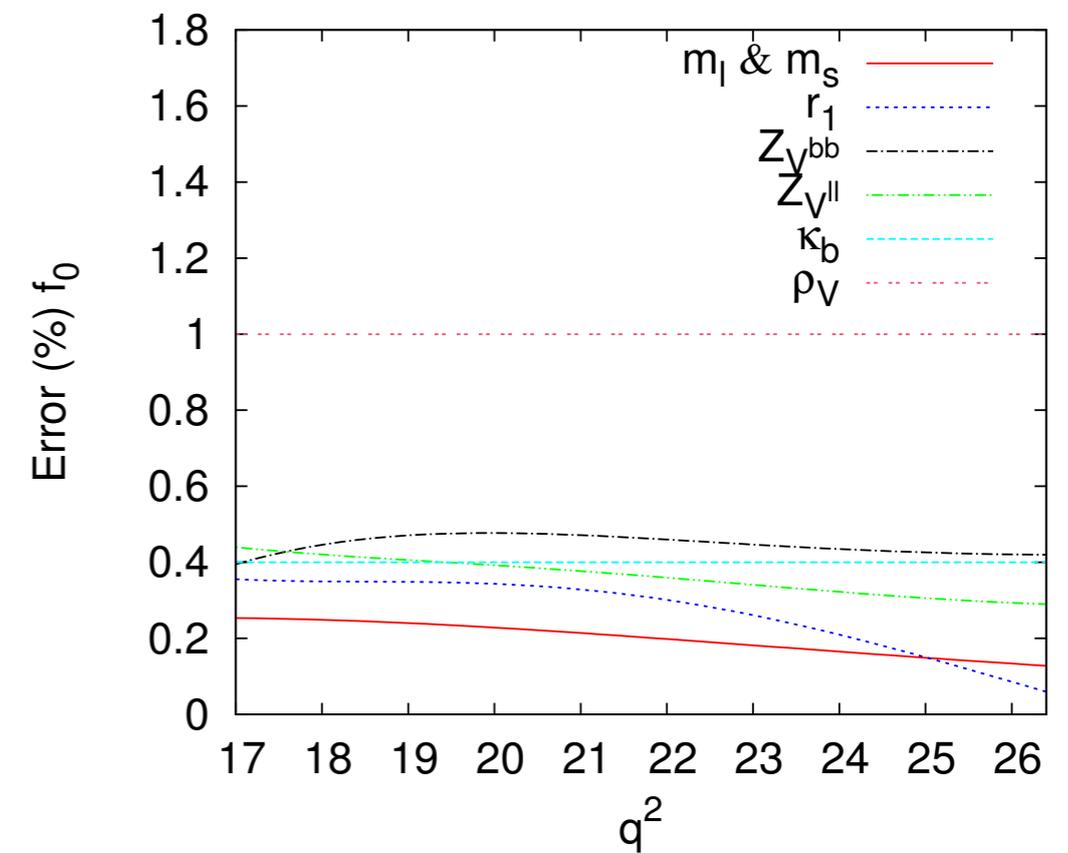
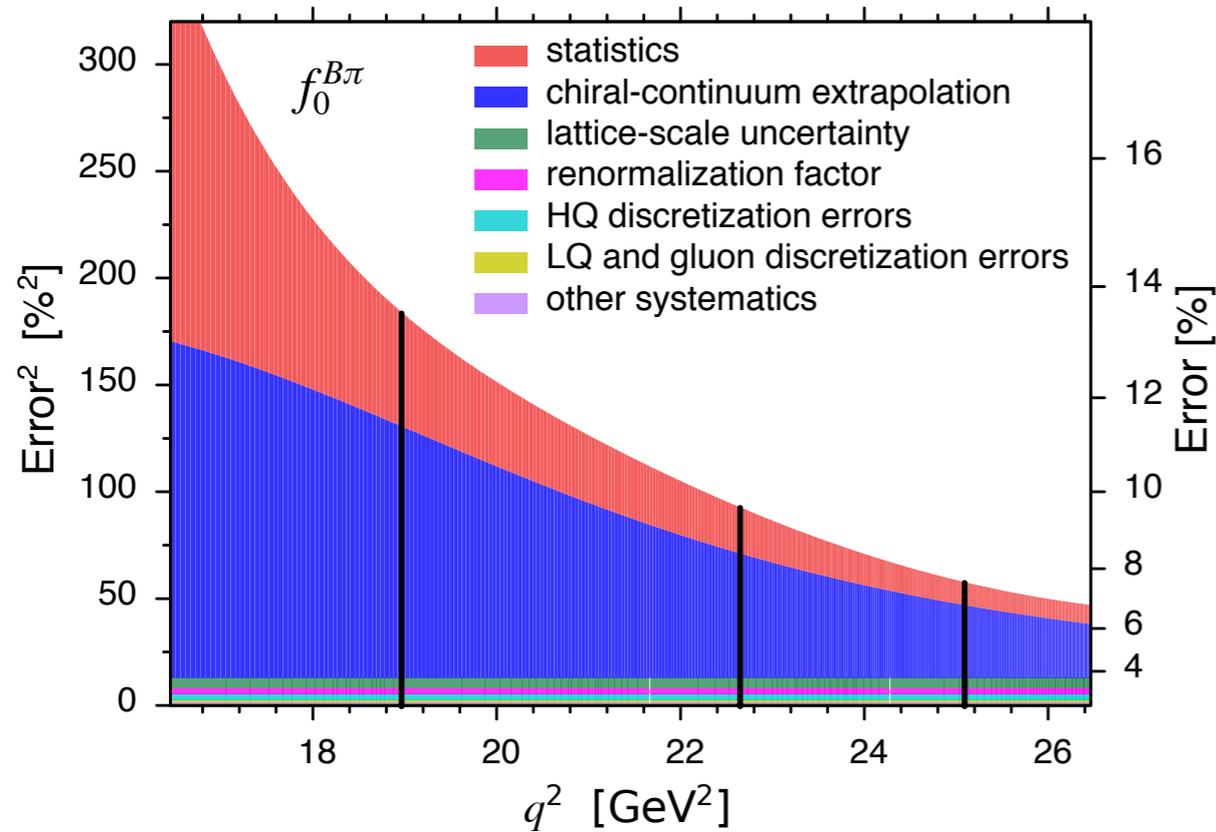
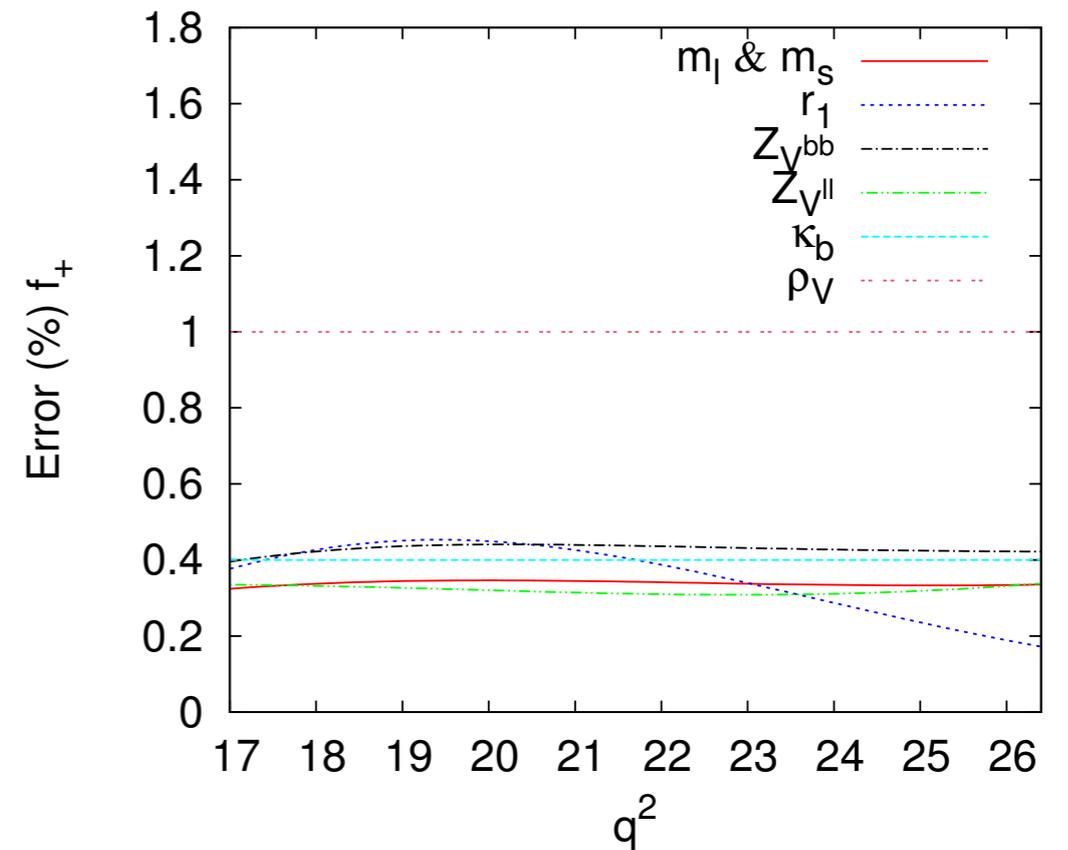
	FNAL/MILC	RBC/UKQCD	HPQCD
ensembles	MILC	RBC/UKQCD	MILC
N_f	2+1	2+1	2+1
a (fm)	4/0.045 – 0.12	2/0.086, 0.11	2/0.09, 0.12
M_π^{\min} [MeV]	220	289	260
$M_\pi^{\min} L$	3.8	4.0	3.8
l quarks	asqtad	DW	asqtad
b quark	RHQ (Fermilab)	RHQ (Columbia)	NRQCD
reference	[1503.07839]	[1501.05373]	[1310.3207]

new results for $B \rightarrow \pi l \nu$

[RBC / UKQCD]



[FNAL / MILC]

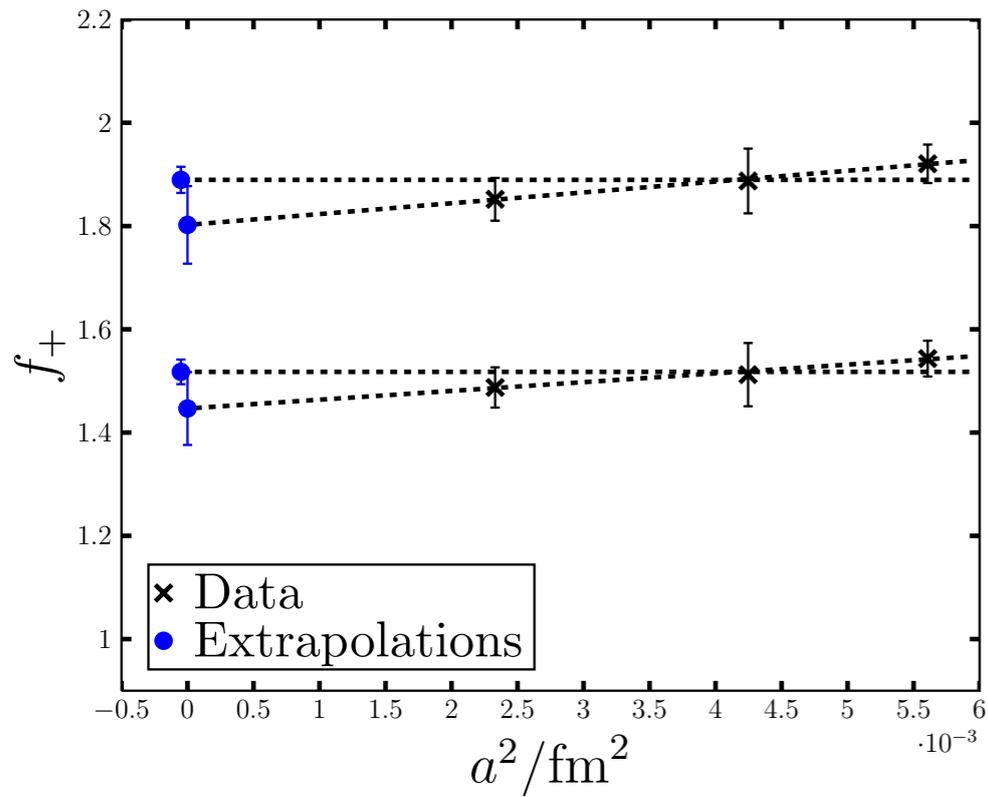


$$B_s \rightarrow Kl\nu$$

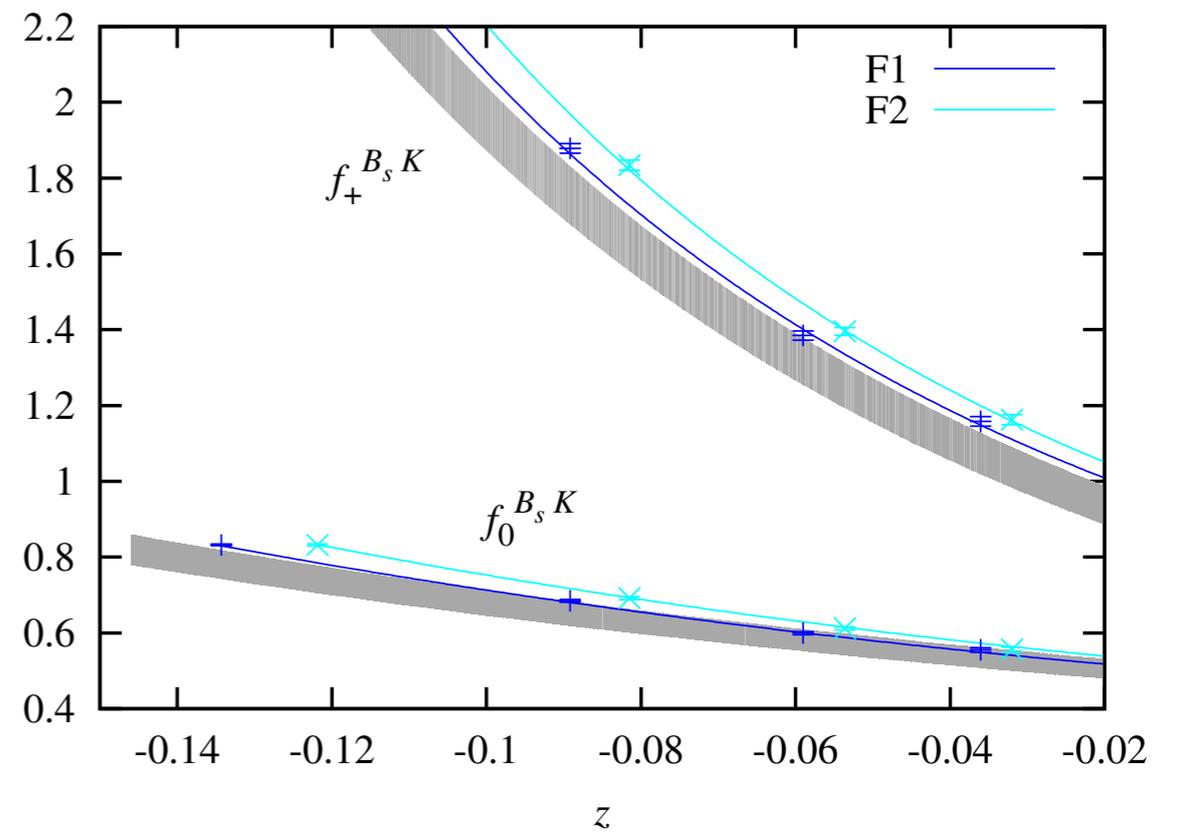
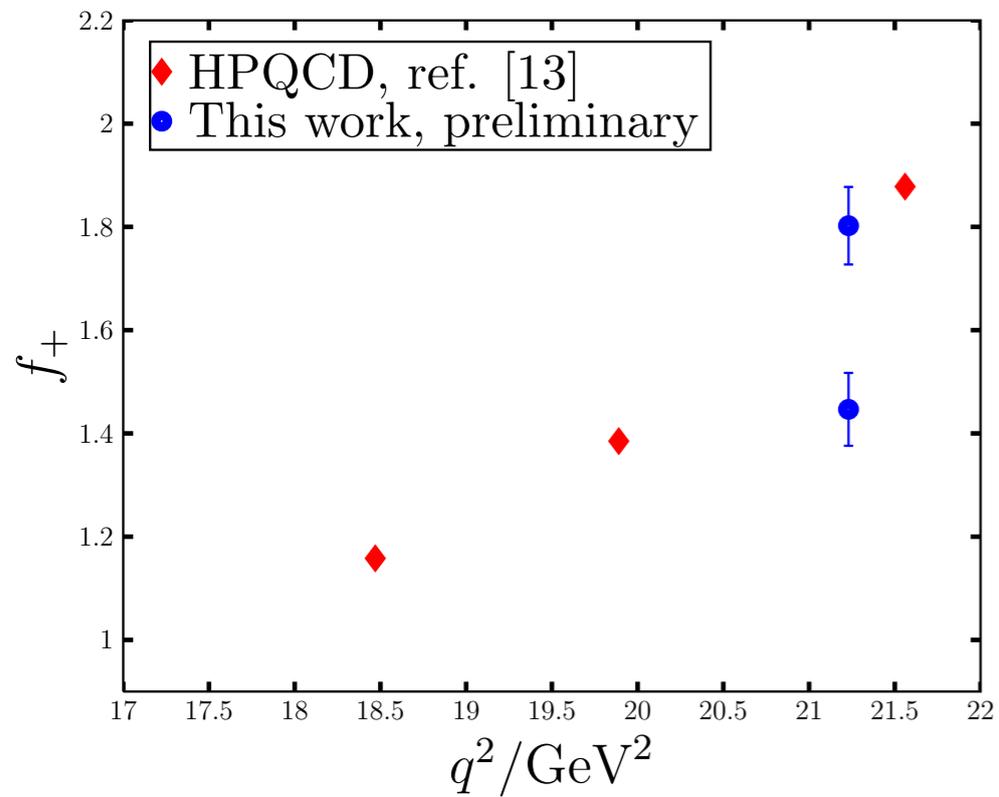
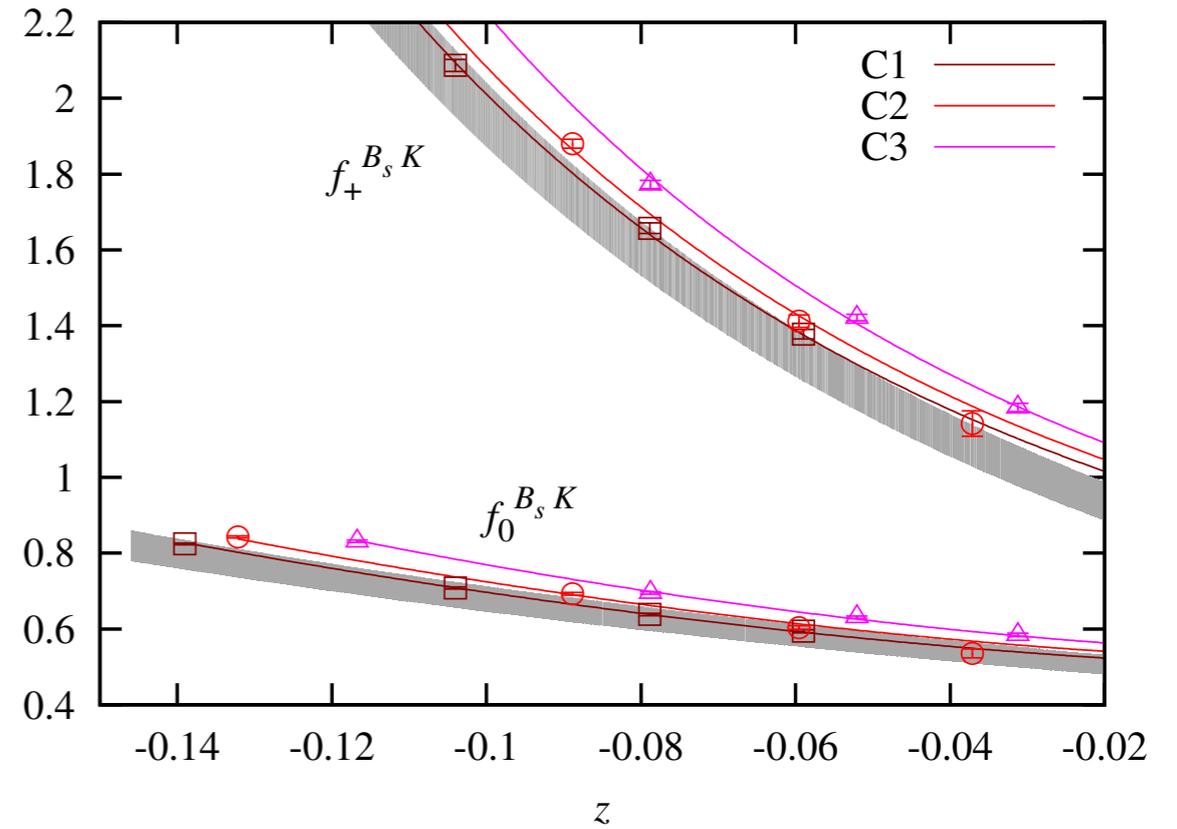
	RBC/UKQCD	HPQCD	ALPHA
ensembles	RBC/UKQCD	MILC	CLS
N_f	2+1	2+1	2
a (fm)	2/0.086, 0.11	2/0.09, 0.12	3/0.0483 – 0.0749
M_π^{\min} [MeV]	289	260	310
$M_\pi^{\min} L$	4.0	3.8	4.0
l quarks	DW	asqtad	NP O(a) improved
b quark	RHQ (Columbia)	NRQCD	npHQET
reference	[1501.05373]	[1406.2279]	[1411.3916]

$B_s \rightarrow Kl\nu$

[ALPHA]



[HPQCD]



impact of fitting, systematics estimation

significant differences in estimates of fit and systematic uncertainties in otherwise very similar computations

well-known example from light-quark physics (both computations use MILC ensembles, relatively minor differences)

MILC 13

$$f_{K^\pm} / f_{\pi^\pm} |_{N_f=2+1+1} = 1.1947 \overset{\text{stat}}{(26)} \overset{\text{CL}}{(33)} \overset{\text{FV}}{(17)} \overset{\text{e.m.}}{(2)}$$

HPQCD 13

$$f_{K^\pm} / f_{\pi^\pm} |_{N_f=2+1+1} = 1.1916 \overset{\text{stat}}{(15)} \overset{\text{CL}}{(12)} \overset{\text{FV}}{(1)} \overset{\text{(misc)}}{(10)}$$